Computer Algebra Independent Integration Tests

Summer 2023 edition

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/199-7.4.2-Exponentials-of-inverse-hyperbolic-cotangent-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [935]. This is test number [199].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (935)	0.00 (0)
Mathematica	97.75 (914)	2.25 (21)
Fricas	89.95 (841)	10.05 (94)
Maple	83.85 (784)	16.15 (151)
Mupad	55.40 (518)	44.60 (417)
Maxima	54.01 (505)	45.99 (430)
Giac	47.49 (444)	52.51 (491)
Sympy	20.75 (194)	79.25 (741)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

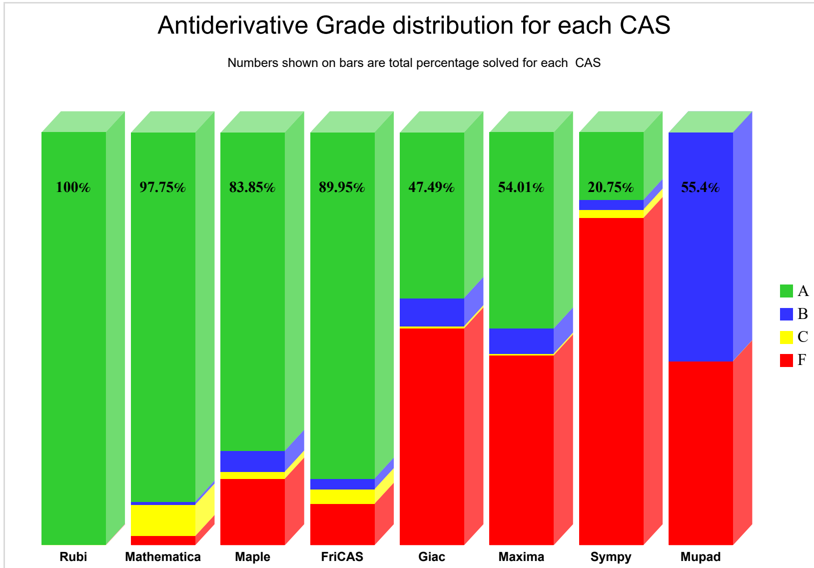
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

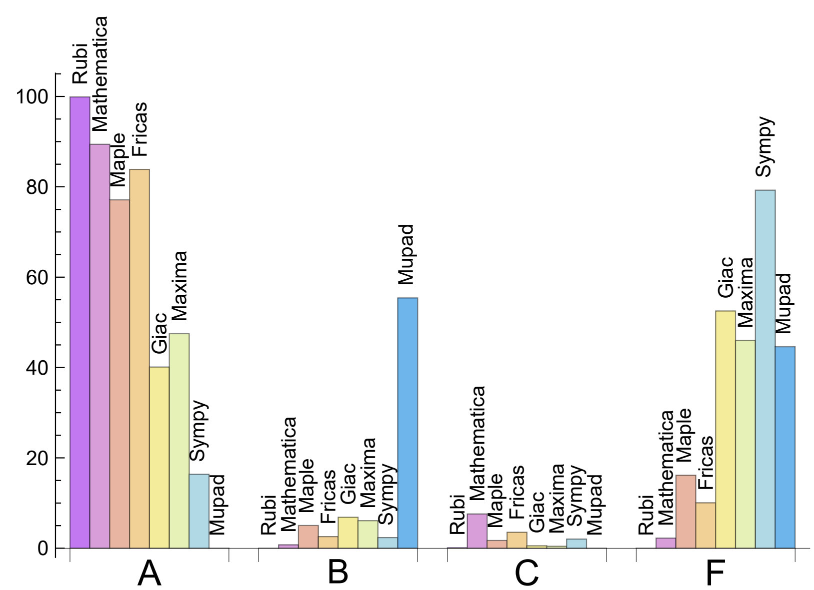
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.893	0.000	0.107	0.000
Mathematica	89.412	0.749	7.594	2.246
Fricas	83.850	2.567	3.529	10.053
Maple	77.112	5.027	1.711	16.150
Maxima	47.487	6.096	0.428	45.989
Giac	40.107	6.845	0.535	52.513
Sympy	16.364	2.353	2.032	79.251
Mupad	0.000	55.401	0.000	44.599

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	21	80.95	19.05	0.00
Fricas	94	100.00	0.00	0.00
Maple	151	100.00	0.00	0.00
Mupad	417	0.00	100.00	0.00
Maxima	430	100.00	0.00	0.00
Giac	491	65.78	0.00	34.22
Sympy	741	69.77	29.96	0.27

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.15
Maxima	0.23
Fricas	0.26
Mathematica	0.28
Giac	0.52
Maple	0.63
Mupad	2.32
Sympy	3.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	80.65	0.74	71.00	0.67
Mupad	97.81	0.96	81.00	0.90
Maxima	115.90	1.13	97.00	1.03
Maple	117.42	1.04	85.00	0.82
Sympy	120.88	1.56	58.00	1.04
Giac	123.98	1.20	91.00	1.01
Rubi	134.16	1.01	111.00	1.00
Fricas	138.95	1.16	99.00	1.03

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

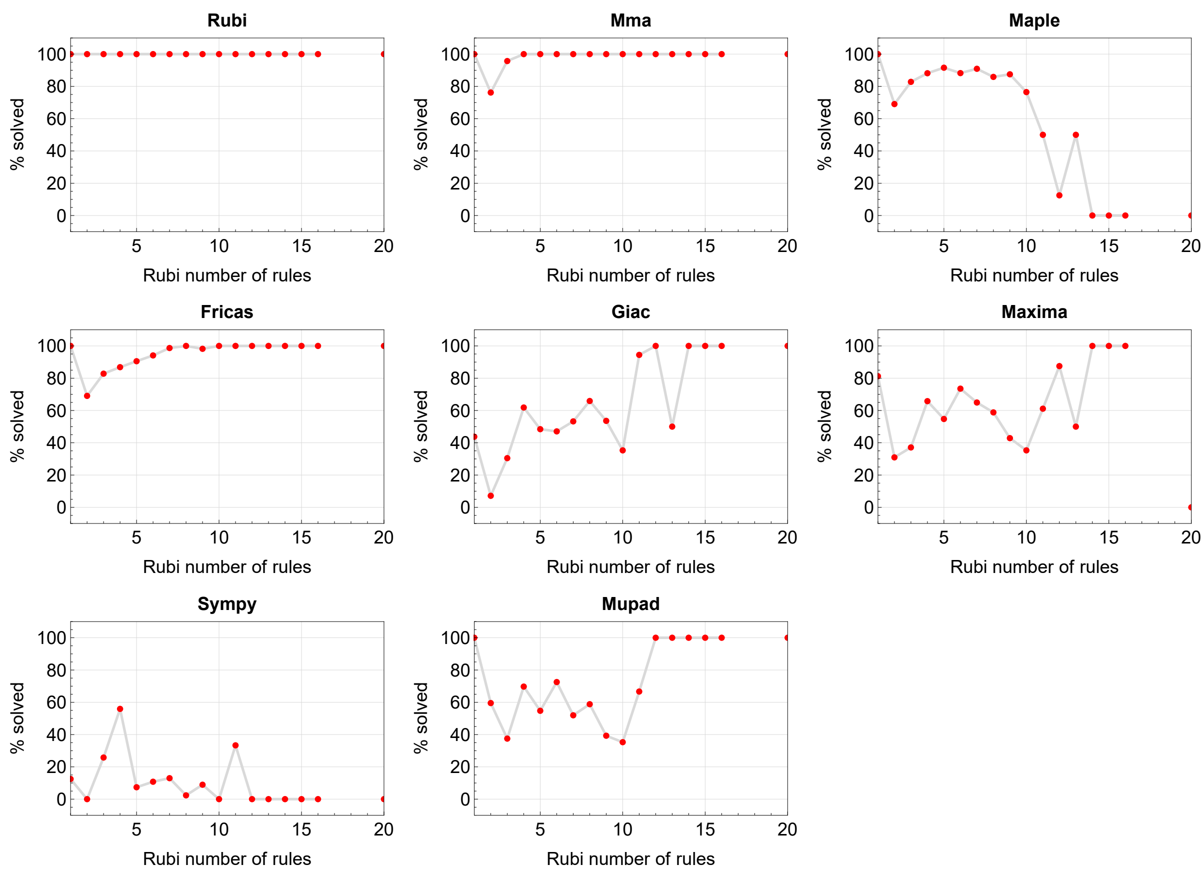


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

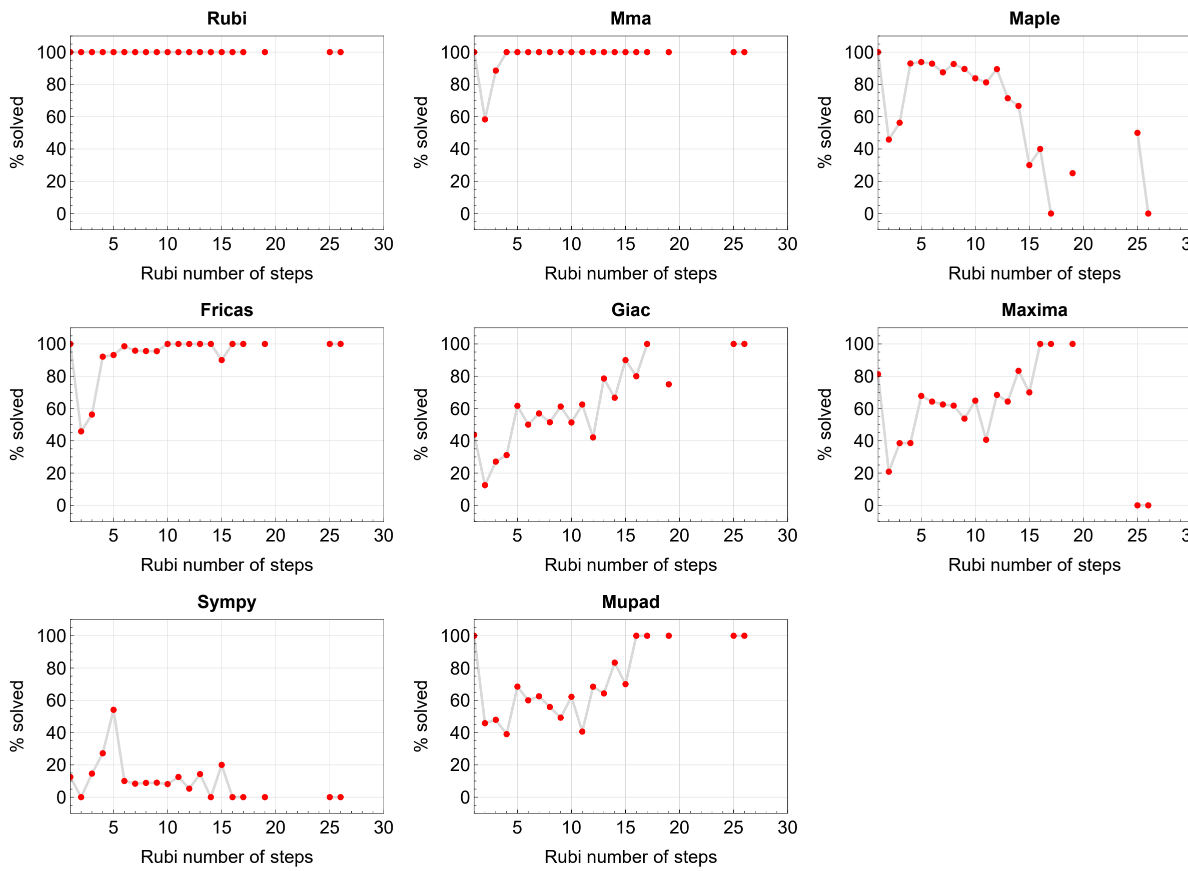


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

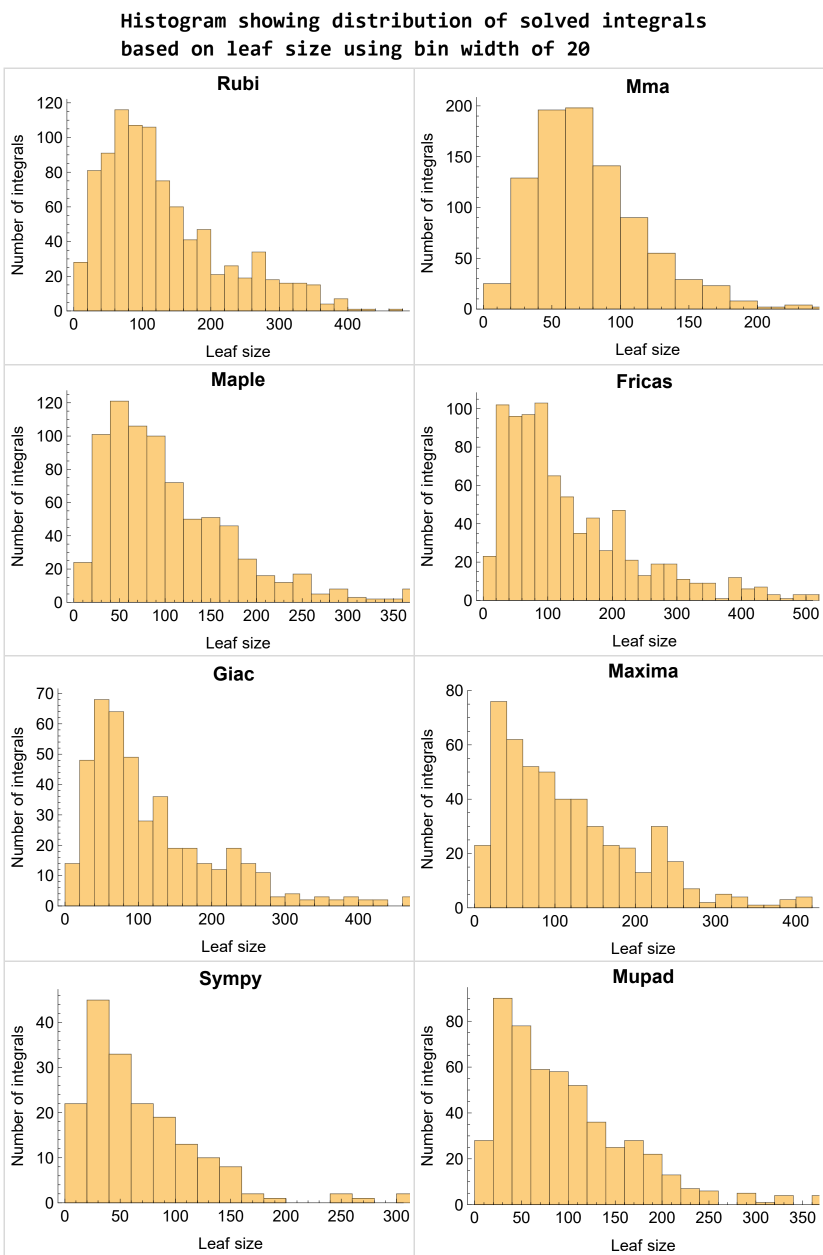


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

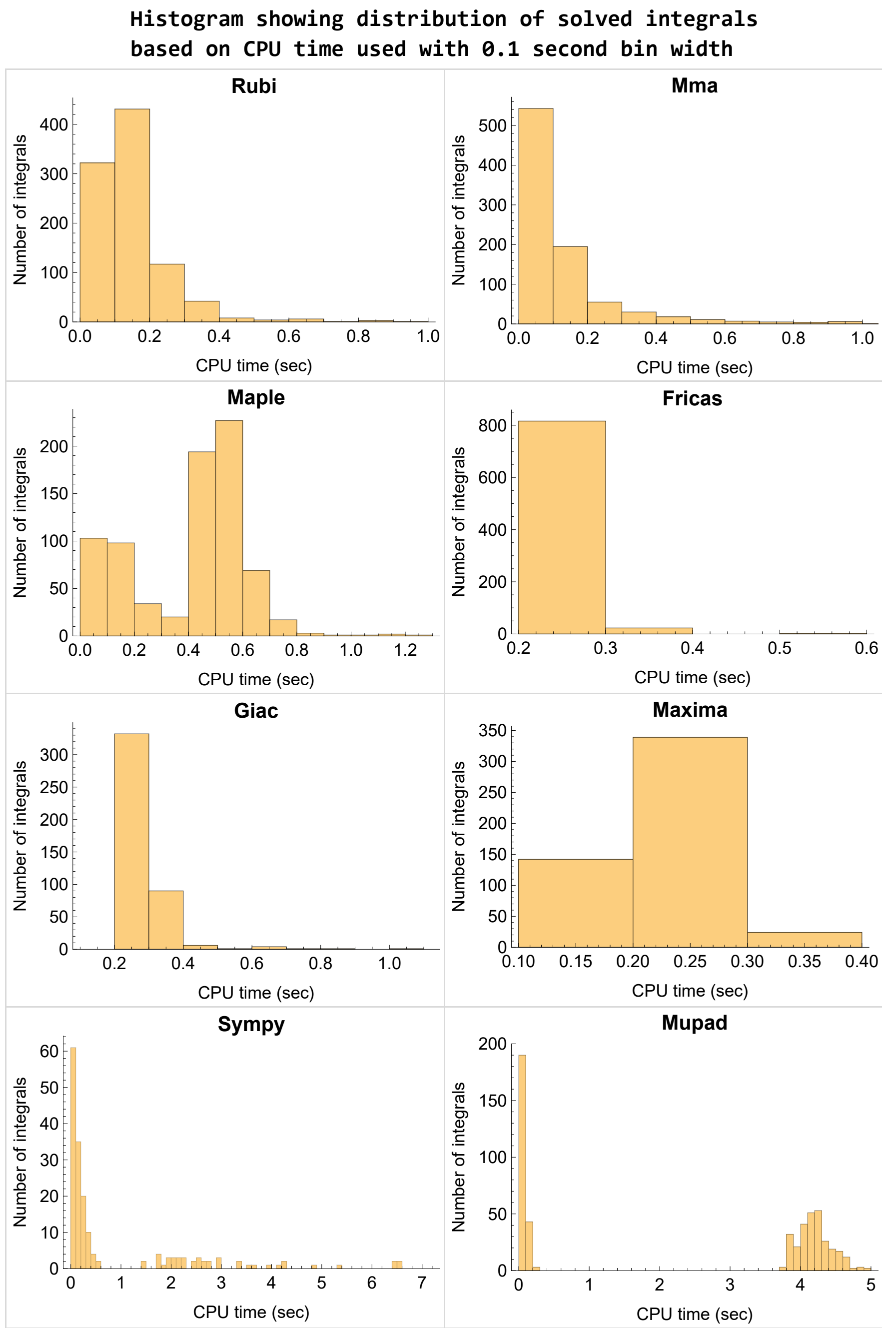


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

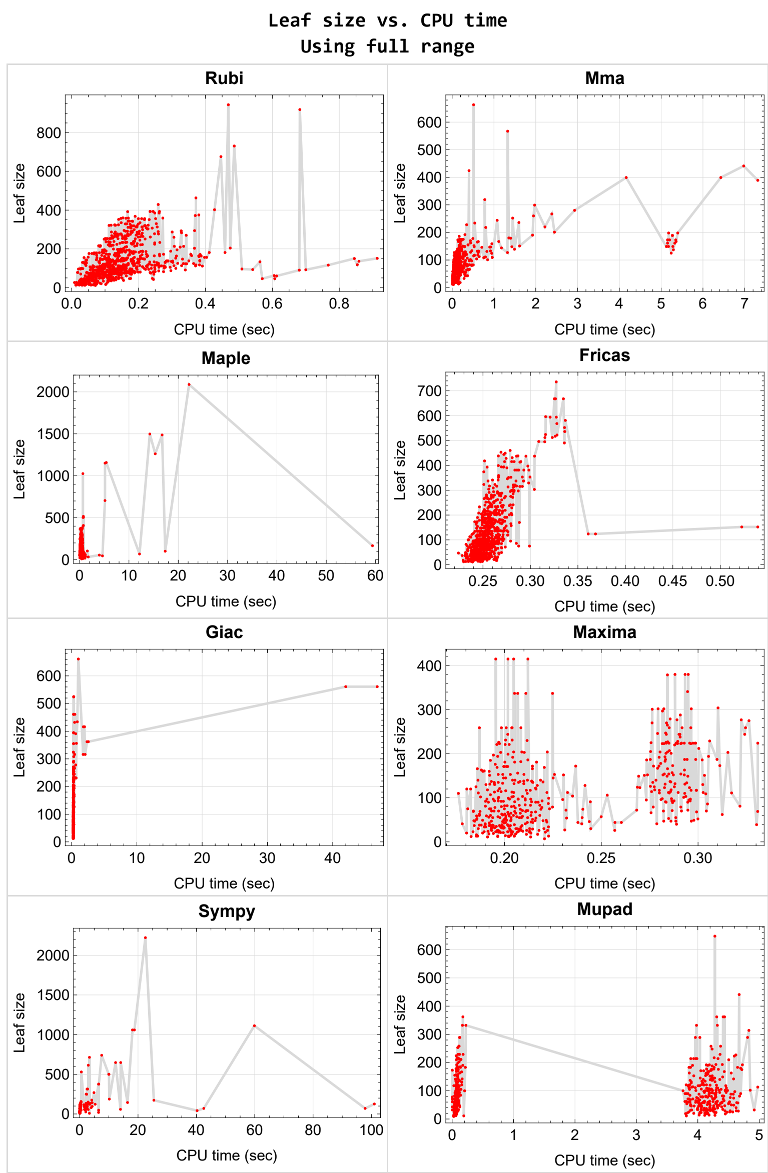


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{61, 70, 79, 88, 133, 135, 136, 138, 332, 731, 734\}$

Maple $\{116, 118\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

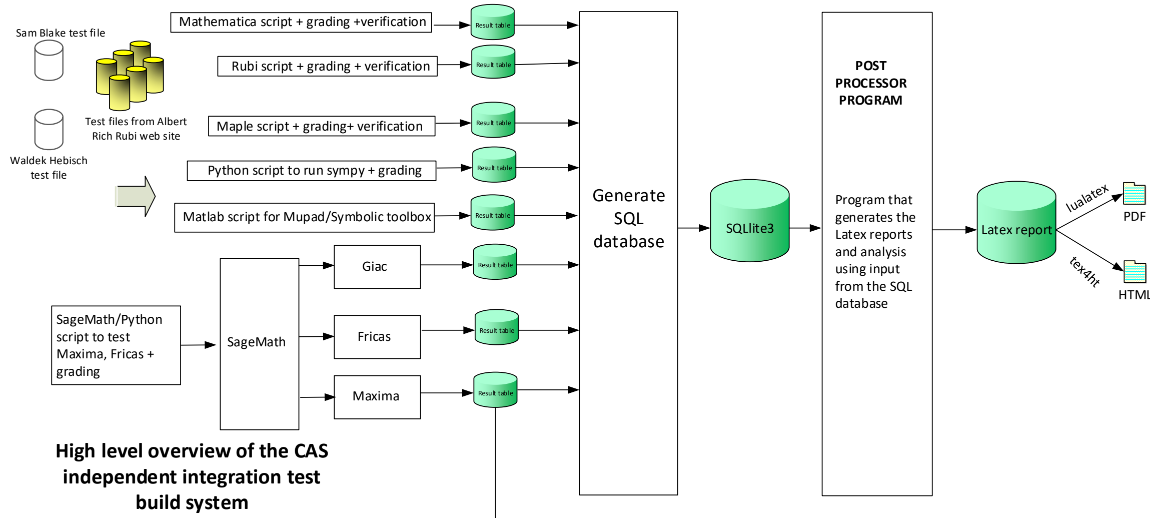
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

B grade { }

C grade { 172 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 65, 66, 68, 69, 71, 77, 78, 80, 81, 83, 84, 86, 87, 89, 95, 96, 97, 98, 99, 101, 102, 104, 105, 106, 107, 113, 114, 120, 121, 122, 124, 127, 132, 134, 137, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 550, 552, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 735, 738, 739, 740, 741, 742, 743, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade { 365, 381, 398, 584, 736, 737, 744 }

C grade { 61, 64, 67, 70, 72, 73, 74, 75, 76, 79, 82, 85, 88, 90, 91, 92, 93, 94, 100, 103, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 123, 125, 126, 128, 129, 130, 131, 133, 135, 136, 138, 172, 267, 268, 269, 429, 430, 431, 432, 451, 452, 453, 454, 458, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 498, 499, 569, 602, 731, 734 }

F normal fail { 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 549, 551, 553, 554, 933, 934, 935 }

F(-1) timedout fail { 545, 546, 547, 548 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 435, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade { 4, 5, 6, 7, 20, 21, 22, 36, 37, 38, 39, 53, 54, 55, 161, 162, 182, 201, 220, 381, 382, 383, 398, 399, 416, 418, 432, 433, 434, 436, 437, 450, 451, 452, 453, 454, 473, 474, 477, 478, 479, 501, 502, 527, 528, 529, 677 }

C grade { 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 132, 134, 137 }

F normal fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 585, 586, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade { 5, 6, 21, 37, 38, 183, 194, 201, 242, 290, 381, 442, 466, 467, 492, 493, 519, 520, 572, 577, 584, 587, 588, 605 }

C grade { 64, 65, 66, 67, 73, 74, 75, 76, 82, 83, 84, 85, 91, 92, 93, 94, 100, 101, 102, 103, 109, 110, 111, 112, 117, 118, 119, 126, 127, 128, 129, 130, 131 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 323, 324, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 673, 674, 675, 676, 677, 712, 713, 714, 715, 716, 732, 733, 740, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 880, 904 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 53, 54, 158, 159, 160, 161, 178, 179, 180, 181, 187, 194, 198, 199, 200, 201, 212, 222, 281, 282, 285, 286, 379, 380, 381, 382, 396, 397, 398, 399, 414, 416, 417, 435, 584, 587, 636, 662 }

C grade { 321, 322, 325, 326 }

F normal fail { 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 30, 31, 32, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 158, 159, 160, 161, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 185, 186, 188, 189, 190, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 222, 226, 228, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 245, 248, 249, 250, 251, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 311, 333, 334, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 382, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 400, 401, 403, 409, 410, 411, 412, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 435, 498, 499, 500, 556, 557, 558, 559, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 581, 582, 585, 587, 588, 589, 590, 591, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 623, 624, 625, 626, 627, 651, 652, 653, 674, 675, 676, 677, 678, 713, 714, 715, 716, 717, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 838, 839, 840, 863, 864, 865, 886, 887, 888, 890, 892, 910, 911, 912, 914, 916 }

B grade { 5, 6, 7, 8, 9, 29, 37, 39, 41, 174, 183, 184, 191, 194, 212, 235, 236, 237, 294, 301, 302, 379, 380, 381, 396, 398, 399, 404, 405, 406, 407, 408, 413, 414, 450, 451, 452, 453, 454, 501, 530, 531, 532, 577, 583, 584, 586, 610, 629, 655, 679, 680, 681, 718, 719, 720, 799, 800, 891, 893, 894, 915, 917, 918 }

C grade { 321, 322, 325, 326, 328 }

F normal fail { 18, 19, 20, 21, 22, 23, 24, 50, 51, 52, 53, 54, 55, 56, 57, 58, 128, 132, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 167, 187, 197, 202, 206, 217, 218, 219, 220, 221, 223, 224, 225, 295, 335, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 376, 377, 378, 383, 418, 429, 430, 431, 432, 433, 434, 436, 438, 440, 442, 443, 444, 445, 455, 457, 462, 466, 467, 468, 470, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 507, 516, 518, 519, 520, 521, 522, 523, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 560, 561, 562, 563, 579, 580, 594, 595, 596, 597, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 682, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 736, 737, 738, 739, 740, 741, 742, 743, 746, 747, 748, 749, 750, 752, 753, 754, 755, 756, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 776, 777, 778, 779, 795, 796, 810, 811, 812, 813, 828, 829, 830, 831, 832, 833, 834, 846, 847, 848, 849, 850, 851, 852, 855, 856, 857, 858, 871, 872, 873, 874, 875, 880, 881, 882, 883, 884, 885, 895, 896, 897, 898, 900, 902, 904, 905, 906, 907, 909, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { 2, 33, 38, 40, 133, 138, 177, 216, 227, 229, 244, 246, 247, 252, 253, 254, 255, 257, 259, 270, 271, 272, 273, 274, 277, 296, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 327, 329, 330, 331, 332, 336, 350, 351, 352, 353, 354, 355, 356, 357, 358, 372, 373, 384, 386, 402, 419, 437, 439, 441, 446, 447, 448, 449, 456, 458, 459, 460, 461, 463, 464, 465, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 487, 502, 503, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 515, 517, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 628, 654, 668, 673, 683, 692, 699, 701, 711, 712, 730, 731, 734, 735, 744, 745, 751, 757, 793, 835, 836, 837, 841, 842, 843, 844, 845, 853, 854, 859, 860, 861, 862, 866, 867, 868, 869, 870, 876, 877, 878, 879, 889, 899, 901, 903, 908, 913, 931 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 369, 370, 371, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 494, 495, 496, 497, 503, 504, 505, 506, 521, 522, 523, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 629, 630, 631, 632, 639, 655, 656, 657, 658, 665, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 881, 882, 885, 905, 906, 909 }

C grade { }

F normal fail { }

F(-1) timedout fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 498, 499, 500, 501, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 633, 634, 635, 636, 637, 638, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 907, 908, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-2) exception fail { }

Sympy

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 169, 170, 171, 172, 173, 174, 175, 188, 189, 190, 191, 192, 193, 195, 196, 207, 208, 209, 210, 211, 212, 213, 214, 215, 235, 236, 237, 238, 239, 240, 241, 242, 261, 262, 263, 264, 265, 266, 267, 268, 269, 288, 292, 301, 302, 303, 304, 320, 324, 330, 332, 333, 334, 341, 342, 343, 344, 345, 387, 388, 389, 390, 391, 392, 393, 394, 395, 404, 405, 406, 407, 408, 409, 410, 411, 412, 421, 422, 423, 424, 425, 427, 428, 564, 565, 566, 567, 568, 569, 570, 571, 572, 585, 588, 589, 598, 599, 600, 601, 602, 603, 604, 605, 780, 781, 782, 783, 784, 785, 786, 787, 788, 797, 798, 799, 800, 801, 802, 803, 804, 805, 814, 815, 816, 817, 818, 819, 820, 821 }

B grade { 134, 167, 168, 176, 187, 194, 305, 426, 502, 528, 581, 582, 583, 584, 586, 587, 623, 624, 625, 626, 651, 652 }

C grade { 137, 327, 328, 369, 370, 446, 447, 448, 449, 552, 740, 767, 770, 838, 839, 840, 863, 864, 865 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 140, 141, 142, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 247, 248, 256, 257, 258, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294, 296, 297, 298, 299, 300, 306, 307, 308, 309, 310, 311, 312, 313, 314, 319, 322, 323, 325, 326, 329, 331, 336, 337, 338, 339, 340, 346, 347, 348, 349, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 396, 397, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 417, 418, 419, 420, 429, 430, 431, 432, 433, 434, 435, 436, 437, 442, 443, 450, 451, 452, 453, 454, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 479, 490, 491, 492, 493, 494, 498, 499, 500, 501, 503, 504, 505, 506, 508, 518, 519, 520, 521, 524, 525, 526, 527, 529, 530, 531, 532, 542, 543, 544, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 573, 574, 575, 576, 577, 578, 579, 590, 591, 592, 593, 594, 595, 596, 597, 606, 607, 608, 609, 610, 611, 612, 613, 617, 618, 619, 620, 627, 628, 629, 630, 631, 632, 636, 637, 638, 646, 647, 648, 653, 654, 655, 656, 657, 658, 664, 665, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 691, 692, 693, 694, 695, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 725, 731, 732, 734, 736, 737, 738, 739, 741, 744, 745, 746, 747, 748, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 833, 841, 842, 843, 844, 845, 859, 866, 867, 868, 869, 870, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 908, 910, 911, 912, 913, 914, 915, 916, 917, 918, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { 104, 138, 139, 143, 144, 147, 216, 226, 227, 228, 233, 234, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 259, 260, 270, 271, 272, 273, 274, 275, 276, 277, 278, 295, 315, 316, 317, 318, 321, 335, 350, 351, 352, 353, 354, 355, 356, 357, 358, 371, 438, 439, 440, 441, 444, 445, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 495, 496, 497, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 522, 523, 533, 534, 535, 536, 537, 538, 539, 540, 541, 545, 580, 614, 615, 616, 621, 622, 633, 634, 635, 639, 640, 641, 642, 643, 644, 645, 649, 650, 659, 660, 661, 662, 663, 666, 667, 687, 688, 689, 690, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 721, 722, 723, 724, 726, 727, 728, 729, 730, 733, 735, 742, 743, 749, 750, 771, 796, 829, 830, 831, 832, 834, 835, 836, 837, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 885, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 909, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

F(-2) exception fail { 372, 378 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	68	117	203	92	0	111	171
N.S.	1	1.00	0.60	1.03	1.78	0.81	0.00	0.97	1.50
time (sec)	N/A	0.091	0.049	0.161	0.210	0.255	0.000	0.278	4.095

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	60	109	166	84	0	0	133
N.S.	1	1.00	0.67	1.21	1.84	0.93	0.00	0.00	1.48
time (sec)	N/A	0.073	0.034	0.118	0.214	0.253	0.000	0.000	0.066

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	100	128	73	0	77	98
N.S.	1	1.00	0.78	1.59	2.03	1.16	0.00	1.22	1.56
time (sec)	N/A	0.059	0.026	0.111	0.217	0.249	0.000	0.273	0.069

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	90	90	64	0	57	58
N.S.	1	1.00	1.14	2.50	2.50	1.78	0.00	1.58	1.61
time (sec)	N/A	0.033	0.020	0.107	0.217	0.255	0.000	0.281	0.052

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	36	132	69	57	0	63	37
N.S.	1	1.00	1.64	6.00	3.14	2.59	0.00	2.86	1.68
time (sec)	N/A	0.044	0.011	0.118	0.331	0.247	0.000	0.274	4.140

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	76	53	46	0	66	55
N.S.	1	1.00	1.12	3.17	2.21	1.92	0.00	2.75	2.29
time (sec)	N/A	0.021	0.015	0.126	0.285	0.253	0.000	0.264	0.091

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	85	91	60	0	143	81
N.S.	1	1.00	1.11	2.24	2.39	1.58	0.00	3.76	2.13
time (sec)	N/A	0.025	0.031	0.134	0.283	0.255	0.000	0.266	4.423

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	93	136	68	0	148	105
N.S.	1	1.00	0.68	1.24	1.81	0.91	0.00	1.97	1.40
time (sec)	N/A	0.048	0.059	0.129	0.286	0.253	0.000	0.268	0.070

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	59	101	172	76	0	226	129
N.S.	1	1.00	0.67	1.15	1.95	0.86	0.00	2.57	1.47
time (sec)	N/A	0.066	0.062	0.138	0.277	0.254	0.000	0.270	0.088

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	43	42	37	47	38
N.S.	1	1.00	1.00	0.91	1.00	0.98	0.86	1.09	0.88
time (sec)	N/A	0.046	0.016	0.389	0.200	0.242	0.059	0.257	0.039

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	34	33	27	38	30
N.S.	1	1.00	1.00	0.94	1.03	1.00	0.82	1.15	0.91
time (sec)	N/A	0.042	0.012	0.395	0.208	0.234	0.059	0.251	0.040

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	26	25	20	30	23
N.S.	1	1.00	1.00	0.92	1.00	0.96	0.77	1.15	0.88
time (sec)	N/A	0.029	0.010	0.385	0.222	0.240	0.052	0.270	0.043

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	16	10	14	13
N.S.	1	1.00	1.00	1.00	0.93	1.14	0.71	1.00	0.93
time (sec)	N/A	0.016	0.011	0.382	0.186	0.239	0.044	0.261	4.112

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	13	10	15	14
N.S.	1	1.00	1.00	1.00	0.93	0.93	0.71	1.07	1.00
time (sec)	N/A	0.030	0.009	0.387	0.185	0.249	0.071	0.271	0.050

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	22	15	20	14
N.S.	1	1.00	1.00	1.00	0.95	1.16	0.79	1.05	0.74
time (sec)	N/A	0.035	0.010	0.418	0.206	0.251	0.095	0.253	4.183

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	30	35	26	32	23
N.S.	1	1.00	1.00	0.91	0.91	1.06	0.79	0.97	0.70
time (sec)	N/A	0.036	0.012	0.431	0.186	0.253	0.119	0.263	0.047

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	38	43	34	40	30
N.S.	1	1.00	1.00	0.92	0.95	1.08	0.85	1.00	0.75
time (sec)	N/A	0.037	0.012	0.441	0.187	0.244	0.111	0.261	0.046

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	75	151	182	112	0	0	154
N.S.	1	1.00	0.64	1.28	1.54	0.95	0.00	0.00	1.31
time (sec)	N/A	0.854	0.060	0.137	0.208	0.271	0.000	0.000	4.126

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	66	142	145	103	0	0	117
N.S.	1	1.00	0.72	1.54	1.58	1.12	0.00	0.00	1.27
time (sec)	N/A	0.700	0.045	0.138	0.225	0.253	0.000	0.000	0.070

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	133	110	92	0	0	59
N.S.	1	1.00	0.87	2.15	1.77	1.48	0.00	0.00	0.95
time (sec)	N/A	0.605	0.036	0.128	0.176	0.246	0.000	0.000	4.086

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	53	363	90	104	0	0	54
N.S.	1	1.00	1.15	7.89	1.96	2.26	0.00	0.00	1.17
time (sec)	N/A	0.570	0.043	0.121	0.284	0.252	0.000	0.000	0.042

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	116	72	74	0	0	57
N.S.	1	1.00	0.80	2.27	1.41	1.45	0.00	0.00	1.12
time (sec)	N/A	0.055	0.062	0.144	0.268	0.262	0.000	0.000	4.506

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	125	110	88	0	0	83
N.S.	1	1.00	0.62	1.37	1.21	0.97	0.00	0.00	0.91
time (sec)	N/A	0.340	0.067	0.144	0.293	0.250	0.000	0.000	0.090

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	66	135	154	96	0	0	152
N.S.	1	1.00	0.71	1.45	1.66	1.03	0.00	0.00	1.63
time (sec)	N/A	0.541	0.077	0.145	0.285	0.271	0.000	0.000	4.198

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	58	66	49	78	57
N.S.	1	1.00	1.00	0.91	1.02	1.16	0.86	1.37	1.00
time (sec)	N/A	0.050	0.036	0.484	0.209	0.247	0.104	0.281	0.043

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	49	57	39	69	49
N.S.	1	1.00	1.00	0.94	1.04	1.21	0.83	1.47	1.04
time (sec)	N/A	0.044	0.029	0.437	0.222	0.238	0.095	0.266	0.037

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	41	49	31	64	38
N.S.	1	1.00	1.00	0.92	1.05	1.26	0.79	1.64	0.97
time (sec)	N/A	0.031	0.024	0.440	0.178	0.232	0.094	0.277	0.044

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	26	38	19	46	25
N.S.	1	1.00	0.96	0.96	0.96	1.41	0.70	1.70	0.93
time (sec)	N/A	0.015	0.017	0.434	0.203	0.252	0.096	0.264	0.041

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	18	8	57	12
N.S.	1	1.00	1.00	1.00	0.92	1.38	0.62	4.38	0.92
time (sec)	N/A	0.030	0.009	0.457	0.183	0.235	0.093	0.272	0.040

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	55	26	40	28
N.S.	1	1.00	1.00	0.97	1.06	1.72	0.81	1.25	0.88
time (sec)	N/A	0.038	0.020	0.492	0.190	0.240	0.146	0.260	0.059

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	48	73	41	62	41
N.S.	1	1.00	1.00	0.93	1.04	1.59	0.89	1.35	0.89
time (sec)	N/A	0.045	0.028	0.520	0.184	0.260	0.188	0.275	4.194

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	56	81	49	74	49
N.S.	1	1.00	1.00	0.94	1.04	1.50	0.91	1.37	0.91
time (sec)	N/A	0.049	0.047	0.521	0.192	0.260	0.202	0.270	0.067

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	68	117	203	91	0	0	172
N.S.	1	1.00	0.60	1.03	1.78	0.80	0.00	0.00	1.51
time (sec)	N/A	0.092	0.050	0.125	0.198	0.255	0.000	0.000	4.212

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	60	109	166	83	0	86	134
N.S.	1	1.00	0.67	1.21	1.84	0.92	0.00	0.96	1.49
time (sec)	N/A	0.071	0.043	0.125	0.186	0.253	0.000	0.272	0.061

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	49	100	130	73	0	71	97
N.S.	1	1.00	0.77	1.56	2.03	1.14	0.00	1.11	1.52
time (sec)	N/A	0.052	0.033	0.123	0.190	0.242	0.000	0.281	0.061

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	91	90	64	0	52	58
N.S.	1	1.00	1.14	2.46	2.43	1.73	0.00	1.41	1.57
time (sec)	N/A	0.028	0.021	0.112	0.192	0.264	0.000	0.279	4.633

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	34	130	70	57	0	59	37
N.S.	1	1.00	1.70	6.50	3.50	2.85	0.00	2.95	1.85
time (sec)	N/A	0.036	0.013	0.112	0.282	0.248	0.000	0.271	0.035

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	77	55	47	0	0	55
N.S.	1	1.00	1.04	3.08	2.20	1.88	0.00	0.00	2.20
time (sec)	N/A	0.021	0.018	0.122	0.288	0.248	0.000	0.000	4.187

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	85	93	60	0	157	82
N.S.	1	1.00	1.02	2.12	2.32	1.50	0.00	3.92	2.05
time (sec)	N/A	0.028	0.032	0.125	0.286	0.252	0.000	0.271	4.179

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	52	93	137	68	0	0	105
N.S.	1	1.00	0.68	1.22	1.80	0.89	0.00	0.00	1.38
time (sec)	N/A	0.048	0.056	0.127	0.286	0.270	0.000	0.000	4.380

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	59	101	173	77	0	258	129
N.S.	1	1.00	0.67	1.15	1.97	0.88	0.00	2.93	1.47
time (sec)	N/A	0.067	0.065	0.133	0.311	0.257	0.000	0.287	4.079

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	43	42	37	47	38
N.S.	1	1.00	1.00	0.93	1.02	1.00	0.88	1.12	0.90
time (sec)	N/A	0.043	0.016	0.394	0.192	0.243	0.072	0.272	3.998

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	34	33	27	38	31
N.S.	1	1.00	1.00	0.97	1.03	1.00	0.82	1.15	0.94
time (sec)	N/A	0.038	0.012	0.385	0.223	0.239	0.080	0.265	0.040

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	26	25	20	30	23
N.S.	1	1.00	1.00	0.96	1.04	1.00	0.80	1.20	0.92
time (sec)	N/A	0.031	0.010	0.390	0.193	0.244	0.068	0.261	0.041

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	16	10	14	13
N.S.	1	1.00	1.00	1.08	1.00	1.23	0.77	1.08	1.00
time (sec)	N/A	0.013	0.011	0.388	0.202	0.240	0.060	0.258	0.030

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	14
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.08
time (sec)	N/A	0.033	0.007	0.395	0.205	0.238	0.072	0.264	0.049

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	23	15	20	14
N.S.	1	1.00	1.00	1.06	1.00	1.28	0.83	1.11	0.78
time (sec)	N/A	0.033	0.009	0.421	0.204	0.262	0.091	0.263	4.022

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	30	35	26	32	24
N.S.	1	1.00	1.00	0.94	0.94	1.09	0.81	1.00	0.75
time (sec)	N/A	0.035	0.011	0.432	0.245	0.252	0.103	0.265	0.048

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	38	38	43	34	40	31
N.S.	1	1.00	1.00	0.95	0.95	1.08	0.85	1.00	0.78
time (sec)	N/A	0.040	0.011	0.436	0.188	0.250	0.127	0.264	4.125

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	83	153	223	92	0	0	192
N.S.	1	1.00	0.61	1.12	1.64	0.68	0.00	0.00	1.41
time (sec)	N/A	0.859	0.061	0.141	0.197	0.253	0.000	0.000	0.075

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	75	145	186	84	0	0	156
N.S.	1	1.00	0.65	1.25	1.60	0.72	0.00	0.00	1.34
time (sec)	N/A	0.767	0.062	0.139	0.212	0.256	0.000	0.000	0.063

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	66	136	151	75	0	0	120
N.S.	1	1.00	0.73	1.51	1.68	0.83	0.00	0.00	1.33
time (sec)	N/A	0.680	0.046	0.146	0.192	0.259	0.000	0.000	4.304

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	127	111	66	0	0	78
N.S.	1	1.00	0.90	2.12	1.85	1.10	0.00	0.00	1.30
time (sec)	N/A	0.613	0.038	0.139	0.186	0.244	0.000	0.000	0.061

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	366	89	74	0	0	54
N.S.	1	1.00	1.20	7.96	1.93	1.61	0.00	0.00	1.17
time (sec)	N/A	0.607	0.042	0.122	0.287	0.256	0.000	0.000	0.049

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	109	72	49	0	0	59
N.S.	1	1.00	0.77	2.06	1.36	0.92	0.00	0.00	1.11
time (sec)	N/A	0.053	0.047	0.139	0.295	0.258	0.000	0.000	0.087

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	56	119	112	61	0	0	118
N.S.	1	1.00	0.64	1.37	1.29	0.70	0.00	0.00	1.36
time (sec)	N/A	0.297	0.095	0.151	0.303	0.246	0.000	0.000	0.114

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	129	157	69	0	0	153
N.S.	1	1.00	0.69	1.34	1.64	0.72	0.00	0.00	1.59
time (sec)	N/A	0.509	0.074	0.148	0.287	0.256	0.000	0.000	4.235

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	75	137	193	77	0	0	190
N.S.	1	1.00	0.56	1.03	1.45	0.58	0.00	0.00	1.43
time (sec)	N/A	0.563	0.039	0.153	0.283	0.251	0.000	0.000	4.162

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.107	5.154	0.000	0.292	0.264	0.000	0.351	0.120

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	192
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.084	5.124	0.000	0.285	0.263	0.000	0.330	4.139

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	399	0	187	103	0	172	157
N.S.	1	1.00	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.075	4.165	0.000	0.294	0.269	0.000	0.322	0.087

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	149	93	0	139	120
N.S.	1	1.00	0.46	0.00	1.05	0.65	0.00	0.98	0.85
time (sec)	N/A	0.045	0.115	0.000	0.295	0.261	0.000	0.308	0.082

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	51	0	111	84	0	108	78
N.S.	1	1.00	0.53	0.00	1.16	0.88	0.00	1.12	0.81
time (sec)	N/A	0.027	0.062	0.000	0.317	0.266	0.000	0.311	4.294

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	173	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.181	0.031	0.000	0.299	0.244	0.000	0.299	0.093

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	148	0	186	185	0	186	87
N.S.	1	1.00	0.55	0.00	0.70	0.69	0.00	0.70	0.33
time (sec)	N/A	0.159	0.170	0.000	0.273	0.254	0.000	0.297	4.291

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	173	0	226	202	0	223	132
N.S.	1	1.00	0.54	0.00	0.71	0.63	0.00	0.70	0.41
time (sec)	N/A	0.186	0.137	0.000	0.283	0.262	0.000	0.301	0.087

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	270	217	0	271	168
N.S.	1	1.00	0.26	0.00	0.76	0.61	0.00	0.76	0.47
time (sec)	N/A	0.218	0.079	0.000	0.284	0.262	0.000	0.312	4.431

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.100	5.161	0.000	0.296	0.267	0.000	0.414	0.178

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	192
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.081	5.133	0.000	0.293	0.255	0.000	0.390	4.188

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	399	0	187	103	0	172	157
N.S.	1	1.00	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.069	6.428	0.000	0.310	0.255	0.000	0.370	0.093

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	152	95	0	141	120
N.S.	1	1.00	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.043	0.133	0.000	0.293	0.261	0.000	0.347	4.200

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	56	0	112	86	0	109	79
N.S.	1	1.00	0.57	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.026	0.050	0.000	0.290	0.264	0.000	0.328	4.128

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	173	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.161	0.037	0.000	0.291	0.254	0.000	0.313	0.059

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	46	0	187	184	0	187	88
N.S.	1	1.00	0.17	0.00	0.70	0.69	0.00	0.70	0.33
time (sec)	N/A	0.163	0.052	0.000	0.293	0.249	0.000	0.320	0.090

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	76	0	229	209	0	225	132
N.S.	1	1.00	0.24	0.00	0.72	0.66	0.00	0.71	0.41
time (sec)	N/A	0.178	0.063	0.000	0.306	0.262	0.000	0.341	0.095

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	277	217	0	271	168
N.S.	1	1.00	0.26	0.00	0.78	0.61	0.00	0.76	0.47
time (sec)	N/A	0.209	0.085	0.000	0.289	0.263	0.000	0.352	4.164

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	275	152	0	254	248
N.S.	1	1.00	0.69	0.00	0.96	0.53	0.00	0.89	0.86
time (sec)	N/A	0.131	5.179	0.000	0.326	0.261	0.000	0.352	4.233

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	238	144	0	223	211
N.S.	1	1.00	0.64	0.00	0.95	0.58	0.00	0.89	0.84
time (sec)	N/A	0.102	5.154	0.000	0.291	0.263	0.000	0.348	0.139

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	441	0	203	136	0	192	176
N.S.	1	1.00	2.07	0.00	0.95	0.64	0.00	0.90	0.83
time (sec)	N/A	0.086	6.976	0.000	0.315	0.262	0.000	0.319	0.099

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	80	0	166	128	0	161	139
N.S.	1	1.00	0.45	0.00	0.94	0.73	0.00	0.91	0.79
time (sec)	N/A	0.052	0.142	0.000	0.284	0.252	0.000	0.323	4.202

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	67	0	131	117	0	141	98
N.S.	1	1.00	0.52	0.00	1.01	0.90	0.00	1.08	0.75
time (sec)	N/A	0.037	0.093	0.000	0.283	0.260	0.000	0.303	0.096

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	320	320	30	0	244	241	0	252	118
N.S.	1	1.00	0.09	0.00	0.76	0.75	0.00	0.79	0.37
time (sec)	N/A	0.218	0.054	0.000	0.324	0.260	0.000	0.302	4.455

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	173	0	204	243	0	217	107
N.S.	1	1.00	0.58	0.00	0.68	0.81	0.00	0.73	0.36
time (sec)	N/A	0.212	0.238	0.000	0.284	0.259	0.000	0.295	4.190

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	351	186	0	244	261	0	243	152
N.S.	1	1.00	0.53	0.00	0.70	0.74	0.00	0.69	0.43
time (sec)	N/A	0.218	0.169	0.000	0.284	0.270	0.000	0.316	0.090

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	288	269	0	291	188
N.S.	1	1.00	0.27	0.00	0.75	0.70	0.00	0.76	0.49
time (sec)	N/A	0.262	0.102	0.000	0.291	0.273	0.000	0.321	4.234

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.114	5.202	0.000	0.285	0.276	0.000	0.318	0.095

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	193
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.086	5.177	0.000	0.296	0.256	0.000	0.316	4.399

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	187	102	0	172	157
N.S.	1	1.00	2.17	0.00	1.04	0.57	0.00	0.96	0.88
time (sec)	N/A	0.074	7.312	0.000	0.299	0.260	0.000	0.306	4.144

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	151	93	0	140	121
N.S.	1	1.00	0.46	0.00	1.06	0.65	0.00	0.99	0.85
time (sec)	N/A	0.043	0.189	0.000	0.303	0.264	0.000	0.308	0.072

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	33	0	111	84	0	108	79
N.S.	1	1.00	0.34	0.00	1.14	0.87	0.00	1.11	0.81
time (sec)	N/A	0.027	0.059	0.000	0.285	0.263	0.000	0.301	4.049

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	173	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.167	0.054	0.000	0.298	0.266	0.000	0.311	4.485

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	33	0	186	177	0	186	88
N.S.	1	1.00	0.12	0.00	0.69	0.66	0.00	0.69	0.33
time (sec)	N/A	0.169	0.055	0.000	0.302	0.258	0.000	0.280	4.182

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	56	0	227	201	0	223	132
N.S.	1	1.00	0.18	0.00	0.71	0.63	0.00	0.70	0.41
time (sec)	N/A	0.180	0.079	0.000	0.284	0.264	0.000	0.297	0.068

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	277	216	0	271	169
N.S.	1	1.00	0.26	0.00	0.78	0.61	0.00	0.76	0.47
time (sec)	N/A	0.251	0.135	0.000	0.322	0.256	0.000	0.309	0.077

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.103	5.349	0.000	0.325	0.268	0.000	0.357	4.362

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	193
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.086	5.281	0.000	0.331	0.266	0.000	0.371	4.295

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	125	0	187	103	0	172	157
N.S.	1	1.00	0.70	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.067	5.241	0.000	0.283	0.272	0.000	0.335	0.064

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	152	95	0	141	121
N.S.	1	1.00	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.044	0.226	0.000	0.282	0.262	0.000	0.324	4.096

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	55	0	112	86	0	109	79
N.S.	1	1.00	0.56	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.034	0.139	0.000	0.285	0.255	0.000	0.280	4.079

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	28	0	224	173	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.203	0.079	0.000	0.297	0.250	0.000	0.305	4.240

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	149	0	187	192	0	187	88
N.S.	1	1.00	0.55	0.00	0.70	0.71	0.00	0.70	0.33
time (sec)	N/A	0.168	0.304	0.000	0.275	0.257	0.000	0.292	0.081

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	174	0	228	209	0	225	132
N.S.	1	1.00	0.55	0.00	0.71	0.66	0.00	0.71	0.41
time (sec)	N/A	0.196	0.233	0.000	0.289	0.263	0.000	0.314	4.280

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	270	217	0	271	169
N.S.	1	1.00	0.26	0.00	0.76	0.61	0.00	0.76	0.47
time (sec)	N/A	0.200	0.158	0.000	0.276	0.251	0.000	0.328	0.065

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	279	119	0	254	253
N.S.	1	1.00	0.69	0.00	0.97	0.41	0.00	0.89	0.88
time (sec)	N/A	0.119	5.398	0.000	0.286	0.252	0.000	0.320	0.091

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	244	111	0	223	217
N.S.	1	1.00	0.64	0.00	0.98	0.44	0.00	0.89	0.87
time (sec)	N/A	0.097	5.327	0.000	0.291	0.251	0.000	0.329	0.087

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	137	0	207	103	0	192	181
N.S.	1	1.00	0.64	0.00	0.97	0.48	0.00	0.90	0.85
time (sec)	N/A	0.084	5.293	0.000	0.285	0.266	0.000	0.308	0.079

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	121	0	172	95	0	161	145
N.S.	1	1.00	0.69	0.00	0.98	0.54	0.00	0.91	0.82
time (sec)	N/A	0.061	0.274	0.000	0.280	0.279	0.000	0.305	4.240

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	31	0	132	86	0	129	103
N.S.	1	1.00	0.24	0.00	1.02	0.66	0.00	0.99	0.79
time (sec)	N/A	0.035	0.082	0.000	0.288	0.258	0.000	0.283	4.197

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	320	320	28	0	244	190	0	252	118
N.S.	1	1.00	0.09	0.00	0.76	0.59	0.00	0.79	0.37
time (sec)	N/A	0.220	0.105	0.000	0.290	0.249	0.000	0.286	4.005

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	31	0	204	185	0	204	106
N.S.	1	1.00	0.10	0.00	0.68	0.62	0.00	0.68	0.35
time (sec)	N/A	0.187	0.100	0.000	0.280	0.261	0.000	0.291	4.057

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	351	101	0	247	209	0	243	153
N.S.	1	1.00	0.29	0.00	0.70	0.60	0.00	0.69	0.44
time (sec)	N/A	0.232	0.157	0.000	0.283	0.264	0.000	0.298	0.079

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	297	217	0	291	188
N.S.	1	1.00	0.27	0.00	0.77	0.56	0.00	0.76	0.49
time (sec)	N/A	0.249	0.193	0.000	0.282	0.267	0.000	0.307	0.080

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	189	705	220	173	0	215	168
N.S.	1	1.00	0.66	2.47	0.77	0.61	0.00	0.75	0.59
time (sec)	N/A	0.181	5.268	5.142	0.288	0.241	0.000	0.302	4.178

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	167	1158	194	168	0	191	142
N.S.	1	1.00	0.65	4.49	0.75	0.65	0.00	0.74	0.55
time (sec)	N/A	0.154	0.625	5.403	0.305	0.253	0.000	0.287	4.582

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	35	1151	167	160	0	168	115
N.S.	1	1.00	0.16	5.16	0.75	0.72	0.00	0.75	0.52
time (sec)	N/A	0.135	0.051	5.152	0.279	0.255	0.000	0.281	0.114

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	402	402	26	2088	0	323	0	261	167
N.S.	1	1.00	0.06	5.19	0.00	0.80	0.00	0.65	0.42
time (sec)	N/A	0.427	0.043	22.190	0.000	0.251	0.000	0.295	0.154

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	39	1487	152	211	0	152	109
N.S.	1	1.00	0.17	6.38	0.65	0.91	0.00	0.65	0.47
time (sec)	N/A	0.264	0.063	16.722	0.274	0.250	0.000	0.289	4.144

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	260	124	1262	178	264	0	175	136
N.S.	1	1.00	0.48	4.85	0.68	1.02	0.00	0.67	0.52
time (sec)	N/A	0.306	0.947	15.330	0.281	0.256	0.000	0.291	0.121

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	133	1498	205	269	0	199	161
N.S.	1	1.00	0.46	5.22	0.71	0.94	0.00	0.69	0.56
time (sec)	N/A	0.301	0.234	14.232	0.275	0.253	0.000	0.289	4.139

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	189	408	149	100	0	144	171
N.S.	1	1.00	1.20	2.60	0.95	0.64	0.00	0.92	1.09
time (sec)	N/A	0.049	5.272	0.764	0.284	0.250	0.000	0.288	4.049

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	165	403	123	95	0	120	145
N.S.	1	1.00	1.27	3.10	0.95	0.73	0.00	0.92	1.12
time (sec)	N/A	0.033	0.603	0.710	0.270	0.249	0.000	0.292	4.039

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	85	503	96	87	0	97	118
N.S.	1	1.00	0.89	5.24	1.00	0.91	0.00	1.01	1.23
time (sec)	N/A	0.021	0.188	0.704	0.281	0.249	0.000	0.290	0.050

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	26	1026	140	86	0	79	82
N.S.	1	1.00	0.17	6.62	0.90	0.55	0.00	0.51	0.53
time (sec)	N/A	0.037	0.044	0.661	0.278	0.241	0.000	0.291	4.287

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	87	517	98	97	0	99	118
N.S.	1	1.00	0.88	5.22	0.99	0.98	0.00	1.00	1.19
time (sec)	N/A	0.030	0.168	0.762	0.282	0.260	0.000	0.283	0.027

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	134	506	124	111	0	122	145
N.S.	1	1.00	1.03	3.89	0.95	0.85	0.00	0.94	1.12
time (sec)	N/A	0.043	0.471	0.780	0.269	0.252	0.000	0.278	4.274

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	429	429	167	0	341	266	0	308	227
N.S.	1	1.00	0.39	0.00	0.79	0.62	0.00	0.72	0.53
time (sec)	N/A	0.259	5.359	0.000	0.295	0.251	0.000	0.326	4.620

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	392	392	319	0	304	255	0	288	190
N.S.	1	1.00	0.81	0.00	0.78	0.65	0.00	0.73	0.48
time (sec)	N/A	0.168	0.780	0.000	0.310	0.262	0.000	0.302	0.173

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	352	352	56	0	265	238	0	0	149
N.S.	1	1.00	0.16	0.00	0.75	0.68	0.00	0.00	0.42
time (sec)	N/A	0.141	0.059	0.000	0.284	0.270	0.000	0.000	4.404

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	919	919	30	0	0	418	0	661	648
N.S.	1	1.00	0.03	0.00	0.00	0.45	0.00	0.72	0.71
time (sec)	N/A	0.682	0.050	0.000	0.000	0.252	0.000	1.037	4.278

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	676	676	46	0	0	374	0	432	162
N.S.	1	1.00	0.07	0.00	0.00	0.55	0.00	0.64	0.24
time (sec)	N/A	0.446	0.072	0.000	0.000	0.251	0.000	0.462	4.133

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	731	731	72	0	0	399	0	461	210
N.S.	1	1.00	0.10	0.00	0.00	0.55	0.00	0.63	0.29
time (sec)	N/A	0.486	0.097	0.000	0.000	0.266	0.000	0.503	4.503

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	201	0	0	0	0	0
N.S.	1	1.00	1.04	4.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.029	0.658	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	228	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.913	0.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	106	0	0	97	0	0
N.S.	1	1.00	0.74	3.03	0.00	0.00	2.77	0.00	0.00
time (sec)	N/A	0.032	0.013	0.482	0.000	0.000	1.488	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	128	0	0	0	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.440	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	115	0	0	0	0	0	0
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	93	0	0	116	0	0
N.S.	1	1.00	0.75	2.58	0.00	0.00	3.22	0.00	0.00
time (sec)	N/A	0.030	0.013	0.493	0.000	0.000	1.444	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	192	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.845	0.306	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	118	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.614	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	98	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	142	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	44	0	0	0	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	107	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	132	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.636	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	148	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.528	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	131	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	80	128	259	125	0	138	214
N.S.	1	1.00	0.61	0.97	1.96	0.95	0.00	1.05	1.62
time (sec)	N/A	0.234	0.219	0.424	0.201	0.255	0.000	0.286	4.329

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	120	221	115	0	118	177
N.S.	1	1.00	0.70	1.14	2.10	1.10	0.00	1.12	1.69
time (sec)	N/A	0.160	0.206	0.423	0.196	0.250	0.000	0.284	0.083

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	112	181	103	0	98	138
N.S.	1	1.00	0.82	1.44	2.32	1.32	0.00	1.26	1.77
time (sec)	N/A	0.097	0.153	0.398	0.205	0.249	0.000	0.292	4.231

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	51	92	132	77	0	58	94
N.S.	1	1.00	1.09	1.96	2.81	1.64	0.00	1.23	2.00
time (sec)	N/A	0.051	0.085	0.060	0.205	0.260	0.000	0.291	4.356

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	247	78	87	0	0	48
N.S.	1	1.00	1.18	4.84	1.53	1.71	0.00	0.00	0.94
time (sec)	N/A	0.142	0.128	0.424	0.195	0.267	0.000	0.000	0.084

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	34	36	23	57	0	49	23
N.S.	1	1.00	1.03	1.09	0.70	1.73	0.00	1.48	0.70
time (sec)	N/A	0.072	0.135	0.397	0.198	0.251	0.000	0.289	4.297

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	42	41	39	77	0	85	39
N.S.	1	1.00	0.63	0.61	0.58	1.15	0.00	1.27	0.58
time (sec)	N/A	0.094	0.140	0.404	0.205	0.252	0.000	0.293	4.322

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	51	50	55	96	0	105	56
N.S.	1	1.00	0.51	0.50	0.55	0.96	0.00	1.05	0.56
time (sec)	N/A	0.164	0.141	0.415	0.198	0.243	0.000	0.333	0.046

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	59	58	71	116	0	125	72
N.S.	1	1.00	0.44	0.44	0.53	0.87	0.00	0.94	0.54
time (sec)	N/A	0.247	0.155	0.396	0.211	0.243	0.000	0.336	0.048

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	29	49	28	124	0	28
N.S.	1	1.00	0.67	0.69	1.17	0.67	2.95	0.00	0.67
time (sec)	N/A	0.050	0.035	0.487	0.200	0.245	0.388	0.000	4.660

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	45	60	60	66	60	60
N.S.	1	1.00	0.62	1.22	1.62	1.62	1.78	1.62	1.62
time (sec)	N/A	0.045	0.025	0.465	0.193	0.240	0.055	0.285	0.042

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	29	37	37	36	37	37
N.S.	1	1.00	0.81	0.78	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.044	0.019	0.486	0.199	0.240	0.040	0.275	0.051

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	29	38	38	37	38	38
N.S.	1	1.00	0.81	0.78	1.03	1.03	1.00	1.03	1.03
time (sec)	N/A	0.045	0.018	0.485	0.193	0.240	0.048	0.260	0.050

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	16	18	18	15	18	15
N.S.	1	1.00	0.85	0.80	0.90	0.90	0.75	0.90	0.75
time (sec)	N/A	0.038	0.012	0.464	0.192	0.229	0.031	0.271	0.031

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	26	26	10	12	12	12	12	9
N.S.	1	1.86	1.86	0.71	0.86	0.86	0.86	0.86	0.64
time (sec)	N/A	0.009	0.011	0.415	0.202	0.230	0.026	0.263	0.025

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	30	29	20	31	29
N.S.	1	1.00	0.94	0.91	0.94	0.91	0.62	0.97	0.91
time (sec)	N/A	0.048	0.019	0.520	0.203	0.252	0.079	0.259	4.625

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	14	26	26	24	34	13
N.S.	1	1.00	1.79	1.00	1.86	1.86	1.71	2.43	0.93
time (sec)	N/A	0.038	0.012	0.445	0.209	0.255	0.112	0.269	4.280

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	21	47	47	49	21	46
N.S.	1	1.00	0.62	0.57	1.27	1.27	1.32	0.57	1.24
time (sec)	N/A	0.048	0.018	0.473	0.193	0.238	0.145	0.264	4.315

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	21	57	57	60	21	56
N.S.	1	1.00	0.62	0.57	1.54	1.54	1.62	0.57	1.51
time (sec)	N/A	0.050	0.019	0.460	0.206	0.254	0.185	0.263	0.098

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	138	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	128	259	126	0	138	214
N.S.	1	1.00	0.76	1.22	2.47	1.20	0.00	1.31	2.04
time (sec)	N/A	0.117	0.226	0.474	0.208	0.255	0.000	0.292	0.109

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	106	221	109	0	84	176
N.S.	1	1.00	0.82	1.36	2.83	1.40	0.00	1.08	2.26
time (sec)	N/A	0.101	0.194	0.416	0.202	0.264	0.000	0.283	4.335

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	112	181	103	0	98	139
N.S.	1	1.00	0.82	1.44	2.32	1.32	0.00	1.26	1.78
time (sec)	N/A	0.140	0.177	0.407	0.217	0.253	0.000	0.282	4.244

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	99	135	81	0	74	97
N.S.	1	1.00	0.82	1.52	2.08	1.25	0.00	1.14	1.49
time (sec)	N/A	0.148	0.116	0.119	0.215	0.251	0.000	0.283	4.602

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	345	95	120	0	35	63
N.S.	1	1.00	0.79	4.31	1.19	1.50	0.00	0.44	0.79
time (sec)	N/A	0.199	0.187	0.393	0.211	0.249	0.000	0.296	0.070

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	36	36	23	77	0	69	23
N.S.	1	1.00	1.09	1.09	0.70	2.33	0.00	2.09	0.70
time (sec)	N/A	0.079	0.158	0.433	0.204	0.257	0.000	0.334	0.038

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	41	41	39	95	0	125	39
N.S.	1	1.00	0.61	0.61	0.58	1.42	0.00	1.87	0.58
time (sec)	N/A	0.097	0.169	0.447	0.201	0.256	0.000	0.339	0.042

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	50	50	55	116	0	145	56
N.S.	1	1.00	0.53	0.53	0.59	1.23	0.00	1.54	0.60
time (sec)	N/A	0.196	0.169	0.410	0.208	0.246	0.000	0.347	4.182

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	58	58	71	134	0	165	72
N.S.	1	1.00	0.46	0.46	0.57	1.07	0.00	1.32	0.58
time (sec)	N/A	0.260	0.184	0.434	0.212	0.244	0.000	0.392	0.048

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	74	153	81	530	0	57
N.S.	1	1.00	0.76	1.12	2.32	1.23	8.03	0.00	0.86
time (sec)	N/A	0.060	0.115	0.521	0.226	0.259	0.583	0.000	4.377

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	45	59	59	63	42	59
N.S.	1	1.00	0.58	0.85	1.11	1.11	1.19	0.79	1.11
time (sec)	N/A	0.052	0.027	0.579	0.202	0.238	0.062	0.256	4.162

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	28	28	29	42	24
N.S.	1	1.00	0.81	0.72	0.88	0.88	0.91	1.31	0.75
time (sec)	N/A	0.042	0.019	0.559	0.208	0.234	0.047	0.265	0.045

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	37	37	37	42	37
N.S.	1	1.00	0.86	0.83	1.06	1.06	1.06	1.20	1.06
time (sec)	N/A	0.044	0.020	0.564	0.209	0.228	0.051	0.257	0.049

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	25	25	24	40	19
N.S.	1	1.00	1.24	0.94	1.47	1.47	1.41	2.35	1.12
time (sec)	N/A	0.036	0.017	0.557	0.197	0.239	0.036	0.269	0.037

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	24	28	26	50	26
N.S.	1	1.00	0.96	0.89	0.89	1.04	0.96	1.85	0.96
time (sec)	N/A	0.031	0.015	0.537	0.207	0.240	0.077	0.266	4.145

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	32	44	49	37	57	42
N.S.	1	1.00	0.75	0.67	0.92	1.02	0.77	1.19	0.88
time (sec)	N/A	0.052	0.025	0.553	0.215	0.243	0.160	0.262	0.068

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	51	51	51	50	25
N.S.	1	1.00	1.00	0.96	2.04	2.04	2.04	2.00	1.00
time (sec)	N/A	0.041	0.013	0.557	0.214	0.246	0.178	0.267	4.568

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	27	65	65	70	42	29
N.S.	1	1.00	0.60	0.52	1.25	1.25	1.35	0.81	0.56
time (sec)	N/A	0.053	0.023	0.540	0.203	0.234	0.201	0.265	4.442

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	27	77	77	80	42	29
N.S.	1	1.00	0.58	0.51	1.45	1.45	1.51	0.79	0.55
time (sec)	N/A	0.049	0.023	0.542	0.206	0.243	0.258	0.259	4.527

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	76	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	72	120	221	114	0	109	176
N.S.	1	1.00	0.57	0.94	1.74	0.90	0.00	0.86	1.39
time (sec)	N/A	0.257	0.240	0.423	0.200	0.244	0.000	0.274	0.091

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	112	181	104	0	90	140
N.S.	1	1.00	0.64	1.12	1.81	1.04	0.00	0.90	1.40
time (sec)	N/A	0.193	0.166	0.419	0.200	0.251	0.000	0.282	4.089

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	99	135	81	0	68	96
N.S.	1	1.00	0.82	1.52	2.08	1.25	0.00	1.05	1.48
time (sec)	N/A	0.112	0.114	0.122	0.207	0.249	0.000	0.276	0.067

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	34	76	55	47	0	33	24
N.S.	1	1.00	1.48	3.30	2.39	2.04	0.00	1.43	1.04
time (sec)	N/A	0.076	0.117	0.426	0.209	0.247	0.000	0.282	4.475

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	36	23	39	0	0	23
N.S.	1	1.00	0.96	1.29	0.82	1.39	0.00	0.00	0.82
time (sec)	N/A	0.073	0.132	0.426	0.202	0.241	0.000	0.000	4.055

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	34	41	39	57	0	45	38
N.S.	1	1.00	0.55	0.66	0.63	0.92	0.00	0.73	0.61
time (sec)	N/A	0.103	0.145	0.435	0.199	0.241	0.000	0.291	0.036

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	43	50	55	77	0	65	55
N.S.	1	1.00	0.45	0.53	0.58	0.81	0.00	0.68	0.58
time (sec)	N/A	0.158	0.144	0.425	0.212	0.254	0.000	0.304	0.045

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	51	58	71	95	0	85	71
N.S.	1	1.00	0.40	0.45	0.55	0.74	0.00	0.66	0.55
time (sec)	N/A	0.184	0.157	0.420	0.209	0.246	0.000	0.331	0.048

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	50	63	68	68	75	63
N.S.	1	1.00	0.62	0.55	0.69	0.75	0.75	0.82	0.69
time (sec)	N/A	0.056	0.025	0.498	0.200	0.237	0.099	0.256	4.192

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	42	52	57	56	64	52
N.S.	1	1.00	0.66	0.58	0.71	0.78	0.77	0.88	0.71
time (sec)	N/A	0.055	0.022	0.524	0.200	0.238	0.089	0.279	0.040

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	39	34	41	45	41	52	41
N.S.	1	1.00	0.71	0.62	0.75	0.82	0.75	0.95	0.75
time (sec)	N/A	0.047	0.020	0.486	0.211	0.256	0.076	0.252	0.051

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	24	35	26
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.92	1.35	1.00
time (sec)	N/A	0.030	0.013	0.421	0.191	0.241	0.075	0.262	0.044

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	15	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.07	1.00
time (sec)	N/A	0.041	0.012	0.475	0.200	0.245	0.030	0.270	0.040

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	24	29	23	20	25	12
N.S.	1	1.00	1.00	2.00	2.42	1.92	1.67	2.08	1.00
time (sec)	N/A	0.040	0.012	0.477	0.208	0.239	0.096	0.266	0.062

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	40	48	46	39	46	31
N.S.	1	1.00	0.97	1.21	1.45	1.39	1.18	1.39	0.94
time (sec)	N/A	0.051	0.026	0.498	0.216	0.248	0.153	0.256	0.072

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	51	63	76	54	51	46
N.S.	1	1.00	0.69	1.00	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.055	0.032	0.500	0.202	0.255	0.182	0.253	0.077

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	44	57	84	113	78	89	65
N.S.	1	1.00	0.64	0.83	1.22	1.64	1.13	1.29	0.94
time (sec)	N/A	0.060	0.037	0.490	0.211	0.236	0.258	0.252	4.427

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	96	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.079	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	86	157	244	114	0	0	199
N.S.	1	1.00	0.57	1.03	1.61	0.75	0.00	0.00	1.31
time (sec)	N/A	0.311	0.286	0.422	0.202	0.252	0.000	0.000	0.094

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	78	148	204	104	0	0	163
N.S.	1	1.00	0.60	1.15	1.58	0.81	0.00	0.00	1.26
time (sec)	N/A	0.255	0.219	0.412	0.205	0.251	0.000	0.000	4.544

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	68	136	156	81	0	0	117
N.S.	1	1.00	0.74	1.48	1.70	0.88	0.00	0.00	1.27
time (sec)	N/A	0.174	0.182	0.129	0.207	0.245	0.000	0.000	0.084

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	248	78	63	0	0	48
N.S.	1	1.00	1.02	4.68	1.47	1.19	0.00	0.00	0.91
time (sec)	N/A	0.141	0.169	0.413	0.219	0.244	0.000	0.000	4.202

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	22	22	0	0	22
N.S.	1	1.00	0.93	0.89	0.79	0.79	0.00	0.00	0.79
time (sec)	N/A	0.076	0.145	0.425	0.197	0.235	0.000	0.000	0.032

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	33	30	48	31	0	22	38
N.S.	1	1.00	1.57	1.43	2.29	1.48	0.00	1.05	1.81
time (sec)	N/A	0.073	0.156	0.424	0.203	0.249	0.000	0.279	4.202

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	65	58	0	0	50
N.S.	1	1.00	0.82	0.74	1.07	0.95	0.00	0.00	0.82
time (sec)	N/A	0.101	0.163	0.411	0.206	0.247	0.000	0.000	4.545

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	57	54	82	77	0	0	51
N.S.	1	1.00	0.61	0.57	0.87	0.82	0.00	0.00	0.54
time (sec)	N/A	0.221	0.172	0.408	0.200	0.248	0.000	0.000	4.154

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	66	63	97	96	0	0	60
N.S.	1	1.00	0.53	0.50	0.78	0.77	0.00	0.00	0.48
time (sec)	N/A	0.317	0.173	0.451	0.203	0.240	0.000	0.000	0.076

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	86	69	99	105	0	147	76
N.S.	1	1.00	0.34	0.27	0.39	0.41	0.00	0.58	0.30
time (sec)	N/A	0.152	0.076	0.411	0.223	0.244	0.000	0.290	4.528

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	78	61	83	94	0	0	102
N.S.	1	1.00	0.40	0.31	0.42	0.48	0.00	0.00	0.52
time (sec)	N/A	0.139	0.062	0.418	0.218	0.244	0.000	0.000	4.446

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	137	70	53	67	83	0	97	60
N.S.	1	1.19	0.61	0.46	0.58	0.72	0.00	0.84	0.52
time (sec)	N/A	0.140	0.054	0.409	0.214	0.259	0.000	0.286	4.397

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	89	57	43	45	64	0	0	50
N.S.	1	1.16	0.74	0.56	0.58	0.83	0.00	0.00	0.65
time (sec)	N/A	0.112	0.036	0.415	0.214	0.253	0.000	0.000	4.320

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	49	43
N.S.	1	1.00	1.48	1.21	0.90	1.72	0.00	1.69	1.48
time (sec)	N/A	0.028	0.025	0.407	0.211	0.248	0.000	0.269	4.388

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	83	0	239	0	94	0
N.S.	1	1.00	0.84	0.70	0.00	2.03	0.00	0.80	0.00
time (sec)	N/A	0.114	0.072	0.445	0.000	0.258	0.000	0.272	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	116	118	0	281	0	60	0
N.S.	1	1.00	0.91	0.92	0.00	2.20	0.00	0.47	0.00
time (sec)	N/A	0.126	0.079	0.446	0.000	0.254	0.000	0.295	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	123	167	0	337	0	78	0
N.S.	1	1.00	0.64	0.87	0.00	1.75	0.00	0.40	0.00
time (sec)	N/A	0.143	0.174	0.438	0.000	0.255	0.000	0.306	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	139	219	0	393	0	105	0
N.S.	1	1.00	0.56	0.88	0.00	1.57	0.00	0.42	0.00
time (sec)	N/A	0.159	0.176	0.426	0.000	0.272	0.000	0.325	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	60	58	205	32
N.S.	1	1.00	0.85	0.52	0.80	1.50	1.45	5.12	0.80
time (sec)	N/A	0.072	0.049	0.501	0.198	0.245	2.210	0.257	4.171

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	49	58	141	32
N.S.	1	1.00	0.85	0.52	0.80	1.22	1.45	3.52	0.80
time (sec)	N/A	0.065	0.047	0.523	0.194	0.245	2.139	0.261	0.037

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	21	32	32	58	71	32
N.S.	1	1.00	0.75	0.52	0.80	0.80	1.45	1.78	0.80
time (sec)	N/A	0.065	0.039	0.521	0.195	0.248	2.023	0.260	0.038

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	30	19	56	44	32
N.S.	1	1.00	0.61	0.53	0.79	0.50	1.47	1.16	0.84
time (sec)	N/A	0.062	0.031	0.483	0.206	0.244	2.057	0.263	0.034

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	30	29	54	32	19
N.S.	1	1.00	0.58	0.56	0.83	0.81	1.50	0.89	0.53
time (sec)	N/A	0.064	0.028	0.509	0.206	0.247	1.721	0.265	4.162

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	21	26	44	56	36	20
N.S.	1	1.00	0.89	0.55	0.68	1.16	1.47	0.95	0.53
time (sec)	N/A	0.068	0.064	0.518	0.197	0.248	1.752	0.259	0.036

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	24	56	58	34	20
N.S.	1	1.00	0.85	0.52	0.60	1.40	1.45	0.85	0.50
time (sec)	N/A	0.070	0.070	0.484	0.213	0.239	1.736	0.265	4.029

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	26	66	58	36	20
N.S.	1	1.00	0.85	0.52	0.65	1.65	1.45	0.90	0.50
time (sec)	N/A	0.071	0.071	0.508	0.196	0.250	1.924	0.270	3.934

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	77	64	106	105	0	130	110
N.S.	1	1.00	0.39	0.32	0.54	0.53	0.00	0.66	0.56
time (sec)	N/A	0.137	0.058	0.421	0.253	0.257	0.000	0.290	4.305

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	69	56	90	94	0	0	102
N.S.	1	1.00	0.50	0.41	0.66	0.69	0.00	0.00	0.74
time (sec)	N/A	0.132	0.048	0.429	0.216	0.240	0.000	0.000	4.296

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	59	48	74	83	0	80	93
N.S.	1	1.00	0.66	0.54	0.83	0.93	0.00	0.90	1.04
time (sec)	N/A	0.149	0.045	0.422	0.240	0.244	0.000	0.284	4.230

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	43	35	41	61	0	0	81
N.S.	1	1.00	1.39	1.13	1.32	1.97	0.00	0.00	2.61
time (sec)	N/A	0.030	0.040	0.418	0.279	0.245	0.000	0.000	4.679

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	250	0	0	0
N.S.	1	1.00	0.64	0.66	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.131	0.076	0.452	0.000	0.268	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	116	136	0	288	0	75	0
N.S.	1	1.00	0.66	0.77	0.00	1.63	0.00	0.42	0.00
time (sec)	N/A	0.134	0.159	0.452	0.000	0.265	0.000	0.305	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	125	174	0	341	0	78	0
N.S.	1	1.00	0.67	0.93	0.00	1.82	0.00	0.42	0.00
time (sec)	N/A	0.160	0.165	0.426	0.000	0.266	0.000	0.303	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	142	226	0	393	0	105	0
N.S.	1	1.00	0.57	0.90	0.00	1.57	0.00	0.42	0.00
time (sec)	N/A	0.161	0.160	0.434	0.000	0.254	0.000	0.304	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	147	278	0	449	0	129	0
N.S.	1	1.00	0.48	0.91	0.00	1.46	0.00	0.42	0.00
time (sec)	N/A	0.178	0.220	0.442	0.000	0.273	0.000	0.339	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	311	86	77	128	105	0	0	110
N.S.	1	1.60	0.44	0.40	0.66	0.54	0.00	0.00	0.57
time (sec)	N/A	0.174	0.074	0.421	0.242	0.248	0.000	0.000	4.323

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	254	78	69	112	94	0	0	102
N.S.	1	1.58	0.48	0.43	0.70	0.58	0.00	0.00	0.63
time (sec)	N/A	0.152	0.060	0.429	0.222	0.248	0.000	0.000	4.345

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	197	70	61	96	83	0	0	94
N.S.	1	1.54	0.55	0.48	0.75	0.65	0.00	0.00	0.73
time (sec)	N/A	0.141	0.053	0.429	0.230	0.246	0.000	0.000	4.299

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	137	60	53	72	64	0	0	82
N.S.	1	1.44	0.63	0.56	0.76	0.67	0.00	0.00	0.86
time (sec)	N/A	0.132	0.047	0.408	0.232	0.257	0.000	0.000	4.398

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	89	50	42	54	50	0	43	71
N.S.	1	1.44	0.81	0.68	0.87	0.81	0.00	0.69	1.15
time (sec)	N/A	0.118	0.037	0.434	0.216	0.246	0.000	0.274	4.413

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	35	29	44	0	0	34
N.S.	1	1.00	0.97	1.21	1.00	1.52	0.00	0.00	1.17
time (sec)	N/A	0.028	0.028	0.412	0.214	0.253	0.000	0.000	3.990

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	78	0	141	0	64	0
N.S.	1	1.00	1.00	1.03	0.00	1.86	0.00	0.84	0.00
time (sec)	N/A	0.142	0.056	0.451	0.000	0.254	0.000	0.277	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	123	0	281	0	0	0
N.S.	1	1.00	0.85	0.90	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.147	0.087	0.433	0.000	0.259	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	125	172	0	341	0	88	0
N.S.	1	1.00	0.65	0.89	0.00	1.77	0.00	0.46	0.00
time (sec)	N/A	0.146	0.121	0.435	0.000	0.260	0.000	0.301	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	88	78	123	204	150	161	112
N.S.	1	1.00	0.64	0.57	0.90	1.49	1.09	1.18	0.82
time (sec)	N/A	0.125	0.089	0.574	0.296	0.248	2.900	0.269	0.094

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	71	109	182	133	134	95
N.S.	1	1.00	0.69	0.61	0.94	1.57	1.15	1.16	0.82
time (sec)	N/A	0.099	0.070	0.566	0.304	0.268	2.749	0.267	0.072

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	71	61	95	146	116	107	78
N.S.	1	1.00	0.75	0.64	1.00	1.54	1.22	1.13	0.82
time (sec)	N/A	0.088	0.052	0.556	0.295	0.266	2.612	0.271	4.072

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	57	79	119	99	77	61
N.S.	1	1.00	0.80	0.75	1.04	1.57	1.30	1.01	0.80
time (sec)	N/A	0.074	0.038	0.533	0.291	0.255	2.577	0.269	4.517

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	45	68	118	82	51	47
N.S.	1	1.00	1.00	0.78	1.17	2.03	1.41	0.88	0.81
time (sec)	N/A	0.070	0.031	0.546	0.299	0.248	1.734	0.267	4.021

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	52	88	66	36	28
N.S.	1	1.00	1.00	0.78	1.41	2.38	1.78	0.97	0.76
time (sec)	N/A	0.070	0.022	0.494	0.300	0.260	1.839	0.266	0.095

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	50	71	146	83	54	47
N.S.	1	1.00	0.65	0.88	1.25	2.56	1.46	0.95	0.82
time (sec)	N/A	0.075	0.029	0.513	0.287	0.264	1.970	0.260	4.259

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	39	64	81	196	104	73	65
N.S.	1	1.00	0.47	0.77	0.98	2.36	1.25	0.88	0.78
time (sec)	N/A	0.083	0.031	0.532	0.322	0.256	2.241	0.263	0.105

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	39	78	101	252	121	93	79
N.S.	1	1.00	0.38	0.75	0.97	2.42	1.16	0.89	0.76
time (sec)	N/A	0.101	0.036	0.550	0.293	0.261	2.466	0.266	0.102

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	84	88	152	105	0	0	110
N.S.	1	1.00	0.23	0.24	0.41	0.29	0.00	0.00	0.30
time (sec)	N/A	0.190	0.060	0.463	0.230	0.271	0.000	0.000	4.466

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	76	80	136	94	0	0	102
N.S.	1	1.00	0.24	0.26	0.44	0.30	0.00	0.00	0.33
time (sec)	N/A	0.169	0.053	0.448	0.220	0.245	0.000	0.000	4.854

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	68	72	120	83	0	0	94
N.S.	1	1.00	0.27	0.28	0.47	0.33	0.00	0.00	0.37
time (sec)	N/A	0.163	0.046	0.467	0.216	0.256	0.000	0.000	4.462

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	57	63	93	63	0	0	81
N.S.	1	1.00	0.29	0.32	0.48	0.32	0.00	0.00	0.42
time (sec)	N/A	0.139	0.040	0.482	0.219	0.243	0.000	0.000	4.418

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	48	55	75	50	0	0	71
N.S.	1	1.00	0.35	0.40	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.117	0.031	0.464	0.223	0.251	0.000	0.000	4.266

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	48	47	48	44	0	42	34
N.S.	1	1.00	0.56	0.55	0.56	0.52	0.00	0.49	0.40
time (sec)	N/A	0.115	0.035	0.460	0.220	0.251	0.000	0.276	4.406

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	35	45	43	0	41	32
N.S.	1	1.00	1.41	1.21	1.55	1.48	0.00	1.41	1.10
time (sec)	N/A	0.030	0.033	0.451	0.222	0.257	0.000	0.272	4.234

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	122	85	0	235	0	0	0
N.S.	1	1.00	1.02	0.71	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.131	0.096	0.461	0.000	0.265	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	130	129	0	285	0	90	0
N.S.	1	1.00	0.71	0.70	0.00	1.55	0.00	0.49	0.00
time (sec)	N/A	0.150	0.315	0.467	0.000	0.282	0.000	0.299	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	41	62	110	57	0	59	94
N.S.	1	1.00	0.41	0.63	1.11	0.58	0.00	0.60	0.95
time (sec)	N/A	0.063	0.043	0.127	0.212	0.250	0.000	0.259	0.089

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	40	57	87	54	0	48	68
N.S.	1	1.00	0.51	0.72	1.10	0.68	0.00	0.61	0.86
time (sec)	N/A	0.045	0.029	0.098	0.223	0.245	0.000	0.268	0.043

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	21	22	50	24	0	15	18
N.S.	1	1.00	1.17	1.22	2.78	1.33	0.00	0.83	1.00
time (sec)	N/A	0.043	0.025	0.039	0.203	0.239	0.000	0.254	4.448

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	39	48	83	51	0	38	63
N.S.	1	1.00	1.11	1.37	2.37	1.46	0.00	1.09	1.80
time (sec)	N/A	0.040	0.023	0.094	0.208	0.243	0.000	0.266	4.011

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	52	70	138	66	0	71	118
N.S.	1	1.00	0.39	0.53	1.04	0.50	0.00	0.53	0.89
time (sec)	N/A	0.081	0.048	0.443	0.210	0.248	0.000	0.271	0.049

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	47	65	112	61	0	60	94
N.S.	1	1.00	0.44	0.61	1.06	0.58	0.00	0.57	0.89
time (sec)	N/A	0.067	0.035	0.443	0.233	0.250	0.000	0.266	0.049

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	52	70	138	66	0	72	118
N.S.	1	1.00	0.73	0.99	1.94	0.93	0.00	1.01	1.66
time (sec)	N/A	0.094	0.042	0.462	0.214	0.257	0.000	0.265	4.030

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	60	112	61	0	60	90
N.S.	1	1.00	0.89	1.13	2.11	1.15	0.00	1.13	1.70
time (sec)	N/A	0.065	0.032	0.478	0.205	0.254	0.000	0.270	4.274

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	15	16	26	15	0	14	26
N.S.	1	1.00	0.68	0.73	1.18	0.68	0.00	0.64	1.18
time (sec)	N/A	0.040	0.025	0.430	0.218	0.236	0.000	0.270	0.060

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	35	31	31	26	21	14
N.S.	1	1.00	0.82	1.59	1.41	1.41	1.18	0.95	0.64
time (sec)	N/A	0.053	0.016	0.438	0.199	0.245	2.091	0.264	0.033

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	64	74	66	0	65	43
N.S.	1	1.00	0.87	1.36	1.57	1.40	0.00	1.38	0.91
time (sec)	N/A	0.093	0.076	0.467	0.208	0.251	0.000	0.272	0.052

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	38	52	44	61	0	49	28
N.S.	1	1.00	1.15	1.58	1.33	1.85	0.00	1.48	0.85
time (sec)	N/A	0.095	0.045	0.459	0.257	0.253	0.000	0.267	4.249

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	36	56	44	44	0	44	28
N.S.	1	1.00	0.80	1.24	0.98	0.98	0.00	0.98	0.62
time (sec)	N/A	0.074	0.047	0.440	0.260	0.253	0.000	0.275	0.033

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	12	11	11	8	22	11
N.S.	1	1.00	0.86	0.57	0.52	0.52	0.38	1.05	0.52
time (sec)	N/A	0.056	0.016	0.437	0.213	0.239	3.339	0.275	0.190

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	64	56	84	0	79	40
N.S.	1	1.00	0.78	1.16	1.02	1.53	0.00	1.44	0.73
time (sec)	N/A	0.112	0.062	0.482	0.209	0.239	0.000	0.272	4.623

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	13	31	0	46	13
N.S.	1	1.00	1.00	0.92	0.54	1.29	0.00	1.92	0.54
time (sec)	N/A	0.054	0.020	0.464	0.223	0.266	0.000	0.264	0.025

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	67	49	55	69	0	0	57
N.S.	1	1.00	0.48	0.35	0.39	0.49	0.00	0.00	0.41
time (sec)	N/A	0.148	0.050	0.444	0.240	0.243	0.000	0.000	4.653

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	56	41	41	61	0	81	49
N.S.	1	1.00	0.61	0.45	0.45	0.66	0.00	0.88	0.53
time (sec)	N/A	0.119	0.032	0.454	0.240	0.250	0.000	0.274	4.585

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	49	43
N.S.	1	1.00	1.48	1.21	0.90	1.72	0.00	1.69	1.48
time (sec)	N/A	0.026	0.014	0.432	0.257	0.246	0.000	0.267	0.003

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	75	70	0	207	0	88	0
N.S.	1	1.00	0.80	0.74	0.00	2.20	0.00	0.94	0.00
time (sec)	N/A	0.142	0.050	0.472	0.000	0.266	0.000	0.276	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	76	78	0	229	0	101	0
N.S.	1	1.00	0.78	0.80	0.00	2.36	0.00	1.04	0.00
time (sec)	N/A	0.136	0.045	0.468	0.000	0.280	0.000	0.289	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	48	45	74	44	134	189	83
N.S.	1	1.00	0.48	0.45	0.73	0.44	1.33	1.87	0.82
time (sec)	N/A	0.155	0.080	0.539	0.219	0.242	2.176	0.271	0.047

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	40	37	60	36	109	142	66
N.S.	1	1.00	0.50	0.46	0.75	0.45	1.36	1.78	0.82
time (sec)	N/A	0.149	0.064	0.545	0.214	0.243	2.139	0.265	0.059

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	31	28	44	27	80	92	46
N.S.	1	1.00	0.54	0.49	0.77	0.47	1.40	1.61	0.81
time (sec)	N/A	0.103	0.056	0.549	0.212	0.245	2.532	0.252	0.051

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	30	19	56	44	32
N.S.	1	1.00	0.61	0.53	0.79	0.50	1.47	1.16	0.84
time (sec)	N/A	0.059	0.010	0.566	0.220	0.247	1.987	0.261	0.002

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	49	82	80	40	31
N.S.	1	1.00	1.00	0.82	1.26	2.10	2.05	1.03	0.79
time (sec)	N/A	0.125	0.036	0.510	0.298	0.254	3.959	0.273	4.096

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	43	62	97	0	48	34
N.S.	1	1.00	1.00	1.02	1.48	2.31	0.00	1.14	0.81
time (sec)	N/A	0.128	0.035	0.517	0.313	0.260	0.000	0.257	0.067

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	52	103	117	0	76	54
N.S.	1	1.00	0.81	0.76	1.51	1.72	0.00	1.12	0.79
time (sec)	N/A	0.145	0.055	0.541	0.294	0.260	0.000	0.267	4.501

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	62	134	133	0	104	74
N.S.	1	1.00	0.71	0.70	1.51	1.49	0.00	1.17	0.83
time (sec)	N/A	0.163	0.067	0.578	0.294	0.276	0.000	0.265	4.583

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	71	70	163	149	0	131	91
N.S.	1	1.00	0.65	0.64	1.48	1.35	0.00	1.19	0.83
time (sec)	N/A	0.158	0.077	0.542	0.293	0.257	0.000	0.256	0.077

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	130	146	0	304	0	0	0
N.S.	1	1.00	0.42	0.47	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.223	0.132	0.451	0.000	0.256	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	122	138	0	288	0	145	0
N.S.	1	1.00	0.47	0.53	0.00	1.10	0.00	0.56	0.00
time (sec)	N/A	0.199	0.103	0.456	0.000	0.270	0.000	0.294	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	114	125	0	272	0	0	0
N.S.	1	1.00	0.54	0.59	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.164	0.084	0.457	0.000	0.261	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	250	0	0	0
N.S.	1	1.00	0.64	0.66	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.143	0.047	0.445	0.000	0.262	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	120	107	0	352	0	0	0
N.S.	1	1.00	0.71	0.63	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.173	0.073	0.445	0.000	0.266	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	120	117	0	390	0	0	0
N.S.	1	1.00	0.70	0.68	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.173	0.065	0.471	0.000	0.275	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	132	144	0	428	0	0	0
N.S.	1	1.00	0.59	0.64	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.181	0.175	0.464	0.000	0.274	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	140	165	0	444	0	0	0
N.S.	1	1.00	0.51	0.60	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.209	0.229	0.493	0.000	0.272	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	148	181	0	460	0	0	0
N.S.	1	1.00	0.46	0.56	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.229	0.294	0.487	0.000	0.279	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	46	37	27	33	0	0	48
N.S.	1	1.00	0.32	0.26	0.19	0.23	0.00	0.00	0.33
time (sec)	N/A	0.106	0.035	0.446	0.231	0.237	0.000	0.000	4.348

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	41	32	22	28	70	0	38
N.S.	1	1.00	0.38	0.30	0.21	0.26	0.65	0.00	0.36
time (sec)	N/A	0.076	0.025	0.431	0.221	0.249	42.547	0.000	4.268

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	48	34	22	45	0	63	35
N.S.	1	1.00	0.46	0.33	0.21	0.43	0.00	0.61	0.34
time (sec)	N/A	0.090	0.033	0.441	0.207	0.252	0.000	0.269	4.251

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	43	29	17	40	0	49	30
N.S.	1	1.00	0.63	0.43	0.25	0.59	0.00	0.72	0.44
time (sec)	N/A	0.073	0.025	0.448	0.217	0.256	0.000	0.262	4.242

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	39	30	20	26	0	0	38
N.S.	1	1.00	0.36	0.28	0.19	0.24	0.00	0.00	0.36
time (sec)	N/A	0.079	0.023	0.426	0.214	0.234	0.000	0.000	4.105

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	34	25	15	21	56	0	21
N.S.	1	1.00	0.49	0.36	0.21	0.30	0.80	0.00	0.30
time (sec)	N/A	0.065	0.019	0.448	0.203	0.249	2.948	0.000	4.114

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	41	29	17	40	0	49	30
N.S.	1	1.00	0.58	0.41	0.24	0.56	0.00	0.69	0.42
time (sec)	N/A	0.081	0.023	0.450	0.211	0.249	0.000	0.272	4.203

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	34	24	12	33	0	33	25
N.S.	1	1.00	1.70	1.20	0.60	1.65	0.00	1.65	1.25
time (sec)	N/A	0.021	0.017	0.457	0.202	0.250	0.000	0.262	4.522

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	26	25	13	21	48	0	21
N.S.	1	1.00	0.36	0.34	0.18	0.29	0.66	0.00	0.29
time (sec)	N/A	0.080	0.021	0.443	0.205	0.246	6.502	0.000	4.160

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	21	22	7	18	19	23	18
N.S.	1	1.00	0.64	0.67	0.21	0.55	0.58	0.70	0.55
time (sec)	N/A	0.070	0.013	0.444	0.221	0.264	6.478	0.270	4.129

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	69	66	0	72	0	0	0
N.S.	1	1.00	0.55	0.52	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.084	0.055	0.460	0.000	0.244	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	63	55	0	66	58	0	0
N.S.	1	1.00	0.70	0.61	0.00	0.73	0.64	0.00	0.00
time (sec)	N/A	0.072	0.035	0.455	0.000	0.248	13.994	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	65	47	0	46	0	0	0
N.S.	1	1.00	0.70	0.51	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.089	0.048	0.453	0.000	0.250	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	41	37	0	26	42	0	0
N.S.	1	1.00	0.71	0.64	0.00	0.45	0.72	0.00	0.00
time (sec)	N/A	0.074	0.024	0.442	0.000	0.253	40.258	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	75	90	0	84	126	41	0
N.S.	1	1.00	0.58	0.69	0.00	0.65	0.97	0.32	0.00
time (sec)	N/A	0.111	0.098	0.451	0.000	0.249	101.000	0.282	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	69	79	0	76	70	32	0
N.S.	1	1.00	0.77	0.88	0.00	0.84	0.78	0.36	0.00
time (sec)	N/A	0.106	0.061	0.468	0.000	0.245	97.850	0.289	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	102	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	185	67	59	83	69	0	0	88
N.S.	1	1.30	0.47	0.42	0.58	0.49	0.00	0.00	0.62
time (sec)	N/A	0.174	0.052	0.457	0.213	0.241	0.000	0.000	4.539

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	137	58	50	69	60	0	69	57
N.S.	1	1.32	0.56	0.48	0.66	0.58	0.00	0.66	0.55
time (sec)	N/A	0.136	0.045	0.441	0.205	0.257	0.000	0.274	4.510

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	89	50	42	54	50	0	43	71
N.S.	1	1.44	0.81	0.68	0.87	0.81	0.00	0.69	1.15
time (sec)	N/A	0.158	0.026	0.438	0.201	0.246	0.000	0.271	0.003

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	80	0	206	0	40	0
N.S.	1	1.00	0.79	0.85	0.00	2.19	0.00	0.43	0.00
time (sec)	N/A	0.230	0.055	0.453	0.000	0.259	0.000	0.274	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	76	90	0	232	0	48	0
N.S.	1	1.00	0.79	0.94	0.00	2.42	0.00	0.50	0.00
time (sec)	N/A	0.180	0.047	0.464	0.000	0.256	0.000	0.271	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	85	75	123	168	175	159	114
N.S.	1	1.00	0.61	0.54	0.88	1.21	1.26	1.14	0.82
time (sec)	N/A	0.181	0.142	0.599	0.283	0.254	4.136	0.269	0.095

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	97	153	134	105	80
N.S.	1	1.00	0.80	0.70	1.00	1.58	1.38	1.08	0.82
time (sec)	N/A	0.175	0.100	0.554	0.296	0.255	3.619	0.260	4.221

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	70	59	95	136	122	105	80
N.S.	1	1.00	0.72	0.61	0.98	1.40	1.26	1.08	0.82
time (sec)	N/A	0.133	0.107	0.546	0.273	0.270	3.504	0.276	0.105

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	57	79	119	99	77	61
N.S.	1	1.00	0.80	0.75	1.04	1.57	1.30	1.01	0.80
time (sec)	N/A	0.094	0.024	0.527	0.277	0.248	2.632	0.263	0.003

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	98	157	122	67	57
N.S.	1	1.00	1.00	0.78	1.32	2.12	1.65	0.91	0.77
time (sec)	N/A	0.159	0.039	0.582	0.286	0.253	5.327	0.272	0.103

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	70	111	176	0	71	61
N.S.	1	1.00	1.00	0.90	1.42	2.26	0.00	0.91	0.78
time (sec)	N/A	0.167	0.046	0.552	0.280	0.257	0.000	0.274	4.100

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	80	152	204	0	106	88
N.S.	1	1.00	0.88	0.75	1.43	1.92	0.00	1.00	0.83
time (sec)	N/A	0.194	0.087	0.625	0.287	0.266	0.000	0.267	4.020

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	89	183	220	0	133	105
N.S.	1	1.00	0.80	0.70	1.44	1.73	0.00	1.05	0.83
time (sec)	N/A	0.196	0.105	0.590	0.283	0.263	0.000	0.276	0.144

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	109	97	212	236	0	160	122
N.S.	1	1.00	0.74	0.66	1.43	1.59	0.00	1.08	0.82
time (sec)	N/A	0.217	0.123	0.614	0.300	0.269	0.000	0.275	4.064

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	73	80	117	77	0	0	74
N.S.	1	1.00	0.26	0.28	0.42	0.27	0.00	0.00	0.26
time (sec)	N/A	0.211	0.053	0.454	0.211	0.251	0.000	0.000	4.275

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	65	72	104	69	0	0	88
N.S.	1	1.00	0.28	0.31	0.45	0.30	0.00	0.00	0.38
time (sec)	N/A	0.248	0.042	0.462	0.235	0.252	0.000	0.000	4.577

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	57	64	91	61	0	0	58
N.S.	1	1.00	0.31	0.35	0.50	0.34	0.00	0.00	0.32
time (sec)	N/A	0.145	0.038	0.471	0.244	0.252	0.000	0.000	4.106

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	48	55	75	50	0	0	71
N.S.	1	1.00	0.35	0.40	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.111	0.019	0.466	0.200	0.261	0.000	0.000	0.002

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	78	80	0	207	0	0	0
N.S.	1	1.00	0.56	0.57	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.146	0.132	0.472	0.000	0.270	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	79	86	0	233	0	0	0
N.S.	1	1.00	0.56	0.61	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.158	0.092	0.487	0.000	0.271	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	90	103	0	262	0	0	0
N.S.	1	1.00	0.47	0.54	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.158	0.095	0.484	0.000	0.265	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	98	114	0	278	0	0	0
N.S.	1	1.00	0.41	0.48	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.185	0.100	0.470	0.000	0.259	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	106	125	0	294	0	0	0
N.S.	1	1.00	0.37	0.44	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.196	0.110	0.492	0.000	0.265	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	116	104	122	185	0	0	223
N.S.	1	1.00	0.42	0.37	0.44	0.67	0.00	0.00	0.80
time (sec)	N/A	0.248	0.087	1.574	0.216	0.259	0.000	0.000	4.594

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	78	61	68	93	0	0	140
N.S.	1	1.00	0.61	0.48	0.54	0.73	0.00	0.00	1.10
time (sec)	N/A	0.132	0.057	1.157	0.216	0.262	0.000	0.000	4.357

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	58	34	37	44	0	0	55
N.S.	1	1.00	1.61	0.94	1.03	1.22	0.00	0.00	1.53
time (sec)	N/A	0.024	0.022	1.142	0.203	0.262	0.000	0.000	4.291

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.134	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	1.923	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	0	0	0	0
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	1.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.586	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	87	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.309	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	33	0	58	187	0	32
N.S.	1	1.00	0.69	0.69	0.00	1.21	3.90	0.00	0.67
time (sec)	N/A	0.083	0.285	1.757	0.000	0.254	10.168	0.000	4.919

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	64	46	0	128	1112	0	113
N.S.	1	1.00	0.62	0.44	0.00	1.23	10.69	0.00	1.09
time (sec)	N/A	0.110	0.346	4.630	0.000	0.251	59.893	0.000	4.976

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	83	68	0	228	0	0	180
N.S.	1	1.00	0.37	0.30	0.00	1.02	0.00	0.00	0.80
time (sec)	N/A	0.248	0.408	12.144	0.000	0.255	0.000	0.000	4.697

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	103	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	101	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	94	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	117	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	138	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.185	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	175	157	224	156	0	248	183
N.S.	1	1.00	1.54	1.38	1.96	1.37	0.00	2.18	1.61
time (sec)	N/A	0.204	0.217	0.091	0.284	0.252	0.000	0.286	4.266

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	167	140	201	146	0	221	163
N.S.	1	1.00	1.90	1.59	2.28	1.66	0.00	2.51	1.85
time (sec)	N/A	0.181	0.186	0.081	0.286	0.255	0.000	0.289	0.123

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	158	131	125	119	0	137	90
N.S.	1	1.00	2.55	2.11	2.02	1.92	0.00	2.21	1.45
time (sec)	N/A	0.101	0.197	0.084	0.288	0.254	0.000	0.279	4.698

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	63	66	48	0	42	60
N.S.	1	1.00	1.15	2.33	2.44	1.78	0.00	1.56	2.22
time (sec)	N/A	0.025	0.045	0.048	0.276	0.254	0.000	0.270	3.874

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	143	116	94	0	0	62
N.S.	1	1.00	0.90	2.04	1.66	1.34	0.00	0.00	0.89
time (sec)	N/A	0.149	0.140	0.147	0.201	0.252	0.000	0.000	3.790

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	94	185	137	134	0	0	104
N.S.	1	1.00	0.90	1.76	1.30	1.28	0.00	0.00	0.99
time (sec)	N/A	0.213	0.070	0.158	0.208	0.253	0.000	0.000	0.107

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	225	153	170	0	63	121
N.S.	1	1.00	0.75	1.63	1.11	1.23	0.00	0.46	0.88
time (sec)	N/A	0.286	0.086	0.171	0.197	0.263	0.000	0.295	3.811

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	112	265	169	204	0	0	137
N.S.	1	1.00	0.65	1.55	0.99	1.19	0.00	0.00	0.80
time (sec)	N/A	0.384	0.103	0.174	0.196	0.253	0.000	0.000	0.112

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	47	57	67	63	58	51
N.S.	1	1.00	0.93	0.77	0.93	1.10	1.03	0.95	0.84
time (sec)	N/A	0.103	0.088	0.637	0.197	0.251	0.186	0.288	0.079

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	32	37	43	39	38	35
N.S.	1	1.00	1.02	0.80	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.112	0.063	0.615	0.184	0.237	0.127	0.258	0.054

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	31	37	45	37	38	35
N.S.	1	1.00	0.97	0.79	0.95	1.15	0.95	0.97	0.90
time (sec)	N/A	0.114	0.058	0.593	0.192	0.247	0.128	0.261	4.275

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	21	15	16	19
N.S.	1	1.00	1.00	1.06	1.00	1.31	0.94	1.00	1.19
time (sec)	N/A	0.102	0.043	0.541	0.185	0.262	0.047	0.259	0.039

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	1.00
time (sec)	N/A	0.071	0.030	0.502	0.191	0.232	0.051	0.257	3.814

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	35	35	40	26	36	34
N.S.	1	1.00	0.81	0.95	0.95	1.08	0.70	0.97	0.92
time (sec)	N/A	0.118	0.037	0.494	0.196	0.238	0.106	0.259	0.064

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	47	55	70	49	42	54
N.S.	1	1.00	0.98	0.89	1.04	1.32	0.92	0.79	1.02
time (sec)	N/A	0.117	0.065	0.490	0.203	0.241	0.163	0.255	3.821

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	56	75	100	73	50	71
N.S.	1	1.00	0.86	0.77	1.03	1.37	1.00	0.68	0.97
time (sec)	N/A	0.125	0.079	0.489	0.192	0.244	0.235	0.285	3.840

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	67	93	126	94	58	90
N.S.	1	1.00	0.82	0.77	1.07	1.45	1.08	0.67	1.03
time (sec)	N/A	0.124	0.096	0.489	0.195	0.250	0.279	0.266	0.093

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	175	157	223	156	0	248	183
N.S.	1	1.00	1.70	1.52	2.17	1.51	0.00	2.41	1.78
time (sec)	N/A	0.099	0.265	0.092	0.281	0.254	0.000	0.298	0.136

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	51	105	151	85	0	75	119
N.S.	1	1.00	0.84	1.72	2.48	1.39	0.00	1.23	1.95
time (sec)	N/A	0.041	0.078	0.088	0.272	0.255	0.000	0.284	3.849

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	154	131	125	114	0	138	90
N.S.	1	1.00	2.44	2.08	1.98	1.81	0.00	2.19	1.43
time (sec)	N/A	0.094	0.174	0.087	0.277	0.250	0.000	0.304	3.813

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	73	145	114	88	0	91	82
N.S.	1	1.00	1.49	2.96	2.33	1.80	0.00	1.86	1.67
time (sec)	N/A	0.151	0.121	0.125	0.286	0.255	0.000	0.276	0.101

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	183	133	128	0	63	100
N.S.	1	1.00	0.67	1.74	1.27	1.22	0.00	0.60	0.95
time (sec)	N/A	0.294	0.176	0.162	0.195	0.252	0.000	0.277	3.760

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	225	153	170	0	63	120
N.S.	1	1.00	0.75	1.63	1.11	1.23	0.00	0.46	0.87
time (sec)	N/A	0.350	0.088	0.178	0.203	0.251	0.000	0.324	0.090

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	112	265	169	204	0	0	137
N.S.	1	1.00	0.68	1.61	1.02	1.24	0.00	0.00	0.83
time (sec)	N/A	0.359	0.097	0.180	0.221	0.252	0.000	0.000	4.232

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	120	305	185	240	0	63	153
N.S.	1	1.00	0.59	1.50	0.91	1.18	0.00	0.31	0.75
time (sec)	N/A	0.474	0.110	0.171	0.193	0.252	0.000	0.352	3.934

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	48	59	67	63	123	51
N.S.	1	1.00	0.80	0.75	0.92	1.05	0.98	1.92	0.80
time (sec)	N/A	0.119	0.083	0.755	0.186	0.250	0.214	0.265	3.883

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	27	31	36	31	59	27
N.S.	1	1.00	0.93	0.90	1.03	1.20	1.03	1.97	0.90
time (sec)	N/A	0.109	0.060	0.701	0.193	0.231	0.096	0.265	0.053

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	34	43	37	98	31
N.S.	1	1.00	1.00	0.79	0.89	1.13	0.97	2.58	0.82
time (sec)	N/A	0.108	0.057	0.673	0.199	0.243	0.110	0.258	3.882

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	24	27	32	26	94	25
N.S.	1	1.00	0.85	0.89	1.00	1.19	0.96	3.48	0.93
time (sec)	N/A	0.107	0.049	0.613	0.193	0.256	0.086	0.271	3.856

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	22	24	23	17	55	24
N.S.	1	1.00	0.88	0.88	0.96	0.92	0.68	2.20	0.96
time (sec)	N/A	0.075	0.041	0.604	0.209	0.252	0.154	0.262	0.074

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	43	49	64	41	74	48
N.S.	1	1.00	0.96	0.81	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.113	0.039	0.575	0.201	0.242	0.153	0.262	0.064

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	56	75	100	73	94	71
N.S.	1	1.00	0.89	0.79	1.06	1.41	1.03	1.32	1.00
time (sec)	N/A	0.121	0.072	0.587	0.181	0.248	0.195	0.272	3.860

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	67	93	126	94	109	90
N.S.	1	1.00	0.80	0.75	1.04	1.42	1.06	1.22	1.01
time (sec)	N/A	0.127	0.087	0.595	0.197	0.240	0.305	0.266	3.793

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	79	78	113	154	114	124	109
N.S.	1	1.00	0.75	0.74	1.08	1.47	1.09	1.18	1.04
time (sec)	N/A	0.126	0.100	0.592	0.195	0.244	0.425	0.272	3.859

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	175	157	223	156	0	265	185
N.S.	1	1.00	1.30	1.16	1.65	1.16	0.00	1.96	1.37
time (sec)	N/A	0.299	0.176	0.169	0.291	0.260	0.000	0.294	0.131

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	167	149	201	143	0	232	163
N.S.	1	1.00	1.58	1.41	1.90	1.35	0.00	2.19	1.54
time (sec)	N/A	0.226	0.147	0.157	0.283	0.256	0.000	0.280	3.862

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	55	134	126	113	0	130	90
N.S.	1	1.00	0.71	1.74	1.64	1.47	0.00	1.69	1.17
time (sec)	N/A	0.176	0.220	0.152	0.275	0.260	0.000	0.275	0.083

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	73	136	114	88	0	85	82
N.S.	1	1.00	1.49	2.78	2.33	1.80	0.00	1.73	1.67
time (sec)	N/A	0.119	0.126	0.131	0.276	0.246	0.000	0.281	0.074

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	28	44	27	0	24	39
N.S.	1	1.00	1.00	1.47	2.32	1.42	0.00	1.26	2.05
time (sec)	N/A	0.042	0.107	0.187	0.200	0.244	0.000	0.262	0.055

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	69	144	120	97	0	0	62
N.S.	1	1.00	0.95	1.97	1.64	1.33	0.00	0.00	0.85
time (sec)	N/A	0.100	0.044	0.219	0.182	0.256	0.000	0.000	3.814

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	94	185	137	134	0	0	105
N.S.	1	1.00	0.90	1.76	1.30	1.28	0.00	0.00	1.00
time (sec)	N/A	0.248	0.076	0.230	0.193	0.258	0.000	0.000	3.865

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	225	153	170	0	59	121
N.S.	1	1.00	0.75	1.63	1.11	1.23	0.00	0.43	0.88
time (sec)	N/A	0.284	0.088	0.244	0.199	0.244	0.000	0.294	3.867

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	51	60	71	56	62	61
N.S.	1	1.00	0.83	0.78	0.92	1.09	0.86	0.95	0.94
time (sec)	N/A	0.106	0.085	0.651	0.187	0.274	0.301	0.275	0.102

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	43	51	62	42	53	51
N.S.	1	1.00	0.85	0.80	0.94	1.15	0.78	0.98	0.94
time (sec)	N/A	0.133	0.074	0.618	0.186	0.249	0.240	0.280	0.085

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	31	40	43	31	42	40
N.S.	1	1.00	0.85	0.78	1.00	1.08	0.78	1.05	1.00
time (sec)	N/A	0.126	0.061	0.579	0.190	0.239	0.210	0.281	4.252

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	23	22	17	25	23
N.S.	1	1.00	1.00	0.87	1.00	0.96	0.74	1.09	1.00
time (sec)	N/A	0.084	0.038	0.553	0.184	0.248	0.126	0.271	0.075

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	20	19	17	21	19
N.S.	1	1.00	1.10	1.00	1.00	0.95	0.85	1.05	0.95
time (sec)	N/A	0.088	0.023	0.498	0.180	0.243	0.093	0.276	3.801

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	34	27	34	36	17
N.S.	1	1.00	1.00	1.56	1.89	1.50	1.89	2.00	0.94
time (sec)	N/A	0.103	0.046	0.519	0.190	0.242	0.129	0.285	0.073

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	48	53	59	56	51	52
N.S.	1	1.00	1.00	0.84	0.93	1.04	0.98	0.89	0.91
time (sec)	N/A	0.114	0.075	0.506	0.186	0.242	0.241	0.265	0.096

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	60	69	93	73	57	68
N.S.	1	1.00	0.96	0.80	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.118	0.095	0.520	0.187	0.242	0.292	0.265	3.879

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	567	191	246	157	0	0	211
N.S.	1	1.00	3.46	1.16	1.50	0.96	0.00	0.00	1.29
time (sec)	N/A	0.380	1.328	0.182	0.279	0.258	0.000	0.000	0.144

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	663	181	225	146	0	0	190
N.S.	1	1.00	4.91	1.34	1.67	1.08	0.00	0.00	1.41
time (sec)	N/A	0.298	0.513	0.171	0.276	0.272	0.000	0.000	3.914

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	424	169	149	120	0	0	117
N.S.	1	1.00	4.04	1.61	1.42	1.14	0.00	0.00	1.11
time (sec)	N/A	0.280	0.404	0.176	0.269	0.276	0.000	0.000	0.110

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	234	371	135	92	0	0	107
N.S.	1	1.00	3.12	4.95	1.80	1.23	0.00	0.00	1.43
time (sec)	N/A	0.172	0.541	0.138	0.277	0.263	0.000	0.000	0.099

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	137	120	69	0	0	87
N.S.	1	1.00	0.85	1.90	1.67	0.96	0.00	0.00	1.21
time (sec)	N/A	0.128	0.146	0.150	0.180	0.257	0.000	0.000	0.062

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	69	138	125	67	0	0	90
N.S.	1	1.00	0.93	1.86	1.69	0.91	0.00	0.00	1.22
time (sec)	N/A	0.076	0.045	0.252	0.185	0.258	0.000	0.000	3.840

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	41	92	42	0	42	41
N.S.	1	1.00	0.73	0.91	2.04	0.93	0.00	0.93	0.91
time (sec)	N/A	0.037	0.027	0.203	0.199	0.247	0.000	0.266	0.077

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	94	218	160	134	0	0	128
N.S.	1	1.00	0.85	1.96	1.44	1.21	0.00	0.00	1.15
time (sec)	N/A	0.109	0.078	0.243	0.192	0.247	0.000	0.000	3.894

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	259	176	170	0	0	144
N.S.	1	1.00	0.75	1.88	1.28	1.23	0.00	0.00	1.04
time (sec)	N/A	0.288	0.095	0.242	0.195	0.259	0.000	0.000	0.096

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	279	109	166	0	437	0	0	0
N.S.	1	1.19	0.46	0.71	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.128	0.126	0.092	0.000	0.295	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	221	101	149	0	415	0	0	0
N.S.	1	1.13	0.52	0.76	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.115	0.089	0.082	0.000	0.273	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	189	89	132	0	381	0	0	0
N.S.	1	1.20	0.57	0.84	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.103	0.078	0.083	0.000	0.293	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	70	105	0	313	0	0	0
N.S.	1	1.00	0.60	0.90	0.00	2.68	0.00	0.00	0.00
time (sec)	N/A	0.158	0.058	0.084	0.000	0.276	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	295	0	0	0
N.S.	1	1.00	0.85	1.12	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	0.119	0.047	0.084	0.000	0.283	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	95	151	0	517	0	0	0
N.S.	1	1.00	0.62	0.99	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.106	0.073	0.264	0.000	0.326	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	122	259	0	594	0	0	0
N.S.	1	1.00	0.57	1.20	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.118	0.103	0.281	0.000	0.327	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	135	328	0	668	0	0	0
N.S.	1	1.00	0.49	1.18	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.148	0.136	0.300	0.000	0.326	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	91	155	0	234	2222	0	0
N.S.	1	1.00	0.64	1.08	0.00	1.64	15.54	0.00	0.00
time (sec)	N/A	0.180	0.160	0.546	0.000	0.257	22.535	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	83	144	0	212	740	0	0
N.S.	1	1.00	0.70	1.22	0.00	1.80	6.27	0.00	0.00
time (sec)	N/A	0.164	0.128	0.549	0.000	0.258	7.542	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	108	0	182	144	0	0
N.S.	1	1.00	0.79	1.14	0.00	1.92	1.52	0.00	0.00
time (sec)	N/A	0.145	0.088	0.559	0.000	0.257	16.411	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	55	103	0	137	173	0	0
N.S.	1	1.00	0.79	1.47	0.00	1.96	2.47	0.00	0.00
time (sec)	N/A	0.138	0.066	0.541	0.000	0.267	25.431	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	98	0	124	0	96	0
N.S.	1	1.00	1.00	1.96	0.00	2.48	0.00	1.92	0.00
time (sec)	N/A	0.150	0.037	0.526	0.000	0.257	0.000	0.294	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	43	150	0	176	0	170	0
N.S.	1	1.00	0.61	2.14	0.00	2.51	0.00	2.43	0.00
time (sec)	N/A	0.139	0.035	0.538	0.000	0.258	0.000	0.351	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	201	0	238	0	245	0
N.S.	1	1.00	0.58	2.12	0.00	2.51	0.00	2.58	0.00
time (sec)	N/A	0.146	0.038	0.544	0.000	0.278	0.000	0.410	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	58	247	0	294	0	316	0
N.S.	1	1.00	0.49	2.09	0.00	2.49	0.00	2.68	0.00
time (sec)	N/A	0.155	0.037	0.537	0.000	0.261	0.000	0.476	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	46	291	0	346	0	391	0
N.S.	1	1.00	0.32	2.01	0.00	2.39	0.00	2.70	0.00
time (sec)	N/A	0.178	0.049	0.540	0.000	0.276	0.000	0.629	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	109	178	0	437	0	0	0
N.S.	1	1.00	0.41	0.66	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.120	0.113	0.084	0.000	0.285	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	101	161	0	415	0	0	0
N.S.	1	1.00	0.43	0.68	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.110	0.090	0.086	0.000	0.280	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	89	144	0	381	0	0	0
N.S.	1	1.00	0.57	0.92	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.188	0.072	0.079	0.000	0.286	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	66	118	0	315	0	0	0
N.S.	1	1.00	0.56	1.00	0.00	2.67	0.00	0.00	0.00
time (sec)	N/A	0.154	0.049	0.073	0.000	0.289	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	95	160	0	512	0	0	0
N.S.	1	1.00	0.62	1.05	0.00	3.37	0.00	0.00	0.00
time (sec)	N/A	0.100	0.058	0.237	0.000	0.315	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	115	259	0	581	0	0	0
N.S.	1	1.00	0.53	1.20	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	0.114	0.115	0.277	0.000	0.337	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	135	326	0	668	0	0	0
N.S.	1	1.00	0.49	1.19	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.132	0.141	0.276	0.000	0.335	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	143	376	0	736	0	0	0
N.S.	1	1.00	0.43	1.12	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.148	0.168	0.282	0.000	0.327	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	101	161	0	415	0	0	0
N.S.	1	1.00	0.46	0.73	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.110	0.110	0.154	0.000	0.283	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	89	144	0	381	0	0	0
N.S.	1	1.00	0.55	0.89	0.00	2.37	0.00	0.00	0.00
time (sec)	N/A	0.099	0.074	0.161	0.000	0.284	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	70	118	0	315	0	0	0
N.S.	1	1.00	0.50	0.84	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.136	0.056	0.151	0.000	0.283	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	297	0	0	0
N.S.	1	1.00	0.82	1.28	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.115	0.041	0.138	0.000	0.281	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	102	0	299	0	0	0
N.S.	1	1.00	0.83	1.31	0.00	3.83	0.00	0.00	0.00
time (sec)	N/A	0.113	0.047	0.142	0.000	0.281	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	91	162	0	522	0	0	0
N.S.	1	1.00	0.60	1.07	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.111	0.077	0.312	0.000	0.328	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	123	264	0	596	0	0	0
N.S.	1	1.00	0.56	1.21	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.118	0.109	0.303	0.000	0.316	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	135	316	0	668	0	0	0
N.S.	1	1.00	0.49	1.14	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.137	0.148	0.334	0.000	0.325	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	223	0	323	0	0	0
N.S.	1	1.00	0.77	1.37	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.205	0.272	0.539	0.000	0.263	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	116	213	0	285	0	0	0
N.S.	1	1.00	0.84	1.54	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.199	0.117	0.517	0.000	0.269	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	193	0	235	0	0	0
N.S.	1	1.00	0.84	1.71	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.185	0.090	0.505	0.000	0.256	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	168	0	219	0	0	0
N.S.	1	1.00	1.00	1.83	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.151	0.057	0.501	0.000	0.267	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	136	0	234	0	0	0
N.S.	1	1.00	1.00	1.43	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.145	0.056	0.500	0.000	0.263	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	136	0	231	0	0	0
N.S.	1	1.00	1.00	1.45	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.159	0.061	0.536	0.000	0.270	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	70	227	0	287	0	0	0
N.S.	1	1.00	0.60	1.96	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.188	0.060	0.496	0.000	0.268	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	79	271	0	359	0	0	0
N.S.	1	1.00	0.54	1.84	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.198	0.073	0.517	0.000	0.266	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	82	315	0	431	0	0	0
N.S.	1	1.00	0.48	1.83	0.00	2.51	0.00	0.00	0.00
time (sec)	N/A	0.224	0.072	0.487	0.000	0.281	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	140	229	0	437	0	0	0
N.S.	1	1.00	0.42	0.68	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.204	0.129	0.165	0.000	0.304	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	132	212	0	415	0	0	0
N.S.	1	1.00	0.48	0.77	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.146	0.124	0.158	0.000	0.290	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	124	195	0	381	0	0	0
N.S.	1	1.00	0.57	0.89	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.124	0.110	0.161	0.000	0.288	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	71	169	0	315	0	0	0
N.S.	1	1.00	0.45	1.07	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.111	0.052	0.148	0.000	0.291	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	299	0	0	0
N.S.	1	1.00	0.48	1.04	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.105	0.056	0.147	0.000	0.283	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	69	149	0	303	0	0	0
N.S.	1	1.00	0.58	1.26	0.00	2.57	0.00	0.00	0.00
time (sec)	N/A	0.169	0.050	0.143	0.000	0.304	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	64	149	0	311	0	0	0
N.S.	1	1.00	0.55	1.27	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.166	0.051	0.139	0.000	0.282	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	90	258	0	524	0	0	0
N.S.	1	1.00	0.45	1.30	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.125	0.070	0.319	0.000	0.316	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	121	290	0	594	0	0	0
N.S.	1	1.00	0.45	1.09	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.133	0.091	0.330	0.000	0.321	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	147	121	0	337	0	0	0
N.S.	1	1.00	0.90	0.74	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.247	0.919	0.066	0.000	0.293	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	148	102	0	317	0	0	0
N.S.	1	1.00	1.19	0.82	0.00	2.56	0.00	0.00	0.00
time (sec)	N/A	0.182	1.428	0.066	0.000	0.285	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	295	0	0	0
N.S.	1	1.00	0.85	1.12	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	0.113	0.044	0.055	0.000	0.279	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	132	88	0	275	0	0	0
N.S.	1	1.00	1.74	1.16	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.163	0.599	0.069	0.000	0.274	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	41	0	58	0	0	47
N.S.	1	1.00	1.22	1.11	0.00	1.57	0.00	0.00	1.27
time (sec)	N/A	0.113	0.197	0.048	0.000	0.253	0.000	0.000	4.438

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	58	47	0	68	0	0	53
N.S.	1	1.00	0.75	0.61	0.00	0.88	0.00	0.00	0.69
time (sec)	N/A	0.147	0.243	0.047	0.000	0.252	0.000	0.000	4.046

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	66	55	0	77	0	0	100
N.S.	1	1.00	0.56	0.47	0.00	0.66	0.00	0.00	0.85
time (sec)	N/A	0.227	0.236	0.048	0.000	0.254	0.000	0.000	4.119

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	74	63	0	84	0	0	108
N.S.	1	1.00	0.47	0.40	0.00	0.53	0.00	0.00	0.68
time (sec)	N/A	0.225	0.286	0.049	0.000	0.260	0.000	0.000	4.153

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	50	127	0	179	0	142	0
N.S.	1	1.00	0.38	0.98	0.00	1.38	0.00	1.09	0.00
time (sec)	N/A	0.280	0.045	0.497	0.000	0.263	0.000	0.291	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	50	119	0	163	0	127	0
N.S.	1	1.00	0.48	1.13	0.00	1.55	0.00	1.21	0.00
time (sec)	N/A	0.261	0.042	0.495	0.000	0.255	0.000	0.300	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	111	0	147	0	112	0
N.S.	1	1.00	0.96	1.39	0.00	1.84	0.00	1.40	0.00
time (sec)	N/A	0.180	0.082	0.493	0.000	0.264	0.000	0.292	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	98	0	124	0	96	0
N.S.	1	1.00	1.00	1.96	0.00	2.48	0.00	1.92	0.00
time (sec)	N/A	0.121	0.027	0.489	0.000	0.266	0.000	0.289	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	98	0	111	95	0	0
N.S.	1	1.00	1.00	2.09	0.00	2.36	2.02	0.00	0.00
time (sec)	N/A	0.241	0.038	0.490	0.000	0.266	4.292	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	0	28	0	0	24
N.S.	1	1.00	0.67	0.64	0.00	0.67	0.00	0.00	0.57
time (sec)	N/A	0.245	0.045	0.484	0.000	0.257	0.000	0.000	4.037

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	36	35	0	36	0	0	32
N.S.	1	1.00	0.52	0.51	0.00	0.52	0.00	0.00	0.46
time (sec)	N/A	0.246	0.052	0.478	0.000	0.252	0.000	0.000	3.900

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	44	43	0	44	0	0	77
N.S.	1	1.00	0.46	0.45	0.00	0.46	0.00	0.00	0.80
time (sec)	N/A	0.249	0.060	0.482	0.000	0.251	0.000	0.000	4.160

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	52	51	0	52	0	0	98
N.S.	1	1.00	0.43	0.42	0.00	0.43	0.00	0.00	0.81
time (sec)	N/A	0.255	0.072	0.478	0.000	0.239	0.000	0.000	4.098

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	252	224	0	568	0	0	0
N.S.	1	1.00	0.81	0.72	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.256	1.449	0.257	0.000	0.328	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	244	202	0	552	0	0	0
N.S.	1	1.00	0.93	0.77	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.235	1.077	0.247	0.000	0.336	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	236	180	0	536	0	0	0
N.S.	1	1.00	1.13	0.86	0.00	2.56	0.00	0.00	0.00
time (sec)	N/A	0.160	1.598	0.243	0.000	0.336	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	95	160	0	512	0	0	0
N.S.	1	1.00	0.62	1.05	0.00	3.37	0.00	0.00	0.00
time (sec)	N/A	0.105	0.059	0.233	0.000	0.323	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	218	159	0	490	0	0	0
N.S.	1	1.00	1.49	1.09	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.203	0.804	0.256	0.000	0.336	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	155	140	0	353	0	0	0
N.S.	1	1.00	1.24	1.12	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.169	0.382	0.247	0.000	0.300	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	162	165	0	381	0	0	0
N.S.	1	1.00	0.95	0.97	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.211	0.442	0.259	0.000	0.298	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	170	187	0	397	0	0	0
N.S.	1	1.00	0.81	0.89	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.324	0.437	0.261	0.000	0.297	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	178	206	0	413	0	0	0
N.S.	1	1.00	0.59	0.68	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.228	0.496	0.262	0.000	0.289	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	93	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	147	133	0	337	0	0	0
N.S.	1	1.00	0.90	0.81	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.263	0.899	0.141	0.000	0.285	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	139	116	0	321	0	0	0
N.S.	1	1.00	1.12	0.94	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.188	1.492	0.137	0.000	0.284	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	297	0	0	0
N.S.	1	1.00	0.82	1.28	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.161	0.044	0.124	0.000	0.287	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	132	100	0	275	0	0	0
N.S.	1	1.00	1.74	1.32	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.176	0.635	0.135	0.000	0.279	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	46	54	0	59	0	0	54
N.S.	1	1.00	0.66	0.77	0.00	0.84	0.00	0.00	0.77
time (sec)	N/A	0.136	0.195	0.124	0.000	0.247	0.000	0.000	4.022

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	58	62	0	69	0	0	62
N.S.	1	1.00	0.51	0.55	0.00	0.61	0.00	0.00	0.55
time (sec)	N/A	0.158	0.273	0.116	0.000	0.269	0.000	0.000	3.997

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	66	70	0	77	0	0	100
N.S.	1	1.00	0.44	0.47	0.00	0.52	0.00	0.00	0.67
time (sec)	N/A	0.214	0.240	0.135	0.000	0.249	0.000	0.000	4.159

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	116	197	0	271	0	0	0
N.S.	1	1.00	0.67	1.15	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.356	0.166	0.490	0.000	0.281	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	108	189	0	259	0	0	0
N.S.	1	1.00	0.73	1.29	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.332	0.117	0.490	0.000	0.264	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	100	181	0	239	0	0	0
N.S.	1	1.00	0.82	1.48	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.232	0.096	0.484	0.000	0.271	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	168	0	219	0	0	0
N.S.	1	1.00	1.00	1.83	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.146	0.029	0.481	0.000	0.273	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	165	0	203	141	0	0
N.S.	1	1.00	1.00	1.92	0.00	2.36	1.64	0.00	0.00
time (sec)	N/A	0.260	0.048	0.523	0.000	0.263	4.276	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	142	0	161	0	0	0
N.S.	1	1.00	0.84	1.73	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.275	0.075	0.499	0.000	0.258	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	79	152	0	181	0	278	0
N.S.	1	1.00	0.70	1.35	0.00	1.60	0.00	2.46	0.00
time (sec)	N/A	0.329	0.092	0.517	0.000	0.251	0.000	0.638	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	87	160	0	201	0	356	0
N.S.	1	1.00	0.77	1.42	0.00	1.78	0.00	3.15	0.00
time (sec)	N/A	0.306	0.146	0.510	0.000	0.255	0.000	0.742	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	95	168	0	213	0	434	0
N.S.	1	1.00	0.58	1.03	0.00	1.31	0.00	2.66	0.00
time (sec)	N/A	0.313	0.161	0.505	0.000	0.252	0.000	0.879	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	167	197	0	353	0	0	0
N.S.	1	1.00	0.55	0.65	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.226	1.115	0.144	0.000	0.291	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	159	180	0	337	0	0	0
N.S.	1	1.00	0.63	0.72	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.220	0.951	0.146	0.000	0.293	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	151	163	0	321	0	0	0
N.S.	1	1.00	0.76	0.82	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.147	1.614	0.143	0.000	0.282	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	299	0	0	0
N.S.	1	1.00	0.48	1.04	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.085	0.047	0.124	0.000	0.280	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	131	151	0	277	0	0	0
N.S.	1	1.00	0.98	1.13	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.182	0.831	0.151	0.000	0.269	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	58	62	0	59	0	0	54
N.S.	1	1.00	0.53	0.57	0.00	0.54	0.00	0.00	0.50
time (sec)	N/A	0.153	0.217	0.154	0.000	0.263	0.000	0.000	4.193

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	70	70	0	69	0	0	62
N.S.	1	1.00	0.47	0.47	0.00	0.46	0.00	0.00	0.41
time (sec)	N/A	0.183	0.274	0.151	0.000	0.258	0.000	0.000	4.247

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	78	78	0	77	0	0	100
N.S.	1	1.00	0.41	0.41	0.00	0.41	0.00	0.00	0.53
time (sec)	N/A	0.296	0.261	0.153	0.000	0.251	0.000	0.000	4.447

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	86	86	0	85	0	0	108
N.S.	1	1.00	0.30	0.30	0.00	0.29	0.00	0.00	0.37
time (sec)	N/A	0.259	0.330	0.139	0.000	0.269	0.000	0.000	4.162

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	155	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.379	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.466	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	113	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	269	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	4.72	0.00	0.00
time (sec)	N/A	0.085	0.033	0.000	0.000	0.000	4.895	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	111	160	415	169	0	218	362
N.S.	1	1.00	0.28	0.41	1.06	0.43	0.00	0.55	0.92
time (sec)	N/A	0.235	0.208	0.478	0.202	0.259	0.000	0.307	4.413

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	95	144	337	148	0	178	289
N.S.	1	1.00	0.30	0.46	1.08	0.47	0.00	0.57	0.92
time (sec)	N/A	0.211	0.203	0.489	0.207	0.248	0.000	0.293	3.960

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	79	128	259	125	0	137	214
N.S.	1	1.00	0.34	0.55	1.11	0.54	0.00	0.59	0.92
time (sec)	N/A	0.161	0.136	0.489	0.204	0.260	0.000	0.287	3.885

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	61	108	171	91	0	90	131
N.S.	1	1.00	0.42	0.74	1.18	0.63	0.00	0.62	0.90
time (sec)	N/A	0.111	0.110	0.062	0.201	0.248	0.000	0.278	0.078

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	22	34	0	0	22
N.S.	1	1.00	1.00	1.77	1.69	2.62	0.00	0.00	1.69
time (sec)	N/A	0.023	0.153	0.496	0.213	0.247	0.000	0.000	3.834

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	47	65	58	0	0	50
N.S.	1	1.00	0.98	0.92	1.27	1.14	0.00	0.00	0.98
time (sec)	N/A	0.046	0.301	0.474	0.208	0.247	0.000	0.000	0.081

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	99	86	0	0	60
N.S.	1	1.00	0.78	0.76	1.16	1.01	0.00	0.00	0.71
time (sec)	N/A	0.078	0.377	0.482	0.197	0.262	0.000	0.000	4.298

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	82	81	132	134	0	0	142
N.S.	1	1.00	0.69	0.68	1.11	1.13	0.00	0.00	1.19
time (sec)	N/A	0.114	0.456	0.485	0.210	0.241	0.000	0.000	0.071

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	47	85	113	113	119	113	113
N.S.	1	1.00	0.56	1.01	1.35	1.35	1.42	1.35	1.35
time (sec)	N/A	0.080	0.045	0.647	0.204	0.244	0.042	0.281	3.900

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	39	61	82	82	87	82	82
N.S.	1	1.00	0.57	0.88	1.19	1.19	1.26	1.19	1.19
time (sec)	N/A	0.074	0.036	0.625	0.201	0.232	0.038	0.260	0.041

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	53	70	70	70	70	70
N.S.	1	1.00	0.60	1.02	1.35	1.35	1.35	1.35	1.35
time (sec)	N/A	0.063	0.029	0.634	0.195	0.236	0.033	0.284	0.037

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	29	38	38	36	38	38
N.S.	1	1.00	0.66	0.83	1.09	1.09	1.03	1.09	1.09
time (sec)	N/A	0.062	0.020	0.592	0.194	0.243	0.028	0.284	0.053

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	14	21	21	20	21	17
N.S.	1	1.00	1.33	0.93	1.40	1.40	1.33	1.40	1.13
time (sec)	N/A	0.034	0.016	0.490	0.199	0.238	0.025	0.268	0.037

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	13	13	13	10	14	14
N.S.	1	1.00	1.12	0.81	0.81	0.81	0.62	0.88	0.88
time (sec)	N/A	0.052	0.061	0.574	0.196	0.235	0.058	0.280	0.039

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	51	63	76	54	51	46
N.S.	1	1.00	0.69	1.00	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.063	0.032	0.574	0.187	0.233	0.147	0.282	3.895

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	76	91	121	85	74	73
N.S.	1	1.00	0.73	0.88	1.06	1.41	0.99	0.86	0.85
time (sec)	N/A	0.077	0.045	0.612	0.214	0.244	0.221	0.267	0.094

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	82	92	140	217	141	91	121
N.S.	1	1.00	0.68	0.76	1.16	1.79	1.17	0.75	1.00
time (sec)	N/A	0.094	0.071	0.618	0.204	0.252	0.334	0.267	4.141

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	111	160	415	170	0	216	362
N.S.	1	1.00	0.28	0.41	1.06	0.43	0.00	0.55	0.92
time (sec)	N/A	0.234	0.207	0.504	0.212	0.257	0.000	0.320	4.308

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	95	144	337	147	0	177	289
N.S.	1	1.00	0.30	0.46	1.08	0.47	0.00	0.57	0.92
time (sec)	N/A	0.196	0.172	0.492	0.225	0.243	0.000	0.298	4.033

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	79	128	259	126	0	138	214
N.S.	1	1.00	0.34	0.55	1.11	0.54	0.00	0.59	0.92
time (sec)	N/A	0.162	0.137	0.489	0.187	0.247	0.000	0.289	3.985

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	61	108	171	91	0	90	133
N.S.	1	1.00	0.42	0.74	1.18	0.63	0.00	0.62	0.92
time (sec)	N/A	0.115	0.103	0.126	0.202	0.248	0.000	0.279	0.073

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	51	0	49	23
N.S.	1	1.00	1.00	1.33	1.28	2.83	0.00	2.72	1.28
time (sec)	N/A	0.035	0.177	0.513	0.189	0.240	0.000	0.285	0.037

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	49	55	77	0	65	55
N.S.	1	1.00	0.78	0.89	1.00	1.40	0.00	1.18	1.00
time (sec)	N/A	0.063	0.342	0.499	0.194	0.245	0.000	0.311	4.370

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	63	97	96	0	0	60
N.S.	1	1.00	0.73	0.69	1.07	1.05	0.00	0.00	0.66
time (sec)	N/A	0.092	0.410	0.504	0.191	0.244	0.000	0.000	4.310

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	82	81	131	124	0	0	76
N.S.	1	1.00	0.65	0.64	1.03	0.98	0.00	0.00	0.60
time (sec)	N/A	0.126	0.489	0.489	0.192	0.250	0.000	0.000	3.981

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	39	75	101	101	109	102	101
N.S.	1	1.00	0.59	1.14	1.53	1.53	1.65	1.55	1.53
time (sec)	N/A	0.071	0.039	0.689	0.197	0.235	0.049	0.265	3.968

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	69	92	92	100	90	92
N.S.	1	1.00	0.60	1.33	1.77	1.77	1.92	1.73	1.77
time (sec)	N/A	0.060	0.032	0.655	0.190	0.242	0.043	0.268	0.047

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	45	59	59	63	78	59
N.S.	1	1.00	0.66	1.29	1.69	1.69	1.80	2.23	1.69
time (sec)	N/A	0.052	0.025	0.654	0.191	0.234	0.036	0.264	0.035

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	47	47	48	64	47
N.S.	1	1.00	2.18	0.94	2.76	2.76	2.82	3.76	2.76
time (sec)	N/A	0.044	0.024	0.624	0.185	0.224	0.034	0.270	0.033

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	32	33	37	36	60	33
N.S.	1	1.00	0.83	0.70	0.72	0.80	0.78	1.30	0.72
time (sec)	N/A	0.039	0.020	0.595	0.193	0.249	0.066	0.259	0.053

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	13	19	19	17	27	12
N.S.	1	1.00	1.92	1.00	1.46	1.46	1.31	2.08	0.92
time (sec)	N/A	0.058	0.014	0.647	0.194	0.239	0.092	0.275	0.055

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	41	41	42	15	40
N.S.	1	1.00	1.00	0.89	2.28	2.28	2.33	0.83	2.22
time (sec)	N/A	0.075	0.088	0.625	0.190	0.232	0.107	0.263	0.068

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	65	102	147	99	91	83
N.S.	1	1.00	0.60	0.75	1.17	1.69	1.14	1.05	0.95
time (sec)	N/A	0.134	0.042	0.621	0.188	0.252	0.253	0.276	0.105

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	92	130	191	129	127	111
N.S.	1	1.00	0.66	0.75	1.07	1.57	1.06	1.04	0.91
time (sec)	N/A	0.101	0.066	0.632	0.195	0.250	0.294	0.267	3.960

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	111	160	415	170	0	196	362
N.S.	1	1.00	0.28	0.41	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.263	0.219	0.487	0.195	0.249	0.000	0.303	4.435

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	95	144	337	147	0	161	289
N.S.	1	1.00	0.30	0.46	1.08	0.47	0.00	0.51	0.92
time (sec)	N/A	0.192	0.207	0.493	0.205	0.257	0.000	0.289	0.124

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	79	128	259	126	0	126	214
N.S.	1	1.00	0.34	0.55	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.151	0.134	0.489	0.204	0.255	0.000	0.287	3.967

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	61	108	171	91	0	82	132
N.S.	1	1.00	0.42	0.74	1.18	0.63	0.00	0.57	0.91
time (sec)	N/A	0.093	0.116	0.066	0.200	0.243	0.000	0.287	3.893

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	23	0	0	23
N.S.	1	1.00	1.00	1.50	1.44	1.44	0.00	0.00	1.44
time (sec)	N/A	0.026	0.165	0.497	0.202	0.256	0.000	0.000	0.033

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	67	50	0	0	55
N.S.	1	1.00	0.87	0.89	1.22	0.91	0.00	0.00	1.00
time (sec)	N/A	0.050	0.304	0.500	0.202	0.238	0.000	0.000	0.058

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	64	65	102	76	0	0	109
N.S.	1	1.00	0.70	0.71	1.12	0.84	0.00	0.00	1.20
time (sec)	N/A	0.083	0.386	0.504	0.204	0.256	0.000	0.000	0.065

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	81	135	104	0	0	148
N.S.	1	1.00	0.63	0.64	1.06	0.82	0.00	0.00	1.17
time (sec)	N/A	0.113	0.471	0.507	0.197	0.262	0.000	0.000	4.235

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	39	61	80	80	87	80	80
N.S.	1	1.00	0.53	0.84	1.10	1.10	1.19	1.10	1.10
time (sec)	N/A	0.067	0.037	0.620	0.204	0.237	0.041	0.259	0.083

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	31	53	70	70	70	70	70
N.S.	1	1.00	0.56	0.96	1.27	1.27	1.27	1.27	1.27
time (sec)	N/A	0.065	0.030	0.631	0.193	0.238	0.037	0.272	0.057

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	29	37	37	36	37	37
N.S.	1	1.00	0.81	0.78	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.058	0.022	0.627	0.197	0.236	0.029	0.267	0.055

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	14	20	20	19	20	17
N.S.	1	1.00	1.31	0.88	1.25	1.25	1.19	1.25	1.06
time (sec)	N/A	0.034	0.017	0.516	0.189	0.238	0.029	0.264	0.038

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	14	12	12	10	14	12
N.S.	1	1.00	1.29	1.00	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.064	0.058	0.592	0.196	0.247	0.070	0.259	0.054

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	51	63	76	54	51	46
N.S.	1	1.00	0.67	1.04	1.29	1.55	1.10	1.04	0.94
time (sec)	N/A	0.074	0.034	0.589	0.199	0.242	0.160	0.276	0.066

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	61	76	91	121	85	74	73
N.S.	1	1.00	0.73	0.90	1.08	1.44	1.01	0.88	0.87
time (sec)	N/A	0.091	0.048	0.610	0.205	0.271	0.240	0.274	4.001

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	80	92	140	217	141	91	121
N.S.	1	1.00	0.67	0.77	1.18	1.82	1.18	0.76	1.02
time (sec)	N/A	0.097	0.068	0.602	0.205	0.271	0.339	0.284	4.101

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	111	160	415	169	0	198	362
N.S.	1	1.00	0.28	0.41	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.244	0.229	0.513	0.205	0.261	0.000	0.305	0.175

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	95	144	337	148	0	162	289
N.S.	1	1.00	0.30	0.46	1.08	0.47	0.00	0.52	0.92
time (sec)	N/A	0.210	0.174	0.513	0.211	0.247	0.000	0.278	0.120

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	79	128	259	125	0	126	214
N.S.	1	1.00	0.34	0.55	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.153	0.145	0.510	0.198	0.247	0.000	0.282	3.932

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	61	108	171	92	0	82	133
N.S.	1	1.00	0.42	0.74	1.18	0.63	0.00	0.57	0.92
time (sec)	N/A	0.136	0.126	0.132	0.200	0.251	0.000	0.282	0.067

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	34	0	49	23
N.S.	1	1.00	1.00	1.33	1.28	1.89	0.00	2.72	1.28
time (sec)	N/A	0.028	0.192	0.518	0.195	0.249	0.000	0.309	4.274

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	47	60	58	0	65	60
N.S.	1	1.00	0.78	0.85	1.09	1.05	0.00	1.18	1.09
time (sec)	N/A	0.050	0.312	0.510	0.218	0.239	0.000	0.313	3.852

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	65	103	86	0	0	116
N.S.	1	1.00	0.73	0.71	1.13	0.95	0.00	0.00	1.27
time (sec)	N/A	0.080	0.415	0.507	0.205	0.244	0.000	0.000	3.847

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	82	81	136	134	0	0	155
N.S.	1	1.00	0.65	0.64	1.07	1.06	0.00	0.00	1.22
time (sec)	N/A	0.111	0.505	0.515	0.214	0.249	0.000	0.000	0.043

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	79	113	0	117	0	0	0
N.S.	1	1.00	0.34	0.49	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.144	0.088	0.510	0.000	0.250	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	71	97	0	95	0	0	0
N.S.	1	1.00	0.39	0.53	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.147	0.068	0.500	0.000	0.241	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	63	81	0	73	0	0	0
N.S.	1	1.00	0.46	0.60	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.127	0.052	0.498	0.000	0.250	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	53	63	0	43	0	0	0
N.S.	1	1.00	0.57	0.68	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.120	0.038	0.497	0.000	0.249	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	44	0	22	0	0	0
N.S.	1	1.00	0.60	0.65	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.089	0.023	0.497	0.000	0.243	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	0	22	0	0	0
N.S.	1	1.00	1.00	1.34	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.127	0.022	0.478	0.000	0.254	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	86	0	0	0
N.S.	1	1.00	0.62	0.92	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.138	0.053	0.484	0.000	0.285	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	136	0	0	0
N.S.	1	1.00	0.45	0.92	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.151	0.069	0.514	0.000	0.248	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	101	241	0	191	0	0	0
N.S.	1	1.00	0.36	0.87	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.168	0.107	0.542	0.000	0.273	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	167	138	192	329	714	164	0
N.S.	1	1.00	0.95	0.78	1.09	1.87	4.06	0.93	0.00
time (sec)	N/A	0.130	0.203	0.661	0.280	0.298	3.360	0.306	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	151	122	173	286	614	141	0
N.S.	1	1.00	0.99	0.80	1.13	1.87	4.01	0.92	0.00
time (sec)	N/A	0.138	0.166	0.592	0.287	0.267	2.966	0.315	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	135	106	154	241	314	116	0
N.S.	1	1.00	1.04	0.82	1.18	1.85	2.42	0.89	0.00
time (sec)	N/A	0.117	0.145	0.589	0.283	0.279	2.530	0.284	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	117	90	131	180	250	85	0
N.S.	1	1.00	1.09	0.84	1.22	1.68	2.34	0.79	0.00
time (sec)	N/A	0.101	0.111	0.596	0.293	0.271	2.240	0.301	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	69	47	134	0	62	0
N.S.	1	1.00	0.88	0.80	0.55	1.56	0.00	0.72	0.00
time (sec)	N/A	0.091	0.063	0.597	0.282	0.256	0.000	0.282	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	82	79	40	153	0	0	0
N.S.	1	1.00	1.39	1.34	0.68	2.59	0.00	0.00	0.00
time (sec)	N/A	0.084	0.049	0.579	0.289	0.258	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	64	31	61	47	0	148	33
N.S.	1	1.00	1.25	0.61	1.20	0.92	0.00	2.90	0.65
time (sec)	N/A	0.096	0.047	0.589	0.199	0.267	0.000	0.326	4.055

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	53	47	80	75	0	0	56
N.S.	1	1.00	0.72	0.64	1.08	1.01	0.00	0.00	0.76
time (sec)	N/A	0.115	0.050	0.614	0.198	0.288	0.000	0.000	4.024

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	64	99	124	0	0	134
N.S.	1	1.00	0.99	0.66	1.02	1.28	0.00	0.00	1.38
time (sec)	N/A	0.092	0.061	0.626	0.202	0.361	0.000	0.000	4.067

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	112	80	118	152	0	0	177
N.S.	1	1.00	0.93	0.67	0.98	1.27	0.00	0.00	1.48
time (sec)	N/A	0.107	0.073	0.612	0.191	0.540	0.000	0.000	4.144

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	71	100	204	95	0	0	0
N.S.	1	1.00	0.38	0.54	1.10	0.51	0.00	0.00	0.00
time (sec)	N/A	0.149	0.069	0.517	0.209	0.265	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	63	100	172	95	0	0	0
N.S.	1	1.00	0.45	0.72	1.24	0.68	0.00	0.00	0.00
time (sec)	N/A	0.145	0.058	0.513	0.214	0.240	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	55	84	140	73	0	0	0
N.S.	1	1.00	0.59	0.90	1.51	0.78	0.00	0.00	0.00
time (sec)	N/A	0.135	0.049	0.510	0.219	0.259	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	58	48	97	42	0	0	0
N.S.	1	1.00	1.26	1.04	2.11	0.91	0.00	0.00	0.00
time (sec)	N/A	0.140	0.038	0.514	0.214	0.259	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	59	67	0	33	0	0	0
N.S.	1	1.00	0.52	0.59	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.111	0.031	0.513	0.000	0.253	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	49	64	0	39	0	0	0
N.S.	1	1.00	0.62	0.81	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.127	0.038	0.506	0.000	0.252	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	51	39	0	39	0	0	90
N.S.	1	1.00	1.09	0.83	0.00	0.83	0.00	0.00	1.91
time (sec)	N/A	0.130	0.091	0.515	0.000	0.244	0.000	0.000	4.189

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	71	169	0	139	0	0	0
N.S.	1	1.00	0.38	0.91	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.148	0.074	0.521	0.000	0.264	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	99	241	0	190	0	0	0
N.S.	1	1.00	0.36	0.87	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.201	0.094	0.565	0.000	0.244	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	79	113	0	117	0	0	0
N.S.	1	1.00	0.34	0.48	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.165	0.072	0.507	0.000	0.245	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	71	97	0	95	0	0	0
N.S.	1	1.00	0.38	0.52	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.160	0.059	0.510	0.000	0.259	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	63	81	0	73	0	0	0
N.S.	1	1.00	0.45	0.58	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.144	0.049	0.556	0.000	0.254	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	53	63	0	43	0	0	0
N.S.	1	1.00	0.56	0.66	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.131	0.041	0.514	0.000	0.252	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	22	0	0	0
N.S.	1	1.00	0.59	0.64	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.089	0.025	0.523	0.000	0.248	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	0	22	0	0	0
N.S.	1	1.00	1.00	1.38	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.155	0.025	0.521	0.000	0.243	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	54	84	0	83	0	0	0
N.S.	1	1.00	0.60	0.93	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.142	0.056	0.520	0.000	0.258	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	81	169	0	137	0	0	0
N.S.	1	1.00	0.44	0.92	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.151	0.080	0.515	0.000	0.269	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	99	241	0	186	0	0	0
N.S.	1	1.00	0.36	0.87	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.196	0.099	0.523	0.000	0.258	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	136	106	154	241	314	117	0
N.S.	1	1.00	1.04	0.81	1.18	1.84	2.40	0.89	0.00
time (sec)	N/A	0.112	0.174	0.638	0.291	0.282	2.742	0.294	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	117	90	130	180	248	85	0
N.S.	1	1.00	1.08	0.83	1.20	1.67	2.30	0.79	0.00
time (sec)	N/A	0.098	0.115	0.629	0.302	0.261	2.410	0.283	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	69	47	134	0	62	0
N.S.	1	1.00	1.15	0.79	0.54	1.54	0.00	0.71	0.00
time (sec)	N/A	0.086	0.062	0.639	0.286	0.256	0.000	0.284	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	100	73	39	151	0	0	0
N.S.	1	1.00	1.67	1.22	0.65	2.52	0.00	0.00	0.00
time (sec)	N/A	0.080	0.075	0.620	0.330	0.255	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	63	31	60	47	0	148	33
N.S.	1	1.00	1.21	0.60	1.15	0.90	0.00	2.85	0.63
time (sec)	N/A	0.096	0.048	0.621	0.205	0.262	0.000	0.314	3.998

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	47	79	75	0	0	56
N.S.	1	1.00	1.05	0.63	1.05	1.00	0.00	0.00	0.75
time (sec)	N/A	0.098	0.059	0.625	0.225	0.299	0.000	0.000	4.066

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	64	98	124	0	0	134
N.S.	1	1.00	0.98	0.65	1.00	1.27	0.00	0.00	1.37
time (sec)	N/A	0.099	0.075	0.622	0.211	0.369	0.000	0.000	4.120

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	112	80	117	152	0	0	177
N.S.	1	1.00	0.93	0.66	0.97	1.26	0.00	0.00	1.46
time (sec)	N/A	0.102	0.097	0.622	0.213	0.523	0.000	0.000	4.261

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	71	100	204	95	0	0	0
N.S.	1	1.00	0.38	0.53	1.08	0.50	0.00	0.00	0.00
time (sec)	N/A	0.146	0.065	0.520	0.222	0.256	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	63	100	172	95	0	0	0
N.S.	1	1.00	0.44	0.70	1.21	0.67	0.00	0.00	0.00
time (sec)	N/A	0.139	0.059	0.524	0.237	0.240	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	84	140	73	0	0	0
N.S.	1	1.00	0.58	0.88	1.47	0.77	0.00	0.00	0.00
time (sec)	N/A	0.132	0.048	0.523	0.212	0.239	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	58	48	97	42	0	0	0
N.S.	1	1.00	1.23	1.02	2.06	0.89	0.00	0.00	0.00
time (sec)	N/A	0.123	0.038	0.519	0.207	0.242	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	58	67	0	33	0	0	0
N.S.	1	1.00	0.52	0.60	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.097	0.032	0.529	0.000	0.249	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	47	62	0	38	0	0	0
N.S.	1	1.00	0.61	0.81	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.121	0.045	0.532	0.000	0.245	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	39	0	39	0	0	58
N.S.	1	1.00	1.11	0.85	0.00	0.85	0.00	0.00	1.26
time (sec)	N/A	0.125	0.101	0.523	0.000	0.252	0.000	0.000	4.394

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	71	169	0	136	0	0	0
N.S.	1	1.00	0.39	0.93	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.201	0.073	0.522	0.000	0.257	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	99	241	0	193	0	0	0
N.S.	1	1.00	0.36	0.88	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.176	0.103	0.521	0.000	0.250	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	47	0	25	0	0	0
N.S.	1	1.00	0.59	0.62	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.156	0.028	0.520	0.000	0.252	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	47	0	25	0	0	0
N.S.	1	1.00	0.58	0.64	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.126	0.025	0.518	0.000	0.263	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	44	0	22	0	0	0
N.S.	1	1.00	0.60	0.65	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.079	0.011	0.518	0.000	0.262	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	18	0	0	0
N.S.	1	1.00	0.59	0.64	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.100	0.025	0.529	0.000	0.254	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	48	0	22	0	0	0
N.S.	1	1.00	0.60	0.66	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.144	0.026	0.530	0.000	0.244	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	96	97	117	184	0	0	0
N.S.	1	1.00	0.70	0.71	0.85	1.34	0.00	0.00	0.00
time (sec)	N/A	0.288	0.201	0.664	0.296	0.259	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	89	93	168	0	84	0
N.S.	1	1.00	0.79	0.79	0.83	1.50	0.00	0.75	0.00
time (sec)	N/A	0.277	0.153	0.654	0.288	0.257	0.000	0.292	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	79	80	70	150	0	72	0
N.S.	1	1.00	0.93	0.94	0.82	1.76	0.00	0.85	0.00
time (sec)	N/A	0.177	0.133	0.651	0.291	0.258	0.000	0.290	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	69	47	134	0	62	0
N.S.	1	1.00	0.88	0.80	0.55	1.56	0.00	0.72	0.00
time (sec)	N/A	0.092	0.045	0.628	0.289	0.249	0.000	0.269	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	97	129	90	191	0	95	0
N.S.	1	1.00	1.29	1.72	1.20	2.55	0.00	1.27	0.00
time (sec)	N/A	0.236	0.133	0.633	0.295	0.273	0.000	0.295	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	104	102	0	209	0	134	0
N.S.	1	1.00	1.27	1.24	0.00	2.55	0.00	1.63	0.00
time (sec)	N/A	0.250	0.151	0.660	0.000	0.260	0.000	0.288	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	76	78	0	148	0	200	0
N.S.	1	1.00	0.97	1.00	0.00	1.90	0.00	2.56	0.00
time (sec)	N/A	0.235	0.171	0.674	0.000	0.250	0.000	0.277	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	82	86	0	164	0	250	0
N.S.	1	1.00	0.83	0.87	0.00	1.66	0.00	2.53	0.00
time (sec)	N/A	0.311	0.181	0.689	0.000	0.261	0.000	0.293	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	95	95	0	180	0	324	0
N.S.	1	1.00	0.73	0.73	0.00	1.38	0.00	2.49	0.00
time (sec)	N/A	0.279	0.198	0.733	0.000	0.262	0.000	0.290	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	87	92	0	58	0	0	0
N.S.	1	1.00	0.38	0.40	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.185	0.064	0.572	0.000	0.242	0.000	0.000	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	74	83	0	49	0	0	0
N.S.	1	1.00	0.40	0.45	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.179	0.049	0.531	0.000	0.270	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	66	76	0	42	0	0	0
N.S.	1	1.00	0.43	0.50	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.151	0.044	0.526	0.000	0.239	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	59	67	0	33	0	0	0
N.S.	1	1.00	0.52	0.59	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.096	0.017	0.528	0.000	0.247	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	54	59	0	28	0	0	0
N.S.	1	1.00	0.47	0.52	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.123	0.034	0.523	0.000	0.264	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	54	65	0	33	0	0	0
N.S.	1	1.00	0.47	0.57	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.162	0.041	0.523	0.000	0.259	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	64	77	0	90	0	0	0
N.S.	1	1.00	0.42	0.50	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.167	0.055	0.535	0.000	0.259	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	76	85	0	98	0	0	0
N.S.	1	1.00	0.39	0.44	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.172	0.062	0.547	0.000	0.255	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	80	93	0	106	0	0	0
N.S.	1	1.00	0.35	0.41	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.178	0.067	0.553	0.000	0.250	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	83	106	0	76	0	0	0
N.S.	1	1.00	0.39	0.50	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.165	0.106	0.548	0.000	0.260	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	74	98	0	69	0	0	0
N.S.	1	1.00	0.43	0.57	0.00	0.40	0.00	0.00	0.00
time (sec)	N/A	0.141	0.070	0.549	0.000	0.263	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	62	86	0	56	0	0	0
N.S.	1	1.00	0.48	0.66	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.197	0.060	0.549	0.000	0.260	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	84	0	86	0	0	0
N.S.	1	1.00	0.60	0.97	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.149	0.055	0.542	0.000	0.265	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	86	0	0	0
N.S.	1	1.00	0.62	0.92	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.126	0.028	0.559	0.000	0.246	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	68	93	0	63	0	0	0
N.S.	1	1.00	0.38	0.53	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.182	0.069	0.566	0.000	0.254	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	79	118	0	92	0	0	0
N.S.	1	1.00	0.37	0.55	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.188	0.071	0.548	0.000	0.255	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	94	138	0	113	0	0	0
N.S.	1	1.00	0.37	0.55	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.190	0.089	0.546	0.000	0.255	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	100	185	0	138	0	0	0
N.S.	1	1.00	0.38	0.71	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.159	0.126	0.549	0.000	0.273	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	84	169	0	122	0	0	0
N.S.	1	1.00	0.39	0.78	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.195	0.107	0.546	0.000	0.258	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	72	169	0	136	0	0	0
N.S.	1	1.00	0.41	0.96	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.185	0.101	0.549	0.000	0.244	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	85	164	0	134	0	0	0
N.S.	1	1.00	0.46	0.89	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.191	0.070	0.553	0.000	0.259	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	62	164	0	134	0	0	0
N.S.	1	1.00	0.45	1.20	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.153	0.074	0.544	0.000	0.246	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	136	0	0	0
N.S.	1	1.00	0.45	0.92	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.140	0.033	0.560	0.000	0.246	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	88	196	0	145	0	0	0
N.S.	1	1.00	0.32	0.72	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.193	0.117	0.534	0.000	0.265	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	99	225	0	174	0	0	0
N.S.	1	1.00	0.32	0.73	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.241	0.112	0.537	0.000	0.256	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	47	0	25	0	0	0
N.S.	1	1.00	0.59	0.62	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.166	0.031	0.540	0.000	0.238	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	47	0	25	0	0	0
N.S.	1	1.00	0.58	0.64	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.141	0.029	0.543	0.000	0.247	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	22	0	0	0
N.S.	1	1.00	0.59	0.64	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.090	0.014	0.549	0.000	0.242	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	42	46	0	20	0	0	0
N.S.	1	1.00	0.60	0.66	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.100	0.026	0.542	0.000	0.240	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	43	48	0	22	0	0	0
N.S.	1	1.00	0.60	0.67	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.148	0.028	0.550	0.000	0.237	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	96	97	117	184	0	0	0
N.S.	1	1.00	0.70	0.71	0.85	1.34	0.00	0.00	0.00
time (sec)	N/A	0.302	0.209	0.712	0.299	0.261	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	89	93	168	0	84	0
N.S.	1	1.00	0.79	0.79	0.83	1.50	0.00	0.75	0.00
time (sec)	N/A	0.263	0.161	0.685	0.296	0.268	0.000	0.298	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	70	150	0	73	0
N.S.	1	1.00	0.94	0.94	0.83	1.79	0.00	0.87	0.00
time (sec)	N/A	0.186	0.135	0.672	0.304	0.265	0.000	0.284	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	69	47	134	0	62	0
N.S.	1	1.00	1.15	0.79	0.54	1.54	0.00	0.71	0.00
time (sec)	N/A	0.090	0.049	0.652	0.297	0.271	0.000	0.287	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	97	121	86	191	0	95	0
N.S.	1	1.00	1.29	1.61	1.15	2.55	0.00	1.27	0.00
time (sec)	N/A	0.237	0.155	0.657	0.305	0.275	0.000	0.289	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	104	102	0	210	0	134	0
N.S.	1	1.00	1.27	1.24	0.00	2.56	0.00	1.63	0.00
time (sec)	N/A	0.245	0.150	0.691	0.000	0.261	0.000	0.294	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	76	79	0	149	0	200	0
N.S.	1	1.00	0.97	1.01	0.00	1.91	0.00	2.56	0.00
time (sec)	N/A	0.245	0.186	0.710	0.000	0.273	0.000	0.283	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	82	87	0	165	0	250	0
N.S.	1	1.00	0.81	0.86	0.00	1.63	0.00	2.48	0.00
time (sec)	N/A	0.277	0.176	0.722	0.000	0.270	0.000	0.282	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	95	95	0	181	0	324	0
N.S.	1	1.00	0.73	0.73	0.00	1.39	0.00	2.49	0.00
time (sec)	N/A	0.302	0.199	0.750	0.000	0.269	0.000	0.276	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	86	92	0	58	0	0	0
N.S.	1	1.00	0.38	0.41	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.196	0.075	0.550	0.000	0.243	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	74	83	0	49	0	0	0
N.S.	1	1.00	0.40	0.45	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.186	0.051	0.546	0.000	0.246	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	65	76	0	42	0	0	0
N.S.	1	1.00	0.43	0.50	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.156	0.045	0.537	0.000	0.254	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	58	67	0	33	0	0	0
N.S.	1	1.00	0.52	0.60	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.101	0.019	0.540	0.000	0.240	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	50	61	0	26	0	0	0
N.S.	1	1.00	0.45	0.54	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.119	0.032	0.546	0.000	0.253	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	54	64	0	33	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.165	0.040	0.546	0.000	0.263	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	63	77	0	88	0	0	0
N.S.	1	1.00	0.41	0.51	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.164	0.053	0.542	0.000	0.261	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	75	85	0	98	0	0	0
N.S.	1	1.00	0.39	0.44	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.165	0.069	0.544	0.000	0.250	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	79	93	0	104	0	0	0
N.S.	1	1.00	0.35	0.41	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.180	0.056	0.545	0.000	0.246	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	129	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	56	62	54	74	0	0	93
N.S.	1	1.00	0.68	0.76	0.66	0.90	0.00	0.00	1.13
time (sec)	N/A	0.169	0.049	0.608	0.232	0.261	0.000	0.000	4.237

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	60	64	57	75	0	0	94
N.S.	1	1.00	0.72	0.77	0.69	0.90	0.00	0.00	1.13
time (sec)	N/A	0.183	0.039	0.559	0.250	0.258	0.000	0.000	4.107

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	110	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.264	0.000	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	75	0	0	0	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	267	0	0	0	0	0	0
N.S.	1	1.00	3.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	2.386	0.000	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	1.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	30	27	49	0	39
N.S.	1	1.00	1.00	1.00	1.67	1.50	2.72	0.00	2.17
time (sec)	N/A	0.030	0.179	1.076	0.213	0.253	0.509	0.000	4.059

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	55	0	79	0	0	106
N.S.	1	1.00	0.76	0.76	0.00	1.10	0.00	0.00	1.47
time (sec)	N/A	0.062	0.325	4.000	0.000	0.249	0.000	0.000	4.223

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	97	101	0	174	0	0	192
N.S.	1	1.00	0.76	0.80	0.00	1.37	0.00	0.00	1.51
time (sec)	N/A	0.107	0.411	17.351	0.000	0.259	0.000	0.000	4.726

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	152	167	0	309	0	0	314
N.S.	1	1.00	0.77	0.85	0.00	1.57	0.00	0.00	1.59
time (sec)	N/A	0.151	0.523	59.376	0.000	0.261	0.000	0.000	4.832

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	280	0	0	0	0	0	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	2.930	0.000	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	101	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.848	0.000	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	81	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.70
time (sec)	N/A	0.039	0.323	0.527	0.000	0.259	0.000	0.000	4.079

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	1.70
time (sec)	N/A	0.098	0.774	0.523	0.000	0.279	0.000	0.000	4.220

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	299	140	0	291	0	0	289
N.S.	1	1.00	1.80	0.84	0.00	1.75	0.00	0.00	1.74
time (sec)	N/A	0.137	1.974	0.519	0.000	0.256	0.000	0.000	4.810

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	260	218	0	453	0	0	441
N.S.	1	1.00	1.09	0.91	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.189	1.946	0.523	0.000	0.269	0.000	0.000	4.674

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	359	133	0	0	0	0	0	0
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.917	0.000	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	127	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.623	0.000	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	82	0	0	81
N.S.	1	1.00	0.93	1.07	0.00	1.78	0.00	0.00	1.76
time (sec)	N/A	0.064	0.369	0.536	0.000	0.255	0.000	0.000	4.129

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.70
time (sec)	N/A	0.038	0.036	0.533	0.000	0.251	0.000	0.000	0.002

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	127	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	1.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	463	463	201	0	0	0	0	0	0
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	2.448	0.000	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	110	93	0	175	0	0	175
N.S.	1	1.00	0.33	0.28	0.00	0.53	0.00	0.00	0.53
time (sec)	N/A	0.253	0.959	0.543	0.000	0.273	0.000	0.000	4.217

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	96	0	180	0	0	175
N.S.	1	1.00	1.07	0.94	0.00	1.76	0.00	0.00	1.72
time (sec)	N/A	0.138	0.959	0.585	0.000	0.269	0.000	0.000	4.717

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	108	86	0	171	0	0	176
N.S.	1	1.00	1.11	0.89	0.00	1.76	0.00	0.00	1.81
time (sec)	N/A	0.115	0.756	0.548	0.000	0.271	0.000	0.000	4.226

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	1.70
time (sec)	N/A	0.083	0.173	0.539	0.000	0.262	0.000	0.000	0.003

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	944	944	220	0	0	0	0	0	0
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	2.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	83	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	38	34	42	0	0	59
N.S.	1	1.00	0.71	0.75	0.67	0.82	0.00	0.00	1.16
time (sec)	N/A	0.088	0.098	1.209	0.199	0.247	0.000	0.000	3.965

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	40	36	44	0	0	59
N.S.	1	1.00	0.69	0.77	0.69	0.85	0.00	0.00	1.13
time (sec)	N/A	0.090	0.076	0.866	0.202	0.247	0.000	0.000	4.010

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.294	0.000	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	67	0	0	0	648	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	12.00	0.00	0.00
time (sec)	N/A	0.067	0.025	0.000	0.000	0.000	14.066	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.272	0.000	0.000	0.000	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	73	0	0	0	648	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	11.78	0.00	0.00
time (sec)	N/A	0.067	0.033	0.000	0.000	0.000	12.314	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	119	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	120	188	380	201	0	461	332
N.S.	1	1.00	0.35	0.55	1.11	0.59	0.00	1.35	0.97
time (sec)	N/A	0.182	0.324	0.109	0.288	0.268	0.000	0.302	0.224

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	104	172	302	179	0	355	258
N.S.	1	1.00	0.39	0.64	1.13	0.67	0.00	1.32	0.96
time (sec)	N/A	0.136	0.264	0.095	0.282	0.267	0.000	0.313	4.215

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	94	156	223	157	0	249	183
N.S.	1	1.00	0.48	0.80	1.15	0.81	0.00	1.28	0.94
time (sec)	N/A	0.093	0.177	0.085	0.282	0.280	0.000	0.295	4.647

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	53	127	117	104	0	130	84
N.S.	1	1.00	0.50	1.19	1.09	0.97	0.00	1.21	0.79
time (sec)	N/A	0.055	0.152	0.079	0.280	0.271	0.000	0.292	0.075

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	56	144	116	93	0	0	62
N.S.	1	1.00	0.54	1.38	1.12	0.89	0.00	0.00	0.60
time (sec)	N/A	0.056	0.137	0.192	0.197	0.252	0.000	0.000	0.081

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	83	218	160	134	0	0	128
N.S.	1	1.00	0.46	1.21	0.89	0.74	0.00	0.00	0.71
time (sec)	N/A	0.083	0.340	0.224	0.189	0.251	0.000	0.000	0.071

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	99	292	194	178	0	0	171
N.S.	1	1.00	0.39	1.15	0.76	0.70	0.00	0.00	0.67
time (sec)	N/A	0.118	0.408	0.243	0.196	0.252	0.000	0.000	0.096

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	115	366	230	274	0	0	210
N.S.	1	1.00	0.35	1.12	0.70	0.84	0.00	0.00	0.64
time (sec)	N/A	0.199	0.515	0.256	0.212	0.259	0.000	0.000	0.069

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	88	114	122	124	115	89
N.S.	1	1.00	1.00	0.69	0.90	0.96	0.98	0.91	0.70
time (sec)	N/A	0.129	0.049	0.848	0.198	0.238	0.371	0.259	0.111

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	64	81	89	88	82	65
N.S.	1	1.00	1.00	0.71	0.90	0.99	0.98	0.91	0.72
time (sec)	N/A	0.127	0.038	0.784	0.205	0.240	0.238	0.277	3.968

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	55	70	78	76	71	56
N.S.	1	1.00	1.00	0.72	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.113	0.033	0.710	0.197	0.254	0.172	0.276	4.013

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	31	35	43	39	36	32
N.S.	1	1.00	1.00	0.79	0.90	1.10	1.00	0.92	0.82
time (sec)	N/A	0.110	0.024	0.641	0.194	0.267	0.092	0.273	0.057

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	23
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.10
time (sec)	N/A	0.069	0.019	0.573	0.204	0.245	0.051	0.282	0.047

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	36	35	40	36	36	33
N.S.	1	1.00	0.78	1.00	0.97	1.11	1.00	1.00	0.92
time (sec)	N/A	0.118	0.038	0.531	0.198	0.241	0.073	0.281	0.057

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	60	69	93	73	57	68
N.S.	1	1.00	1.00	0.80	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.130	0.056	0.563	0.211	0.244	0.202	0.266	0.099

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	84	97	137	102	80	94
N.S.	1	1.00	0.75	0.76	0.88	1.25	0.93	0.73	0.85
time (sec)	N/A	0.147	0.084	0.546	0.207	0.254	0.333	0.284	4.189

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	98	108	145	233	156	96	142
N.S.	1	1.00	0.68	0.74	1.00	1.61	1.08	0.66	0.98
time (sec)	N/A	0.164	0.120	0.539	0.201	0.260	0.469	0.279	4.214

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	126	180	380	201	0	461	332
N.S.	1	1.00	0.37	0.52	1.11	0.59	0.00	1.34	0.97
time (sec)	N/A	0.173	0.291	0.104	0.295	0.258	0.000	0.311	4.318

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	110	173	302	179	0	355	258
N.S.	1	1.00	0.41	0.64	1.12	0.67	0.00	1.32	0.96
time (sec)	N/A	0.139	0.236	0.093	0.297	0.263	0.000	0.301	4.371

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	94	157	223	156	0	248	183
N.S.	1	1.00	0.48	0.81	1.14	0.80	0.00	1.27	0.94
time (sec)	N/A	0.098	0.198	0.081	0.289	0.255	0.000	0.297	0.215

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	57	129	118	106	0	130	84
N.S.	1	1.00	0.75	1.70	1.55	1.39	0.00	1.71	1.11
time (sec)	N/A	0.045	0.159	0.142	0.286	0.267	0.000	0.300	0.138

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	69	185	133	128	0	0	100
N.S.	1	1.00	0.48	1.28	0.92	0.89	0.00	0.00	0.69
time (sec)	N/A	0.074	0.162	0.149	0.199	0.251	0.000	0.000	0.203

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	78	225	153	170	0	63	121
N.S.	1	1.00	0.43	1.24	0.85	0.94	0.00	0.35	0.67
time (sec)	N/A	0.092	0.361	0.214	0.210	0.288	0.000	0.314	0.157

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	101	299	192	204	0	0	160
N.S.	1	1.00	0.40	1.17	0.75	0.80	0.00	0.00	0.63
time (sec)	N/A	0.120	0.435	0.227	0.204	0.254	0.000	0.000	4.164

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	117	373	226	248	0	0	203
N.S.	1	1.00	0.36	1.13	0.69	0.75	0.00	0.00	0.62
time (sec)	N/A	0.165	0.529	0.247	0.197	0.258	0.000	0.000	0.115

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	116	80	103	111	112	184	81
N.S.	1	1.00	1.00	0.69	0.89	0.96	0.97	1.59	0.70
time (sec)	N/A	0.155	0.047	0.940	0.209	0.237	0.341	0.280	0.090

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	71	92	100	100	160	72
N.S.	1	1.00	1.00	0.71	0.92	1.00	1.00	1.60	0.72
time (sec)	N/A	0.125	0.038	0.837	0.198	0.234	0.257	0.275	4.241

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	47	59	67	65	136	48
N.S.	1	1.00	1.00	0.75	0.94	1.06	1.03	2.16	0.76
time (sec)	N/A	0.114	0.030	0.730	0.201	0.254	0.157	0.269	0.119

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	40	46	56	53	112	43
N.S.	1	1.00	1.00	0.78	0.90	1.10	1.04	2.20	0.84
time (sec)	N/A	0.107	0.024	0.697	0.199	0.234	0.105	0.263	0.126

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	32	35	26	66	32
N.S.	1	1.00	1.00	0.88	0.97	1.06	0.79	2.00	0.97
time (sec)	N/A	0.074	0.025	0.645	0.200	0.257	0.146	0.285	0.078

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	43	49	64	41	74	48
N.S.	1	1.00	1.00	0.81	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.124	0.042	0.629	0.189	0.244	0.128	0.263	3.879

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	56	75	100	83	93	71
N.S.	1	1.00	0.89	0.79	1.06	1.41	1.17	1.31	1.00
time (sec)	N/A	0.126	0.047	0.601	0.193	0.254	0.171	0.270	3.919

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	82	107	163	114	130	104
N.S.	1	1.00	0.80	0.74	0.96	1.47	1.03	1.17	0.94
time (sec)	N/A	0.144	0.073	0.593	0.193	0.243	0.336	0.267	0.121

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	98	108	135	207	144	170	131
N.S.	1	1.00	0.67	0.74	0.92	1.42	0.99	1.16	0.90
time (sec)	N/A	0.160	0.111	0.603	0.195	0.252	0.479	0.270	0.155

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	120	189	380	201	0	524	332
N.S.	1	1.00	0.35	0.55	1.11	0.59	0.00	1.53	0.97
time (sec)	N/A	0.169	0.347	0.111	0.293	0.278	0.000	0.320	3.979

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	110	173	302	179	0	394	258
N.S.	1	1.00	0.41	0.64	1.12	0.67	0.00	1.46	0.96
time (sec)	N/A	0.130	0.247	0.095	0.279	0.257	0.000	0.308	0.109

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	94	157	223	156	0	264	183
N.S.	1	1.00	0.48	0.81	1.14	0.80	0.00	1.35	0.94
time (sec)	N/A	0.098	0.198	0.087	0.296	0.257	0.000	0.300	3.881

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	55	127	117	107	0	121	84
N.S.	1	1.00	0.51	1.18	1.08	0.99	0.00	1.12	0.78
time (sec)	N/A	0.068	0.147	0.076	0.299	0.259	0.000	0.282	3.872

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	57	138	121	67	0	0	86
N.S.	1	1.00	0.54	1.31	1.15	0.64	0.00	0.00	0.82
time (sec)	N/A	0.063	0.139	0.199	0.197	0.252	0.000	0.000	0.060

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	85	213	163	119	0	0	137
N.S.	1	1.00	0.47	1.19	0.91	0.66	0.00	0.00	0.77
time (sec)	N/A	0.096	0.344	0.229	0.188	0.247	0.000	0.000	3.875

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	101	287	197	161	0	0	178
N.S.	1	1.00	0.40	1.13	0.77	0.63	0.00	0.00	0.70
time (sec)	N/A	0.123	0.437	0.238	0.194	0.249	0.000	0.000	0.063

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	117	361	231	205	0	0	217
N.S.	1	1.00	0.36	1.10	0.70	0.62	0.00	0.00	0.66
time (sec)	N/A	0.161	0.534	0.244	0.201	0.254	0.000	0.000	4.266

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	64	81	89	88	82	67
N.S.	1	1.00	1.00	0.71	0.90	0.99	0.98	0.91	0.74
time (sec)	N/A	0.129	0.043	0.778	0.190	0.247	0.251	0.275	4.029

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	55	70	78	76	71	56
N.S.	1	1.00	1.00	0.72	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.136	0.032	0.729	0.184	0.240	0.170	0.278	0.106

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	37	43	39	38	35
N.S.	1	1.00	1.00	0.80	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.133	0.025	0.663	0.194	0.233	0.093	0.274	0.078

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	25
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.19
time (sec)	N/A	0.067	0.019	0.625	0.198	0.232	0.055	0.272	0.065

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	34	38	36	36	33
N.S.	1	1.00	0.80	1.03	0.97	1.09	1.03	1.03	0.94
time (sec)	N/A	0.118	0.039	0.543	0.200	0.244	0.075	0.273	0.103

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	60	69	92	75	57	68
N.S.	1	1.00	0.96	0.82	0.95	1.26	1.03	0.78	0.93
time (sec)	N/A	0.129	0.059	0.555	0.184	0.244	0.201	0.266	4.024

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	104	84	97	137	102	80	94
N.S.	1	1.00	0.96	0.78	0.90	1.27	0.94	0.74	0.87
time (sec)	N/A	0.148	0.088	0.543	0.194	0.257	0.341	0.263	0.121

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	124	108	145	233	156	96	142
N.S.	1	1.00	0.87	0.76	1.01	1.63	1.09	0.67	0.99
time (sec)	N/A	0.162	0.107	0.566	0.199	0.246	0.474	0.266	0.163

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	126	180	379	201	0	525	332
N.S.	1	1.00	0.37	0.52	1.10	0.59	0.00	1.53	0.97
time (sec)	N/A	0.181	0.306	0.105	0.284	0.255	0.000	0.331	0.168

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	110	173	301	179	0	395	258
N.S.	1	1.00	0.41	0.64	1.12	0.67	0.00	1.47	0.96
time (sec)	N/A	0.150	0.250	0.103	0.276	0.259	0.000	0.302	0.114

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	94	157	224	156	0	264	183
N.S.	1	1.00	0.48	0.81	1.15	0.80	0.00	1.35	0.94
time (sec)	N/A	0.098	0.197	0.087	0.289	0.274	0.000	0.289	0.093

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	57	130	118	103	0	122	84
N.S.	1	1.00	0.75	1.71	1.55	1.36	0.00	1.61	1.11
time (sec)	N/A	0.046	0.169	0.143	0.278	0.260	0.000	0.282	0.064

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	69	173	140	96	0	59	114
N.S.	1	1.00	0.48	1.20	0.97	0.67	0.00	0.41	0.79
time (sec)	N/A	0.074	0.158	0.146	0.186	0.246	0.000	0.292	3.920

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	78	207	161	135	0	59	141
N.S.	1	1.00	0.43	1.14	0.89	0.75	0.00	0.33	0.78
time (sec)	N/A	0.096	0.360	0.243	0.189	0.265	0.000	0.311	0.051

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	101	281	199	179	0	0	183
N.S.	1	1.00	0.40	1.11	0.79	0.71	0.00	0.00	0.72
time (sec)	N/A	0.139	0.438	0.231	0.202	0.246	0.000	0.000	3.857

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	117	355	231	275	0	0	224
N.S.	1	1.00	0.36	1.09	0.71	0.84	0.00	0.00	0.69
time (sec)	N/A	0.175	0.532	0.251	0.204	0.254	0.000	0.000	0.065

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	94	112	0	96	0	0	0
N.S.	1	1.00	0.29	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.112	0.092	0.095	0.000	0.242	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	73	96	0	74	0	0	0
N.S.	1	1.00	0.31	0.41	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.103	0.067	0.058	0.000	0.251	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	62	80	0	42	0	0	0
N.S.	1	1.00	0.42	0.55	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.090	0.052	0.052	0.000	0.249	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	38	50	0	17	0	0	0
N.S.	1	1.00	0.57	0.75	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.112	0.030	0.045	0.000	0.265	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	43	57	0	24	0	0	0
N.S.	1	1.00	0.60	0.79	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.081	0.036	0.043	0.000	0.265	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	71	102	0	68	0	0	0
N.S.	1	1.00	0.41	0.59	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.103	0.083	0.057	0.000	0.259	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	97	175	0	137	0	0	0
N.S.	1	1.00	0.37	0.67	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.123	0.131	0.056	0.000	0.267	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	119	247	0	205	0	0	0
N.S.	1	1.00	0.33	0.69	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.170	0.184	0.059	0.000	0.258	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	150	250	0	438	1059	561	0
N.S.	1	1.00	0.40	0.67	0.00	1.18	2.85	1.51	0.00
time (sec)	N/A	0.370	0.248	0.606	0.000	0.287	18.755	41.994	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	134	234	0	394	500	416	0
N.S.	1	1.00	0.46	0.80	0.00	1.34	1.70	1.41	0.00
time (sec)	N/A	0.368	0.199	0.622	0.000	0.272	10.045	1.810	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	115	202	0	317	376	266	0
N.S.	1	1.00	0.54	0.95	0.00	1.49	1.77	1.25	0.00
time (sec)	N/A	0.307	0.172	0.550	0.000	0.257	6.572	0.424	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	0
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.248	0.134	0.526	0.000	0.255	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	68	160	0	216	0	0	0
N.S.	1	1.00	0.61	1.44	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.224	0.169	0.553	0.000	0.267	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	95	214	0	280	0	0	0
N.S.	1	1.00	0.77	1.74	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.344	0.141	0.579	0.000	0.262	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	105	298	0	352	0	0	0
N.S.	1	1.00	0.52	1.47	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.367	0.173	0.562	0.000	0.279	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	133	382	0	496	0	0	0
N.S.	1	1.00	0.47	1.35	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.329	0.197	0.572	0.000	0.309	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	95	112	0	96	0	0	0
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.131	0.093	0.059	0.000	0.248	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	97	112	0	96	0	0	0
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.140	0.082	0.050	0.000	0.267	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	78	96	0	72	0	0	0
N.S.	1	1.00	0.33	0.41	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.113	0.064	0.054	0.000	0.250	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	57	69	0	44	0	0	0
N.S.	1	1.00	0.39	0.47	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.097	0.055	0.049	0.000	0.254	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	51	65	0	27	0	0	0
N.S.	1	1.00	0.47	0.60	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.090	0.042	0.049	0.000	0.250	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	57	85	0	49	0	0	0
N.S.	1	1.00	0.50	0.74	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.094	0.068	0.050	0.000	0.241	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	64	102	0	81	0	0	0
N.S.	1	1.00	0.37	0.60	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.109	0.096	0.051	0.000	0.243	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	96	175	0	138	0	0	0
N.S.	1	1.00	0.36	0.66	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.136	0.129	0.058	0.000	0.265	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	116	247	0	207	0	0	0
N.S.	1	1.00	0.32	0.69	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.154	0.189	0.056	0.000	0.280	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	95	112	0	96	0	0	0
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.130	0.083	0.052	0.000	0.252	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	75	96	0	74	0	0	0
N.S.	1	1.00	0.32	0.40	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.127	0.068	0.051	0.000	0.247	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	63	80	0	44	0	0	0
N.S.	1	1.00	0.43	0.54	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.148	0.048	0.051	0.000	0.240	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	39	52	0	19	0	0	0
N.S.	1	1.00	0.57	0.76	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.081	0.028	0.050	0.000	0.249	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	43	59	0	26	0	0	0
N.S.	1	1.00	0.60	0.82	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.089	0.037	0.046	0.000	0.244	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	70	103	0	66	0	0	0
N.S.	1	1.00	0.41	0.60	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.114	0.079	0.052	0.000	0.266	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	95	175	0	135	0	0	0
N.S.	1	1.00	0.36	0.67	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.128	0.171	0.056	0.000	0.256	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	119	247	0	201	0	0	0
N.S.	1	1.00	0.33	0.69	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.162	0.183	0.057	0.000	0.280	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	150	250	0	438	1059	561	0
N.S.	1	1.00	0.40	0.67	0.00	1.17	2.82	1.50	0.00
time (sec)	N/A	0.380	0.260	0.575	0.000	0.266	18.111	46.812	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	134	234	0	394	500	416	0
N.S.	1	1.00	0.46	0.80	0.00	1.34	1.71	1.42	0.00
time (sec)	N/A	0.328	0.201	0.569	0.000	0.269	9.966	1.980	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	115	202	0	316	376	266	0
N.S.	1	1.00	0.54	0.95	0.00	1.48	1.77	1.25	0.00
time (sec)	N/A	0.335	0.168	0.540	0.000	0.265	6.474	0.447	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	0
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.255	0.132	0.511	0.000	0.255	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	68	154	0	212	0	0	0
N.S.	1	1.00	0.61	1.38	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.269	0.172	0.529	0.000	0.270	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	95	202	0	279	0	0	0
N.S.	1	1.00	0.77	1.63	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.334	0.133	0.536	0.000	0.267	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	105	286	0	351	0	0	0
N.S.	1	1.00	0.54	1.47	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.315	0.159	0.546	0.000	0.277	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	131	370	0	495	0	0	0
N.S.	1	1.00	0.49	1.37	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.343	0.194	0.552	0.000	0.315	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	95	112	0	96	0	0	0
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.118	0.100	0.054	0.000	0.256	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	97	112	0	96	0	0	0
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.129	0.088	0.053	0.000	0.256	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	79	96	0	74	0	0	0
N.S.	1	1.00	0.34	0.41	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.115	0.072	0.053	0.000	0.254	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	57	69	0	42	0	0	0
N.S.	1	1.00	0.39	0.47	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.104	0.057	0.052	0.000	0.250	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	47	67	0	25	0	0	0
N.S.	1	1.00	0.44	0.63	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.093	0.039	0.051	0.000	0.257	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	55	87	0	47	0	0	0
N.S.	1	1.00	0.49	0.77	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.099	0.059	0.053	0.000	0.258	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	63	102	0	81	0	0	0
N.S.	1	1.00	0.38	0.61	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.128	0.093	0.053	0.000	0.248	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	95	175	0	137	0	0	0
N.S.	1	1.00	0.36	0.66	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.134	0.133	0.059	0.000	0.275	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	108	247	0	208	0	0	0
N.S.	1	1.00	0.30	0.69	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.147	0.236	0.060	0.000	0.246	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	52	63	44	72	0	0	0
N.S.	1	1.00	0.65	0.79	0.55	0.90	0.00	0.00	0.00
time (sec)	N/A	0.205	0.044	0.039	0.238	0.262	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	24	0	0	46
N.S.	1	1.00	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.179	0.032	0.046	0.000	0.241	0.000	0.000	4.050

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	43	52	0	21	0	0	45
N.S.	1	1.00	0.61	0.73	0.00	0.30	0.00	0.00	0.63
time (sec)	N/A	0.123	0.035	0.059	0.000	0.246	0.000	0.000	4.063

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	38	50	0	17	0	0	0
N.S.	1	1.00	0.57	0.75	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.077	0.016	0.043	0.000	0.247	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	41	50	0	21	0	0	0
N.S.	1	1.00	0.59	0.71	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.176	0.032	0.049	0.000	0.239	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	53	0	21	0	0	63
N.S.	1	1.00	1.00	1.15	0.00	0.46	0.00	0.00	1.37
time (sec)	N/A	0.173	0.029	0.051	0.000	0.254	0.000	0.000	4.319

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	93	134	0	222	0	128	0
N.S.	1	1.00	0.58	0.84	0.00	1.39	0.00	0.80	0.00
time (sec)	N/A	0.381	0.221	0.579	0.000	0.251	0.000	0.291	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	84	124	0	204	0	116	0
N.S.	1	1.00	0.68	1.01	0.00	1.66	0.00	0.94	0.00
time (sec)	N/A	0.340	0.173	0.585	0.000	0.267	0.000	0.287	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	77	114	0	188	0	106	0
N.S.	1	1.00	0.79	1.16	0.00	1.92	0.00	1.08	0.00
time (sec)	N/A	0.218	0.077	0.576	0.000	0.252	0.000	0.295	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	0
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.239	0.037	0.579	0.000	0.259	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	82	145	0	252	0	127	0
N.S.	1	1.00	0.70	1.24	0.00	2.15	0.00	1.09	0.00
time (sec)	N/A	0.382	0.193	0.575	0.000	0.256	0.000	0.361	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	78	142	0	177	0	194	0
N.S.	1	1.00	0.70	1.28	0.00	1.59	0.00	1.75	0.00
time (sec)	N/A	0.391	0.136	0.608	0.000	0.249	0.000	0.376	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	151	0	201	0	231	0
N.S.	1	1.00	0.63	1.10	0.00	1.47	0.00	1.69	0.00
time (sec)	N/A	0.383	0.141	0.581	0.000	0.252	0.000	0.690	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	94	159	0	217	0	316	0
N.S.	1	1.00	0.60	1.02	0.00	1.39	0.00	2.03	0.00
time (sec)	N/A	0.404	0.144	0.586	0.000	0.263	0.000	2.078	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	102	167	0	233	0	362	0
N.S.	1	1.00	0.56	0.92	0.00	1.29	0.00	2.00	0.00
time (sec)	N/A	0.411	0.168	0.575	0.000	0.265	0.000	2.364	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	71	89	0	48	0	0	0
N.S.	1	1.00	0.38	0.48	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.213	0.058	0.090	0.000	0.251	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	63	82	0	41	0	0	0
N.S.	1	1.00	0.41	0.54	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.205	0.050	0.050	0.000	0.249	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	56	73	0	32	0	0	0
N.S.	1	1.00	0.50	0.65	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.139	0.036	0.049	0.000	0.246	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	51	65	0	27	0	0	0
N.S.	1	1.00	0.47	0.60	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.085	0.020	0.049	0.000	0.251	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	51	67	0	32	0	0	0
N.S.	1	1.00	0.47	0.62	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.212	0.045	0.052	0.000	0.253	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	61	82	0	85	0	0	0
N.S.	1	1.00	0.41	0.56	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.218	0.059	0.051	0.000	0.258	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	73	90	0	93	0	0	0
N.S.	1	1.00	0.39	0.48	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.212	0.072	0.059	0.000	0.251	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	77	98	0	101	0	0	0
N.S.	1	1.00	0.35	0.44	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.212	0.079	0.054	0.000	0.246	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	91	106	0	109	0	0	0
N.S.	1	1.00	0.34	0.40	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.213	0.072	0.056	0.000	0.259	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	49	65	46	73	0	0	0
N.S.	1	1.00	0.60	0.80	0.57	0.90	0.00	0.00	0.00
time (sec)	N/A	0.190	0.051	0.039	0.244	0.243	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	24	0	0	46
N.S.	1	1.00	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.188	0.035	0.046	0.000	0.254	0.000	0.000	3.942

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	44	52	0	21	0	0	45
N.S.	1	1.00	0.61	0.72	0.00	0.29	0.00	0.00	0.62
time (sec)	N/A	0.129	0.031	0.043	0.000	0.241	0.000	0.000	4.012

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	39	52	0	19	0	0	0
N.S.	1	1.00	0.57	0.76	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.076	0.016	0.046	0.000	0.240	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	50	0	21	0	0	0
N.S.	1	1.00	0.58	0.72	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.179	0.030	0.048	0.000	0.244	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	53	0	21	0	0	63
N.S.	1	1.00	0.98	1.13	0.00	0.45	0.00	0.00	1.34
time (sec)	N/A	0.176	0.029	0.046	0.000	0.244	0.000	0.000	4.109

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	93	134	0	222	0	128	0
N.S.	1	1.00	0.57	0.82	0.00	1.36	0.00	0.79	0.00
time (sec)	N/A	0.387	0.213	0.577	0.000	0.264	0.000	0.299	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	84	125	0	204	0	117	0
N.S.	1	1.00	0.68	1.01	0.00	1.65	0.00	0.94	0.00
time (sec)	N/A	0.365	0.161	0.574	0.000	0.256	0.000	0.297	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	114	0	188	0	106	0
N.S.	1	1.00	1.01	1.15	0.00	1.90	0.00	1.07	0.00
time (sec)	N/A	0.238	0.076	0.578	0.000	0.255	0.000	0.282	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	0
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.260	0.037	0.546	0.000	0.255	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	82	145	0	252	0	127	0
N.S.	1	1.00	0.70	1.24	0.00	2.15	0.00	1.09	0.00
time (sec)	N/A	0.372	0.187	0.633	0.000	0.253	0.000	0.369	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	78	143	0	176	0	194	0
N.S.	1	1.00	0.70	1.28	0.00	1.57	0.00	1.73	0.00
time (sec)	N/A	0.370	0.131	0.589	0.000	0.255	0.000	0.373	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	86	150	0	200	0	231	0
N.S.	1	1.00	0.61	1.07	0.00	1.43	0.00	1.65	0.00
time (sec)	N/A	0.383	0.135	0.592	0.000	0.249	0.000	0.671	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	94	159	0	216	0	316	0
N.S.	1	1.00	0.60	1.02	0.00	1.38	0.00	2.03	0.00
time (sec)	N/A	0.397	0.139	0.601	0.000	0.255	0.000	1.773	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	102	167	0	232	0	362	0
N.S.	1	1.00	0.56	0.92	0.00	1.28	0.00	2.00	0.00
time (sec)	N/A	0.458	0.150	0.610	0.000	0.264	0.000	2.566	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	71	89	0	48	0	0	0
N.S.	1	1.00	0.38	0.48	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.222	0.062	0.095	0.000	0.241	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	62	82	0	41	0	0	0
N.S.	1	1.00	0.41	0.54	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.209	0.053	0.054	0.000	0.244	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	55	73	0	32	0	0	0
N.S.	1	1.00	0.49	0.65	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.143	0.038	0.052	0.000	0.241	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	47	67	0	25	0	0	0
N.S.	1	1.00	0.44	0.63	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.090	0.019	0.049	0.000	0.247	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	51	66	0	32	0	0	0
N.S.	1	1.00	0.47	0.61	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.196	0.047	0.057	0.000	0.259	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	60	82	0	83	0	0	0
N.S.	1	1.00	0.41	0.56	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.203	0.057	0.060	0.000	0.250	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	72	90	0	91	0	0	0
N.S.	1	1.00	0.39	0.48	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.206	0.069	0.055	0.000	0.243	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	76	98	0	99	0	0	0
N.S.	1	1.00	0.34	0.44	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.206	0.061	0.056	0.000	0.242	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	90	106	0	107	0	0	0
N.S.	1	1.00	0.34	0.40	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.206	0.072	0.055	0.000	0.252	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	123	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	94	0	0	0	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	180	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	1.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	146	0	0	0	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	0.710	0.000	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	112	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.647	0.000	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [129] had the largest ratio of [1.42900000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	10	0.600
2	A	7	6	1.00	10	0.600
3	A	6	6	1.00	8	0.750
4	A	5	5	1.00	6	0.833
5	A	6	6	1.00	10	0.600
6	A	3	3	1.00	10	0.300
7	A	3	3	1.00	10	0.300
8	A	7	5	1.00	10	0.500
9	A	5	4	1.00	10	0.400
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	10	0.300
13	A	4	3	1.00	8	0.375
14	A	4	3	1.00	12	0.250
15	A	4	3	1.00	12	0.250
16	A	4	3	1.00	12	0.250
17	A	4	3	1.00	12	0.250
18	A	14	9	1.00	12	0.750
19	A	12	8	1.00	10	0.800
20	A	8	7	1.00	8	0.875
21	A	8	7	1.00	12	0.583
22	A	5	5	1.00	12	0.417
23	A	9	7	1.00	12	0.583
24	A	10	9	1.00	12	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	3	1.00	12	0.250
26	A	4	3	1.00	12	0.250
27	A	4	3	1.00	10	0.300
28	A	4	3	1.00	8	0.375
29	A	4	3	1.00	12	0.250
30	A	4	3	1.00	12	0.250
31	A	4	3	1.00	12	0.250
32	A	4	3	1.00	12	0.250
33	A	8	6	1.00	12	0.500
34	A	7	6	1.00	12	0.500
35	A	6	6	1.00	10	0.600
36	A	5	5	1.00	8	0.625
37	A	6	6	1.00	12	0.500
38	A	3	3	1.00	12	0.250
39	A	3	3	1.00	12	0.250
40	A	7	5	1.00	12	0.417
41	A	5	4	1.00	12	0.333
42	A	4	3	1.00	12	0.250
43	A	4	3	1.00	12	0.250
44	A	4	3	1.00	10	0.300
45	A	4	3	1.00	8	0.375
46	A	4	3	1.00	12	0.250
47	A	4	3	1.00	12	0.250
48	A	4	3	1.00	12	0.250
49	A	4	3	1.00	12	0.250
50	A	19	9	1.00	12	0.750
51	A	14	9	1.00	12	0.750
52	A	12	8	1.00	10	0.800
53	A	8	7	1.00	8	0.875
54	A	8	7	1.00	12	0.583
55	A	5	5	1.00	12	0.417
56	A	9	7	1.00	12	0.583
57	A	10	9	1.00	12	0.750
58	A	14	11	1.00	12	0.917
59	A	11	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	10	8	1.00	14	0.571
61	A	9	8	1.00	14	0.571
62	A	7	7	1.00	12	0.583
63	A	6	6	1.00	10	0.600
64	A	17	14	1.00	14	1.000
65	A	13	10	1.00	14	0.714
66	A	14	11	1.00	14	0.786
67	A	15	12	1.00	14	0.857
68	A	11	8	1.00	14	0.571
69	A	10	8	1.00	14	0.571
70	A	9	8	1.00	14	0.571
71	A	7	7	1.00	12	0.583
72	A	6	6	1.00	10	0.600
73	A	17	14	1.00	14	1.000
74	A	13	10	1.00	14	0.714
75	A	14	11	1.00	14	0.786
76	A	15	12	1.00	14	0.857
77	A	12	9	1.00	14	0.643
78	A	11	9	1.00	14	0.643
79	A	10	9	1.00	14	0.643
80	A	8	7	1.00	12	0.583
81	A	7	6	1.00	10	0.600
82	A	19	16	1.00	14	1.143
83	A	14	11	1.00	14	0.786
84	A	15	11	1.00	14	0.786
85	A	16	12	1.00	14	0.857
86	A	11	8	1.00	14	0.571
87	A	10	8	1.00	14	0.571
88	A	9	8	1.00	14	0.571
89	A	7	7	1.00	12	0.583
90	A	6	6	1.00	10	0.600
91	A	17	14	1.00	14	1.000
92	A	13	10	1.00	14	0.714
93	A	14	11	1.00	14	0.786
94	A	15	12	1.00	14	0.857

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	11	8	1.00	14	0.571
96	A	10	8	1.00	14	0.571
97	A	9	8	1.00	14	0.571
98	A	7	7	1.00	12	0.583
99	A	6	6	1.00	10	0.600
100	A	17	14	1.00	14	1.000
101	A	13	10	1.00	14	0.714
102	A	14	11	1.00	14	0.786
103	A	15	12	1.00	14	0.857
104	A	12	9	1.00	14	0.643
105	A	11	9	1.00	14	0.643
106	A	10	9	1.00	14	0.643
107	A	8	7	1.00	12	0.583
108	A	7	6	1.00	10	0.600
109	A	19	16	1.00	14	1.143
110	A	14	11	1.00	14	0.786
111	A	15	11	1.00	14	0.786
112	A	16	12	1.00	14	0.857
113	A	16	11	1.00	12	0.917
114	A	14	10	1.00	10	1.000
115	A	13	9	1.00	8	1.125
116	A	25	13	1.00	12	1.083
117	A	14	10	1.00	12	0.833
118	A	15	11	1.00	12	0.917
119	A	16	12	1.00	12	1.000
120	A	6	5	1.00	12	0.417
121	A	4	4	1.00	10	0.400
122	A	3	3	1.00	8	0.375
123	A	4	4	1.00	12	0.333
124	A	3	3	1.00	12	0.250
125	A	4	4	1.00	12	0.333
126	A	19	15	1.00	14	1.071
127	A	17	14	1.00	12	1.167
128	A	16	13	1.00	10	1.300
129	A	39	20	1.00	14	1.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	25	11	1.00	14	0.786
131	A	26	12	1.00	14	0.857
132	A	5	5	1.00	12	0.417
133	A	9	5	1.00	12	0.417
134	A	4	4	1.00	12	0.333
135	A	4	3	1.00	10	0.300
136	A	4	3	1.00	12	0.250
137	A	4	4	1.00	12	0.333
138	A	9	5	1.00	12	0.417
139	A	2	2	1.00	14	0.143
140	A	2	2	1.00	14	0.143
141	A	2	2	1.00	14	0.143
142	A	2	2	1.00	14	0.143
143	A	2	2	1.00	14	0.143
144	A	2	2	1.00	14	0.143
145	A	2	2	1.00	12	0.167
146	A	2	2	1.00	12	0.167
147	A	2	2	1.00	14	0.143
148	A	2	2	1.00	12	0.167
149	A	5	5	1.00	12	0.417
150	A	3	3	1.00	10	0.300
151	A	2	2	1.00	8	0.250
152	A	4	4	1.00	12	0.333
153	A	2	2	1.00	12	0.167
154	A	3	3	1.00	12	0.250
155	A	4	4	1.00	12	0.333
156	A	4	4	1.00	12	0.333
157	A	4	4	1.00	16	0.250
158	A	9	8	1.00	16	0.500
159	A	8	8	1.00	16	0.500
160	A	7	7	1.00	16	0.438
161	A	6	6	1.00	14	0.429
162	A	7	7	1.00	16	0.438
163	A	3	3	1.00	16	0.188
164	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	6	6	1.00	16	0.375
166	A	8	6	1.00	16	0.375
167	A	5	4	1.00	18	0.222
168	A	4	3	1.00	18	0.167
169	A	4	3	1.00	18	0.167
170	A	4	3	1.00	18	0.167
171	A	4	3	1.00	18	0.167
172	C	1	1	1.86	16	0.062
173	A	4	3	1.00	18	0.167
174	A	3	3	1.00	18	0.167
175	A	4	3	1.00	18	0.167
176	A	4	3	1.00	18	0.167
177	A	5	4	1.00	18	0.222
178	A	8	7	1.00	18	0.389
179	A	7	6	1.00	18	0.333
180	A	8	8	1.00	18	0.444
181	A	8	8	1.00	16	0.500
182	A	9	8	1.00	18	0.444
183	A	3	3	1.00	18	0.167
184	A	4	4	1.00	18	0.222
185	A	6	6	1.00	18	0.333
186	A	7	6	1.00	18	0.333
187	A	5	4	1.00	18	0.222
188	A	4	3	1.00	18	0.167
189	A	5	4	1.00	18	0.222
190	A	4	3	1.00	18	0.167
191	A	3	3	1.00	18	0.167
192	A	4	3	1.00	16	0.188
193	A	4	3	1.00	18	0.167
194	A	3	3	1.00	18	0.167
195	A	4	3	1.00	18	0.167
196	A	4	3	1.00	18	0.167
197	A	3	3	1.00	18	0.167
198	A	9	7	1.00	18	0.389
199	A	8	7	1.00	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	7	7	1.00	16	0.438
201	A	5	5	1.00	18	0.278
202	A	3	3	1.00	18	0.167
203	A	4	4	1.00	18	0.222
204	A	6	6	1.00	18	0.333
205	A	7	6	1.00	18	0.333
206	A	4	4	1.00	18	0.222
207	A	4	3	1.00	18	0.167
208	A	4	3	1.00	18	0.167
209	A	4	3	1.00	18	0.167
210	A	4	3	1.00	16	0.188
211	A	3	3	1.00	18	0.167
212	A	4	4	1.00	18	0.222
213	A	5	4	1.00	18	0.222
214	A	5	4	1.00	18	0.222
215	A	5	4	1.00	18	0.222
216	A	3	3	1.00	18	0.167
217	A	10	8	1.00	18	0.444
218	A	9	8	1.00	18	0.444
219	A	8	8	1.00	16	0.500
220	A	6	6	1.00	18	0.333
221	A	3	3	1.00	18	0.167
222	A	3	3	1.00	18	0.167
223	A	5	5	1.00	18	0.278
224	A	6	5	1.00	18	0.278
225	A	7	5	1.00	18	0.278
226	A	7	6	1.00	18	0.333
227	A	6	6	1.00	18	0.333
228	A	5	5	1.19	18	0.278
229	A	4	4	1.16	18	0.222
230	A	1	1	1.00	18	0.056
231	A	5	5	1.00	18	0.278
232	A	5	5	1.00	18	0.278
233	A	6	5	1.00	18	0.278
234	A	7	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	4	1.00	20	0.200
236	A	5	4	1.00	20	0.200
237	A	5	4	1.00	20	0.200
238	A	5	4	1.00	20	0.200
239	A	5	4	1.00	20	0.200
240	A	5	4	1.00	20	0.200
241	A	5	4	1.00	20	0.200
242	A	5	4	1.00	20	0.200
243	A	6	6	1.00	20	0.300
244	A	5	5	1.00	20	0.250
245	A	4	4	1.00	20	0.200
246	A	1	1	1.00	20	0.050
247	A	6	5	1.00	20	0.250
248	A	6	5	1.00	20	0.250
249	A	6	5	1.00	20	0.250
250	A	7	5	1.00	20	0.250
251	A	8	5	1.00	20	0.250
252	A	8	6	1.60	20	0.300
253	A	7	6	1.58	20	0.300
254	A	6	6	1.54	20	0.300
255	A	5	5	1.44	20	0.250
256	A	4	4	1.44	20	0.200
257	A	1	1	1.00	20	0.050
258	A	4	4	1.00	20	0.200
259	A	5	5	1.00	20	0.250
260	A	6	5	1.00	20	0.250
261	A	10	6	1.00	20	0.300
262	A	9	6	1.00	20	0.300
263	A	8	6	1.00	20	0.300
264	A	7	6	1.00	20	0.300
265	A	6	6	1.00	20	0.300
266	A	5	5	1.00	20	0.250
267	A	6	6	1.00	20	0.300
268	A	7	6	1.00	20	0.300
269	A	8	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	9	6	1.00	20	0.300
271	A	8	6	1.00	20	0.300
272	A	7	6	1.00	20	0.300
273	A	6	6	1.00	20	0.300
274	A	5	5	1.00	20	0.250
275	A	4	4	1.00	20	0.200
276	A	1	1	1.00	20	0.050
277	A	5	5	1.00	20	0.250
278	A	6	5	1.00	20	0.250
279	A	7	6	1.00	9	0.667
280	A	6	5	1.00	8	0.625
281	A	3	3	1.00	11	0.273
282	A	6	6	1.00	10	0.600
283	A	8	6	1.00	11	0.546
284	A	7	5	1.00	10	0.500
285	A	8	8	1.00	13	0.615
286	A	7	7	1.00	12	0.583
287	A	3	3	1.00	11	0.273
288	A	4	4	1.00	10	0.400
289	A	8	8	1.00	13	0.615
290	A	7	7	1.00	12	0.583
291	A	5	5	1.00	11	0.454
292	A	3	3	1.00	10	0.300
293	A	9	9	1.00	13	0.692
294	A	3	3	1.00	12	0.250
295	A	3	3	1.00	21	0.143
296	A	5	4	1.00	21	0.190
297	A	4	4	1.00	19	0.210
298	A	1	1	1.00	18	0.056
299	A	5	5	1.00	21	0.238
300	A	5	5	1.00	21	0.238
301	A	5	4	1.00	23	0.174
302	A	5	4	1.00	23	0.174
303	A	5	4	1.00	21	0.190
304	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	6	1.00	23	0.261
306	A	6	6	1.00	23	0.261
307	A	7	7	1.00	23	0.304
308	A	8	7	1.00	23	0.304
309	A	9	7	1.00	23	0.304
310	A	10	7	1.00	23	0.304
311	A	9	7	1.00	23	0.304
312	A	7	6	1.00	21	0.286
313	A	6	5	1.00	20	0.250
314	A	8	8	1.00	23	0.348
315	A	8	8	1.00	23	0.348
316	A	9	9	1.00	23	0.391
317	A	10	9	1.00	23	0.391
318	A	11	9	1.00	23	0.391
319	A	6	6	1.00	13	0.462
320	A	5	5	1.00	12	0.417
321	A	5	5	1.00	15	0.333
322	A	4	4	1.00	14	0.286
323	A	5	5	1.00	13	0.385
324	A	4	4	1.00	12	0.333
325	A	4	4	1.00	15	0.267
326	A	1	1	1.00	14	0.071
327	A	4	4	1.00	13	0.308
328	A	3	3	1.00	12	0.250
329	A	6	6	1.00	15	0.400
330	A	5	5	1.00	14	0.357
331	A	5	5	1.00	13	0.385
332	A	4	4	1.00	12	0.333
333	A	6	6	1.00	15	0.400
334	A	5	5	1.00	14	0.357
335	A	4	4	1.00	23	0.174
336	A	6	5	1.30	23	0.217
337	A	5	5	1.32	21	0.238
338	A	4	4	1.44	20	0.200
339	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
340	A	5	5	1.00	23	0.217
341	A	9	7	1.00	23	0.304
342	A	9	7	1.00	23	0.304
343	A	8	7	1.00	21	0.333
344	A	7	6	1.00	20	0.300
345	A	9	8	1.00	23	0.348
346	A	9	8	1.00	23	0.348
347	A	10	9	1.00	23	0.391
348	A	11	9	1.00	23	0.391
349	A	12	9	1.00	23	0.391
350	A	8	6	1.00	23	0.261
351	A	7	6	1.00	23	0.261
352	A	6	6	1.00	21	0.286
353	A	5	5	1.00	20	0.250
354	A	6	6	1.00	23	0.261
355	A	6	6	1.00	23	0.261
356	A	7	7	1.00	23	0.304
357	A	8	7	1.00	23	0.304
358	A	9	7	1.00	23	0.304
359	A	6	6	1.00	24	0.250
360	A	4	4	1.00	24	0.167
361	A	1	1	1.00	22	0.045
362	A	3	3	1.00	24	0.125
363	A	3	3	1.00	24	0.125
364	A	3	3	1.00	18	0.167
365	A	3	3	1.00	18	0.167
366	A	3	3	1.00	18	0.167
367	A	3	3	1.00	16	0.188
368	A	3	3	1.00	18	0.167
369	A	3	3	1.00	18	0.167
370	A	4	4	1.00	18	0.222
371	A	6	6	1.00	18	0.333
372	A	3	3	1.00	20	0.150
373	A	3	3	1.00	20	0.150
374	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	3	3	1.00	20	0.150
376	A	3	3	1.00	20	0.150
377	A	4	4	1.00	20	0.200
378	A	5	4	1.00	20	0.200
379	A	9	9	1.00	20	0.450
380	A	8	8	1.00	20	0.400
381	A	7	7	1.00	20	0.350
382	A	3	3	1.00	18	0.167
383	A	7	7	1.00	20	0.350
384	A	8	7	1.00	20	0.350
385	A	9	7	1.00	20	0.350
386	A	10	7	1.00	20	0.350
387	A	5	4	1.00	22	0.182
388	A	5	4	1.00	22	0.182
389	A	5	4	1.00	22	0.182
390	A	6	5	1.00	22	0.227
391	A	5	4	1.00	20	0.200
392	A	5	4	1.00	22	0.182
393	A	5	4	1.00	22	0.182
394	A	5	4	1.00	22	0.182
395	A	5	4	1.00	22	0.182
396	A	8	8	1.00	22	0.364
397	A	4	4	1.00	22	0.182
398	A	8	8	1.00	22	0.364
399	A	8	8	1.00	20	0.400
400	A	8	7	1.00	22	0.318
401	A	9	7	1.00	22	0.318
402	A	10	7	1.00	22	0.318
403	A	11	7	1.00	22	0.318
404	A	5	4	1.00	22	0.182
405	A	6	5	1.00	22	0.227
406	A	5	4	1.00	22	0.182
407	A	5	4	1.00	22	0.182
408	A	5	4	1.00	20	0.200
409	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	5	4	1.00	22	0.182
411	A	5	4	1.00	22	0.182
412	A	5	4	1.00	22	0.182
413	A	10	8	1.00	22	0.364
414	A	9	8	1.00	22	0.364
415	A	8	8	1.00	22	0.364
416	A	7	7	1.00	20	0.350
417	A	2	2	1.00	22	0.091
418	A	6	6	1.00	22	0.273
419	A	8	7	1.00	22	0.318
420	A	9	7	1.00	22	0.318
421	A	5	4	1.00	22	0.182
422	A	5	4	1.00	22	0.182
423	A	5	4	1.00	22	0.182
424	A	5	4	1.00	20	0.200
425	A	5	4	1.00	22	0.182
426	A	6	5	1.00	22	0.227
427	A	5	4	1.00	22	0.182
428	A	5	4	1.00	22	0.182
429	A	11	9	1.00	22	0.409
430	A	10	9	1.00	22	0.409
431	A	9	9	1.00	22	0.409
432	A	8	8	1.00	20	0.400
433	A	6	6	1.00	22	0.273
434	A	6	6	1.00	22	0.273
435	A	3	3	1.00	22	0.136
436	A	7	7	1.00	22	0.318
437	A	9	7	1.00	22	0.318
438	A	8	7	1.19	22	0.318
439	A	7	7	1.13	22	0.318
440	A	7	7	1.20	22	0.318
441	A	5	5	1.00	22	0.227
442	A	4	4	1.00	22	0.182
443	A	8	7	1.00	22	0.318
444	A	9	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	10	8	1.00	22	0.364
446	A	12	9	1.00	24	0.375
447	A	11	9	1.00	24	0.375
448	A	10	9	1.00	24	0.375
449	A	9	9	1.00	24	0.375
450	A	8	8	1.00	24	0.333
451	A	9	9	1.00	24	0.375
452	A	10	9	1.00	24	0.375
453	A	11	9	1.00	24	0.375
454	A	12	9	1.00	24	0.375
455	A	8	8	1.00	24	0.333
456	A	8	7	1.00	24	0.292
457	A	6	5	1.00	24	0.208
458	A	5	5	1.00	24	0.208
459	A	8	7	1.00	24	0.292
460	A	9	8	1.00	24	0.333
461	A	10	8	1.00	24	0.333
462	A	11	8	1.00	24	0.333
463	A	7	7	1.00	24	0.292
464	A	6	6	1.00	24	0.250
465	A	6	6	1.00	24	0.250
466	A	4	4	1.00	24	0.167
467	A	4	4	1.00	24	0.167
468	A	9	8	1.00	24	0.333
469	A	10	9	1.00	24	0.375
470	A	11	10	1.00	24	0.417
471	A	14	10	1.00	24	0.417
472	A	13	10	1.00	24	0.417
473	A	12	10	1.00	24	0.417
474	A	11	9	1.00	24	0.375
475	A	11	9	1.00	24	0.375
476	A	12	10	1.00	24	0.417
477	A	12	10	1.00	24	0.417
478	A	13	10	1.00	24	0.417
479	A	14	10	1.00	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
480	A	9	8	1.00	24	0.333
481	A	8	8	1.00	24	0.333
482	A	7	7	1.00	24	0.292
483	A	6	6	1.00	24	0.250
484	A	6	6	1.00	24	0.250
485	A	5	5	1.00	24	0.208
486	A	5	5	1.00	24	0.208
487	A	9	8	1.00	24	0.333
488	A	10	9	1.00	24	0.375
489	A	3	3	1.00	25	0.120
490	A	6	5	1.00	25	0.200
491	A	5	5	1.00	23	0.217
492	A	4	4	1.00	22	0.182
493	A	4	4	1.00	25	0.160
494	A	2	2	1.00	25	0.080
495	A	3	3	1.00	25	0.120
496	A	4	4	1.00	25	0.160
497	A	5	4	1.00	25	0.160
498	A	11	9	1.00	27	0.333
499	A	10	9	1.00	27	0.333
500	A	9	9	1.00	25	0.360
501	A	8	8	1.00	24	0.333
502	A	8	8	1.00	27	0.296
503	A	7	6	1.00	27	0.222
504	A	7	6	1.00	27	0.222
505	A	7	6	1.00	27	0.222
506	A	7	6	1.00	27	0.222
507	A	11	8	1.00	27	0.296
508	A	10	8	1.00	27	0.296
509	A	9	8	1.00	25	0.320
510	A	8	7	1.00	24	0.292
511	A	8	7	1.00	27	0.259
512	A	5	4	1.00	27	0.148
513	A	6	5	1.00	27	0.185
514	A	7	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	8	6	1.00	27	0.222
516	A	4	4	1.00	27	0.148
517	A	6	5	1.00	27	0.185
518	A	5	5	1.00	25	0.200
519	A	4	4	1.00	24	0.167
520	A	4	4	1.00	27	0.148
521	A	3	3	1.00	27	0.111
522	A	4	4	1.00	27	0.148
523	A	5	5	1.00	27	0.185
524	A	14	10	1.00	27	0.370
525	A	13	10	1.00	27	0.370
526	A	12	10	1.00	25	0.400
527	A	11	9	1.00	24	0.375
528	A	11	9	1.00	27	0.333
529	A	9	8	1.00	27	0.296
530	A	10	9	1.00	27	0.333
531	A	11	9	1.00	27	0.333
532	A	11	9	1.00	27	0.333
533	A	9	8	1.00	27	0.296
534	A	8	8	1.00	27	0.296
535	A	7	7	1.00	25	0.280
536	A	6	6	1.00	24	0.250
537	A	6	5	1.00	27	0.185
538	A	4	3	1.00	27	0.111
539	A	5	4	1.00	27	0.148
540	A	6	5	1.00	27	0.185
541	A	4	3	1.00	27	0.111
542	A	5	5	1.00	20	0.250
543	A	3	3	1.00	22	0.136
544	A	5	5	1.00	22	0.227
545	A	3	3	1.00	24	0.125
546	A	3	3	1.00	24	0.125
547	A	3	3	1.00	24	0.125
548	A	3	3	1.00	24	0.125
549	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
550	A	3	3	1.00	23	0.130
551	A	3	3	1.00	23	0.130
552	A	7	7	1.00	22	0.318
553	A	3	3	1.00	20	0.150
554	A	3	3	1.00	22	0.136
555	A	9	9	1.00	22	0.409
556	A	13	5	1.00	20	0.250
557	A	11	5	1.00	20	0.250
558	A	9	5	1.00	20	0.250
559	A	7	5	1.00	18	0.278
560	A	1	1	1.00	20	0.050
561	A	2	2	1.00	20	0.100
562	A	3	2	1.00	20	0.100
563	A	4	2	1.00	20	0.100
564	A	4	3	1.00	22	0.136
565	A	4	3	1.00	22	0.136
566	A	4	3	1.00	22	0.136
567	A	4	3	1.00	22	0.136
568	A	3	3	1.00	20	0.150
569	A	3	3	1.00	22	0.136
570	A	5	4	1.00	22	0.182
571	A	5	4	1.00	22	0.182
572	A	5	4	1.00	22	0.182
573	A	13	5	1.00	22	0.227
574	A	11	5	1.00	22	0.227
575	A	9	5	1.00	22	0.227
576	A	7	5	1.00	20	0.250
577	A	1	1	1.00	22	0.045
578	A	2	2	1.00	22	0.091
579	A	3	2	1.00	22	0.091
580	A	4	2	1.00	22	0.091
581	A	4	3	1.00	22	0.136
582	A	4	3	1.00	22	0.136
583	A	4	3	1.00	22	0.136
584	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	4	3	1.00	20	0.150
586	A	3	3	1.00	22	0.136
587	A	3	3	1.00	22	0.136
588	A	5	4	1.00	22	0.182
589	A	5	4	1.00	22	0.182
590	A	13	5	1.00	22	0.227
591	A	11	5	1.00	22	0.227
592	A	9	5	1.00	22	0.227
593	A	7	5	1.00	20	0.250
594	A	1	1	1.00	22	0.045
595	A	2	2	1.00	22	0.091
596	A	3	2	1.00	22	0.091
597	A	4	2	1.00	22	0.091
598	A	4	3	1.00	22	0.136
599	A	4	3	1.00	22	0.136
600	A	4	3	1.00	22	0.136
601	A	3	3	1.00	20	0.150
602	A	3	3	1.00	22	0.136
603	A	5	4	1.00	22	0.182
604	A	5	4	1.00	22	0.182
605	A	5	4	1.00	22	0.182
606	A	13	5	1.00	22	0.227
607	A	11	5	1.00	22	0.227
608	A	9	5	1.00	22	0.227
609	A	7	5	1.00	20	0.250
610	A	1	1	1.00	22	0.045
611	A	2	2	1.00	22	0.091
612	A	3	2	1.00	22	0.091
613	A	4	2	1.00	22	0.091
614	A	4	3	1.00	22	0.136
615	A	4	3	1.00	22	0.136
616	A	4	3	1.00	22	0.136
617	A	4	3	1.00	22	0.136
618	A	3	2	1.00	22	0.091
619	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	5	4	1.00	22	0.182
621	A	5	4	1.00	22	0.182
622	A	5	4	1.00	22	0.182
623	A	10	7	1.00	24	0.292
624	A	9	7	1.00	24	0.292
625	A	8	7	1.00	24	0.292
626	A	7	7	1.00	24	0.292
627	A	6	6	1.00	24	0.250
628	A	5	5	1.00	24	0.208
629	A	4	4	1.00	24	0.167
630	A	5	5	1.00	24	0.208
631	A	6	5	1.00	24	0.208
632	A	7	5	1.00	24	0.208
633	A	4	3	1.00	24	0.125
634	A	4	3	1.00	24	0.125
635	A	4	3	1.00	24	0.125
636	A	3	3	1.00	24	0.125
637	A	4	3	1.00	24	0.125
638	A	4	3	1.00	24	0.125
639	A	3	3	1.00	24	0.125
640	A	5	4	1.00	24	0.167
641	A	5	4	1.00	24	0.167
642	A	4	3	1.00	24	0.125
643	A	4	3	1.00	24	0.125
644	A	4	3	1.00	24	0.125
645	A	4	3	1.00	24	0.125
646	A	3	2	1.00	24	0.083
647	A	3	3	1.00	24	0.125
648	A	5	4	1.00	24	0.167
649	A	5	4	1.00	24	0.167
650	A	5	4	1.00	24	0.167
651	A	8	7	1.00	24	0.292
652	A	7	7	1.00	24	0.292
653	A	6	6	1.00	24	0.250
654	A	5	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
655	A	4	4	1.00	24	0.167
656	A	5	5	1.00	24	0.208
657	A	6	5	1.00	24	0.208
658	A	7	5	1.00	24	0.208
659	A	4	3	1.00	24	0.125
660	A	4	3	1.00	24	0.125
661	A	4	3	1.00	24	0.125
662	A	3	3	1.00	24	0.125
663	A	4	3	1.00	24	0.125
664	A	4	3	1.00	24	0.125
665	A	3	3	1.00	24	0.125
666	A	5	4	1.00	24	0.167
667	A	5	4	1.00	24	0.167
668	A	4	3	1.00	25	0.120
669	A	4	3	1.00	23	0.130
670	A	3	2	1.00	22	0.091
671	A	4	3	1.00	25	0.120
672	A	4	3	1.00	25	0.120
673	A	8	7	1.00	27	0.259
674	A	7	7	1.00	27	0.259
675	A	6	6	1.00	25	0.240
676	A	6	6	1.00	24	0.250
677	A	9	9	1.00	27	0.333
678	A	9	9	1.00	27	0.333
679	A	7	7	1.00	27	0.259
680	A	8	8	1.00	27	0.296
681	A	9	8	1.00	27	0.296
682	A	4	3	1.00	27	0.111
683	A	4	3	1.00	27	0.111
684	A	4	3	1.00	25	0.120
685	A	4	3	1.00	24	0.125
686	A	4	3	1.00	27	0.111
687	A	4	3	1.00	27	0.111
688	A	4	3	1.00	27	0.111
689	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	4	3	1.00	27	0.111
691	A	4	3	1.00	25	0.120
692	A	4	3	1.00	25	0.120
693	A	4	3	1.00	25	0.120
694	A	5	4	1.00	23	0.174
695	A	5	4	1.00	22	0.182
696	A	4	3	1.00	25	0.120
697	A	4	3	1.00	25	0.120
698	A	4	3	1.00	25	0.120
699	A	4	3	1.00	25	0.120
700	A	4	3	1.00	25	0.120
701	A	5	4	1.00	25	0.160
702	A	5	4	1.00	25	0.160
703	A	5	4	1.00	23	0.174
704	A	5	4	1.00	22	0.182
705	A	4	3	1.00	25	0.120
706	A	4	3	1.00	25	0.120
707	A	4	3	1.00	27	0.111
708	A	4	3	1.00	25	0.120
709	A	3	2	1.00	24	0.083
710	A	4	3	1.00	27	0.111
711	A	4	3	1.00	27	0.111
712	A	8	7	1.00	27	0.259
713	A	7	7	1.00	27	0.259
714	A	6	6	1.00	25	0.240
715	A	6	6	1.00	24	0.250
716	A	9	9	1.00	27	0.333
717	A	9	9	1.00	27	0.333
718	A	7	7	1.00	27	0.259
719	A	8	8	1.00	27	0.296
720	A	9	8	1.00	27	0.296
721	A	4	3	1.00	27	0.111
722	A	4	3	1.00	27	0.111
723	A	4	3	1.00	25	0.120
724	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
725	A	4	3	1.00	27	0.111
726	A	4	3	1.00	27	0.111
727	A	4	3	1.00	27	0.111
728	A	4	3	1.00	27	0.111
729	A	4	3	1.00	27	0.111
730	A	5	4	1.00	27	0.148
731	A	8	6	1.00	27	0.222
732	A	4	3	1.00	25	0.120
733	A	4	3	1.00	27	0.111
734	A	8	6	1.00	27	0.222
735	A	5	4	1.00	27	0.148
736	A	3	3	1.00	22	0.136
737	A	3	3	1.00	22	0.136
738	A	3	3	1.00	20	0.150
739	A	2	2	1.00	8	0.250
740	A	1	1	1.00	22	0.045
741	A	2	2	1.00	22	0.091
742	A	3	2	1.00	22	0.091
743	A	4	2	1.00	22	0.091
744	A	3	3	1.00	24	0.125
745	A	3	3	1.00	24	0.125
746	A	3	3	1.00	24	0.125
747	A	1	1	1.00	24	0.042
748	A	2	2	1.00	24	0.083
749	A	3	2	1.00	24	0.083
750	A	4	2	1.00	24	0.083
751	A	7	6	1.00	27	0.222
752	A	4	4	1.00	27	0.148
753	A	1	1	1.00	25	0.040
754	A	1	1	1.00	24	0.042
755	A	5	5	1.00	27	0.185
756	A	8	6	1.00	27	0.222
757	A	6	4	1.00	27	0.148
758	A	2	2	1.00	27	0.074
759	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	2	2	1.00	24	0.083
761	A	15	6	1.00	27	0.222
762	A	3	3	1.00	22	0.136
763	A	3	3	1.00	23	0.130
764	A	3	3	1.00	23	0.130
765	A	4	4	1.00	22	0.182
766	A	3	3	1.00	22	0.136
767	A	4	4	1.00	22	0.182
768	A	3	3	1.00	20	0.150
769	A	3	3	1.00	22	0.136
770	A	4	4	1.00	22	0.182
771	A	3	3	1.00	22	0.136
772	A	14	8	1.00	20	0.400
773	A	12	8	1.00	20	0.400
774	A	10	8	1.00	20	0.400
775	A	9	9	1.00	18	0.500
776	A	6	6	1.00	20	0.300
777	A	8	6	1.00	20	0.300
778	A	10	6	1.00	20	0.300
779	A	12	6	1.00	20	0.300
780	A	5	4	1.00	22	0.182
781	A	5	4	1.00	22	0.182
782	A	5	4	1.00	22	0.182
783	A	5	4	1.00	22	0.182
784	A	5	4	1.00	20	0.200
785	A	5	4	1.00	22	0.182
786	A	5	4	1.00	22	0.182
787	A	5	4	1.00	22	0.182
788	A	5	4	1.00	22	0.182
789	A	14	8	1.00	22	0.364
790	A	12	8	1.00	22	0.364
791	A	10	8	1.00	22	0.364
792	A	8	8	1.00	20	0.400
793	A	7	6	1.00	22	0.273
794	A	9	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
795	A	10	6	1.00	22	0.273
796	A	12	6	1.00	22	0.273
797	A	5	4	1.00	22	0.182
798	A	5	4	1.00	22	0.182
799	A	5	4	1.00	22	0.182
800	A	5	4	1.00	22	0.182
801	A	5	4	1.00	20	0.200
802	A	5	4	1.00	22	0.182
803	A	5	4	1.00	22	0.182
804	A	5	4	1.00	22	0.182
805	A	5	4	1.00	22	0.182
806	A	14	8	1.00	22	0.364
807	A	12	8	1.00	22	0.364
808	A	10	8	1.00	22	0.364
809	A	9	9	1.00	20	0.450
810	A	6	6	1.00	22	0.273
811	A	8	6	1.00	22	0.273
812	A	10	6	1.00	22	0.273
813	A	12	6	1.00	22	0.273
814	A	5	4	1.00	22	0.182
815	A	5	4	1.00	22	0.182
816	A	5	4	1.00	22	0.182
817	A	5	4	1.00	20	0.200
818	A	5	4	1.00	22	0.182
819	A	5	4	1.00	22	0.182
820	A	5	4	1.00	22	0.182
821	A	5	4	1.00	22	0.182
822	A	14	8	1.00	22	0.364
823	A	12	8	1.00	22	0.364
824	A	10	8	1.00	22	0.364
825	A	8	8	1.00	20	0.400
826	A	7	6	1.00	22	0.273
827	A	9	8	1.00	22	0.364
828	A	10	6	1.00	22	0.273
829	A	12	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
830	A	4	3	1.00	22	0.136
831	A	4	3	1.00	22	0.136
832	A	4	3	1.00	22	0.136
833	A	4	3	1.00	22	0.136
834	A	4	3	1.00	22	0.136
835	A	4	3	1.00	22	0.136
836	A	4	3	1.00	22	0.136
837	A	4	3	1.00	22	0.136
838	A	15	11	1.00	24	0.458
839	A	13	11	1.00	24	0.458
840	A	11	11	1.00	24	0.458
841	A	9	9	1.00	24	0.375
842	A	7	7	1.00	24	0.292
843	A	7	7	1.00	24	0.292
844	A	9	8	1.00	24	0.333
845	A	11	8	1.00	24	0.333
846	A	4	3	1.00	24	0.125
847	A	4	3	1.00	24	0.125
848	A	4	3	1.00	24	0.125
849	A	4	3	1.00	24	0.125
850	A	4	3	1.00	24	0.125
851	A	4	3	1.00	24	0.125
852	A	4	3	1.00	24	0.125
853	A	4	3	1.00	24	0.125
854	A	4	3	1.00	24	0.125
855	A	4	3	1.00	24	0.125
856	A	4	3	1.00	24	0.125
857	A	4	3	1.00	24	0.125
858	A	4	3	1.00	24	0.125
859	A	4	3	1.00	24	0.125
860	A	4	3	1.00	24	0.125
861	A	4	3	1.00	24	0.125
862	A	4	3	1.00	24	0.125
863	A	15	11	1.00	24	0.458
864	A	13	11	1.00	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
865	A	11	11	1.00	24	0.458
866	A	9	9	1.00	24	0.375
867	A	7	7	1.00	24	0.292
868	A	7	7	1.00	24	0.292
869	A	9	8	1.00	24	0.333
870	A	11	8	1.00	24	0.333
871	A	4	3	1.00	24	0.125
872	A	4	3	1.00	24	0.125
873	A	4	3	1.00	24	0.125
874	A	4	3	1.00	24	0.125
875	A	4	3	1.00	24	0.125
876	A	4	3	1.00	24	0.125
877	A	4	3	1.00	24	0.125
878	A	4	3	1.00	24	0.125
879	A	4	3	1.00	24	0.125
880	A	4	3	1.00	25	0.120
881	A	4	3	1.00	25	0.120
882	A	3	2	1.00	23	0.087
883	A	4	3	1.00	22	0.136
884	A	4	3	1.00	25	0.120
885	A	3	3	1.00	25	0.120
886	A	9	8	1.00	27	0.296
887	A	8	7	1.00	27	0.259
888	A	7	6	1.00	25	0.240
889	A	9	9	1.00	24	0.375
890	A	9	9	1.00	27	0.333
891	A	7	6	1.00	27	0.222
892	A	8	7	1.00	27	0.259
893	A	10	8	1.00	27	0.296
894	A	11	8	1.00	27	0.296
895	A	4	3	1.00	27	0.111
896	A	4	3	1.00	27	0.111
897	A	4	3	1.00	25	0.120
898	A	4	3	1.00	24	0.125
899	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	4	3	1.00	27	0.111
901	A	4	3	1.00	27	0.111
902	A	4	3	1.00	27	0.111
903	A	4	3	1.00	27	0.111
904	A	4	3	1.00	27	0.111
905	A	4	3	1.00	27	0.111
906	A	3	2	1.00	25	0.080
907	A	4	3	1.00	24	0.125
908	A	4	3	1.00	27	0.111
909	A	3	3	1.00	27	0.111
910	A	9	8	1.00	27	0.296
911	A	8	7	1.00	27	0.259
912	A	7	6	1.00	25	0.240
913	A	9	9	1.00	24	0.375
914	A	9	9	1.00	27	0.333
915	A	7	6	1.00	27	0.222
916	A	8	7	1.00	27	0.259
917	A	10	8	1.00	27	0.296
918	A	11	8	1.00	27	0.296
919	A	4	3	1.00	27	0.111
920	A	4	3	1.00	27	0.111
921	A	4	3	1.00	25	0.120
922	A	4	3	1.00	24	0.125
923	A	4	3	1.00	27	0.111
924	A	4	3	1.00	27	0.111
925	A	4	3	1.00	27	0.111
926	A	4	3	1.00	27	0.111
927	A	4	3	1.00	27	0.111
928	A	4	4	1.00	20	0.200
929	A	5	5	1.00	22	0.227
930	A	7	5	1.00	22	0.227
931	A	6	6	1.00	24	0.250
932	A	4	4	1.00	24	0.167
933	A	3	3	1.00	22	0.136
934	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
935	A	3	3	1.00	23	0.130

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 114

$$\int e^{\coth^{-1}(ax)}x^3\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] 3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4+2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2*
(1-1/a^2/x^2)^(1/2)/a^2+1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6304, 849, 821, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)}x^3\,dx=\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}$$

```
[In] Int[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2)
 + (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}+\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^4}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a^4}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(16+9ax+8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

```
[In] Integrate[E^ArcCoth[a*x]*x^3,x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3+8a^2x^2+9ax+16)(ax-1)}{24a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{8a^3\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax-15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-8((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24a\ln\left(\frac{a^2x+\sqrt{a^2}\cdots}{\cdots}\right.\right.}{24\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^4\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*a^3*x^3+8*a^2*x^2+9*a*x+16)*(a*x-1)/a^4/((a*x-1)/(a*x+1))^(1/2)+3/8
/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^
(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int e^{\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{(6\,a^4x^4+14\,a^3x^3+17\,a^2x^2+25\,ax+16)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 + 14*a^3*x^3 + 17*a^2*x^2 + 25*a*x + 16)*sqrt((a*x - 1)/(a
*x + 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^3\,dx=\int\frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3,x)
```

```
[Out] Integral(x**3/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(94) = 188$.

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{24}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-49\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-39\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(2*(9*((a*x - 1)/(a*x + 1))^(7/2) - 49*((a*x - 1)/(a*x + 1))^(5/2) +
 31*((a*x - 1)/(a*x + 1))^(3/2) - 39*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1
)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 9*log(sqrt((a*x - 1)/(a*x + 1))
 + 1)/a^5 - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2x\left(\frac{3x}{a\mathrm{sgn}(ax+1)}+\frac{4}{a^2\mathrm{sgn}(ax+1)}\right)+\frac{9}{a^3\mathrm{sgn}(ax+1)}\right)x+\frac{16}{a^4\mathrm{sgn}(ax+1)}\right)-\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8a^3|a|\mathrm{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*x^2 - 1)*((2*x*(3*x/(a*sgn(a*x + 1)) + 4/(a^2*sgn(a*x + 1)))
+ 9/(a^3*sgn(a*x + 1)))*x + 16/(a^4*sgn(a*x + 1))) - 3/8*log(abs(-x*abs(a)
+ sqrt(a^2*x^2 - 1)))/(a^3*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.50

$$\int e^{\coth^{-1}(ax)}x^3\,dx=\frac{\frac{13\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{49\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}+\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}$$

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((13*((a*x - 1)/(a*x + 1))^(1/2))/4 - (31*((a*x - 1)/(a*x + 1))^(3/2))/12 +
 (49*((a*x - 1)/(a*x + 1))^(5/2))/12 - (3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(
a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (
a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (3*atanh(((a*
x - 1)/(a*x + 1))^(1/2)))/(4*a^4)
```

3.2 $\int e^{\coth^{-1}(ax)} x^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 90

$$\int e^{\coth^{-1}(ax)} x^2 \, dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2+1/2*x^2*
(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6304, 849, 821, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)} x^2 \, dx = \frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a} + \frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2} + \frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[In] Int[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{4}{a^2}+\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(4+3ax+2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

```
[In] Integrate[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + 3*a*x + 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(2a^2x^2+3ax+4)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{2a^2\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$\frac{(ax-1)\left(3\sqrt{a^2x^2-1}\sqrt{a^2}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a+6\sqrt{a^2}\sqrt{(ax-1)(ax+1)}+6a\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1}}{\sqrt{a^2}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\,a^3\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*a^2*x^2+3*a*x+4)*(a*x-1)/a^3/((a*x-1)/(a*x+1))^(1/2)+1/2/a^2*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-
1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \frac{(2\,a^3x^3+5\,a^2x^2+7\,ax+4)\sqrt{\frac{ax-1}{ax+1}}+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="fricas")
[Out] 1/6*((2*a^3*x^3 + 5*a^2*x^2 + 7*a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) + 3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2,x)
[Out] Integral(x**2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(74) = 148.

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.84

$$\int e^{\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="maxima")
[Out] -1/6*a*(2*(3*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
9*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4
/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^4 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \frac{3\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3} + \left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3 + \frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3} - \frac{3\,a^3\,(a\,x-1)}{a\,x+1}} + \frac{\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + ((a*x
- 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x -
 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x +
 1))^(1/2))/a^3
```

3.3 $\int e^{\coth^{-1}(ax)} x \, dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int e^{\coth^{-1}(ax)} x \, dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2+x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a
^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6304, 849, 821, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)} x \, dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2} + \frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}$$

```
[In] Int[E^ArcCoth[a*x]*x,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{-\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}
\end{aligned}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(2+ax) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

```
[In] Integrate[E^ArcCoth[a*x]*x,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/
(2*a^2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.59

method	result	size
risch	$\frac{(ax+2)(ax-1)}{2a^2\sqrt{\frac{ax-1}{ax+1}}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{2a\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	100
default	$\frac{(ax-1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+2\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+2a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^2\sqrt{a^2}}$	152

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x+2)*(a*x-1)/a^2/((a*x-1)/(a*x+1))^(1/2)+1/2/a*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1
/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{(a^2x^2+3\,ax+2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 3*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(
a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x,x)
```

```
[Out] Integral(x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{1}{2}\,a\left(\frac{2\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(((a*x - 1)/(a*x + 1))^(3/2) - 3*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^3 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x}{a\mathrm{sgn}\,(ax+1)}+\frac{2}{a^2\mathrm{sgn}\,(ax+1)}\right) - \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,a|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="giac")
[Out] 1/2*sqrt(a^2*x^2 - 1)*(x/(a*sgn(a*x + 1)) + 2/(a^2*sgn(a*x + 1))) - 1/2*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}-\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}$$

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (3*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(
a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(
a*x + 1))^(1/2))/a^2
```

3.4 $\int e^{\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int e^{\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6303, 821, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[E^ArcCoth[a*x],x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}x + a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int e^{\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x+\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

```
[In] Integrate[E^ArcCoth[a*x],x]
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(32) = 64.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

method	result	size
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	90
default	$\frac{(ax-1)\left(a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	97

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x-1)/((a*x-1)/(a*x+1))^(1/2)+ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))
/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int e^{\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int e^{\coth^{-1}(ax)}\,dx = -\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}}{a\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x
^2 - 1)/(a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int e^{\coth^{-1}(ax)}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.5 $\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx = -\csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)$$

```
[Out] -arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6304, 858, 222, 272, 65, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx = \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right) - \csc^{-1}(ax)$$

```
[In] Int[E^ArcCoth[a*x]/x,x]
```

```
[Out] -ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) \\
&= -\csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = -\arcsin\left(\frac{1}{ax}\right) + \log\left(x\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)$$

```
[In] Integrate[E^ArcCoth[a*x]/x,x]
[Out] -ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 6.00

method	result	size
default	$-\frac{(ax-1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)-\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	132

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -(a*x-1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(
1/2)-a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2
)/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = 2\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
[Out] 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(1/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(20) = 40.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = \frac{2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} - \frac{a\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] 2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - a*log(abs(-x*abs(a)
+ sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx = 2 \operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + 2 \operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
```

3.6 $\int \frac{e^{\coth^{-1}(ax)}}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} dx = a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

```
[Out] -a*arccsc(a*x)+a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6304, 655, 222}

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} dx = a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

```
[In] Int[E^ArcCoth[a*x]/x^2,x]
```

```
[Out] a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= a\sqrt{1-\frac{1}{a^2x^2}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = a\left(\sqrt{1-\frac{1}{a^2x^2}} - \arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[E^ArcCoth[a*x]/x^2,x]
[Out] a*(Sqrt[1 - 1/(a^2*x^2)] - ArcSin[1/(a*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(22) = 44.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.17

method	result
risch	$\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}} - \frac{a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,x\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)/x/((a*x-1)/(a*x+1))^(1/2)-a*arctan(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a
*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + (ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} + \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*x
 - 1)/(a*x + 1))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(22) = 44.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,a\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}\left(ax+1\right)} + \frac{2\,|a|}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)\operatorname{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) + 2*abs(a)/(((x*abs(
a) - sqrt(a^2*x^2 - 1))^2 + 1)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (2*a*((a*x - 1)/(a*x + 1))^(1/2))/(
(a*x - 1)/(a*x + 1) + 1)
```

3.7 $\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx = \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] -1/2*a^2*arccsc(a*x)+1/2*a*(2*a+1/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6304, 794, 222}

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx = \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[In] Int[E^ArcCoth[a*x]/x^3,x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a + x^(-1)))/2 - (a^2*ArcCsc[a*x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(1+2ax)-ax\arcsin\left(\frac{1}{ax}\right)\right)}{2x}$$

```
[In] Integrate[E^ArcCoth[a*x]/x^3,x]
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a*x) - a*x*ArcSin[1/(a*x)]))/(2*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(32) = 64.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

method	result
risch	$\frac{(ax-1)(2ax+1)}{2x^{2}\sqrt{\frac{ax-1}{ax+1}}}-\frac{a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{(ax-1)(ax+1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-2\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}+2\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,ax+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+a^{2}x^{2}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+2\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{\cdots}}{\sqrt{a^{2}}}\cdots\right)\cdots\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,x^{2}\sqrt{a^{2}\cdots}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x-1)*(2*a*x+1)/x^2/((a*x-1)/(a*x+1))^(1/2)-1/2*a^2*arctan(1/(a^2*x^2
-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx=\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(2\,a^2x^2+3\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (2*a^2*x^2 + 3*a*x + 1)*
sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx=\int \frac{1}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.39

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] (a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*((a*x - 1)/(a*x + 1))^(3/2) + 3*a
*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^
2 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(32) = 64$.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.76

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \frac{a^2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} - \frac{\left(x|a| - \sqrt{a^2x^2-1}\right)^3a^2 - 2\left(x|a| - \sqrt{a^2x^2-1}\right)^2a|a| - \left(x|a| - \sqrt{a^2x^2-1}\right)a^2 - 2\,a|a|}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^2\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - ((x*abs(a) - sqrt(
a^2*x^2 - 1))^3*a^2 - 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*abs(a) - (x*abs(
a) - sqrt(a^2*x^2 - 1))*a^2 - 2*a*abs(a))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^
2 + 1)^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3} \, dx = a^2 \sqrt{\frac{a x - 1}{a x + 1}} + \frac{\sqrt{\frac{a x-1}{a x+1}}}{2 x^2} + a^2 \operatorname{atan}\left(\sqrt{\frac{a x - 1}{a x + 1}}\right) + \frac{3 a \sqrt{\frac{a x-1}{a x+1}}}{2 x}$$

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] a^2*((a*x - 1)/(a*x + 1))^(1/2) + ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) + a^2
*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (3*a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x
)
```

3.8 $\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = a^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

```
[Out] -1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)+a^3*(1-1/a^2/x^2)^(1/2)+1/
2*a^2*(1-1/a^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6304, 811, 655, 201, 222}

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{2}a^3\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] a^3*Sqrt[1 - 1/(a^2*x^2)] - (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[1 -
 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(a^2\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + a^2\text{Subst}\left(\int \left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\\
&\quad - a^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + a^2\text{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\\
&\quad - a^3\csc^{-1}(ax) + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(2+3ax+4a^2x^2)}{x^2} - 3a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x + 4*a^2*x^2))/x^2 - 3*a^2*ArcSin[1/(a
*x)]))/6
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(ax-1)(4a^2x^2+3ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}} - \frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\frac{a^2x+\sqrt{a^2}}{\sqrt{}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x-1)*(4*a^2*x^2+3*a*x+2)/x^3/((a*x-1)/(a*x+1))^(1/2)-1/2*a^3*arctan(
1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + (4\,a^3x^3+7\,a^2x^2+5\,ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*a^3*x^3 + 7*a^2*x^2 +
 5*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**4,x)
[Out] Integral(1/(x**4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(63) = 126.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.81

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{3\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (3*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 9*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{a^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} - \frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5a^3-12\left(x|a|-\sqrt{a^2x^2-1}\right)^2a^2|a|-3\left(x|a|-\sqrt{a^2x^2-1}\right)a^3-4\,a^2|a|}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] a^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/3*(3*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^5*a^3 - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^2*abs(a)
- 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*a^3 - 4*a^2*abs(a))/(((x*abs(a) - sqrt(a
^2*x^2 - 1))^2 + 1)^3*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3} + a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{7\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x} + \frac{5\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3)
 + a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (7*a^2*((a*x - 1)/(a*x + 1))^(1/
2))/(6*x) + (5*a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

3.9 $\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 88

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx=\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{3}{8}a^4\csc^{-1}(ax)$$

```
[Out] -3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a+9/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2
/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6304, 847, 794, 222}

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx=-\frac{3}{8}a^4\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)$$

```
[In] Int[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a + 9/x))/24 + (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - (3*a^4*ArcCsc[a*x])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{1}{4}a^2\text{Subst}\left(\int \frac{x^2\left(-\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{12}a^4\text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}+\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}\\
&\quad + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{3}{8}a^4\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(6+8ax+9a^2x^2+16a^3x^3\right)}{x^3}-9a^3\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^3 - 9*a^
3*ArcSin[1/(a*x)]))/24
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(ax-1)\left(16a^3x^3+9a^2x^2+8ax+6\right)}{24x^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{8\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+24\ln\left(\frac{a^2x+\sqrt{a\cdots}}{\cdots}\right.\cdots}{24\sqrt{\cdots}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(a*x-1)*(16*a^3*x^3+9*a^2*x^2+8*a*x+6)/x^4/((a*x-1)/(a*x+1))^(1/2)-3/8
*a^4*arctan(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(16\,a^4x^4+25\,a^3x^3+17\,a^2x^2+14\,ax+6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) + (16*a^4*x^4 + 25*a^3*x
^3 + 17*a^2*x^2 + 14*a*x + 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Integral(1/(x**5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{9\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (9*a^3*((a*x - 1)/(a*x + 1)
)^(7/2) + 49*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 31*a^3*((a*x - 1)/(a*x + 1))
^(3/2) + 39*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
- 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{3\,a^4\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{4\,\mathrm{sgn}\,(ax+1)} - \frac{9\left(x|a| - \sqrt{a^2x^2-1}\right)^7 a^4 + 33\left(x|a| - \sqrt{a^2x^2-1}\right)^5 a^4 - 48\left(x|a| - \sqrt{a^2x^2-1}\right)^4 a^3|a| - 33\left(x|a| - \sqrt{a^2x^2-1}\right)^{\cdots}}{12\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^4 \mathrm{sgn}\cdots}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/12*(9*(x*abs
(a) - sqrt(a^2*x^2 - 1))^7*a^4 + 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*a^4 -
48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a) - 33*(x*abs(a) - sqrt(a^2*x^
2 - 1))^3*a^4 - 64*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a) - 9*(x*abs(a
) - sqrt(a^2*x^2 - 1))*a^4 - 16*a^3*abs(a))/(((x*abs(a) - sqrt(a^2*x^2 - 1)
)^2 + 1)^4*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.47

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4} + \frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4} + \frac{17\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2} + \frac{25\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x} + \frac{7\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

```
[In] int(1/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4)
 + (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 + (17*a^2*((a*x - 1)/(a*x +
1))^(1/2))/(24*x^2) + (25*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (7*a*((
a*x - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

3.10 $\int e^{2\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

```
[Out] 2*x/a^3+x^2/a^2+2/3*x^3/a+1/4*x^4+2*ln(-a*x+1)/a^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2\log(1-ax)}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}x^3\,dx\\
&= -\int \frac{x^3(1+ax)}{1-ax}\,dx\\
&= -\int \left(-\frac{2}{a^3}-\frac{2x}{a^2}-\frac{2x^2}{a}-x^3-\frac{2}{a^3(-1+ax)}\right)dx\\
&= \frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3,x]
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{x^{2}}{a^{2}}+\frac{x^{4}}{4}+\frac{2x}{a^{3}}+\frac{2x^{3}}{3a}+\frac{2\ln(ax-1)}{a^{4}}$	39
risch	$\frac{x^{2}}{a^{2}}+\frac{x^{4}}{4}+\frac{2x}{a^{3}}+\frac{2x^{3}}{3a}+\frac{2\ln(ax-1)}{a^{4}}$	39
default	$\frac{\frac{1}{4}a^{3}x^{4}+\frac{2}{3}a^{2}x^{3}+a\,x^{2}+2x}{a^{3}}+\frac{2\ln(ax-1)}{a^{4}}$	42
parallelrisch	$\frac{3a^{4}x^{4}+8a^{3}x^{3}+12a^{2}x^{2}+24ax+24\ln(ax-1)}{12a^{4}}$	43
meijerg	$\frac{\frac{ax\left(15a^{3}x^{3}+20a^{2}x^{2}+30ax+60\right)}{60}+\ln(-ax+1)}{a^{4}}-\frac{-\frac{ax\left(4a^{2}x^{2}+6ax+12\right)}{12}-\ln(-ax+1)}{a^{4}}$	73

```
[In] int(1/(a*x-1)*(a*x+1)*x^3,x,method=_RETURNVERBOSE)
[Out] x^2/a^2+1/4*x^4+2*x/a^3+2/3*x^3/a+2/a^4*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}x^{3}\,dx=\frac{3\,a^{4}x^{4}+8\,a^{3}x^{3}+12\,a^{2}x^{2}+24\,ax+24\,\log\left(ax-1\right)}{12\,a^{4}}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="fricas")
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}x^{3}\,dx=\frac{x^{4}}{4}+\frac{2x^{3}}{3a}+\frac{x^{2}}{a^{2}}+\frac{2x}{a^{3}}+\frac{2\log\left(ax-1\right)}{a^{4}}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3,x)
[Out] x**4/4 + 2*x**3/(3*a) + x**2/a**2 + 2*x/a**3 + 2*log(a*x - 1)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^3x^4 + 8\,a^2x^3 + 12\,ax^2 + 24\,x}{12\,a^3} + \frac{2\,\log\left(ax - 1\right)}{a^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 + 2*log(a*x - 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax - 1|\right)}{a^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 + 2*log(abs(a*x - 1)
)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{2\,\ln\left(a\,x - 1\right)}{a^4} + \frac{2\,x}{a^3} + \frac{x^4}{4} + \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

```
[In] int((x^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*log(a*x - 1))/a^4 + (2*x)/a^3 + x^4/4 + (2*x^3)/(3*a) + x^2/a^2
```

3.11 $\int e^{2\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

```
[Out] 2*x/a^2+x^2/a+1/3*x^3+2*ln(-a*x+1)/a^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{2\log(1-ax)}{a^3}+\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}x^2\,dx\\ &= -\int \frac{x^2(1+ax)}{1-ax}\,dx\\ &= -\int \left(-\frac{2}{a^2}-\frac{2x}{a}-x^2-\frac{2}{a^2(-1+ax)}\right)dx\\ &= \frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2,x]
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
norman	$\frac{x^2}{a}+\frac{x^3}{3}+\frac{2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	31
risch	$\frac{x^2}{a}+\frac{x^3}{3}+\frac{2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	31
default	$\frac{\frac{1}{3}a^2x^3+a\,x^2+2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	34
parallelrisch	$\frac{a^3x^3+3a^2x^2+6ax+6\ln(ax-1)}{3a^3}$	34
meijerg	$-\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)}{a^3}+\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^3}$	57

[In] int(1/(a*x-1)*(a*x+1)*x^2,x,method=_RETURNVERBOSE)

[Out] x^2/a+1/3*x^3+2*x/a^2+2/a^3*ln(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{a^3x^3+3\,a^2x^2+6\,ax+6\,\log{(ax-1)}}{3\,a^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="fricas")

[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{x^3}{3}+\frac{x^2}{a}+\frac{2x}{a^2}+\frac{2\log{(ax-1)}}{a^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)*x**2,x)

[Out] x**3/3 + x**2/a + 2*x/a**2 + 2*log(a*x - 1)/a**3

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^2x^3 + 3\,ax^2 + 6\,x}{3\,a^2} + \frac{2\,\log\left(ax - 1\right)}{a^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="maxima")
[Out] 1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 + 2*log(a*x - 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 + 3\,a^2x^2 + 6\,ax}{3\,a^3} + \frac{2\,\log\left(|ax - 1|\right)}{a^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="giac")
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 + 2*log(abs(a*x - 1))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{2\,\ln\left(a\,x - 1\right)}{a^3} + \frac{2\,x}{a^2} + \frac{x^3}{3} + \frac{x^2}{a}$$

```
[In] int((x^2*(a*x + 1))/(a*x - 1),x)
[Out] (2*log(a*x - 1))/a^3 + (2*x)/a^2 + x^3/3 + x^2/a
```

3.12 $\int e^{2\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

```
[Out] 2*x/a+1/2*x^2+2*ln(-a*x+1)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6261, 78}

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x,x]
```

```
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}x\,dx\\ &= -\int \frac{x(1+ax)}{1-ax}\,dx\\ &= -\int \left(-\frac{2}{a}-x-\frac{2}{a(-1+ax)}\right)dx\\ &= \frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x,x]
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
norman	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
risch	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
parallelrisch	$\frac{a^2x^2+4ax+4\ln(ax-1)}{2a^2}$	26
default	$\frac{\frac{1}{2}a\,x^2+2x}{a}+\frac{2\ln(ax-1)}{a^2}$	27
meijerg	$\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^2}-\frac{-ax-\ln(-ax+1)}{a^2}$	43

```
[In] int(1/(a*x-1)*(a*x+1)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+2*x/a+2/a^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2+4\,ax+4\,\log{(ax-1)}}{2\,a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="fricas")
[Out] 1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}+\frac{2x}{a}+\frac{2\log{(ax-1)}}{a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x)
[Out] x**2/2 + 2*x/a + 2*log(a*x - 1)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x\,dx = \frac{ax^2+4\,x}{2\,a} + \frac{2\,\log\left(ax-1\right)}{a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="maxima")
[Out] 1/2*(a*x^2 + 4*x)/a + 2*log(a*x - 1)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}x\,dx = \frac{a^2x^2+4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax-1|\right)}{a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="giac")
[Out] 1/2*(a^2*x^2 + 4*a*x)/a^2 + 2*log(abs(a*x - 1))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}x\,dx = \frac{2\,\ln\left(a\,x-1\right)}{a^2} + \frac{2\,x}{a} + \frac{x^2}{2}$$

```
[In] int((x*(a*x + 1))/(a*x - 1),x)
[Out] (2*log(a*x - 1))/a^2 + (2*x)/a + x^2/2
```

3.13 $\int e^{2\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(1-ax)}{a}$$

```
[Out] x+2*ln(-a*x+1)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6302, 6260, 45}

$$\int e^{2\coth^{-1}(ax)}\,dx = \frac{2\log(1-ax)}{a} + x$$

```
[In] Int[E^(2*ArcCoth[a*x]),x]
```

```
[Out] x + (2*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6260

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\,dx\\ &= -\int \frac{1+ax}{1-ax}\,dx\\ &= -\int \left(-1-\frac{2}{-1+ax}\right)dx\\ &= x+\frac{2\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\,dx = x+\frac{2\log(1-ax)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x]),x]
[Out] x + (2*Log[1 - a*x])/a
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$x+\frac{2\ln(ax-1)}{a}$	14
norman	$x+\frac{2\ln(ax-1)}{a}$	14
risch	$x+\frac{2\ln(ax-1)}{a}$	14
parallelrisch	$\frac{ax+2\ln(ax-1)}{a}$	17
meijerg	$\frac{\ln(-ax+1)}{a}-\frac{-ax-\ln(-ax+1)}{a}$	32

```
[In] int((a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
[Out] x+2/a*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{2\coth^{-1}(ax)}\,dx = \frac{ax + 2\log(ax-1)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="fricas")
[Out] (a*x + 2*log(a*x - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(ax-1)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1),x)
[Out] x + 2*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(ax-1)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="maxima")
[Out] x + 2*log(a*x - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(|ax-1|)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="giac")
[Out] x + 2*log(abs(a*x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\,\ln(a\,x-1)}{a}$$

```
[In] int((a*x + 1)/(a*x - 1),x)
[Out] x + (2*log(a*x - 1))/a
```

3.14 $\int \frac{e^{2\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{e^{2\coth^{-1}(ax)}}{x} \, dx = -\log(x) + 2\log(1-ax)$$

```
[Out] -ln(x)+2*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{x} \, dx = 2\log(1-ax) - \log(x)$$

```
[In] Int[E^(2*ArcCoth[a*x])/x,x]
```

```
[Out] -Log[x] + 2*Log[1 - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{x}\,dx \\
&= -\int \frac{1+ax}{x(1-ax)}\,dx \\
&= -\int \left(\frac{1}{x}-\frac{2a}{-1+ax}\right)dx \\
&= -\log(x)+2\log(1-ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log(x)+2\log(1-ax)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/x,x]
[Out] -Log[x] + 2*Log[1 - a*x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$-\ln\left(x\right)+2\ln\left(ax-1\right)$	14
norman	$-\ln\left(x\right)+2\ln\left(ax-1\right)$	14
parallelrisch	$-\ln\left(x\right)+2\ln\left(ax-1\right)$	14
risch	$-\ln\left(x\right)+2\ln\left(-ax+1\right)$	15
meijerg	$2\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)$	21

```
[In] int(1/(a*x-1)*(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(ax-1\right) - \log\left(x\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="fricas")
[Out] 2*log(a*x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log\left(x\right) + 2\log\left(x-\frac{1}{a}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x)
[Out] -log(x) + 2*log(x - 1/a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(ax-1\right) - \log\left(x\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="maxima")
[Out] 2*log(a*x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(|ax-1|\right) - \log\left(|x|\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="giac")
[Out] 2*log(abs(a*x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\ln\left(3-3\,a\,x\right) - \ln\left(x\right)$$

```
[In] int((a*x + 1)/(x*(a*x - 1)),x)
[Out] 2*log(3 - 3*a*x) - log(x)
```

3.15 $\int \frac{e^{2 \coth^{-1}(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{e^{2 \coth^{-1}(ax)}}{x^2} \, dx = \frac{1}{x} - 2a \log(x) + 2a \log(1 - ax)$$

```
[Out] 1/x-2*a*ln(x)+2*a*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$\int \frac{e^{2 \coth^{-1}(ax)}}{x^2} \, dx = -2a \log(x) + 2a \log(1 - ax) + \frac{1}{x}$$

```
[In] Int[E^(2*ArcCoth[a*x])/x^2,x]
```

```
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{x^2}\,dx\\ &= -\int \frac{1+ax}{x^2(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x^2}+\frac{2a}{x}-\frac{2a^2}{-1+ax}\right)dx\\ &= \frac{1}{x}-2a\log(x)+2a\log(1-ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x}-2a\log(x)+2a\log(1-ax)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^2,x]
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{1}{x}-2a\ln\left(x\right)+2a\ln\left(ax-1\right)$	19
norman	$\frac{1}{x}-2a\ln\left(x\right)+2a\ln\left(ax-1\right)$	19
risch	$\frac{1}{x}-2a\ln\left(x\right)+2a\ln\left(-ax+1\right)$	20
parallelrisch	$-\frac{2a\ln(x)x-2a\ln(ax-1)x-1}{x}$	24
meijerg	$-a(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right))+a\left(\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)+\frac{1}{ax}\right)$	48

```
[In] int(1/(a*x-1)*(a*x+1)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x-2*a*ln(x)+2*a*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,ax\log\left(ax-1\right)-2\,ax\log\left(x\right)+1}{x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="fricas")
[Out] (2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2a\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{1}{x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**2,x)
[Out] 2*a*(-log(x) + log(x - 1/a)) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\log\left(ax-1\right)-2\,a\log\left(x\right)+\frac{1}{x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="maxima")
[Out] 2*a*log(a*x - 1) - 2*a*log(x) + 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2} \, dx = 2\,a\log\left(|ax-1|\right) - 2\,a\log\left(|x|\right) + \frac{1}{x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="giac")
[Out] 2*a*log(abs(a*x - 1)) - 2*a*log(abs(x)) + 1/x
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2} \, dx = \frac{1}{x} - 4\,a\,\mathrm{atanh}(2\,a\,x-1)$$

```
[In] int((a*x + 1)/(x^2*(a*x - 1)),x)
[Out] 1/x - 4*a*atanh(2*a*x - 1)
```

3.16 $\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} + \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1-ax)$$

```
[Out] 1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = -2a^2\log(x) + 2a^2\log(1-ax) + \frac{2a}{x} + \frac{1}{2x^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])/x^3,x]
```

```
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{x^3}\,dx \\
&= -\int \frac{1+ax}{x^3(1-ax)}\,dx \\
&= -\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{-1+ax}\right)dx \\
&= \frac{1}{2x^2}+\frac{2a}{x}-2a^2\log(x)+2a^2\log(1-ax)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2x^2}+\frac{2a}{x}-2a^2\log(x)+2a^2\log(1-ax)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^3,x]
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{1}{2}+2ax}{x^2}-2a^2\ln(x)+2a^2\ln(ax-1)$
default	$\frac{1}{2x^2}+\frac{2a}{x}-2a^2\ln(x)+2a^2\ln(ax-1)$
risch	$\frac{\frac{1}{2}+2ax}{x^2}+2a^2\ln(-ax+1)-2a^2\ln(x)$
parallelrisch	$-\frac{4a^2\ln(x)x^2-4a^2\ln(ax-1)x^2-1-4ax}{2x^2}$
meijerg	$a^2\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)-a^2\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\right.$

```
[In] int(1/(a*x-1)*(a*x+1)/x^3,x,method=_RETURNVERBOSE)
[Out] (1/2+2*a*x)/x^2-2*a^2*ln(x)+2*a^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx=\frac{4\,a^2x^2\log(ax-1)-4\,a^2x^2\log(x)+4\,ax+1}{2\,x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="fricas")
[Out] 1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx=2a^2\left(-\log(x)+\log\left(x-\frac{1}{a}\right)\right)+\frac{4ax+1}{2x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**3,x)
[Out] 2*a**2*(-log(x) + log(x - 1/a)) + (4*a*x + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(ax-1\right) - 2\,a^2\log\left(x\right) + \frac{4\,ax+1}{2\,x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="maxima")
[Out] 2*a^2*log(a*x - 1) - 2*a^2*log(x) + 1/2*(4*a*x + 1)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(|ax-1|\right) - 2\,a^2\log\left(|x|\right) + \frac{4\,ax+1}{2\,x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="giac")
[Out] 2*a^2*log(abs(a*x - 1)) - 2*a^2*log(abs(x)) + 1/2*(4*a*x + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{2\,a\,x+\frac{1}{2}}{x^2} - 4\,a^2\,\mathrm{atanh}(2\,a\,x-1)$$

```
[In] int((a*x + 1)/(x^3*(a*x - 1)),x)
[Out] (2*a*x + 1/2)/x^2 - 4*a^2*atanh(2*a*x - 1)
```

3.17 $\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

```
[Out] 1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=-2a^3\log(x)+2a^3\log(1-ax)+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{3x^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])/x^4,x]
```

```
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{x^4}\,dx\\ &= -\int \frac{1+ax}{x^4(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}-\frac{2a^4}{-1+ax}\right)dx\\ &= \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^4,x]
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(ax-1)$
default	$\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\ln(x)+2a^3\ln(ax-1)$
risch	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(-ax+1)$
parallelrisch	$-\frac{6a^3\ln(x)x^3-6a^3\ln(ax-1)x^3-1-6a^2x^2-3ax}{3x^3}$
meijerg	$-a^3\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)+a^3\left(\ln(-ax+1)-\ln(x)-\ln(-a)\right.$

[In] int(1/(a*x-1)*(a*x+1)/x^4,x,method=_RETURNVERBOSE)

[Out] (1/3+2*a^2*x^2+a*x)/x^3-2*a^3*ln(x)+2*a^3*ln(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=\frac{6\,a^3x^3\log(ax-1)-6\,a^3x^3\log(x)+6\,a^2x^2+3\,ax+1}{3\,x^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="fricas")

[Out] 1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^3

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=2a^3\left(-\log(x)+\log\left(x-\frac{1}{a}\right)\right)+\frac{6a^2x^2+3ax+1}{3x^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)/x**4,x)

[Out] 2*a**3*(-log(x) + log(x - 1/a)) + (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2\,a^3\log\left(ax-1\right) - 2\,a^3\log\left(x\right) + \frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*log(a*x - 1) - 2*a^3*log(x) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2\,a^3\log\left(|ax-1|\right) - 2\,a^3\log\left(|x|\right) + \frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*log(abs(a*x - 1)) - 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x
^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^2\,x^2+a\,x+\frac{1}{3}}{x^3} - 4\,a^3\,\mathrm{atanh}(2\,a\,x-1)$$

```
[In] int((a*x + 1)/(x^4*(a*x - 1)),x)
```

```
[Out] (a*x + 2*a^2*x^2 + 1/3)/x^3 - 4*a^3*atanh(2*a*x - 1)
```

3.18 $\int e^{3\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] 11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3-4*(1-1/a^2/x^2)^(1/2)/a^2/(a-1/x)+14/
3*x*(1-1/a^2/x^2)^(1/2)/a^2+3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6304, 6874, 665, 277, 270, 272, 44, 65, 214}

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^4\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{4}{a^3(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3}-\frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3}\\
&\quad-\frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}-\frac{3\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&\quad-\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^3}\\
&\quad-\frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3a^2}-\frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\\
&\quad-\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}+\frac{4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 \\
&\quad + \frac{4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3} + \frac{3\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a} \\
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{11\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$
\int e^{3\coth^{-1}(ax)}x^2\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-52+19ax+7a^2x^2+2a^3x^3\right)}{-1+ax} + 33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 + a
*x) + 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

method	result
risch	$\frac{(2a^2x^2+9ax+28)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}} + \frac{\left(\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a^2\sqrt{a^2}} - \frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^4\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-1)(ax+1))}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*a^2*x^2+9*a*x+28)*(a*x-1)/a^3/((a*x-1)/(a*x+1))^(1/2)+(11/2/a^2*ln(a
^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^4/(x-1/a)*((x-1/a)^2*a^
2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \frac{33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - 33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) + (2\,a^4x^4+9\,a^3x^3+26\,a^2x^2-33\,ax-52)\ldots}{6\,(a^4x-a^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(33*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 33*(a*x - 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (2*a^4*x^4 + 9*a^3*x^3 + 26*a^2*x^2 - 33*a*x
- 52)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.54

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(\frac{75\,(ax-1)}{ax+1}-\frac{88\,(ax-1)^2}{(ax+1)^2}+\frac{33\,(ax-1)^3}{(ax+1)^3}-12\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-\ldots\right)}{a^4}\ldots\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(75*(a*x - 1)/(a*x + 1) - 88*(a*x - 1)^2/(a*x + 1)^2 + 33*(a*x -
1)^3/(a*x + 1)^3 - 12)/(a^4*((a*x - 1)/(a*x + 1))^(7/2) - 3*a^4*((a*x - 1)/
(a*x + 1))^(5/2) + 3*a^4*((a*x - 1)/(a*x + 1))^(3/2) - a^4*sqrt((a*x - 1)/(
a*x + 1))) - 33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 33*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^4)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")

[Out] undef

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}-\frac{\frac{88\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{11\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{25\,(a\,x-1)}{a\,x+1}+4}{a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

[In] int(x^2/((a*x - 1)/(a*x + 1))^(3/2),x)

[Out] (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3 - ((88*(a*x - 1)^2)/(3*(a*x + 1)^2) - (11*(a*x - 1)^3)/(a*x + 1)^3 - (25*(a*x - 1))/(a*x + 1) + 4)/(a^3*((a*x - 1)/(a*x + 1))^(1/2) - 3*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^3*((a*x - 1)/(a*x + 1))^(5/2) - a^3*((a*x - 1)/(a*x + 1))^(7/2))

3.19 $\int e^{3\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 92

$$\int e^{3\coth^{-1}(ax)}x\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a-1/x)+3*x*(1
-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6304, 6874, 665, 272, 44, 65, 214, 270}

$$\int e^{3\coth^{-1}(ax)}x\,dx=\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a^2(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int e^{3\coth^{-1}(ax)}x\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-14+5ax+a^2x^2)}{-1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 + 5*a*x + a^2*x^2))/(-1 + a*x) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(ax+6)(ax-1)}{2a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^3\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+10\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+10\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(a}{\sqrt{a^2}}\right.$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x+6)*(a*x-1)/a^2/((a*x-1)/(a*x+1))^(1/2)+(9/2/a*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^3/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}x\,dx$$
$$=\frac{9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^3x^3+6\,a^2x^2-9\,ax-14)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3x-a^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(9*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x - 1)*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1) + (a^3*x^3 + 6*a^2*x^2 - 9*a*x - 14)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\,dx=\int\frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x,x)
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.58

$$\int e^{3\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{2}a\left(\frac{2\left(\frac{15(ax-1)}{ax+1}-\frac{9(ax-1)^2}{(ax+1)^2}-4\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-2a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="maxima")
[Out] 1/2*a*(2*(15*(a*x - 1)/(a*x + 1) - 9*(a*x - 1)^2/(a*x + 1)^2 - 4)/(a^3*((a*
x - 1)/(a*x + 1))^(5/2) - 2*a^3*((a*x - 1)/(a*x + 1))^(3/2) + a^3*sqrt((a*x
 - 1)/(a*x + 1))) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}x\,dx=\int\frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int e^{3\coth^{-1}(ax)}x\,dx = \frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2} - \frac{\frac{9\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{15\,(a\,x-1)}{a\,x+1} + 4}{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 2\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} + a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(x/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - ((9*(a*x - 1)^2)/(a*x + 1)^2 -
 (15*(a*x - 1))/(a*x + 1) + 4)/(a^2*((a*x - 1)/(a*x + 1))^(1/2) - 2*a^2*((a
*x - 1)/(a*x + 1))^(3/2) + a^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.20 $\int e^{3\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int e^{3\coth^{-1}(ax)}\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*(1-1/a^2/x^2)^(1/2)/(a-1/x)+x*(1-1/a^2/x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6303, 6874, 665, 270, 272, 65, 214}

$$\int e^{3\coth^{-1}(ax)}\,dx=\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}$$

```
[In] Int[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^2\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\ &= -\text{Subst}\left(\int \left(\frac{4}{a(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}}\right)\,dx, x, \frac{1}{x}\right) \end{aligned}$$

$$= -\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a} - \frac{4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a}$$
$$- \text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}x - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{2a}$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}x + (3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}x + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{3\coth^{-1}(ax)}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(-5+ax)}{-1+ax} + \frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + a*x))/(-1 + a*x) + (3*Log[a*(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}} + \frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}} - \frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^2\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-}{a\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/((a*x-1)/(a*x+1))^(1/2)+(3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/
2))/(a^2)^(1/2)-4/a^2/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((
a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int e^{3\coth^{-1}(ax)}\,dx = \frac{3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-4\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x-a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 4*a*x - 5)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*x - a)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int e^{3\coth^{-1}(ax)}\,dx = -a\left(\frac{2\left(\frac{3\,(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*sq
rt((a*x - 1)/(a*x + 1))) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*log
(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int e^{3\coth^{-1}(ax)}\,dx = \frac{2\,a\,x + 12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 10}{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*a*x + 12*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 10)/(2*a*((a*x - 1)/(a*x + 1))^(1/2))
```

3.21 $\int \frac{e^{3\coth^{-1}(ax)}}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{3\coth^{-1}(ax)}}{x} dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

```
[Out] arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a-1/x)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6304, 6874, 222, 665, 272, 65, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{x} dx = \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) - \frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax)$$

```
[In] Int[E^(3*ArcCoth[a*x])/x,x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right)\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -\left(4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}\,x}{-1+ax} + \arcsin\left(\frac{1}{ax}\right) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/x,x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(42) = 84.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 7.89

method	result
default	$\frac{\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-2\sqrt{a^2x^2-1}\,\sqrt{a}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^
2-1)^(1/2))+ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3
*x^2+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2-2*(a^2*x^2-1)^(1/2)*(a^2)^
(1/2)*a*x-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*ln((a^2*x+(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-2*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a
^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\frac{2\,(ax-1)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + (ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) + 4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(2*(a*x - 1)*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x - 1)*log(sqrt((a*x -
 1)/(a*x + 1)) + 1) + (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + 4*(a*x
 + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a} + \frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a + 4/(a*sqrt((a*x - 1)/(a*x + 1
))))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{4}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
- 4/((a*x - 1)/(a*x + 1))^(1/2)
```

3.22 $\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -3a\sqrt{1-\frac{1}{a^2x^2}}-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)$$

```
[Out] 3*a*arccsc(a*x)-2*(a+1/x)^2/a/(1-1/a^2/x^2)^(1/2)-3*a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6304, 867, 683, 655, 222}

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}-3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

```
[In] Int[E^(3*ArcCoth[a*x])/x^2,x]
```

```
[Out] -3*a*Sqrt[1 - 1/(a^2*x^2)] - (2*(a + x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
 3*a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -3a\sqrt{1-\frac{1}{a^2x^2}}-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -3a\sqrt{1-\frac{1}{a^2x^2}}-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(1-5ax)}{-1+ax} + 3a\arcsin\left(\frac{1}{ax}\right)$$

[In] Integrate[E^(3*ArcCoth[a*x])/x^2,x]

[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x))/(-1 + a*x) + 3*a*ArcSin[1/(a*x)]

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+5\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -(a*x-1)/x/((a*x-1)/(a*x+1))^(1/2)+(3*a*arctan(1/(a^2*x^2-1)^(1/2))-4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -\frac{6\left(a^2x^2-ax\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(5\,a^2x^2+4\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")

[Out] -(6*(a^2*x^2 - a*x)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**2,x)
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
[Out] -2*a*((3*(a*x - 1)/(a*x + 1) + 2)/(((a*x - 1)/(a*x + 1))^(3/2) + sqrt((a*x
- 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-6\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{5\,a}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] 1/(x*((a*x - 1)/(a*x + 1))^(1/2)) - 6*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) -
 (5*a)/((a*x - 1)/(a*x + 1))^(1/2)
```

3.23 $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}+\frac{9}{2}a^2\csc^{-1}(ax)$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)/(a-1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a-1/x)+9/2*
a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6304, 1647, 1607, 12, 807, 679, 222}

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{9}{2}a^2\csc^{-1}(ax)-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}$$

```
[In] Int[E^(3*ArcCoth[a*x])/x^3,x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a - x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a - x^(-1))) + (9*a^2*ArcCsc[a*x
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(-ax-x^2)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} + (3a)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(1+5ax-14a^2x^2)}{x(-1+ax)} + 9a\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^3,x]
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x - 14*a^2*x^2))/(x*(-1 + a*x)) + 9*a*A
rcSin[1/(a*x)]))/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{(ax-1)(6ax+1)}{2x^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}-\frac{4a\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{-6\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{5}x^{5}+6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}+21\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}+9a^{4}x^{4}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+6\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-1)*(6*a*x+1)/x^2/((a*x-1)/(a*x+1))^(1/2)+(9/2*a^2*arctan(1/(a^2*x
^2-1)^(1/2))-4*a/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((a*x-1
)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{18\left(a^{3}x^{3}-a^{2}x^{2}\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(14\,a^{3}x^{3}+9\,a^{2}x^{2}-6\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^{3}-x^{2}\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*(a^3*x^3 - a^2*x^2)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (14*a^3*x^
3 + 9*a^2*x^2 - 6*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{15\,(ax-1)a}{ax+1}+\frac{9\,(ax-1)^2a}{(ax+1)^2}+4\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (15*(a*x - 1)*a/(a*x + 1) + 9*(a*
x - 1)^2*a/(a*x + 1)^2 + 4*a)/(((a*x - 1)/(a*x + 1))^(5/2) + 2*((a*x - 1)/(
a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2\,x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-\frac{7\,a^2}{\sqrt{\frac{a\,x-1}{a\,x+1}}}-9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{5\,a}{2\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 1/(2*x^2*((a*x - 1)/(a*x + 1))^(1/2)) - (7*a^2)/((a*x - 1)/(a*x + 1))^(1/2)
 - 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (5*a)/(2*x*((a*x - 1)/(a*x + 1
))^(1/2))
```

3.24 $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{11}{2}a^3\csc^{-1}(ax)$$

```
[Out] 11/2*a^3*arccsc(a*x)-(a+1/x)^3/(1-1/a^2/x^2)^(1/2)-1/3*a*(3*a+1/x)^2*(1-1/a
^2/x^2)^(1/2)-1/6*a^2*(28*a+3/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6304, 1647, 1607, 12, 866, 1649, 1668, 794, 222}

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \frac{11}{2}a^3\csc^{-1}(ax) - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] -((a + x^(-1))^3/Sqrt[1 - 1/(a^2*x^2)]) - (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a + x
^(-1))^2)/3 - (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a + 3/x))/6 + (11*a^3*ArcCsc[a
*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

```
&& GtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(-ax^2-x^3\right)}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2 (3a^2+ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5-\frac{3x}{a}\right)(3a^2+ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{1}{2}(11a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{11}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(2+7ax+19a^2x^2-52a^3x^3)}{x^2(-1+ax)} + 33a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^4,x]
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^2*(-1
+ a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{(ax-1)(28a^2x^2+9ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}} + \frac{\left(\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2} - \frac{4a^2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{-30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+93\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^5x^5+30\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{}}{\sqrt{a^2}}\right.}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/6*(a*x-1)*(28*a^2*x^2+9*a*x+2)/x^3/((a*x-1)/(a*x+1))^(1/2)+(11/2*a^3*arc
tan(1/(a^2*x^2-1)^(1/2))-4*a^2/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(
a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{66\,(a^4x^4-a^3x^3)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(52\,a^4x^4+33\,a^3x^3-26\,a^2x^2-9\,ax-2)\sqrt{\frac{ax-1}{ax+1}}}{6\,(ax^4-x^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
[Out] -1/6*(66*(a^4*x^4 - a^3*x^3)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (52*a^4*x^
4 + 33*a^3*x^3 - 26*a^2*x^2 - 9*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4
- x^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**4,x)
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{75\,(ax-1)a^2}{ax+1}+\frac{88\,(ax-1)^2a^2}{(ax+1)^2}+\frac{33\,(ax-1)^3a^2}{(ax+1)^3}+12\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (75*(a*x - 1)*a^2/(a*x + 1
) + 88*(a*x - 1)^2*a^2/(a*x + 1)^2 + 33*(a*x - 1)^3*a^2/(a*x + 1)^3 + 12*a^
2)/(((a*x - 1)/(a*x + 1))^(7/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + 3*((a*x -
 1)/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{4\,a^3+\frac{88\,a^3\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}+\frac{11\,a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{25\,a^3\,\left(a\,x-1\right)}{a\,x+1}}{\sqrt{\frac{a\,x-1}{a\,x+1}}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] - (4*a^3 + (88*a^3*(a*x - 1)^2)/(3*(a*x + 1)^2) + (11*a^3*(a*x - 1)^3)/(a*x
 + 1)^3 + (25*a^3*(a*x - 1))/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/2) + 3*((
a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + ((a*x - 1)/(a*x
 + 1))^(7/2)) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.25 $\int e^{4 \coth^{-1}(ax)} x^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int e^{4 \coth^{-1}(ax)} x^3 \, dx = \frac{12x}{a^3} + \frac{4x^2}{a^2} + \frac{4x^3}{3a} + \frac{x^4}{4} + \frac{4}{a^4(1-ax)} + \frac{16\log(1-ax)}{a^4}$$

```
[Out] 12*x/a^3+4*x^2/a^2+4/3*x^3/a+1/4*x^4+4/a^4/(-a*x+1)+16*ln(-a*x+1)/a^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 90}

$$\int e^{4 \coth^{-1}(ax)} x^3 \, dx = \frac{4}{a^4(1-ax)} + \frac{16\log(1-ax)}{a^4} + \frac{12x}{a^3} + \frac{4x^2}{a^2} + \frac{4x^3}{3a} + \frac{x^4}{4}$$

```
[In] Int[E^(4*ArcCoth[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{4\text{arctanh}(ax)}x^3\,dx\\ &= \int \frac{x^3(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3+\frac{4}{a^3(-1+ax)^2}+\frac{16}{a^3(-1+ax)}\right)dx\\ &= \frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^3,x]
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x^4}{4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}-\frac{4}{a^4(ax-1)}+\frac{16\ln(ax-1)}{a^4}$
norman	$\frac{\frac{13x^4}{12}+\frac{a\,x^5}{4}+\frac{8x^2}{a^2}+\frac{8x^3}{3a}-\frac{16}{a^4}}{ax-1}+\frac{16\ln(ax-1)}{a^4}$
default	$\frac{\frac{1}{4}a^3x^4+\frac{4}{3}a^2x^3+4a\,x^2+12x}{a^3}-\frac{4}{a^4(ax-1)}+\frac{16\ln(ax-1)}{a^4}$
parallelrisch	$\frac{3a^5x^5+13a^4x^4+32a^3x^3-192+96a^2x^2+192a\ln(ax-1)x-192\ln(ax-1)}{12a^4(ax-1)}$
meijerg	$\frac{\frac{ax\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+5\ln(-ax+1)}{a^4}-\frac{2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a^4}+\frac{ax\left(-2a^2x^2-6a\right.}{-4ax+4}$

[In] int(1/(a*x-1)^2*(a*x+1)^2*x^3,x,method=_RETURNVERBOSE)

[Out] 1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3-4/a^4/(a*x-1)+16/a^4*ln(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^5x^5+13\,a^4x^4+32\,a^3x^3+96\,a^2x^2-144\,ax+192\,(ax-1)\log{(ax-1)}-48}{12\,(a^5x-a^4)}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="fricas")

[Out] 1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*x - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{x^4}{4}-\frac{4}{a^5x-a^4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{16\log{(ax-1)}}{a^4}$$

[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**3,x)

[Out] x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*log(a*x - 1)/a**4

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = -\frac{4}{a^5x-a^4} + \frac{3\,a^3x^4+16\,a^2x^3+48\,ax^2+144\,x}{12\,a^3} + \frac{16\,\log\left(ax-1\right)}{a^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="maxima")
[Out] -4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 + 1
6*log(a*x - 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{(ax-1)^4\left(\frac{28}{ax-1}+\frac{114}{(ax-1)^2}+\frac{300}{(ax-1)^3}+3\right)}{12\,a^4} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^4} - \frac{4}{(ax-1)a^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="giac")
[Out] 1/12*(a*x - 1)^4*(28/(a*x - 1) + 114/(a*x - 1)^2 + 300/(a*x - 1)^3 + 3)/a^4
 - 16*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a^4 - 4/((a*x - 1)*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{16\,\ln\left(a\,x-1\right)}{a^4} - \frac{4}{a\,\left(a^4\,x-a^3\right)} + \frac{12\,x}{a^3} + \frac{x^4}{4} + \frac{4\,x^3}{3\,a} + \frac{4\,x^2}{a^2}$$

```
[In] int((x^3*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (16*log(a*x - 1))/a^4 - 4/(a*(a^4*x - a^3)) + (12*x)/a^3 + x^4/4 + (4*x^3)/
(3*a) + (4*x^2)/a^2
```

3.26 $\int e^{4\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 47

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

```
[Out] 8*x/a^2+2*x^2/a+1/3*x^3+4/a^3/(-a*x+1)+12*ln(-a*x+1)/a^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

```
[In] Int[E^(4*ArcCoth[a*x])*x^2,x]
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
```

```
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}x^2\,dx \\
&= \int \frac{x^2(1+ax)^2}{(1-ax)^2}\,dx \\
&= \int \left(\frac{8}{a^2}+\frac{4x}{a}+x^2+\frac{4}{a^2(-1+ax)^2}+\frac{12}{a^2(-1+ax)}\right)dx \\
&= \frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^2,x]
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^3}{3}+\frac{2x^2}{a}+\frac{8x}{a^2}-\frac{4}{a^3(ax-1)}+\frac{12\ln(ax-1)}{a^3}$
norman	$\frac{\frac{5x^3}{3}+\frac{a\,x^4}{3}+\frac{6x^2}{a}-\frac{12}{a^3}}{ax-1}+\frac{12\ln(ax-1)}{a^3}$
default	$\frac{\frac{1}{3}a^2x^3+2a\,x^2+8x}{a^2}-\frac{4}{a^3(ax-1)}+\frac{12\ln(ax-1)}{a^3}$
parallelrisch	$\frac{a^4x^4+5a^3x^3-36+18a^2x^2+36a\ln(ax-1)x-36\ln(ax-1)}{3a^3(ax-1)}$
meijerg	$-\frac{-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)}{a^3}+\frac{\frac{2ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+6\ln(-ax+1)}{a^3}-\frac{-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)}{a^3}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+2*x^2/a+8*x/a^2-4/a^3/(a*x-1)+12/a^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{a^4x^4+5\,a^3x^3+18\,a^2x^2-24\,ax+36\,(ax-1)\log\left(ax-1\right)-12}{3\,(a^4x-a^3)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="fricas")
[Out] 1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
- 12)/(a^4*x - a^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{x^3}{3}-\frac{4}{a^4x-a^3}+\frac{2x^2}{a}+\frac{8x}{a^2}+\frac{12\log\left(ax-1\right)}{a^3}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**2,x)
[Out] x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=-\frac{4}{a^4x-a^3}+\frac{a^2x^3+6\,ax^2+24\,x}{3\,a^2}+\frac{12\,\log\left(ax-1\right)}{a^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="maxima")
[Out] -4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{(ax-1)^3\left(\frac{9}{ax-1}+\frac{39}{(ax-1)^2}+1\right)}{3\,a^3}-\frac{12\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^3}-\frac{4}{(ax-1)a^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="giac")
[Out] 1/3*(a*x - 1)^3*(9/(a*x - 1) + 39/(a*x - 1)^2 + 1)/a^3 - 12*log(abs(a*x - 1
)/((a*x - 1)^2*abs(a)))/a^3 - 4/((a*x - 1)*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{12\,\ln\left(a\,x-1\right)}{a^3}-\frac{4}{a\,\left(a^3\,x-a^2\right)}+\frac{8\,x}{a^2}+\frac{x^3}{3}+\frac{2\,x^2}{a}$$

```
[In] int((x^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (12*log(a*x - 1))/a^3 - 4/(a*(a^3*x - a^2)) + (8*x)/a^2 + x^3/3 + (2*x^2)/a
```

3.27 $\int e^{4\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}$$

```
[Out] 4*x/a+1/2*x^2+4/a^2/(-a*x+1)+8*ln(-a*x+1)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6261, 78}

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

```
[In] Int[E^(4*ArcCoth[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{4\text{arctanh}(ax)}x\,dx\\ &= \int \frac{x(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{4}{a}+x+\frac{4}{a(-1+ax)^2}+\frac{8}{a(-1+ax)}\right)dx\\ &= \frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*x,x]
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{x^2}{2}+\frac{4x}{a}-\frac{4}{a^2(ax-1)}+\frac{8\ln(ax-1)}{a^2}$	36
default	$\frac{\frac{1}{2}a\,x^2+4x}{a}-\frac{4}{a^2(ax-1)}+\frac{8\ln(ax-1)}{a^2}$	39
norman	$\frac{\frac{7x^2}{2}+\frac{a\,x^3}{2}-\frac{8x}{a}}{ax-1}+\frac{8\ln(ax-1)}{a^2}$	39
parallelrisch	$\frac{a^3x^3+7a^2x^2+16a\ln(ax-1)x-16ax-16\ln(ax-1)}{2(ax-1)a^2}$	51
meijerg	$\frac{\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)}{a^2}-\frac{2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a^2}+\frac{\frac{ax}{-ax+1}+\ln(-ax+1)}{a^2}$	98

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+4*x/a-4/a^2/(a*x-1)+8/a^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{a^3x^3+7\,a^2x^2-8\,ax+16\,(ax-1)\log{(ax-1)}-8}{2\,(a^3x-a^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="fricas")
[Out] 1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
a^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}-\frac{4}{a^3x-a^2}+\frac{4x}{a}+\frac{8\log{(ax-1)}}{a^2}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x,x)
[Out] x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{ax^2+8\,x}{2\,a} - \frac{4}{a^3x-a^2} + \frac{8\,\log\left(ax-1\right)}{a^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{\frac{(ax-1)^2\left(\frac{10}{ax-1}+1\right)}{a} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{8}{(ax-1)a}}{2\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="giac")
```

```
[Out] 1/2*((a*x - 1)^2*(10/(a*x - 1) + 1)/a - 16*log(abs(a*x - 1)/((a*x - 1)^2*ab
s(a)))/a - 8/((a*x - 1)*a))/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{8\,\ln\left(a\,x-1\right)}{a^2} + \frac{4\,x}{a} + \frac{x^2}{2} + \frac{4}{a\,\left(a-a^2\,x\right)}$$

```
[In] int((x*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (8*log(a*x - 1))/a^2 + (4*x)/a + x^2/2 + 4/(a*(a - a^2*x))
```

3.28 $\int e^{4 \coth^{-1}(ax)} dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int e^{4 \coth^{-1}(ax)} dx = x + \frac{4}{a(1-ax)} + \frac{4\log(1-ax)}{a}$$

```
[Out] x+4/a/(-a*x+1)+4*ln(-a*x+1)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6260, 45}

$$\int e^{4 \coth^{-1}(ax)} dx = \frac{4}{a(1-ax)} + \frac{4\log(1-ax)}{a} + x$$

```
[In] Int[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6260

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{4\text{arctanh}(ax)}\,dx\\ &= \int \frac{(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(1+\frac{4}{(-1+ax)^2}+\frac{4}{-1+ax}\right)dx\\ &= x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\,dx = x-\frac{4}{a(-1+ax)}+\frac{4\log(1-ax)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x]),x]
[Out] x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$x-\frac{4}{a(ax-1)}+\frac{4\ln(ax-1)}{a}$	26
risch	$x-\frac{4}{a(ax-1)}+\frac{4\ln(ax-1)}{a}$	26
norman	$\frac{a\,x^2-5x}{ax-1}+\frac{4\ln(ax-1)}{a}$	30
parallelrisch	$\frac{a^2x^2+4a\ln(ax-1)x-5-4\ln(ax-1)}{(ax-1)a}$	39
meijerg	$-\frac{-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)}{a}+\frac{\frac{2ax}{-ax+1}+2\ln(-ax+1)}{a}+\frac{x}{-ax+1}$	69

```
[In] int(1/(a*x-1)^2*(a*x+1)^2,x,method=_RETURNVERBOSE)
[Out] x-4/a/(a*x-1)+4/a*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int e^{4\coth^{-1}(ax)}\,dx = \frac{a^2x^2 - ax + 4\,(ax-1)\log{(ax-1)} - 4}{a^2x - a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{4\coth^{-1}(ax)}\,dx = x - \frac{4}{a^2x - a} + \frac{4\log{(ax-1)}}{a}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2,x)
[Out] x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\,dx = x + \frac{4\,\log{(ax-1)}}{a} - \frac{4}{a^2x - a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="maxima")
[Out] x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int e^{4\coth^{-1}(ax)}\,dx = \frac{ax-1}{a} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4}{(ax-1)a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="giac")
[Out] (a*x - 1)/a - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4/((a*x - 1)*a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\,dx = x - \frac{4}{a\,(a\,x-1)} + \frac{4\,\ln(a\,x-1)}{a}$$

```
[In] int((a*x + 1)^2/(a*x - 1)^2,x)
[Out] x - 4/(a*(a*x - 1)) + (4*log(a*x - 1))/a
```

3.29 $\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx = \frac{4}{1-ax} + \log(x)$$

```
[Out] 4/(-a*x+1)+ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx = \frac{4}{1-ax} + \log(x)$$

```
[In] Int[E^(4*ArcCoth[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
```

```
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{x} \, dx \\
&= \int \frac{(1+ax)^2}{x(1-ax)^2} \, dx \\
&= \int \left(\frac{1}{x} + \frac{4a}{(-1+ax)^2}\right) \, dx \\
&= \frac{4}{1-ax} + \log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx = \frac{4}{1-ax} + \log(x)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/x,x]
[Out] 4/(1 - a*x) + Log[x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
default	$\ln(x) - \frac{4}{ax-1}$	13
norman	$-\frac{4ax}{ax-1} + \ln(x)$	15
risch	$-\frac{4}{ax-1} + \ln(-x)$	15
parallelrisch	$\frac{a\ln(x)x-4ax-\ln(x)}{ax-1}$	23
meijerg	$\frac{3ax}{-ax+1} + \frac{2ax}{-2ax+2} + 1 + \ln(x) + \ln(-a)$	33

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-4/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \frac{(ax-1)\log(x)-4}{ax-1}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="fricas")
[Out] ((a*x - 1)*log(x) - 4)/(a*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \log(x) - \frac{4}{ax-1}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x,x)
[Out] log(x) - 4/(a*x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = -\frac{4}{ax-1} + \log(x)$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="maxima")
[Out] -4/(a*x - 1) + log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a} + \frac{4}{(ax-1)a}\right)$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="giac")
```

```
[Out] -a*(log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - log(abs(-1/(a*x - 1) - 1))/a
 + 4/((a*x - 1)*a))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \ln(x) - \frac{4}{a\,x-1}$$

```
[In] int((a*x + 1)^2/(x*(a*x - 1)^2),x)
```

```
[Out] log(x) - 4/(a*x - 1)
```

3.30 $\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

```
[Out] -1/x+4*a/(-a*x+1)+4*a*ln(x)-4*a*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = \frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

```
[In] Int[E^(4*ArcCoth[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
```

]

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{x^2}\,dx \\
&= \int \frac{(1+ax)^2}{x^2(1-ax)^2}\,dx \\
&= \int \left(\frac{1}{x^2}+\frac{4a}{x}+\frac{4a^2}{(-1+ax)^2}-\frac{4a^2}{-1+ax}\right)dx \\
&= -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^2,x]
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result
default	$-\frac{1}{x}+4a\ln(x)-\frac{4a}{ax-1}-4a\ln(ax-1)$
risch	$\frac{-5ax+1}{(ax-1)x}+4a\ln(-x)-4a\ln(ax-1)$
norman	$\frac{-5a^2x^2+1}{(ax-1)x}+4a\ln(x)-4a\ln(ax-1)$
parallelrisch	$\frac{4a^2\ln(x)x^2-4a^2\ln(ax-1)x^2-5a^2x^2+1-4a\ln(x)x+4a\ln(ax-1)x}{(ax-1)x}$
meijerg	$\frac{a^2x}{-ax+1}+2a\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)-a\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)\right.$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -\frac{5\,ax+4\,(a^2x^2-ax)\log(ax-1)-4\,(a^2x^2-ax)\log(x)-1}{ax^2-x}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="fricas")
[Out] -(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(a
*x^2 - x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 4a\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-5ax+1}{ax^2-x}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**2,x)
[Out] 4*a*(log(x) - log(x - 1/a)) + (-5*a*x + 1)/(a*x**2 - x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -4\,a\log(ax-1)+4\,a\log(x)-\frac{5\,ax-1}{ax^2-x}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="maxima")
[Out] -4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 4\,a\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a}{ax-1}+\frac{a}{\frac{1}{ax-1}+1}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="giac")
[Out] 4*a*log(abs(-1/(a*x - 1) - 1)) - 4*a/(a*x - 1) + a/(1/(a*x - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 8\,a\,\mathrm{atanh}(2\,a\,x-1)+\frac{5\,a\,x-1}{x-a\,x^2}$$

```
[In] int((a*x + 1)^2/(x^2*(a*x - 1)^2),x)
[Out] 8*a*atanh(2*a*x - 1) + (5*a*x - 1)/(x - a*x^2)
```

3.31 $\int \frac{e^{4\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3} dx = -\frac{1}{2x^2} - \frac{4a}{x} + \frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax)$$

```
[Out] -1/2/x^2-4*a/x+4*a^2/(-a*x+1)+8*a^2*ln(x)-8*a^2*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3} dx = \frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax) - \frac{4a}{x} - \frac{1}{2x^2}$$

```
[In] Int[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{x^3}\,dx \\ &= \int \frac{(1+ax)^2}{x^3(1-ax)^2}\,dx \\ &= \int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{(-1+ax)^2}-\frac{8a^3}{-1+ax}\right)dx \\ &= -\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^3,x]
[Out] -1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{2x^2}-\frac{4a}{x}+8a^2\ln\left(x\right)-\frac{4a^2}{ax-1}-8a^2\ln\left(ax-1\right)$
norman	$\frac{\frac{1}{2}-8a^3x^3+\frac{7}{2}ax}{(ax-1)x^2}+8a^2\ln\left(x\right)-8a^2\ln\left(ax-1\right)$
risch	$\frac{-8a^2x^2+\frac{7}{2}ax+\frac{1}{2}}{x^2(ax-1)}-8a^2\ln\left(ax-1\right)+8a^2\ln\left(-x\right)$
parallelrisch	$\frac{16a^3\ln(x)x^3-16a^3\ln(ax-1)x^3-16a^3x^3-16a^2\ln(x)x^2+16a^2\ln(ax-1)x^2+1+7ax}{2x^2(ax-1)}$
meijerg	$a^2\left(\frac{2ax}{-2ax+2}-\ln\left(-ax+1\right)+1+\ln\left(x\right)+\ln\left(-a\right)\right)-2a^2\left(-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)-1-2\ln$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int\frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx=-\frac{16\,a^2x^2-7\,ax+16\,(a^3x^3-a^2x^2)\log\left(ax-1\right)-16\,(a^3x^3-a^2x^2)\log\left(x\right)-1}{2\,(ax^3-x^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x^
3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int\frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx=8a^2\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-16a^2x^2+7ax+1}{2ax^3-2x^2}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**3,x)
```

```
[Out] 8*a**2*(log(x) - log(x - 1/a)) + (-16*a**2*x**2 + 7*a*x + 1)/(2*a*x**3 - 2*
x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -8\,a^2\log\left(ax-1\right) + 8\,a^2\log\left(x\right) - \frac{16\,a^2x^2 - 7\,ax - 1}{2\,(ax^3 - x^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="maxima")
[Out] -8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 8\,a^2\log\left(\left|-\frac{1}{ax-1} - 1\right|\right) - \frac{4\,a^2}{ax-1} + \frac{9\,a^2 + \frac{10\,a^2}{ax-1}}{2\left(\frac{1}{ax-1} + 1\right)^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="giac")
[Out] 8*a^2*log(abs(-1/(a*x - 1) - 1)) - 4*a^2/(a*x - 1) + 1/2*(9*a^2 + 10*a^2/(a
*x - 1))/(1/(a*x - 1) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 16\,a^2\,\mathrm{atanh}(2\,a\,x-1) + \frac{-8\,a^2\,x^2 + \frac{7\,a\,x}{2} + \frac{1}{2}}{a\,x^3 - x^2}$$

```
[In] int((a*x + 1)^2/(x^3*(a*x - 1)^2),x)
[Out] 16*a^2*atanh(2*a*x - 1) + ((7*a*x)/2 - 8*a^2*x^2 + 1/2)/(a*x^3 - x^2)
```

3.32 $\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx=-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+4*a^3/(-a*x+1)+12*a^3*ln(x)-12*a^3*ln(-a*x+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx=\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

```
[In] Int[E^(4*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{x^4}\,dx\\
&= \int \frac{(1+ax)^2}{x^4(1-ax)^2}\,dx\\
&= \int \left(\frac{1}{x^4}+\frac{4a}{x^3}+\frac{8a^2}{x^2}+\frac{12a^3}{x}+\frac{4a^4}{(-1+ax)^2}-\frac{12a^4}{-1+ax}\right)dx\\
&= -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^4,x]
[Out] -1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+12a^3\ln(x)-\frac{4a^3}{ax-1}-12a^3\ln(ax-1)$
norman	$\frac{\frac{1}{3}-12a^4x^4+\frac{5}{3}ax+6a^2x^2}{(ax-1)x^3}+12a^3\ln(x)-12a^3\ln(ax-1)$
risch	$\frac{-12a^3x^3+6a^2x^2+\frac{5}{3}ax+\frac{1}{3}}{x^3(ax-1)}+12a^3\ln(-x)-12a^3\ln(ax-1)$
parallelrisch	$\frac{36\ln(x)x^4a^4-36\ln(ax-1)x^4a^4-36a^4x^4-36a^3\ln(x)x^3+36a^3\ln(ax-1)x^3+1+18a^2x^2+5ax}{3x^3(ax-1)}$
meijerg	$-a^3\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)+2a^3\left(\frac{4ax}{-4ax+4}-3\ln(-ax+\right.$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{36\,a^3x^3-18\,a^2x^2-5\,ax+36\,(a^4x^4-a^3x^3)\log(ax-1)-36\,(a^4x^4-a^3x^3)\log(x)-1}{3\,(ax^4-x^3)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="fricas")
[Out] -1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
 - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 12a^3\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-36a^3x^3+18a^2x^2+5ax+1}{3ax^4-3x^3}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**4,x)
[Out] 12*a**3*(log(x) - log(x - 1/a)) + (-36*a**3*x**3 + 18*a**2*x**2 + 5*a*x + 1
)/(3*a*x**4 - 3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -12\,a^3\log\left(ax-1\right)+12\,a^3\log\left(x\right)-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax-1}{3\left(ax^4-x^3\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] -12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x
 - 1)/(a*x^4 - x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 12\,a^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^3}{ax-1}+\frac{31\,a^3+\frac{69\,a^3}{ax-1}+\frac{39\,a^3}{(ax-1)^2}}{3\left(\frac{1}{ax-1}+1\right)^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*a^3*log(abs(-1/(a*x - 1) - 1)) - 4*a^3/(a*x - 1) + 1/3*(31*a^3 + 69*a^3/
(a*x - 1) + 39*a^3/(a*x - 1)^2)/(1/(a*x - 1) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 24\,a^3\,\mathrm{atanh}(2\,a\,x-1)+\frac{-12\,a^3\,x^3+6\,a^2\,x^2+\frac{5\,a\,x}{3}+\frac{1}{3}}{a\,x^4-x^3}$$

```
[In] int((a*x + 1)^2/(x^4*(a*x - 1)^2),x)
```

```
[Out] 24*a^3*atanh(2*a*x - 1) + ((5*a*x)/3 + 6*a^2*x^2 - 12*a^3*x^3 + 1/3)/(a*x^4
 - x^3)
```

3.33 $\int e^{-\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] 3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2*
(1-1/a^2/x^2)^(1/2)/a^2-1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6304, 849, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}$$

```
[In] Int[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2
) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}-\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^4}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a^4}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^2}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-16+9ax-8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

```
[In] Integrate[x^3/E^ArcCoth[a*x],x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3-8a^2x^2+9ax-16)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{24a^4}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8a^3\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\Big(6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-8((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-24\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}-15\ln\Big(\frac{a^2x+\sqrt{a^2x^2}}{\sqrt{a^2}}}{24\sqrt{(ax-1)(ax+1)}\,a^4\sqrt{a^2}}$

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*a^3*x^3-8*a^2*x^2+9*a*x-16)*(a*x+1)/a^4*((a*x-1)/(a*x+1))^(1/2)+3/8
/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^
(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \frac{(6\,a^4x^4-2\,a^3x^3+a^2x^2-7\,ax-16)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 7*a*x - 16)*sqrt((a*x - 1)/(a*x +
1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1))/a^4
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=\int x^3\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(94) = 188$.

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.78

$$\int e^{-\coth^{-1}(ax)}x^3\,dx =$$
$$-\frac{1}{24}\,a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-31\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^5}\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*a*(2*(39*((a*x - 1)/(a*x + 1))^(7/2) - 31*((a*x - 1)/(a*x + 1))^(5/2)
 + 49*((a*x - 1)/(a*x + 1))^(3/2) - 9*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x -
1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^5 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.51

$$\int e^{-\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}-\frac{\frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{49\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{12}-\frac{13\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}$$

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((3*((a*x - 1)/(a*x + 1))^
(1/2))/4 - (49*((a*x - 1)/(a*x + 1))^(3/2))/12 + (31*((a*x - 1)/(a*x + 1))^
(5/2))/12 - (13*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^2)/
(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)
^4 - (4*a^4*(a*x - 1))/(a*x + 1))
```

3.34 $\int e^{-\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] -1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2-1/2*x^2
*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6304, 849, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}x^2\,dx$$
$$=-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[In] Int[x^2/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{4}{a^2}-\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(4-3ax+2a^2x^2) - 3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

```
[In] Integrate[x^2/E^ArcCoth[a*x],x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 - 3*a*x + 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(2a^2x^2-3ax+4)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{6a^3} - \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2a^2\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+6a\ln\left(\frac{a^2x+\cdots}{\cdots}\right.\right.}{6\sqrt{(ax-1)(ax+1)}\,a^3\sqrt{a^2}}$

```
[In] int(x^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*a^2*x^2-3*a*x+4)*(a*x+1)/a^3*((a*x-1)/(a*x+1))^(1/2)-1/2/a^2*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-
1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{(2\,a^3x^3 - a^2x^2 + ax + 4)\sqrt{\frac{ax-1}{ax+1}} - 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) + 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{6\,a^3}$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - a^2*x^2 + a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt
((a*x - 1)/(a*x + 1)) + 1) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \int x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(74) = 148.

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.84

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1} - \frac{3\,(ax-1)^2a^4}{(ax+1)^2} + \frac{(ax-1)^3a^4}{(ax+1)^3} - a^4} + \frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^4} - \frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{a^4}\right)$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(9*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
3*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4
/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^4 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{1}{6}\sqrt{a^2x^2-1}\left(x\left(\frac{2\,x\mathrm{sgn}(ax+1)}{a}-\frac{3\,\mathrm{sgn}(ax+1)}{a^2}\right)+\frac{4\,\mathrm{sgn}(ax+1)}{a^3}\right) + \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a^2|a|}$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*(x*(2*x*sgn(a*x + 1)/a - 3*sgn(a*x + 1)/a^2) + 4*sgn(
a*x + 1)/a^3) + 1/2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a
^2*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + 3*((a*x
- 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x -
 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - atanh(((a*x - 1)/(a*x +
 1))^(1/2))/a^3
```

3.35 $\int e^{-\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int e^{-\coth^{-1}(ax)}x\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a
^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6304, 849, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}$$

```
[In] Int[x/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/a) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}
\end{aligned}
$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int e^{-\coth^{-1}(ax)}x\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-2+ax) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

```
[In] Integrate[x/E^ArcCoth[a*x],x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])
/(2*a^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.56

method	result	siz
risch	$\frac{(ax-2)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2a\sqrt{a^2}\,(ax-1)}$	100
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-2\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+2a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)\right)}{2\sqrt{(ax-1)(ax+1)}\,a^2\sqrt{a^2}}$	152

```
[In] int(x*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x-2)*(a*x+1)/a^2*((a*x-1)/(a*x+1))^(1/2)+1/2/a*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1
/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.03

$$\int e^{-\coth^{-1}(ax)}x\,dx$$
$$=-\frac{1}{2}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*((a*x - 1)/(a*x + 1))^(3/2) - sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^3 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int e^{-\coth^{-1}(ax)}x\,dx = \frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x\mathrm{sgn}(ax+1)}{a}-\frac{2\,\mathrm{sgn}(ax+1)}{a^2}\right) - \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a|a|}$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*(x*sgn(a*x + 1)/a - 2*sgn(a*x + 1)/a^2) - 1/2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int e^{-\coth^{-1}(ax)}x\,dx = \frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

```
[In] int(x*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] atanh(((a*x - 1)/(a*x + 1))^(1/2))/a^2 - (((a*x - 1)/(a*x + 1))^(1/2) - 3*(
(a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(
a*x - 1))/(a*x + 1))
```

3.36 $\int e^{-\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int e^{-\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6303, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\,dx=x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[In] Int[E^(-ArcCoth[a*x]),x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a} \\
&= \sqrt{1-\frac{1}{a^2x^2}}x - a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) \\
&= \sqrt{1-\frac{1}{a^2x^2}}x - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}\,dx = \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

```
[In] Integrate[E^(-ArcCoth[a*x]),x]
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(33) = 66.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.46

method	result	size
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a} - \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}\,(ax-1)}$	91
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)-\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	99

```
[In] int(((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x+1)*((a*x-1)/(a*x+1))^(1/2)-ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))
/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\,dx = \int \sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int e^{-\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{a}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2
- 1)*sgn(a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.37 $\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

```
[Out] arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6304, 858, 222, 272, 65, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) + \csc^{-1}(ax)$$

```
[In] Int[1/(E^ArcCoth[a*x]*x),x]
```

```
[Out] ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\
&= \csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} dx = \arcsin\left(\frac{1}{ax}\right) + \log\left(x\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*x),x]
[Out] ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 6.50

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}+a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)-\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	130

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^
2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)
)/(a^2)^(1/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1))^(1/2)
/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} dx = -2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
[Out] -2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) - \frac{a\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] -2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - a*log(abs(-x*abs(a)
 + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x,x)
[Out] 2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.38 $\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

```
[Out] -a*arccsc(a*x)-a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6304, 655, 222}

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\csc^{-1}(ax)\right) - a\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*x^2),x]
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\ &= -a\sqrt{1-\frac{1}{a^2x^2}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\ &= -a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -a\left(\sqrt{1-\frac{1}{a^2x^2}} + \arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^2),x]
[Out] -(a*(Sqrt[1 - 1/(a^2*x^2)] + ArcSin[1/(a*x)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(23) = 46.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

method	result
risch	$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}-\frac{a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,x\sqrt{a^2}}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(a*x+1)/x*((a*x-1)/(a*x+1))^(1/2)-a*arctan(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(
a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} - \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) - arctan(sqrt((a*
x - 1)/(a*x + 1))))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2} \, dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (2*a*((a*x - 1)/(a*x + 1))^(1/2))/(
(a*x - 1)/(a*x + 1) + 1)
```

3.39 $\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] 1/2*a^2*arccsc(a*x)+1/2*a*(2*a-1/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6304, 794, 222}

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[In] Int[1/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a - x^(-1)))/2 + (a^2*ArcCsc[a*x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = \frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+2ax)+ax\arcsin\left(\frac{1}{ax}\right)\right)}{2x}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^3),x]
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + 2*a*x) + a*x*ArcSin[1/(a*x)]))/(2*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(34) = 68.

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

method	result
risch	$\frac{(ax+1)(2ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^2}+\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2ax-2}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+2\ln\left(\frac{a^2x+\sqrt{}}{}\right.\right.}{2\sqrt{(ax-1)(ax+1)}\,x^2\sqrt{}}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x+1)*(2*a*x-1)/x^2*((a*x-1)/(a*x+1))^(1/2)+1/2*a^2*arctan(1/(a^2*x^2
-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (2\,a^2x^2 + ax - 1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^2*x^2 + a*x - 1)*s
qrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.32

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(a*arctan(sqrt((a*x - 1)/(a*x + 1))) - (3*a*((a*x - 1)/(a*x + 1))^(3/2) +
a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)
^2 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -a^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1) + \frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3 a^2\mathrm{sgn}(ax+1) + 2\left(x|a|-\sqrt{a^2x^2-1}\right)^2 a|a|\mathrm{sgn}(ax+1) - \left(x|a|-\sqrt{a^2x^2-1}\right)a^2\mathrm{sg}}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + ((x*abs(a) - sqrt
(a^2*x^2 - 1))^3*a^2*sgn(a*x + 1) + 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*ab
s(a)*sgn(a*x + 1) - (x*abs(a) - sqrt(a^2*x^2 - 1))*a^2*sgn(a*x + 1) + 2*a*a
bs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = a^2\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2} - a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^3,x)
```

```
[Out] a^2*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) - a^2
*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x)
```

3.40 $\int \frac{e^{-\coth^{-1}(ax)}}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4} dx = -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)$$

```
[Out] 1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)-a^3*(1-1/a^2/x^2)^(1/2)+1/2
*a^2*(1-1/a^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6304, 811, 655, 201, 222}

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4} dx = -\frac{1}{2}a^3\csc^{-1}(ax) + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} - a^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[
1 - 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(a^2\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + a^2\text{Subst}\left(\int \left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\\
&\quad - a^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + a^2\text{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\\
&\quad - a^3\csc^{-1}(ax) + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}(2-3ax+4a^2x^2)}{6x^2} - \frac{1}{2}a^3\arcsin\left(\frac{1}{ax}\right)$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -1/6*(a*Sqrt[1 - 1/(a^2*x^2)]*(2 - 3*a*x + 4*a^2*x^2))/x^2 - (a^3*ArcSin[1/
(a*x)])/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(ax+1)\left(4a^2x^2-3ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6x^3} - \frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-3a^3x^3\sqrt{a^2}\right.}{6\sqrt{(ax-1)(a}}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x+1)*(4*a^2*x^2-3*a*x+2)/x^3*((a*x-1)/(a*x+1))^(1/2)-1/2*a^3*arctan
(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \left(4\,a^3x^3+a^2x^2-ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (4*a^3*x^3 + a^2*x^2 - a
*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(64) = 128$.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.80

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (9*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4} \, dx = a^3 \operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3} - \frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} - \frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x} + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^4,x)
```

```
[Out] a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) - ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3)
 - (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 - (a^2*((a*x - 1)/(a*x + 1))^(1/2)
)/(6*x) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

3.41 $\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx = \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3}{8}a^4\csc^{-1}(ax)$$

```
[Out] 3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a-9/x)*(1-1/a^2/x^2)^(1/2)-1/4*a*(1-1/a^2/
x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6304, 847, 794, 222}

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx = \frac{3}{8}a^4\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)$$

```
[In] Int[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a - 9/x))/24 - (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + (3*a^4*ArcCsc[a*x])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{4}a^2\text{Subst}\left(\int \frac{x^2\left(\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{12}a^4\text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}-\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}\\
&\quad+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3}{8}a^4\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(-6+8ax-9a^2x^2+16a^3x^3)}{x^3}+9a^3\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^3 + 9*a
^3*ArcSin[1/(a*x)]))/24
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(ax+1)(16a^3x^3-9a^2x^2+8ax-6)\sqrt{\frac{ax-1}{ax+1}}}{24x^4}+\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4-9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+24\ln\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(a*x+1)*(16*a^3*x^3-9*a^2*x^2+8*a*x-6)/x^4*((a*x-1)/(a*x+1))^(1/2)+3/8
*a^4*arctan(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(16\,a^4x^4+7\,a^3x^3-a^2x^2+2\,ax-6)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (16*a^4*x^4 + 7*a^3*x
^3 - a^2*x^2 + 2*a*x - 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^5} dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**5,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(74) = 148.

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.97

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx = -\frac{1}{12}\left(9 a^3 \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{39 a^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 31 a^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 49 a^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 9 a^3 \sqrt{\frac{ax-1}{ax+1}}}{\frac{4(ax-1)}{ax+1} + \frac{6(ax-1)^2}{(ax+1)^2} + \frac{4(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right) a$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (39*a^3*((a*x - 1)/(a*x +
1))^(7/2) + 31*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 49*a^3*((a*x - 1)/(a*x + 1
))^(3/2) + 9*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
 - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.93

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx = -\frac{3}{4} a^4 \arctan\left(-x|a| + \sqrt{a^2x^2-1}\right) \operatorname{sgn}(ax+1) + \frac{9\left(x|a| - \sqrt{a^2x^2-1}\right)^7 a^4 \operatorname{sgn}(ax+1) + 33\left(x|a| - \sqrt{a^2x^2-1}\right)^5 a^4 \operatorname{sgn}(ax+1) + 48\left(x|a| - \sqrt{a^2x^2-1}\right)^4 a$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + 1/12*(9*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^7*a^4*sgn(a*x + 1) + 33*(x*abs(a) - sqrt(a^2*x^2
- 1))^5*a^4*sgn(a*x + 1) + 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a)*s
gn(a*x + 1) - 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*a^4*sgn(a*x + 1) + 64*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a)*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(
a^2*x^2 - 1))*a^4*sgn(a*x + 1) + 16*a^3*abs(a)*sgn(a*x + 1))/((x*abs(a) - s
qrt(a^2*x^2 - 1))^2 + 1)^4
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.47

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4} - \frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4} - \frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2} + \frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x} + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^5,x)
```

```
[Out] (2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 - ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4)
 - (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - (a^2*((a*x - 1)/(a*x + 1))
^(1/2))/(24*x^2) + (7*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (a*((a*x -
1)/(a*x + 1))^(1/2))/(12*x^3)
```

3.42 $\int e^{-2\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = -\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

```
[Out] -2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{2\log(ax+1)}{a^4}-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

```
[In] Int[x^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\operatorname{arctanh}(ax)}x^3\,dx\\
&= -\int \frac{x^3(1-ax)}{1+ax}\,dx\\
&= -\int \left(\frac{2}{a^3}-\frac{2x}{a^2}+\frac{2x^2}{a}-x^3-\frac{2}{a^3(1+ax)}\right)dx\\
&= -\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = -\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

```
[In] Integrate[x^3/E^(2*ArcCoth[a*x]),x]
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

method	result	size
norman	$-\frac{2x}{a^{3}}+\frac{x^{2}}{a^{2}}-\frac{2x^{3}}{3a}+\frac{x^{4}}{4}+\frac{2\ln(ax+1)}{a^{4}}$	39
risch	$-\frac{2x}{a^{3}}+\frac{x^{2}}{a^{2}}-\frac{2x^{3}}{3a}+\frac{x^{4}}{4}+\frac{2\ln(ax+1)}{a^{4}}$	39
default	$\frac{2\ln(ax+1)}{a^{4}}+\frac{\frac{1}{4}a^{3}x^{4}-\frac{2}{3}a^{2}x^{3}+a\,x^{2}-2x}{a^{3}}$	42
parallelrisch	$\frac{3a^{4}x^{4}-8a^{3}x^{3}+12a^{2}x^{2}-24ax+24\ln(ax+1)}{12a^{4}}$	43
meijerg	$\frac{-\frac{ax\left(-15a^{3}x^{3}+20a^{2}x^{2}-30ax+60\right)}{60}+\ln(ax+1)}{a^{4}}-\frac{\frac{ax\left(4a^{2}x^{2}-6ax+12\right)}{12}-\ln(ax+1)}{a^{4}}$	71

[In] int(x^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] -2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^{3}\,dx=\frac{3\,a^{4}x^{4}-8\,a^{3}x^{3}+12\,a^{2}x^{2}-24\,ax+24\,\log\left(ax+1\right)}{12\,a^{4}}$$

[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}x^{3}\,dx=\frac{x^{4}}{4}-\frac{2x^{3}}{3a}+\frac{x^{2}}{a^{2}}-\frac{2x}{a^{3}}+\frac{2\log\left(ax+1\right)}{a^{4}}$$

[In] integrate(x**3*(a*x-1)/(a*x+1),x)

[Out] x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 + 2*log(a*x + 1)/a**4

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^3x^4 - 8\,a^2x^3 + 12\,ax^2 - 24\,x}{12\,a^3} + \frac{2\,\log\left(ax+1\right)}{a^4}$$

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 + 2*log(a*x + 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax+1|\right)}{a^4}$$

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x)/a^4 + 2*log(abs(a*x + 1)
)/a^4
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{2\,\ln\left(a\,x+1\right)}{a^4} - \frac{2\,x}{a^3} + \frac{x^4}{4} - \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

```
[In] int((x^3*(a*x - 1))/(a*x + 1),x)
[Out] (2*log(a*x + 1))/a^4 - (2*x)/a^3 + x^4/4 - (2*x^3)/(3*a) + x^2/a^2
```

3.43 $\int e^{-2\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

```
[Out] 2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=-\frac{2\log(ax+1)}{a^3}+\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}$$

```
[In] Int[x^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}x^2\,dx \\
&= -\int \frac{x^2(1-ax)}{1+ax}\,dx \\
&= -\int \left(-\frac{2}{a^2}+\frac{2x}{a}-x^2+\frac{2}{a^2(1+ax)}\right)dx \\
&= \frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

```
[In] Integrate[x^2/E^(2*ArcCoth[a*x]),x]
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{2x}{a^2} - \frac{x^2}{a} + \frac{x^3}{3} - \frac{2\ln(ax+1)}{a^3}$	32
risch	$\frac{2x}{a^2} - \frac{x^2}{a} + \frac{x^3}{3} - \frac{2\ln(ax+1)}{a^3}$	32
default	$-\frac{2\ln(ax+1)}{a^3} + \frac{\frac{1}{3}a^2x^3 - a\,x^2 + 2x}{a^2}$	35
parallelrisch	$-\frac{-a^3x^3+3a^2x^2-6ax+6\ln(ax+1)}{3a^3}$	35
meijerg	$\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^3} - \frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^3}$	55

```
[In] int(x^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] 2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 - 3\,a^2x^2 + 6\,ax - 6\,\log\left(ax+1\right)}{3\,a^3}$$

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} - \frac{x^2}{a} + \frac{2x}{a^2} - \frac{2\log\left(ax+1\right)}{a^3}$$

```
[In] integrate(x**2*(a*x-1)/(a*x+1),x)
[Out] x**3/3 - x**2/a + 2*x/a**2 - 2*log(a*x + 1)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^2x^3 - 3\,ax^2 + 6\,x}{3\,a^2} - \frac{2\,\log\left(ax+1\right)}{a^3}$$

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 - 2*log(a*x + 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 - 3\,a^2x^2 + 6\,ax}{3\,a^3} - \frac{2\,\log\left(|ax+1|\right)}{a^3}$$

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x + 1))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{2\,x}{a^2} - \frac{2\,\ln\left(a\,x+1\right)}{a^3} + \frac{x^3}{3} - \frac{x^2}{a}$$

```
[In] int((x^2*(a*x - 1))/(a*x + 1),x)
[Out] (2*x)/a^2 - (2*log(a*x + 1))/a^3 + x^3/3 - x^2/a
```

3.44 $\int e^{-2\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int e^{-2\coth^{-1}(ax)}x\,dx=-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}$$

```
[Out] -2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6261, 78}

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

```
[In] Int[x/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)} x\,dx \\ &= -\int \frac{x(1-ax)}{1+ax}\,dx \\ &= -\int \left(\frac{2}{a} - x - \frac{2}{a(1+ax)}\right) dx \\ &= -\frac{2x}{a} + \frac{x^2}{2} + \frac{2\log(1+ax)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)} x\,dx = -\frac{2x}{a} + \frac{x^2}{2} + \frac{2\log(1+ax)}{a^2}$$

```
[In] Integrate[x/E^(2*ArcCoth[a*x]),x]
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
norman	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
risch	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
parallelrisch	$\frac{a^2x^2-4ax+4\ln(ax+1)}{2a^2}$	26
default	$\frac{2\ln(ax+1)}{a^2}+\frac{\frac{1}{2}a\,x^2-2x}{a}$	27
meijerg	$\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^2}-\frac{ax-\ln(ax+1)}{a^2}$	40

```
[In] int(x*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2-4\,ax+4\,\log(ax+1)}{2\,a^2}$$

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}-\frac{2x}{a}+\frac{2\log(ax+1)}{a^2}$$

```
[In] integrate(x*(a*x-1)/(a*x+1),x)
[Out] x**2/2 - 2*x/a + 2*log(a*x + 1)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{ax^2 - 4\,x}{2\,a} + \frac{2\,\log\left(ax + 1\right)}{a^2}$$

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/2*(a*x^2 - 4*x)/a + 2*log(a*x + 1)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{a^2x^2 - 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax + 1|\right)}{a^2}$$

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/2*(a^2*x^2 - 4*a*x)/a^2 + 2*log(abs(a*x + 1))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{2\,\ln\left(a\,x + 1\right)}{a^2} - \frac{2\,x}{a} + \frac{x^2}{2}$$

```
[In] int((x*(a*x - 1))/(a*x + 1),x)
[Out] (2*log(a*x + 1))/a^2 - (2*x)/a + x^2/2
```

3.45 $\int e^{-2\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(1+ax)}{a}$$

```
[Out] x-2*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6302, 6260, 45}

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(ax+1)}{a}$$

```
[In] Int[E^(-2*ArcCoth[a*x]),x]
```

```
[Out] x - (2*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6260

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\,dx\\ &= -\int \frac{1-ax}{1+ax}\,dx\\ &= -\int \left(-1+\frac{2}{1+ax}\right)dx\\ &= x-\frac{2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x-\frac{2\log(1+ax)}{a}$$

```
[In] Integrate[E^(-2*ArcCoth[a*x]),x]
[Out] x - (2*Log[1 + a*x])/a
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$x-\frac{2\ln(ax+1)}{a}$	14
norman	$x-\frac{2\ln(ax+1)}{a}$	14
risch	$x-\frac{2\ln(ax+1)}{a}$	14
parallelrisch	$-\frac{-ax+2\ln(ax+1)}{a}$	19
meijerg	$\frac{ax-\ln(ax+1)}{a}-\frac{\ln(ax+1)}{a}$	29

```
[In] int((a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] x-2*ln(a*x+1)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int e^{-2\coth^{-1}(ax)}\,dx = \frac{ax - 2\,\log(ax+1)}{a}$$

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a*x - 2*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(ax+1)}{a}$$

```
[In] integrate((a*x-1)/(a*x+1),x)
[Out] x - 2*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\log(ax+1)}{a}$$

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] x - 2*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\log(|ax+1|)}{a}$$

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="giac")
[Out] x - 2*log(abs(a*x + 1))/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\ln(a\,x+1)}{a}$$

```
[In] int((a*x - 1)/(a*x + 1),x)
[Out] x - (2*log(a*x + 1))/a
```

3.46 $\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log(1+ax)$$

```
[Out] -ln(x)+2*ln(a*x+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\log(ax+1) - \log(x)$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{x}\,dx\\ &= -\int \frac{1-ax}{x(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x}-\frac{2a}{1+ax}\right)dx\\ &= -\log(x)+2\log(1+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\log(x)+2\log(1+ax)$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x),x]
[Out] -Log[x] + 2*Log[1 + a*x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\ln(x)+2\ln(ax+1)$	14
norman	$-\ln(x)+2\ln(ax+1)$	14
parallelrisch	$-\ln(x)+2\ln(ax+1)$	14
risch	$2\ln(-ax-1)-\ln(x)$	15
meijerg	$2\ln(ax+1)-\ln(x)-\ln(a)$	18

```
[In] int((a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+2*ln(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} dx = 2 \log(ax+1) - \log(x)$$

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="fricas")
[Out] 2*log(a*x + 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} dx = -\log(x) + 2\log\left(x + \frac{1}{a}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/x,x)
[Out] -log(x) + 2*log(x + 1/a)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} dx = 2 \log(ax+1) - \log(x)$$

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] 2*log(a*x + 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} dx = 2 \log(|ax+1|) - \log(|x|)$$

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] 2*log(abs(a*x + 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} dx = 2 \ln(-3\,a\,x-3) - \ln(x)$$

```
[In] int((a*x - 1)/(x*(a*x + 1)),x)
[Out] 2*log(- 3*a*x - 3) - log(x)
```

3.47 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx = \frac{1}{x} + 2a\log(x) - 2a\log(1 + ax)$$

```
[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx = 2a\log(x) - 2a\log(ax + 1) + \frac{1}{x}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{x^2}\,dx\\ &= -\int \frac{1-ax}{x^2(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x^2}-\frac{2a}{x}+\frac{2a^2}{1+ax}\right)dx\\ &= \frac{1}{x}+2a\log(x)-2a\log(1+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x}+2a\log(x)-2a\log(1+ax)$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^2),x]
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{1}{x}+2a\ln\left(x\right)-2a\ln\left(ax+1\right)$	19
norman	$\frac{1}{x}+2a\ln\left(x\right)-2a\ln\left(ax+1\right)$	19
risch	$\frac{1}{x}+2a\ln\left(-x\right)-2a\ln\left(ax+1\right)$	21
parallelrisch	$\frac{2a\ln(x)x-2a\ln(ax+1)x+1}{x}$	23
meijerg	$a(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right))-a\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{ax}\right)$	43

```
[In] int((a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -\frac{2\,ax\log\left(ax+1\right) - 2\,ax\log\left(x\right) - 1}{x}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
[Out] -(2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = 2a\left(\log\left(x\right) - \log\left(x + \frac{1}{a}\right)\right) + \frac{1}{x}$$

```
[In] integrate((a*x-1)/(a*x+1)/x**2,x)
[Out] 2*a*(log(x) - log(x + 1/a)) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\log\left(ax+1\right) + 2\,a\log\left(x\right) + \frac{1}{x}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
[Out] -2*a*log(a*x + 1) + 2*a*log(x) + 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\log\left(|ax+1|\right) + 2\,a\log\left(|x|\right) + \frac{1}{x}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] -2*a*log(abs(a*x + 1)) + 2*a*log(abs(x)) + 1/x
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 4\,a\,\mathrm{atanh}(2\,a\,x+1)$$

```
[In] int((a*x - 1)/(x^2*(a*x + 1)),x)
[Out] 1/x - 4*a*atanh(2*a*x + 1)
```

3.48 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)$$

```
[Out] 1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx=-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{x^3}\,dx \\
&= -\int \frac{1-ax}{x^3(1+ax)}\,dx \\
&= -\int \left(\frac{1}{x^3}-\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{1+ax}\right)dx \\
&= \frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^3),x]
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	siz
norman	$\frac{\frac{1}{2}-2ax}{x^2}-2a^2\ln(x)+2a^2\ln(ax+1)$	30
default	$\frac{1}{2x^2}-\frac{2a}{x}-2a^2\ln(x)+2a^2\ln(ax+1)$	31
risch	$\frac{\frac{1}{2}-2ax}{x^2}+2a^2\ln(-ax-1)-2a^2\ln(x)$	31
parallelrisch	$-\frac{4a^2\ln(x)x^2-4a^2\ln(ax+1)x^2-1+4ax}{2x^2}$	36
meijerg	$a^2\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)-a^2\left(-\ln(ax+1)+\ln(x)+\ln(a)-\frac{1}{2a^2x^2}+\frac{1}{ax}\right)$	62

```
[In] int((a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
[Out] (1/2-2*a*x)/x^2-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = \frac{4\,a^2x^2\log(ax+1)-4\,a^2x^2\log(x)-4\,ax+1}{2\,x^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
[Out] 1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2a^2\left(-\log(x)+\log\left(x+\frac{1}{a}\right)\right)+\frac{-4ax+1}{2x^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/x**3,x)
[Out] 2*a**2*(-log(x) + log(x + 1/a)) + (-4*a*x + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(ax+1\right) - 2\,a^2\log\left(x\right) - \frac{4\,ax-1}{2\,x^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
[Out] 2*a^2*log(a*x + 1) - 2*a^2*log(x) - 1/2*(4*a*x - 1)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(|ax+1|\right) - 2\,a^2\log\left(|x|\right) - \frac{4\,ax-1}{2\,x^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
[Out] 2*a^2*log(abs(a*x + 1)) - 2*a^2*log(abs(x)) - 1/2*(4*a*x - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 4\,a^2\,\mathrm{atanh}(2\,a\,x+1) - \frac{2\,a\,x-\frac{1}{2}}{x^2}$$

```
[In] int((a*x - 1)/(x^3*(a*x + 1)),x)
[Out] 4*a^2*atanh(2*a*x + 1) - (2*a*x - 1/2)/x^2
```

3.49 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4} dx = \frac{1}{3x^3} - \frac{a}{x^2} + \frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1+ax)$$

```
[Out] 1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4} dx = 2a^3\log(x) - 2a^3\log(ax+1) + \frac{2a^2}{x} - \frac{a}{x^2} + \frac{1}{3x^3}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{x^4}\,dx \\
&= -\int \frac{1-ax}{x^4(1+ax)}\,dx \\
&= -\int \left(\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^2}{x^2}-\frac{2a^3}{x}+\frac{2a^4}{1+ax}\right)dx \\
&= \frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^4),x]
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln\left(x\right)-2a^3\ln\left(ax+1\right)$
default	$\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\ln\left(x\right)-2a^3\ln\left(ax+1\right)$
risch	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln\left(-x\right)-2a^3\ln\left(ax+1\right)$
parallelrisch	$\frac{6a^3\ln(x)x^3-6a^3\ln(ax+1)x^3+1+6a^2x^2-3ax}{3x^3}$
meijerg	$a^3\left(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right)-\frac{1}{2a^2x^2}+\frac{1}{ax}\right)-a^3\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{3x^3a^3}+\right.$

```
[In] int((a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
[Out] (1/3+2*a^2*x^2-a*x)/x^3+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -\frac{6\,a^3x^3\log\left(ax+1\right)-6\,a^3x^3\log\left(x\right)-6\,a^2x^2+3\,ax-1}{3\,x^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
[Out] -1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x^
3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = 2a^3\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{6a^2x^2-3ax+1}{3x^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/x**4,x)
[Out] 2*a**3*(log(x) - log(x + 1/a)) + (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -2\,a^3\log\left(ax+1\right) + 2\,a^3\log\left(x\right) + \frac{6\,a^2x^2 - 3\,ax + 1}{3\,x^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
[Out] -2*a^3*log(a*x + 1) + 2*a^3*log(x) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -2\,a^3\log\left(|ax+1|\right) + 2\,a^3\log\left(|x|\right) + \frac{6\,a^2x^2 - 3\,ax + 1}{3\,x^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
[Out] -2*a^3*log(abs(a*x + 1)) + 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/
x^3
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^2\,x^2 - a\,x + \frac{1}{3}}{x^3} - 4\,a^3\,\mathrm{atanh}(2\,a\,x + 1)$$

```
[In] int((a*x - 1)/(x^4*(a*x + 1)),x)
[Out] (2*a^2*x^2 - a*x + 1/3)/x^3 - 4*a^3*atanh(2*a*x + 1)
```

3.50 $\int e^{-3\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 136

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}-\frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{51\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] 51/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-4*(1-1/a^2/x^2)^(1/2)/a^3/(a+1/x)-6*x
*(1-1/a^2/x^2)^(1/2)/a^3+19/8*x^2*(1-1/a^2/x^2)^(1/2)/a^2-x^3*(1-1/a^2/x^2)
^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6304, 6874, 272, 44, 65, 214, 277, 270, 665}

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{51\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}$$

```
[In] Int[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x)
/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

```
/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8
*a^4)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^5\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^4\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^4x\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^4(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{3\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{4\sqrt{1-\frac{1}{a^2x^2}}x}{a^3} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{a}$$
$$-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^4}$$
$$+\frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^2}$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}x}{a^3} + \frac{2\sqrt{1-\frac{1}{a^2x^2}}x^2}{a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{a}$$
$$+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^4}$$
$$-\frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{8a^2} + \frac{4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2}$$
$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^4}$$
$$-\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a^4} + \frac{2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{a}$$
$$+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{6\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^4} + \frac{3\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^2}$$
$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2}$$
$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{51\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-80-29ax+11a^2x^2-6a^3x^3+2a^4x^4)}{1+ax}+51\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{8a^4}$$

```
[In] Integrate[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-80 - 29*a*x + 11*a^2*x^2 - 6*a^3*x^3 + 2*a^4*
x^4))/(1 + a*x) + 51*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(8*a^4)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(2a^3x^3-8a^2x^2+19ax-48)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8a^4}+\frac{\left(\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{8a^3\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^5\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\Big(2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+21\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-8\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+42\sqrt{a}$

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*a^3*x^3-8*a^2*x^2+19*a*x-48)*(a*x+1)/a^4*((a*x-1)/(a*x+1))^(1/2)+(51
/8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^5/(x+1/a)*(a
^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(
a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{(2\,a^4x^4-6\,a^3x^3+11\,a^2x^2-29\,ax-80)\sqrt{\frac{ax-1}{ax+1}}+51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{8\,a^4}$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^4*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt((a*x - 1)/(a*x
 + 1)) + 51*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.64

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{1}{8}a\left(\frac{2\left(77\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-149\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-35\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{51\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{51\log\left(\sqrt{\cdots}\right.}{a^5}\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] -1/8*a*(2*(77*((a*x - 1)/(a*x + 1))^(7/2) - 149*((a*x - 1)/(a*x + 1))^(5/2)
 + 123*((a*x - 1)/(a*x + 1))^(3/2) - 35*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x
- 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x
 + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 51*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^5 + 51*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5 + 32*sqrt((a*x -
1)/(a*x + 1))/a^5)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.41

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \frac{51\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4} - \frac{\frac{35\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{123\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{149\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}-\frac{77\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}} - \frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^4}$$

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (51*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((35*((a*x - 1)/(a*x + 1)
)^(1/2))/4 - (123*((a*x - 1)/(a*x + 1))^(3/2))/4 + (149*((a*x - 1)/(a*x + 1
))^(5/2))/4 - (77*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^2
)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x +
1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (4*((a*x - 1)/(a*x + 1))^(1/2))/a^4
```

3.51 $\int e^{-3\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] -11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+4*(1-1/a^2/x^2)^(1/2)/a^2/(a+1/x)+14
/3*x*(1-1/a^2/x^2)^(1/2)/a^2-3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2
/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6304, 6874, 277, 270, 272, 44, 65, 214, 665}

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[In] Int[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x)
/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^3
)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^3(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right) dx, x, \frac{1}{x}\right)$$

$$= \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{3\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^3} - \frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3a^2} + \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3} - \frac{4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3} - \frac{3\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(52+19ax-7a^2x^2+2a^3x^3)}{1+ax} - 33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

```
[In] Integrate[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*x
) - 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
risch	$\frac{(2a^2x^2-9ax+28)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{6a^3} + \frac{\left(-\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a^2\sqrt{a^2}} + \frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-1)(ax-\right.}{}$

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*a^2*x^2-9*a*x+28)*(a*x+1)/a^3*((a*x-1)/(a*x+1))^(1/2)+(-11/2/a^2*ln(
a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+4/a^4/(x+1/a)*(a^2*(x+1/a)
^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \frac{(2\,a^3x^3-7\,a^2x^2+19\,ax+52)\sqrt{\frac{ax-1}{ax+1}}-33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] 1/6*((2*a^3*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt((a*x - 1)/(a*x + 1)) - 33*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
a^3
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.60

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-52\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}-\frac{24\sqrt{\ldots}}{a\ldots}$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] -1/6*a*(2*(39*((a*x - 1)/(a*x + 1))^(5/2) - 52*((a*x - 1)/(a*x + 1))^(3/2)
+ 21*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*
a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^4 - 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4 - 24*sqrt
((a*x - 1)/(a*x + 1))/a^4)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.34

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \frac{7\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{52\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+13\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^3}-\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (7*((a*x - 1)/(a*x + 1))^(1/2) - (52*((a*x - 1)/(a*x + 1))^(3/2))/3 + 13*((
a*x - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a
*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (4*((a*x - 1)/(a*x
+ 1))^(1/2))/a^3 - (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3
```

3.52 $\int e^{-3\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 90

$$\int e^{-3\coth^{-1}(ax)}x\,dx = -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a+1/x)-3*x*(1
-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6304, 6874, 272, 44, 65, 214, 270, 665}

$$\int e^{-3\coth^{-1}(ax)}x\,dx = \frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2} + \frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a} - \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}$$

```
[In] Int[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^2(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-14-5ax+a^2x^2)}{1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

```
[In] Integrate[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 - 5*a*x + a^2*x^2))/(1 + a*x) + 9*Log[(1 +
 Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

method	result
risch	$\frac{(ax-6)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^2}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^3\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-10\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+10\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)}}{\sqrt{a^2}}\right.\right.$

```
[In] int(x*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x-6)*(a*x+1)/a^2*((a*x-1)/(a*x+1))^(1/2)+(9/2/a*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(
1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{(a^2x^2-5\,ax-14)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - 5*a*x - 14)*sqrt((a*x - 1)/(a*x + 1)) + 9*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.68

$$\int e^{-3\coth^{-1}(ax)}x\,dx = -\frac{1}{2}\,a\left(\frac{2\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}+\frac{8\sqrt{\frac{ax-1}{ax+1}}}{a^3}\right)$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(7*((a*x - 1)/(a*x + 1))^(3/2) - 5*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 9*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^3 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3 + 8
*sqrt((a*x - 1)/(a*x + 1))/a^3)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^2}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}-7\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

```
[In] int(x*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - (4*((a*x - 1)/(a*x + 1))^(1/2)
)/a^2 - (5*((a*x - 1)/(a*x + 1))^(1/2) - 7*((a*x - 1)/(a*x + 1))^(3/2))/(a^
2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1))
```

3.53 $\int e^{-3\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -3*arctanh((1-1/a^2/x^2)^(1/2))/a+4*(1-1/a^2/x^2)^(1/2)/(a+1/x)+x*(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6303, 6874, 270, 272, 65, 214, 665}

$$\int e^{-3\coth^{-1}(ax)}\,dx=-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}$$

```
[In] Int[E^(-3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}-\frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}x + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}x - (3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}x - \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int e^{-3\coth^{-1}(ax)}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(5+ax)}{1+ax} - \frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

```
[In] Integrate[E^(-3*ArcCoth[a*x]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(5 + a*x))/(1 + a*x) - (3*Log[a*(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(54) = 108.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.12

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{}\right.}{a\sqrt{a^2}\,(ax-1)\sqrt{(ax-1)(ax+}}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)*((a*x-1)/(a*x+1))^(1/2)+(-3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1
/2))/(a^2)^(1/2)+4/a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*(
(a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int e^{-3\coth^{-1}(ax)}\,dx = \frac{(ax+5)\sqrt{\frac{ax-1}{ax+1}} - 3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] ((a*x + 5)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2),x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(54) = 108.

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int e^{-3\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2} + \frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + 3*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- 4*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\,dx=\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (4*((a*x -
1)/(a*x + 1))^(1/2))/a - (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.54 $\int \frac{e^{-3\coth^{-1}(ax)}}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x} dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

```
[Out] -arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a+1/x)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6304, 6874, 222, 272, 65, 214, 665}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x} dx = \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) - \frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax)$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}-\frac{4}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= 4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$- \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + \text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax} - \arcsin\left(\frac{1}{ax}\right) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x),x]
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - ArcSin[1/(a*x)] + Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(42) = 84.

Time = 0.12 (sec) , antiderivative size = 366, normalized size of antiderivative = 7.96

method	result
default	$-\frac{\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+2\sqrt{a^2x^2-}\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x
^2-1)^(1/2))-ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^
3*x^2+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+2*(a^2*x^2-1)^(1/2)*(a^2)
^(1/2)*a*x+2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-2*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+(a^2*x^2-1)^(1/2)*(a
^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-a*ln((a^2*x+(a^2)^(1/2)*(
(a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -4\sqrt{\frac{ax-1}{ax+1}} + 2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -4*sqrt((a*x - 1)/(a*x + 1)) + 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(42) = 84.

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$$
$$= a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a} - \frac{4\sqrt{\frac{ax-1}{ax+1}}}{a}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a - 4*sqrt((a*x - 1)/(a*x + 1))/a
)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 4\sqrt{\frac{a\,x-1}{a\,x+1}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
- 4*((a*x - 1)/(a*x + 1))^(1/2)
```

3.55 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2} dx = 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)$$

```
[Out] 3*a*arccsc(a*x)+2*(a-1/x)^2/a/(1-1/a^2/x^2)^(1/2)+3*a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6304, 867, 683, 655, 222}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2} dx = \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\sqrt{1-\frac{1}{a^2x^2}} + 3a\csc^{-1}(ax)$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] 3*a*Sqrt[1 - 1/(a^2*x^2)] + (2*(a - x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
3*a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(1+5ax)}{1+ax} + 3a\arcsin\left(\frac{1}{ax}\right)$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^2),x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x))/(1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(49) = 98.

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}+\frac{\left(3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-5\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] (a*x+1)/x*((a*x-1)/(a*x+1))^(1/2)+(3*a*arctan(1/(a^2*x^2-1)^(1/2))+4/(x+1/a
)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-
1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = -\frac{6\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
[Out] -(6*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (5*a*x + 1)*sqrt((a*x - 1)/(a*x
 + 1)))/x
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**2,x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\left(2\sqrt{\frac{ax-1}{ax+1}} + \frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} - 3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
[Out] 2*a*(2*sqrt((a*x - 1)/(a*x + 1)) + sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*
x + 1) + 1) - 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}} + 5\,a\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 6\,a\,x\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{x}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^2,x)
[Out] (((a*x - 1)/(a*x + 1))^(1/2) + 5*a*x*((a*x - 1)/(a*x + 1))^(1/2) - 6*a*x*at
an(((a*x - 1)/(a*x + 1))^(1/2)))/x
```

3.56 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}-\frac{9}{2}a^2\csc^{-1}(ax)$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)/(a+1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a+1/x)-9/2*
a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6304, 1647, 1607, 12, 807, 679, 222}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{2}a^2\csc^{-1}(ax)-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a + x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a + x^(-1))) - (9*a^2*ArcCsc[a*x
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(ax-x^2)\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - (3a)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(1-5ax-14a^2x^2)}{x(1+ax)} - 9a\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^3),x]
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x - 14*a^2*x^2))/(x*(1 + a*x)) - 9*a*Ar
cSin[1/(a*x)]))/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{(ax+1)(6ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^{2}}+\frac{\left(-\frac{9a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}-\frac{4a\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\frac{\left(-6\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{5}x^{5}+6\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{5}x^{4}+6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}-21\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}-9a^{4}x^{4}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}\cdots}}\right.\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+1)*(6*a*x-1)/x^2*((a*x-1)/(a*x+1))^(1/2)+(-9/2*a^2*arctan(1/(a^2*
x^2-1)^(1/2))-4*a/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-
1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^{3}}\,dx=\frac{18\,a^{2}x^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(14\,a^{2}x^{2}+5\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^{2}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(18*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (14*a^2*x^2 + 5*a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^{3}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 4\,a\sqrt{\frac{ax-1}{ax+1}} - \frac{7\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 5\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] (9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) - 4*a*sqrt((a*x - 1)/(a*x + 1)) - (7
*a*((a*x - 1)/(a*x + 1))^(3/2) + 5*a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1
)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = 9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{5\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}} + 7\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2} + \frac{2\,(a\,x-1)}{a\,x+1} + 1} - 4\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^3,x)
```

```
[Out] 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (5*a^2*((a*x - 1)/(a*x + 1))^(1/2
) + 7*a^2*((a*x - 1)/(a*x + 1))^(3/2))/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x -
 1))/(a*x + 1) + 1) - 4*a^2*((a*x - 1)/(a*x + 1))^(1/2)
```

3.57 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{11}{2}a^3\csc^{-1}(ax)$$

```
[Out] 11/2*a^3*arccsc(a*x)+(a-1/x)^3/(1-1/a^2/x^2)^(1/2)+1/6*a^2*(28*a-3/x)*(1-1/
a^2/x^2)^(1/2)+1/3*a*(3*a-1/x)^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6304, 1647, 1607, 12, 866, 1649, 1668, 794, 222}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx=\frac{11}{2}a^3\csc^{-1}(ax)+\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a - 3/x))/6 + (a - x^(-1))^3/Sqrt[1 - 1/(a^2
*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a - x^(-1))^2)/3 + (11*a^3*ArcCsc[a*x]
)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

```
&& GtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\,(ax^2-x^3)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2 (3a^2-ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5+\frac{3x}{a}\right)(3a^2-ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{1}{2}(11a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{11}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(2-7ax+19a^2x^2+52a^3x^3)}{x^2(1+ax)} + 33a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^4),x]
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^2*(1 +
 a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(ax+1)(28a^2x^2-9ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6x^3} + \frac{\left(\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2} + \frac{4a^2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-30\sqrt{a^2x^2-1}\sqrt{a^2}\,a^6x^6+30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-93\sqrt{a^2x^2-1}\sqrt{a^2}\,a^5x^5-33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^5x^5+30\ln\left(\frac{a^2x+\sqrt{a^2x^2}}{\sqrt{a^2}}\right.\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x+1)*(28*a^2*x^2-9*a*x+2)/x^3*((a*x-1)/(a*x+1))^(1/2)+(11/2*a^3*arct
an(1/(a^2*x^2-1)^(1/2))+4*a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a
*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{66\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \left(52\,a^3x^3 + 19\,a^2x^2 - 7\,ax + 2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (52*a^3*x^3 + 19*a^2*x
^2 - 7*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 12\,a^2\sqrt{\frac{ax-1}{ax+1}} - \frac{39\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 52\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 21\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 12*a^2*sqrt((a*x - 1)/(a*x
 + 1)) - (39*a^2*((a*x - 1)/(a*x + 1))^(5/2) + 52*a^2*((a*x - 1)/(a*x + 1))
^(3/2) + 21*a^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x
- 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")

[Out] undef

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.59

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{52\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+13\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} + 4\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^4,x)

[Out] (7*a^3*((a*x - 1)/(a*x + 1))^(1/2) + (52*a^3*((a*x - 1)/(a*x + 1))^(3/2))/3 + 13*a^3*((a*x - 1)/(a*x + 1))^(5/2))/((3*(a*x - 1)^2)/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 4*a^3*((a*x - 1)/(a*x + 1))^(1/2) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))

3.58 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{51}{8}a^4\csc^{-1}(ax)$$

```
[Out] -51/8*a^4*arccsc(a*x)-a*(a-1/x)^3/(1-1/a^2/x^2)^(1/2)-27/4*a^4*(1-1/a^2/x^2
)^(1/2)-9/8*a^3*(2*a-3/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2/x^2)^(1/2)/x^3
-a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6304, 1647, 1607, 12, 866, 1649, 1829, 27, 757, 655, 222}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\frac{51}{8}a^4\csc^{-1}(ax)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-27*a^4*Sqrt[1 - 1/(a^2*x^2)])/4 - (9*a^3*Sqrt[1 - 1/(a^2*x^2)]*(2*a - 3/x
))/8 - (a*(a - x^(-1))^3)/Sqrt[1 - 1/(a^2*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)])
/(4*x^3) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/x^2 - (51*a^4*ArcCsc[a*x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(ax^3-x^4\right)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\left(-3a^3+a^2x-ax^2\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{1}{4}a^2\text{Subst}\left(\int \frac{12a-28x+\frac{27x^2}{a}-\frac{12x^3}{a^2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\text{Subst}\left(\int \frac{-\frac{36}{a}+\frac{108x}{a^2}-\frac{81x^2}{a^3}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\text{Subst}\left(\int -\frac{9(2a-3x)^2}{a^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{1}{4}(3a)\text{Subst}\left(\int \frac{(2a-3x)^2}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$
$$- \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{-17+\frac{18x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}} - \frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$+ \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{1}{8}\left(51a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}} - \frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{51}{8}a^4\csc^{-1}(ax)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}(-2+6ax-11a^2x^2+29a^3x^3+80a^4x^4)}{8x^3(1+ax)} - \frac{51}{8}a^4\arcsin\left(\frac{1}{ax}\right)$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^5),x]
[Out] -1/8*(a*Sqrt[1 - 1/(a^2*x^2)]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a^
4*x^4))/(x^3*(1 + a*x)) - (51*a^4*ArcSin[1/(a*x)])/8
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(ax+1)\left(48a^3x^3-19a^2x^2+8ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{8x^4}+\frac{\left(-\frac{51a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}-\frac{4a^3\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\frac{\left(-56\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^7x^7+56\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-163\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6-51\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^6x^6+56\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/8*(a*x+1)*(48*a^3*x^3-19*a^2*x^2+8*a*x-2)/x^4*((a*x-1)/(a*x+1))^(1/2)+(-
51/8*a^4*arctan(1/(a^2*x^2-1)^(1/2))-4*a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/
a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{102\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (80\,a^4x^4 + 29\,a^3x^3 - 11\,a^2x^2 + 6\,ax - 2)\sqrt{\frac{ax-1}{ax+1}}}{8\,x^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas")
[Out] 1/8*(102*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (80*a^4*x^4 + 29*a^3*x
^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**5,x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{4}\left(51\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 16\,a^3\sqrt{\frac{ax-1}{ax+1}} - \frac{77\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 149\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 123\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 35\,a^3}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima")
[Out] 1/4*(51*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 16*a^3*sqrt((a*x - 1)/(a*x
+ 1)) - (77*a^3*((a*x - 1)/(a*x + 1))^(7/2) + 149*a^3*((a*x - 1)/(a*x + 1))
^(5/2) + 123*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 35*a^3*sqrt((a*x - 1)/(a*x +
 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a
*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + 1))*a
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.43

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{51\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4} - 4\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{\frac{35\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4} + \frac{123\,a^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4} + \frac{149\,a^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4} + \frac{77\,a^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{\frac{6\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{(a\,x-1)^4}{(a\,x+1)^4} + \frac{4\,(a\,x-1)}{a\,x+1} + 1}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^5,x)
[Out] (51*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - 4*a^4*((a*x - 1)/(a*x + 1))^
(1/2) - ((35*a^4*((a*x - 1)/(a*x + 1))^(1/2))/4 + (123*a^4*((a*x - 1)/(a*x
+ 1))^(3/2))/4 + (149*a^4*((a*x - 1)/(a*x + 1))^(5/2))/4 + (77*a^4*((a*x -
1)/(a*x + 1))^(7/2))/4)/((6*(a*x - 1)^2)/(a*x + 1)^2 + (4*(a*x - 1)^3)/(a*x
 + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + (4*(a*x - 1))/(a*x + 1) + 1)
```

3.59 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5+\frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] 611/1920*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4+269/960*(1-1/a/x)^(3/4)*(1+1
/a/x)^(1/4)*x^2/a^3+11/48*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2+9/40*(1-1
/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5+3
1/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+31/128*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{31\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{611x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{1920a^4}+\frac{269x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{960a^3}+\frac{11x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{48a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{9x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{40a}$$

```
[In] Int[E^(ArcCoth[a*x]/2)*x^4,x]
```

```
[Out] (611*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(1920*a^4) + (269*(1 - 1/(a
*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(3/4)*(1
+ 1/(a*x))^(1/4)*x^3)/(48*a^2) + (9*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^6\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{1}{5}\text{Subst}\left(\int\frac{\frac{9}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5+\frac{1}{20}\text{Subst}\left(\int\frac{-\frac{55}{4a^2}-\frac{27x}{2a^3}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{1}{60}\text{Subst}\left(\int\frac{\frac{269}{8a^3}+\frac{55x}{2a^4}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5+\frac{1}{120}\text{Subst}\left(\int\frac{-\frac{611}{16a^4}-\frac{269x}{8a^5}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{1}{120}\text{Subst}\left(\int\frac{465}{32a^5x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{31\text{Subst}\left(\int\frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{256a^5}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{31\text{Subst}\left(\int\frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^5}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5+\frac{31\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} + \frac{31\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{\frac{24576e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5} + \frac{62976e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{64640e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{34000e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{9620e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 930\arctan\left(\right.}{3840a^5}$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^4,x]
```

```
[Out] ((24576*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (62976*E^(ArcCoth
[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^(ArcCoth[a*x]/2))/(-1 + E^
(2*ArcCoth[a*x]))^3 + (34000*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^
2 + (9620*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 930*ArcTan[E^(Arc
Coth[a*x]/2)] - 465*Log[1 - E^(ArcCoth[a*x]/2)] + 465*Log[1 + E^(ArcCoth[a*
x]/2)])/(3840*a^5)
```

Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\left(384\,a^5x^5+816\,a^4x^4+872\,a^3x^3+978\,a^2x^2+1149\,ax+611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+465\,\mathrm{lo}}{3840\,a^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 + 816*a^4*x^4 + 872*a^3*x^3 + 978*a^2*x^2 + 1149*a*x
 + 611)*((a*x - 1)/(a*x + 1))^(3/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 465*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x***4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{1}{3840}\,a\left(\frac{4\left(465\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\,\mathrm{arct}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(465*((a*x - 1)/(a*x + 1))^(19/4) - 696*((a*x - 1)/(a*x + 1))^
(15/4) + 5090*((a*x - 1)/(a*x + 1))^(11/4) - 1120*((a*x - 1)/(a*x + 1))^(7/
```

```
4) + 2405*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/
(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 465*
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{1120\,(ax-1)\left(\frac{ax}{ax}\right.}{ax+1}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 + 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(1120*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5090*(a*x -
 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 696*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(3/4)/(a*x + 1)^3 - 465*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/
(a*x + 1)^4 - 2405*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{481\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{192}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{6}+\frac{509\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{96}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40}+\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}} - \frac{31\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{31\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((481*((a*x - 1)/(a*x + 1))^(3/4))/192 - (7*((a*x - 1)/(a*x + 1))^(7/4))/6
+ (509*((a*x - 1)/(a*x + 1))^(11/4))/96 - (29*((a*x - 1)/(a*x + 1))^(15/4))
```

```
/40 + (31*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*
x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)
^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (31*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(128*a^5)
```

3.60 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4+\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] 83/192*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+29/96*(1-1/a/x)^(3/4)*(1+1/a/x
)^(1/4)*x^2/a^2+7/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(3
/4)*(1+1/a/x)^(1/4)*x^4+11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+1
1/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{6306, 101, 156, 12, 95, 218, 212, 209\}$

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{11\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{11\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{83x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{192a^3} + \frac{29x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{96a^2} + \frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} + \frac{7x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a}$$

```
[In] Int[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] (83*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(192*a^3) + (29*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(96*a^2) + (7*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
 (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^5\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4+\frac{1}{12}\text{Subst}\left(\int\frac{-\frac{29}{4a^2}-\frac{7x}{a^3}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4-\frac{1}{24}\text{Subst}\left(\int\frac{\frac{83}{8a^3}+\frac{29x}{4a^4}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4+\frac{1}{24}\text{Subst}\left(\int-\frac{33}{16a^4x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4-\frac{11\text{Subst}\left(\int\frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)}{128a^4}$$

$$=\frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4-\frac{11\text{Subst}\left(\int\frac{1}{-1+x^4}dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32a^4}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 + \frac{11\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{11\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 + \frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{11\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

Mathematica [A] (verified)

Time = 5.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{3200e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{2512e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{980e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 66\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 33\log}{384a^4}$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] ((1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (3200*E^(ArcCoth[a
*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^(ArcCoth[a*x]/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (980*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 66*
ArcTan[E^(ArcCoth[a*x]/2)] - 33*Log[1 - E^(ArcCoth[a*x]/2)] + 33*Log[1 + E^
(ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\,(48\,a^4x^4+104\,a^3x^3+114\,a^2x^2+141\,ax+83)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{384\,a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="fricas")
[Out] 1/384*(2*(48*a^4*x^4 + 104*a^3*x^3 + 114*a^2*x^2 + 141*a*x + 83)*((a*x - 1)
/(a*x + 1))^(3/4) - 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x -
 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**3,x)
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{1}{384}\,a\left(\frac{4\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-279\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+107\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\,\log\left(\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="maxima")
[Out] 1/384*a*(4*(33*((a*x - 1)/(a*x + 1))^(15/4) - 279*((a*x - 1)/(a*x + 1))^(11
/4) + 107*((a*x - 1)/(a*x + 1))^(7/4) - 245*((a*x - 1)/(a*x + 1))^(3/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{107\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^5 + 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5
+ 4*(107*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 279*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 33*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(3/4)/(a*x + 1)^3 - 245*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a
*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{245\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{96}-\frac{107\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{96}+\frac{93\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}} - \frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}+\frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((245*((a*x - 1)/(a*x + 1))^(3/4))/96 - (107*((a*x - 1)/(a*x + 1))^(7/4))/9
6 + (93*((a*x - 1)/(a*x + 1))^(11/4))/32 - (11*((a*x - 1)/(a*x + 1))^(15/4)
)/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1
)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (11*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (11*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(64*a^4)
```

3.61 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2}+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3+\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 11/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2+5/12*(1-1/a/x)^(3/4)*(1+1/a/x)^
(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3+3/8*arctan((1+1/a/x)^(1
/4)/(1-1/a/x)^(1/4))/a^3+3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{6306, 101, 156, 12, 95, 218, 212, 209\}$

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{3\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{3\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{11x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2} + \frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} + \frac{5x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

```
[In] Int[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) + (5*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
+ (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^4\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}-\frac{5x}{2a^3}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{9}{8a^3x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^4}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.23

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(-1070609085 - 946471617e^{2\coth^{-1}(ax)} + 369641285e^{4\coth^{-1}(ax)} + 351173641e^{6\coth^{-1}(ax)} - \right.}{}$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] -1/1909440*(-1070609085 - 946471617*E^(2*ArcCoth[a*x]) + 369641285*E^(4*Arc
Coth[a*x]) + 351173641*E^(6*ArcCoth[a*x]) - 23818496*E^(8*ArcCoth[a*x]) + 1
070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 732349800*E^
(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 63506
7810*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])]
- 384831720*E^(6*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[
a*x])] + 60913125*E^(8*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*Ar
cCoth[a*x])] + 1280*E^(8*ArcCoth[a*x])*(821 + 1346*E^(2*ArcCoth[a*x]) + 557
*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(2*A
rcCoth[a*x])] + 10240*E^(8*ArcCoth[a*x])*(23 + 42*E^(2*ArcCoth[a*x]) + 19*E
^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^
(2*ArcCoth[a*x])] + 20480*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2,
 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])] + 40960*E^(10*ArcCoth[a*x
])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth
[a*x])] + 20480*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4},
 {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((7*ArcCoth[a*x])/2))
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\,(8\,a^3x^3+18\,a^2x^2+21\,ax+11)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{48\,a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 18*a^2*x^2 + 21*a*x + 11)*((a*x - 1)/(a*x + 1))^(3/4)
- 18*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}\,a\left(\frac{4\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}} - 6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}} + 29\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1} - \frac{3\,(ax-1)^2a^4}{(ax+1)^2} + \frac{(ax-1)^3a^4}{(ax+1)^3} - a^4} + \frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4} - \frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^4} + \frac{9\,\mathrm{lo}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(9*((a*x - 1)/(a*x + 1))^(11/4) - 6*((a*x - 1)/(a*x + 1))^(7/4)
+ 29*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^
2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 9*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4} - \frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^4} + \frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right|\right)}{a^4} - \frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - \frac{9}{}\right.}{a^4(}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 9*(a*x - 1)^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 29*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/4),x)
[Out] ((29*((a*x - 1)/(a*x + 1))^(3/4))/12 - ((a*x - 1)/(a*x + 1))^(7/4)/2 + (3*(
(a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a
^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*x
- 1)/(a*x + 1))^(1/4)))/(8*a^3) + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8
*a^3)
```

3.62 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx=\frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] 1/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)
*x^2+1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6306, 98, 96, 95, 218, 212, 209}

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx=\frac{\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

```
[In] Int[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(3/4)*(1
 + 1/(a*x))^(5/4)*x^2)/2 + ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/
(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^3\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+5e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x,x]
[Out] ((2*E^(ArcCoth[a*x]/2)*(-1 + 5*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x]
))^2 + ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2+5\,ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 + 5*a*x + 3)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x
 - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x
- 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}\,a\left(\frac{4\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(((a*x - 1)/(a*x + 1))^(7/4) - 5*((a*x - 1)/(a*x + 1))^(3/4))/(2*(
a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.98

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^3 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*((a*x
 - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5*((a*x - 1)/(a*x + 1))^(3/4)
)/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((5*((a*x - 1)/(a*x + 1))^(3/4))/2 - ((a*x - 1)/(a*x + 1))^(7/4)/2)/(a^2 +
(a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - atan(((a*x -
 1)/(a*x + 1))^(1/4))/(4*a^2) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2)
```

3.63 $\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 96

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
+arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6305, 96, 95, 218, 212, 209}

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

```
[In] Int[E^(ArcCoth[a*x]/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x + \frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + \arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

```
[In] Integrate[E^(ArcCoth[a*x]/2),x]
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + ArcTan[E^(ArcCoth[a*x]/
2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4),x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{2\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4),x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/4),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
- 1)/(a*x + 1))^(1/4))/a + atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

3.64 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

```
[Out] 2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/
x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/
(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2
^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6306, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)+2\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^(ArcCoth[a*x]/2)/x,x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$+ 4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$- 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$+ \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$+ \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$+ \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$+ \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$- \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{5}e^{\frac{5}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},e^{4\coth^{-1}(ax)}\right)$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x,x]
[Out] (8*E^((5*ArcCoth[a*x])/2)*Hypergeometric2F1[5/8, 1, 13/8, E^(4*ArcCoth[a*x]
)])/5
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}x}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2} + 2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2} + 2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2} + 2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2} + 2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] (1/2*I - 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
- (1/2*I + 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)) + (1/2*I + 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)) - (1/2*I - 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4)) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))
^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.}{a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax}{ax}\right.\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) +\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1-\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i})$$

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 - 1i) - 2
*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2
i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1i)
```

3.65 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 267

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}-\frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

```
[Out] a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)+1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)-1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6306, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (a*ArcTan[1 - (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 - 1/
(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*Log[1 + Sqrt[1 - 1/(a*x)]/
Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*
Sqrt[2]) - (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - a\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$+\frac{1}{2}a\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$+\frac{a\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$+\frac{a\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$-\frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}+\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$-\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} + \frac{\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*((2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + ArcTan[1 - Sqrt[2]*E^(
ArcCoth[a*x]/2)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]/Sqrt[2] +
 Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]) - Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]))
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^2}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{(-a^4)^{\frac{1}{4}}\,x\log\left(a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + (-a^4)^{\frac{3}{4}}\right) - i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + i\,(-a^4)^{\frac{3}{4}}\right) + i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{2\,x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) + (-a^4)^(3/4)) - I
*(-a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^4)^(3/4)) + I*(
-a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^4)^(3/4)) - (-a^4
)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) - (-a^4)^(3/4)) + 2*(a*x + 1)
*((a*x - 1)/(a*x + 1))^(3/4))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) + 2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) - \sqrt{2}\,\mathrm{lo}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
```

```
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (-1)^(1/4)*a*at
anh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) + (2*a*((a*x - 1)/(a*x + 1))^(3
/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.66 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 319

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

```
[Out] 1/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(
5/4)+1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6306, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}+\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[In] Int[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(3/4)*
(1 + 1/(a*x))^(5/4))/2 - (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\\
&\quad + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{8}a\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\\
&\quad + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)
\end{aligned}$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{4}a^2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}$$

$$+ \frac{1}{8}a^2\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{8}a^2\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$+ \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(-\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{40e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} + 2\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 2\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right) - \sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*((-32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (40*E^(ArcCoth[
a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[
1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] - Sqrt[2]*Log[1 + Sqrt[2]*
E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}x^3}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + (-a^8)^{\frac{3}{4}}\right) - i\,(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + i\,(-a^8)^{\frac{3}{4}}\right) + i\,(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^6\left(\frac{ax-1}{ax+1}\right.\right.}{8\,x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*((-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) + (-a^8)^(3/4)) -
 I*(-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^8)^(3/4)) +
 I*(-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^8)^(3/4)) -
 (-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) - (-a^8)^(3/4)) + 2*(
3*a^2*x^2 + 5*a*x + 2)*((a*x - 1)/(a*x + 1))^(3/4))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) + 2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) - \sqrt{}\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1))*a + 8*(a*((a*x - 1)/(a*x + 1))^(7/4) + 5*a*((a*x - 1)
/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$= \frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a
*x + 1) + 5*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{5\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{2}+\frac{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4} - \frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] ((5*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (a^2*((a*x - 1)/(a*x + 1))^(7/4))/
2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*a^
2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - ((-1)^(1/4)*a^2*atanh((
-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

3.67 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 356

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx &= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} \\ &\quad+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} \\ &\quad-\frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\ &\quad+\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} \\ &\quad-\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\end{aligned}$$

```
[Out] 3/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/12*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^
(5/4)+1/3*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)/x+3/16*a^3*arctan(-1+(1-1/a/x
)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/4)*
2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1
```

```
/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/32*a^3*ln(1+(1-1/a/x
)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{3a^3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}$$

```
[In] Int[E^(ArcCoth[a*x]/2)/x^4,x]
```

```
[Out] (3*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 + (a^3*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(5/4))/12 + (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(
3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x
))^(1/4)])/(8*Sqrt[2]) + (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (3*a^
3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2 \sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\left(-1-\frac{x}{2a}\right)\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int\frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x}-\frac{1}{16}\left(3a^2\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x}+\frac{1}{4}\left(3a^3\right)\text{Subst}\left(\int\frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x}+\frac{1}{4}\left(3a^3\right)\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x}$$

$$+\frac{1}{16}\left(3a^3\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{16}\left(3a^3\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\right.$$

$$
\begin{aligned}
&= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} \\
&+ \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} \\
&+ \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} - \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} \\
&+ \frac{(3a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\
&- \frac{(3a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} \\
&+ \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\
&+ \frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} \\
&- \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(9+6e^{2\coth^{-1}(ax)}+29e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 9\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^4,x]
[Out] (a^3*((8*E^(ArcCoth[a*x]/2)*(9 + 6*E^(2*ArcCoth[a*x]) + 29*E^(4*ArcCoth[a*x
])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x] - 2*
Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^4}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{9\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(27\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+27\left(-a^{12}\right)^{\frac{3}{4}}\right)-9i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(27\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+27i\left(-a^{12}\right)^{\frac{3}{4}}\right)+9i\left(-a^{1}\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
[Out] 1/48*(9*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(a*x + 1))^(1/4) + 27*(-a^1
2)^(3/4)) - 9*I*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(a*x + 1))^(1/4) +
27*I*(-a^12)^(3/4)) + 9*I*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(a*x + 1)
)^(1/4) - 27*I*(-a^12)^(3/4)) - 9*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(
a*x + 1))^(1/4) - 27*(-a^12)^(3/4)) + 2*(11*a^3*x^3 + 21*a^2*x^2 + 18*a*x +
 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**4,x)
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
[Out] 1/96*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))*a^2 + 8*(9*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 6*a^2
*((a*x - 1)/(a*x + 1))^(7/4) + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x
- 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*
a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) - 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(3/4)/(a*x + 1) + 9*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(
a*x + 1)^2 + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} + \frac{3\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{3\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}$$

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] ((29*a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (a^3*((a*x - 1)/(a*x + 1))^(7/4)
)/2 + (3*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2
+ (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + (3*(-1)^(1/4)*a^
3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (3*(-1)^(1/4)*a^3*atanh
((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

3.68 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] 557/640*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4+157/320*(1-1/a/x)^(1/4)*(1+1/
a/x)^(3/4)*x^2/a^3+5/16*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2+11/40*(1-1/
a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5-23
7/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+237/128*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx=-\frac{237\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{557x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{640a^4}+\frac{157x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{320a^3}+\frac{5x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{16a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{40a}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (557*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(640*a^4) + (157*(1 - 1/(a*
x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(1/4)*(1 +
1/(a*x))^(3/4)*x^3)/(16*a^2) + (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^6\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{1}{5}\text{Subst}\left(\int\frac{\frac{11}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{1}{20}\text{Subst}\left(\int\frac{-\frac{75}{4a^2}-\frac{33x}{2a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{1}{60}\text{Subst}\left(\int\frac{\frac{471}{8a^3}+\frac{75x}{2a^4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{1}{120}\text{Subst}\left(\int\frac{-\frac{1671}{16a^4}-\frac{471x}{8a^5}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{1}{120}\text{Subst}\left(\int\frac{3555}{32a^5x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{237\text{Subst}\left(\int\frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{256a^5}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{237\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^5}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{237\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{\;}}{\sqrt[4]{\;}}\right)}{128a^5}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} + \frac{237\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{8192e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5} + \frac{22016e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{23936e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{14032e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{5500e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 2370\arctan(}{1280a^5}$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] ((8192*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (22016*E^((3*A
rcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((3*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^3 + (14032*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (5500*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))
- 2370*ArcTan[E^(ArcCoth[a*x]/2)] - 1185*Log[1 - E^(ArcCoth[a*x]/2)] + 1185
*Log[1 + E^(ArcCoth[a*x]/2)])/(1280*a^5)
```

Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\left(128\,a^5x^5+304\,a^4x^4+376\,a^3x^3+514\,a^2x^2+871\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1185\,\mathrm{l}}{1280\,a^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 + 304*a^4*x^4 + 376*a^3*x^3 + 514*a^2*x^2 + 871*a*x
+ 557)*((a*x - 1)/(a*x + 1))^(1/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1185*log(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{1}{1280}\,a\left(\frac{4\left(395\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,\mathrm{ar}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(395*((a*x - 1)/(a*x + 1))^(17/4) - 1440*((a*x - 1)/(a*x + 1))
^(13/4) + 3710*((a*x - 1)/(a*x + 1))^(9/4) - 1992*((a*x - 1)/(a*x + 1))^(5/
4) + 1375*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 11
85*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$=\frac{1}{1280}a\left(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1992\,(ax-1)\left(\right.}{ax+1}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 - 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1992*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1440*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 395*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/
4)/(a*x + 1)^4 - 1375*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$=\frac{\frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{2}+\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}+\frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((275*((a*x - 1)/(a*x + 1))^(1/4))/64 - (249*((a*x - 1)/(a*x + 1))^(5/4))/4
0 + (371*((a*x - 1)/(a*x + 1))^(9/4))/32 - (9*((a*x - 1)/(a*x + 1))^(13/4))
/2 + (79*((a*x - 1)/(a*x + 1))^(17/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*x
 + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)^
4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (237*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (237*atanh(((a*x - 1)/(a*x + 1)
)^(1/4)))/(128*a^5)
```

3.69 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3}+\frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2}+\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] 63/64*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+15/32*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a^2+3/8*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(1/4
)*(1+1/a/x)^(3/4)*x^4-123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+12
3/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{6306, 101, 156, 12, 95, 304, 209, 212\}$

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{123\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{123\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{63x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{64a^3} + \frac{15x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{32a^2} + \frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{3x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (63*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(64*a^3) + (15*(1 - 1/(a*x))
^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(32*a^2) + (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a
*x))^(3/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (
123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^5\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{9}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}-\frac{9x}{a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{24}\text{Subst}\left(\int \frac{\frac{189}{8a^3}+\frac{45x}{4a^4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{24}\text{Subst}\left(\int -\frac{369}{16a^4x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{123\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{128a^4}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{123\text{Subst}\left(\int \frac{x^2}{-1+x^4}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32a^4}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{123\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} - \frac{123\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{}}{\sqrt[4]{}}\right.}{64a^4}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{123\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{512e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{1152e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{1008e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{532e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 246\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 123}{128a^4}$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] ((512*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (1152*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((3*ArcCoth[a*x])/2))/
(-1 + E^(2*ArcCoth[a*x]))^2 + (532*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x])) - 246*ArcTan[E^(ArcCoth[a*x]/2)] - 123*Log[1 - E^(ArcCoth[a*x]/2)
] + 123*Log[1 + E^(ArcCoth[a*x]/2)])/(128*a^4)
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)

[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(16\,a^4x^4 + 40\,a^3x^3 + 54\,a^2x^2 + 93\,ax + 63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) -}{128\,a^4}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="fricas")

[Out] 1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 54*a^2*x^2 + 93*a*x + 63)*((a*x - 1)/(a*x + 1))^(1/4) + 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**3,x)

[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(3/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{128}a\left(\frac{4\left(41\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-183\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+147\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/128*a*(4*(41*((a*x - 1)/(a*x + 1))^(13/4) - 183*((a*x - 1)/(a*x + 1))^(9/
4) + 147*((a*x - 1)/(a*x + 1))^(5/4) - 133*((a*x - 1)/(a*x + 1))^(1/4))/(4*
(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5
/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 123*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{128}a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{147(ax-1)\left(\frac{ax-1}{ax+1}\right.}{ax+1}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="giac")
```

```
[Out] 1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^5 - 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
5 - 4*(147*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 183*(a*x - 1)^
2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 41*(a*x - 1)^3*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1)^3 - 133*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/
(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx=\frac{\frac{133\left(\frac{ax-1}{ax+1}\right)^{1/4}}{32}-\frac{147\left(\frac{ax-1}{ax+1}\right)^{5/4}}{32}+\frac{183\left(\frac{ax-1}{ax+1}\right)^{9/4}}{32}-\frac{41\left(\frac{ax-1}{ax+1}\right)^{13/4}}{32}}{a^4+\frac{6a^4(ax-1)^2}{(ax+1)^2}-\frac{4a^4(ax-1)^3}{(ax+1)^3}+\frac{a^4(ax-1)^4}{(ax+1)^4}-\frac{4a^4(ax-1)}{ax+1}}+\frac{123\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{64\,a^4}+\frac{123\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{64\,a^4}$$

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((133*((a*x - 1)/(a*x + 1))^(1/4))/32 - (147*((a*x - 1)/(a*x + 1))^(5/4))/3
2 + (183*((a*x - 1)/(a*x + 1))^(9/4))/32 - (41*((a*x - 1)/(a*x + 1))^(13/4)
)/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1
)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (123*a
tan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (123*atanh(((a*x - 1)/(a*x + 1
))^(1/4)))/(64*a^4)
```

3.70 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}+\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3-\frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 23/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2+7/12*(1-1/a/x)^(1/4)*(1+1/a/x)^
(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3-17/8*arctan((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^3+17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{6306, 101, 156, 12, 95, 304, 209, 212\}$

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{17\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{17\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{23x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2} + \frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{7x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (23*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) + (7*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 + (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^4\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3-\frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\end{aligned}$$

$$= \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{23}{4a^2}-\frac{7x}{2a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{51}{8a^3x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\text{Subst}\left(\int \frac{x^2}{-1+x^4}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{17\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{17\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.43 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.23

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(-1357846875 - 1400453615e^{2\coth^{-1}(ax)} + 276606275e^{4\coth^{-1}(ax)} + 438715415e^{6\coth^{-1}(ax)}\right.}{}$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] -1/4213440*(-1357846875 - 1400453615*E^(2*ArcCoth[a*x]) + 276606275*E^(4*Ar
cCoth[a*x]) + 438715415*E^(6*ArcCoth[a*x]) - 12962560*E^(8*ArcCoth[a*x]) +
1357846875*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 818519240*E
^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] - 9977
22110*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])]
 - 501106760*E^(6*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth
[a*x])] + 137997475*E^(8*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*
ArcCoth[a*x])] + 1792*E^(8*ArcCoth[a*x])*(965 + 1618*E^(2*ArcCoth[a*x]) + 6
85*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 11/4}, {1, 1, 23/4}, E^(
2*ArcCoth[a*x])] + 14336*E^(8*ArcCoth[a*x])*(25 + 46*E^(2*ArcCoth[a*x]) + 2
1*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 11/4}, {1, 1, 1, 23/4}
, E^(2*ArcCoth[a*x])] + 28672*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2
, 2, 2, 11/4}, {1, 1, 1, 1, 23/4}, E^(2*ArcCoth[a*x])] + 57344*E^(10*ArcCot
h[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 11/4}, {1, 1, 1, 1, 23/4}, E^(2*A
rcCoth[a*x])] + 28672*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2,
 11/4}, {1, 1, 1, 1, 23/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5*ArcCoth[a*x])/2
))
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)

[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{2\left(8\,a^3x^3+22\,a^2x^2+37\,ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-51\,\log\left(\left(\frac{ax-}{ax+}\right.\right.}{48\,a^3}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="fricas")

[Out] 1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 37*a*x + 23)*((a*x - 1)/(a*x + 1))^(1/4) + 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**2,x)

[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}a\left(\frac{4\left(17\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(17*((a*x - 1)/(a*x + 1))^(9/4) - 30*((a*x - 1)/(a*x + 1))^(5/4)
 + 45*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)
^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 +
51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 - 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 17*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 45*((a*x - 1)/(a*x + 1))^(1/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx=\frac{\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((15*((a*x - 1)/(a*x + 1))^(1/4))/4 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2 + (
17*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2
- (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*atan((
(a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (17*atanh(((a*x - 1)/(a*x + 1))^(1/4
)))/(8*a^3)
```

3.71 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2-\frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] 3/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)
*x^2-9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6306, 98, 96, 95, 304, 209, 212}

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=-\frac{9\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(1/4)*
(1 + 1/(a*x))^(7/4)*x^2)/2 - (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2
)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{3\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 + \frac{9\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - \frac{9\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-3+7e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - 9\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x,x]
[Out] ((2*E^((3*ArcCoth[a*x])/2)*(-3 + 7*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[
a*x]))^2 - 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4
*a^2)
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2+7\,ax+5)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 + 7*a*x + 5)*((a*x - 1)/(a*x + 1))^(1/4) + 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}\,a\left(\frac{4\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} - 7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1} - \frac{(ax-1)^2a^3}{(ax+1)^2} - a^3} + \frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} + \frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^3} - \frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{a^3}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(3*((a*x - 1)/(a*x + 1))^(5/4) - 7*((a*x - 1)/(a*x + 1))^(1/4))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} + \frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^3} - \frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right|\right)}{a^3} - \frac{4\left(\frac{3\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1} - 7\left(\frac{ax}{ax}\right.\right.}{a^3\left(\frac{ax-1}{ax+1} - 1\right)^2}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="giac")
```

```
[Out] 1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^3 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*(
3*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 7*((a*x - 1)/(a*x + 1))
^(1/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

```
[In] int(x/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((7*((a*x - 1)/(a*x + 1))^(1/4))/2 - (3*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^
2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + (9*atan(
((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) + (9*atanh(((a*x - 1)/(a*x + 1))^(1/4
)))/(4*a^2)
```

3.72 $\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x-\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
/a+3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6305, 96, 95, 304, 209, 212}

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=-\frac{3\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a + (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{6\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (8*E^((3*ArcCoth[a*x])/2)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[
3/4, 2, 7/4, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{2\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$=\frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 - 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{a}+\frac{3\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{a}$$

```
[In] int(1/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/a + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.73 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

```
[Out] -2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)
/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6306, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right) - 2\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}} + \frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{7}e^{\frac{7}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},e^{4\coth^{-1}(ax)}\right)$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x,x]
[Out] (8*E^((7*ArcCoth[a*x])/2)*Hypergeometric2F1[7/8, 1, 15/8, E^(4*ArcCoth[a*x]
)])/7
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] (1/2*I + 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
- (1/2*I - 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)) + (1/2*I - 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)) - (1/2*I + 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4)) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))
^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.}{a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax-}\right.\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(1-\mathrm{i})$$

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
```

3.74 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 268

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

```
[Out] a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)-3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6306, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{3a\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - (3*a*ArcTan[1 - (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (3*a*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(2*Sqrt[2]) + (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (3a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}(3a)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}(3a)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{(3a)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{(3a)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{(3a)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{(3a)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -8ae^{\frac{3}{2}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}} + \text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^2,x]
[Out] -8*a*E^((3*ArcCoth[a*x])/2)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometri
c2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])])
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^2}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{3\left(-a^4\right)^{\frac{1}{4}} x \log \left(3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 3\left(-a^4\right)^{\frac{1}{4}}\right) + 3i\left(-a^4\right)^{\frac{1}{4}} x \log \left(3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 3i\left(-a^4\right)^{\frac{1}{4}}\right) - 3i\left(-a^4\right)^{\frac{1}{4}} x \log \left(3\,a\right.}{2\,x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
[Out] 1/2*(3*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) + 3*(-a^4)^(1/4))
 + 3*I*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) + 3*I*(-a^4)^(1/4
)) - 3*I*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) - 3*I*(-a^4)^(1
/4)) - 3*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) - 3*(-a^4)^(1/4
)) + 2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**2,x)
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) + 6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) + 3\sqrt{2}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
```

```
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}$$
$$-{(-1)}^{1/4}\,a\,\mathrm{atan}\left({(-1)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3\mathrm{i}-{(-1)}^{1/4}\,a\,\mathrm{atanh}\left({(-1)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3\mathrm{i}$$

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1) - (-1)^(1/4)*a*
atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i - (-1)^(1/4)*a*atan((-1)^(
1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i
```

3.75 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 319

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}+\frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

```
[Out] 3/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(
7/4)+9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+9/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}} + \frac{9a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4} + \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} - \frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} + \frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (3*a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(7/4))/2 - (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}-\frac{1}{4}(3a)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}\\
&\quad+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}-\frac{1}{8}(9a)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}\\
&\quad+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)
\end{aligned}
$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}(9a^2)\,\text{Subst}\left(\int\frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{4}(9a^2)\,\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{4}(9a^2)\,\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{8}(9a^2)\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{8}(9a^2)\,\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{(9a^2)\,\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{(9a^2)\,\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{(9a^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{(9a^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}+\frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.24

$$\int\frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) - 3\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + 2\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] - 3*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])] + 2*Hyperg
eometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^3}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.66

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3} dx = \frac{9\left(-a^8\right)^{\frac{1}{4}} x^2 \log\left(9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 9\left(-a^8\right)^{\frac{1}{4}}\right) + 9i\left(-a^8\right)^{\frac{1}{4}} x^2 \log\left(9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 9i\left(-a^8\right)^{\frac{1}{4}}\right) - 9i\left(-a^8\right)^{\frac{1}{4}} x^2 \log}{8\,x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 9*(-a^8)^(1
/4)) + 9*I*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 9*I*(-a
^8)^(1/4)) - 9*I*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4) - 9
*I*(-a^8)^(1/4)) - 9*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4)
 - 9*(-a^8)^(1/4)) + 2*(5*a^2*x^2 + 7*a*x + 2)*((a*x - 1)/(a*x + 1))^(1/4))
/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3} dx = \int \frac{1}{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}} dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*a*((a*x - 1)/(a*x + 1))^(5/4) + 7*
a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x +
1)^2 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1) + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1
)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{7a^2\left(\frac{ax-1}{ax+1}\right)^{1/4}}{2}+\frac{3a^2\left(\frac{ax-1}{ax+1}\right)^{5/4}}{2}}{\frac{(ax-1)^2}{(ax+1)^2}+\frac{2(ax-1)}{ax+1}+1}-\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)\,9\mathrm{i}}{4}-\frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)\,9\mathrm{i}}{4}$$

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] ((7*a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (3*a^2*((a*x - 1)/(a*x + 1))^(5/4)
)/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4 - ((-1)^(1/4)*a^2*at
anh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4
```

3.76 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 356

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} + \frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}$$

```
[Out] 17/24*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/4*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)
^(7/4)+1/3*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)/x+17/16*a^3*arctan(-1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^(1/
4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/
```

```
(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+17/32*a^3*ln(1+(1-
1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}} + \frac{17a^3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4} + \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} - \frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}} + \frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (17*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/24 + (a^3*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(7/4))/4 + (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/
(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(
a*x))^(1/4)])/(8*Sqrt[2]) - (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a
*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (
17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}+\frac{1}{3}a^2\text{Subst}\left(\int\frac{\left(-1-\frac{3x}{2a}\right)\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}-\frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)$$

$$=\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}-\frac{1}{16}\left(17a^2\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$=\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}+\frac{1}{4}\left(17a^3\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{2-x^4}}dx,x,\sqrt[4]{1-\frac{1}{ax}}\right)$$

$$=\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}+\frac{1}{4}\left(17a^3\right)\text{Subst}\left(\int\frac{1}{1+x^4}dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$=\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}+\frac{1}{8}\left(17a^3\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{8}\left(17a^3\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}$$

$$+\frac{1}{16}(17a^3)\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{16}(17a^3)\,\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}$$
$$-\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}+\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}$$
$$+\frac{(17a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$
$$-\frac{(17a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}$$
$$-\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$
$$-\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}+\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(17+30e^{2\coth^{-1}(ax)}+45e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 51\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (a^3*((8*E^((3*ArcCoth[a*x])/2)*(17 + 30*E^(2*ArcCoth[a*x]) + 45*E^(4*ArcCo
th[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*
x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^4}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{51\,(-a^{12})^{\frac{1}{4}}\,x^3\log\left(17\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+17\,(-a^{12})^{\frac{1}{4}}\right)+51i\,(-a^{12})^{\frac{1}{4}}\,x^3\log\left(17\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+17i\,(-a^{12})^{\frac{1}{4}}\right)-51i}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(51*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 17*(-a^
12)^(1/4)) + 51*I*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ 17*I*(-a^12)^(1/4)) - 51*I*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x - 1)/(a*x +
 1))^(1/4) - 17*I*(-a^12)^(1/4)) - 51*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x -
1)/(a*x + 1))^(1/4) - 17*(-a^12)^(1/4)) + 2*(23*a^3*x^3 + 37*a^2*x^2 + 22*a
*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**4,x)
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.78

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(17*a^2*((a*x - 1)/(a*x +
 1))^(9/4) + 30*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 45*a^2*((a*x - 1)/(a*x +
1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3
/(a*x + 1)^3 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1) + 17*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1)^2 + 45*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{15\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{5\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} - \frac{(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,17\mathrm{i}}{8} - \frac{(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,17\mathrm{i}}{8}$$

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] ((15*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (5*a^3*((a*x - 1)/(a*x + 1))^(5/4
))/2 + (17*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)^
2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^
3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8 - ((-1)^(1/4)*a^3*ata
nh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8
```

3.77 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 287

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}}+\frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] -26111/1920*(1+1/a/x)^(1/4)/a^5/(1-1/a/x)^(1/4)+5533/1920*(1+1/a/x)^(1/4)*x
/a^4/(1-1/a/x)^(1/4)+1189/960*(1+1/a/x)^(1/4)*x^2/a^3/(1-1/a/x)^(1/4)+181/2
40*(1+1/a/x)^(1/4)*x^3/a^2/(1-1/a/x)^(1/4)+21/40*(1+1/a/x)^(1/4)*x^4/a/(1-1
/a/x)^(1/4)+1/5*(1+1/a/x)^(1/4)*x^5/(1-1/a/x)^(1/4)+1003/128*arctan((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^5+1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1
/4))/a^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6306, 100, 156, 160, 12, 95, 218, 212, 209}

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{1003\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} - \frac{26111\sqrt[4]{\frac{1}{ax}+1}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533x\sqrt[4]{\frac{1}{ax}+1}}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189x^2\sqrt[4]{\frac{1}{ax}+1}}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181x^3\sqrt[4]{\frac{1}{ax}+1}}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}+\frac{21x^4\sqrt[4]{\frac{1}{ax}+1}}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-26111*(1 + 1/(a*x))^(1/4))/(1920*a^5*(1 - 1/(a*x))^(1/4)) + (5533*(1 + 1/
(a*x))^(1/4)*x)/(1920*a^4*(1 - 1/(a*x))^(1/4)) + (1189*(1 + 1/(a*x))^(1/4)*
x^2)/(960*a^3*(1 - 1/(a*x))^(1/4)) + (181*(1 + 1/(a*x))^(1/4)*x^3)/(240*a^2
*(1 - 1/(a*x))^(1/4)) + (21*(1 + 1/(a*x))^(1/4)*x^4)/(40*a*(1 - 1/(a*x))^(1
/4)) + ((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) + (1003*ArcTan[(1
+ 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (1003*ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^6\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}+\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{-\frac{1189}{8a^3}-\frac{543x}{4a^4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{120}\text{Subst}\left(\int \frac{\frac{5533}{16a^4}+\frac{1189x}{4a^5}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{120}\text{Subst}\left(\int \frac{-\frac{15045}{32a^5} - \frac{5533x}{16a^6}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{60}a\text{Subst}\left(\int \frac{15045}{64a^6x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1003\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{256a^5}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1003\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^5}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1003\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} + \frac{1003\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} + \frac{1003\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{-8e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^5} + \frac{122e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{233e^{\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{1661e^{\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{4117e}{192(-1-}}{a^5}$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-8*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]
))^5) + (122*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]))^4) + (233*E^(
```

```
ArcCoth[a*x]/2))/(6*(-1 + E^(2*ArcCoth[a*x]))^3) + (1661*E^(ArcCoth[a*x]/2)
)/(48*(-1 + E^(2*ArcCoth[a*x]))^2) + (4117*E^(ArcCoth[a*x]/2))/(192*(-1 + E
^(2*ArcCoth[a*x]))) + (1003*ArcTan[E^(ArcCoth[a*x]/2)])/128 - (1003*Log[1 -
 E^(ArcCoth[a*x]/2)])/256 + (1003*Log[1 + E^(ArcCoth[a*x]/2)])/256)/a^5
```

Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{2}\coth^{-1}(ax)} x^4\, dx = -\frac{30090\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} -}{3840\,(a^6x -}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="fricas")
[Out] -1/3840*(30090*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 15045*(a*x -
 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*(a*x - 1)*log(((a*x - 1)/(
a*x + 1))^(1/4) - 1) - 2*(384*a^6*x^6 + 1392*a^5*x^5 + 2456*a^4*x^4 + 3826*
a^3*x^3 + 7911*a^2*x^2 - 20578*a*x - 26111)*((a*x - 1)/(a*x + 1))^(3/4))/(a
^6*x - a^5)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)} x^4\, dx = \int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**4,x)
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.96

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{4\left(\frac{58985\,(ax-1)}{ax+1}-\frac{125920\,(ax-1)^2}{(ax+1)^2}+\frac{137930\,(ax-1)^3}{(ax+1)^3}-\frac{72216\,(ax-1)^4}{(ax+1)^4}+\frac{15045\,(ax-1)^5}{(ax+1)^5}-7680\right)}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{21}{4}}-5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}+10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\cdots\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(58985*(a*x - 1)/(a*x + 1) - 125920*(a*x - 1)^2/(a*x + 1)^2 +
137930*(a*x - 1)^3/(a*x + 1)^3 - 72216*(a*x - 1)^4/(a*x + 1)^4 + 15045*(a*x
 - 1)^5/(a*x + 1)^5 - 7680)/(a^6*((a*x - 1)/(a*x + 1))^(21/4) - 5*a^6*((a*x
 - 1)/(a*x + 1))^(17/4) + 10*a^6*((a*x - 1)/(a*x + 1))^(13/4) - 10*a^6*((a*
x - 1)/(a*x + 1))^(9/4) + 5*a^6*((a*x - 1)/(a*x + 1))^(5/4) - a^6*((a*x - 1
)/(a*x + 1))^(1/4)) + 30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{15045\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\cdots\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 + 30720/(a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(49120*(a*x - 1)*((
a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1
))^(3/4)/(a*x + 1)^2 + 33816*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^3 - 7365*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^4 - 20585*((
a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=\frac{1003\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{\frac{787\,(a\,x-1)^2}{6\,(a\,x+1)^2}-\frac{13793\,(a\,x-1)^3}{96\,(a\,x+1)^3}+\frac{3009\,(a\,x-1)^4}{40\,(a\,x+1)^4}-\frac{1003\,(a\,x-1)^5}{64\,(a\,x+1)^5}-\frac{11797\,(a\,x-1)}{192\,(a\,x+1)}+8}{a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-5\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+10\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}-10\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}+5\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}-a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{21/4}}$$

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (1003*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (1003*atan(((a*x - 1)
/(a*x + 1))^(1/4)))/(128*a^5) - ((787*(a*x - 1)^2)/(6*(a*x + 1)^2) - (13793
*(a*x - 1)^3)/(96*(a*x + 1)^3) + (3009*(a*x - 1)^4)/(40*(a*x + 1)^4) - (100
3*(a*x - 1)^5)/(64*(a*x + 1)^5) - (11797*(a*x - 1))/(192*(a*x + 1)) + 8)/(a
^5*((a*x - 1)/(a*x + 1))^(1/4) - 5*a^5*((a*x - 1)/(a*x + 1))^(5/4) + 10*a^5
*((a*x - 1)/(a*x + 1))^(9/4) - 10*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 5*a^5*
((a*x - 1)/(a*x + 1))^(17/4) - a^5*((a*x - 1)/(a*x + 1))^(21/4))
```

3.78 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx=-\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] -2467/192*(1+1/a/x)^(1/4)/a^4/(1-1/a/x)^(1/4)+521/192*(1+1/a/x)^(1/4)*x/a^3
/(1-1/a/x)^(1/4)+113/96*(1+1/a/x)^(1/4)*x^2/a^2/(1-1/a/x)^(1/4)+17/24*(1+1/
a/x)^(1/4)*x^3/a/(1-1/a/x)^(1/4)+1/4*(1+1/a/x)^(1/4)*x^4/(1-1/a/x)^(1/4)+47
5/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used

$= \{6306, 100, 156, 160, 12, 95, 218, 212, 209\}$

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{475\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{475\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} - \frac{2467\sqrt[4]{\frac{1}{ax}+1}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521x\sqrt[4]{\frac{1}{ax}+1}}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113x^2\sqrt[4]{\frac{1}{ax}+1}}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{17x^3\sqrt[4]{\frac{1}{ax}+1}}{24a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-2467*(1 + 1/(a*x))^(1/4))/(192*a^4*(1 - 1/(a*x))^(1/4)) + (521*(1 + 1/(a*
x))^(1/4)*x)/(192*a^3*(1 - 1/(a*x))^(1/4)) + (113*(1 + 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 - 1/(a*x))^(1/4)) + (17*(1 + 1/(a*x))^(1/4)*x^3)/(24*a*(1 - 1/(a
*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^5\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}+\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{24}\text{Subst}\left(\int \frac{\frac{1425}{16a^4}+\frac{521x}{8a^5}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{12}a\text{Subst}\left(\int -\frac{1425}{32a^5x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{475\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{128a^4}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{475\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32a^4}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{475\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{475\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{475\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{-3072e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{5248e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{7376e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{6292e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 2850\arctan}{384a^4}$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-3072*E^(ArcCoth[a*x]/2) + (1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*
x]))^4 + (5248*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (7376*E^(A
rcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (6292*E^(ArcCoth[a*x]/2))/(-1
 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(ArcCoth[a*x]/2)] - 1425*Log[1 - E^(
ArcCoth[a*x]/2)] + 1425*Log[1 + E^(ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx =$$
$$-\frac{2850\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{384\,(a^5x-a^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="fricas")
```

```
[Out] -1/384*(2850*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 1425*(a*x - 1)
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1425*(a*x - 1)*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1) - 2*(48*a^5*x^5 + 184*a^4*x^4 + 362*a^3*x^3 + 747*a^2*x^2
- 1946*a*x - 2467)*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*x - a^4)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**3,x)
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{1}{384}\,a\left(\frac{4\left(\frac{4645\,(ax-1)}{ax+1}-\frac{7483\,(ax-1)^2}{(ax+1)^2}+\frac{5415\,(ax-1)^3}{(ax+1)^3}-\frac{1425\,(ax-1)^4}{(ax+1)^4}-768\right)}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+6\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="maxima")
[Out] 1/384*a*(4*(4645*(a*x - 1)/(a*x + 1) - 7483*(a*x - 1)^2/(a*x + 1)^2 + 5415*
(a*x - 1)^3/(a*x + 1)^3 - 1425*(a*x - 1)^4/(a*x + 1)^4 - 768)/(a^5*((a*x -
1)/(a*x + 1))^(17/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 6*a^5*((a*x - 1
)/(a*x + 1))^(9/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(5/4) + a^5*((a*x - 1)/(a*
x + 1))^(1/4)) - 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}\,a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{3072}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^5 + 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^5 + 3072/(a^5*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(2875*(a*x - 1)*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1) - 2343*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4
)/(a*x + 1)^2 + 657*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1
573*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.84

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{475\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{\frac{7483\,(a\,x-1)^2}{96\,(a\,x+1)^2} - \frac{1805\,(a\,x-1)^3}{32\,(a\,x+1)^3} + \frac{475\,(a\,x-1)^4}{32\,(a\,x+1)^4} - \frac{4645\,(a\,x-1)}{96\,(a\,x+1)} + 8}{a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - 4\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4} + 6\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4} - 4\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4} + a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}$$

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (475*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (475*atan(((a*x - 1)/(a
*x + 1))^(1/4)))/(64*a^4) - ((7483*(a*x - 1)^2)/(96*(a*x + 1)^2) - (1805*(a
*x - 1)^3)/(32*(a*x + 1)^3) + (475*(a*x - 1)^4)/(32*(a*x + 1)^4) - (4645*(a
*x - 1))/(96*(a*x + 1)) + 8)/(a^4*((a*x - 1)/(a*x + 1))^(1/4) - 4*a^4*((a*x
 - 1)/(a*x + 1))^(5/4) + 6*a^4*((a*x - 1)/(a*x + 1))^(9/4) - 4*a^4*((a*x -
1)/(a*x + 1))^(13/4) + a^4*((a*x - 1)/(a*x + 1))^(17/4))
```

3.79 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 213

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=-\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] -287/24*(1+1/a/x)^(1/4)/a^3/(1-1/a/x)^(1/4)+61/24*(1+1/a/x)^(1/4)*x/a^2/(1-
1/a/x)^(1/4)+13/12*(1+1/a/x)^(1/4)*x^2/a/(1-1/a/x)^(1/4)+1/3*(1+1/a/x)^(1/4
)*x^3/(1-1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3+55/8
*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used

$= \{6306, 100, 156, 160, 12, 95, 218, 212, 209\}$

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{55\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{55\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{287\sqrt[4]{\frac{1}{ax}+1}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61x\sqrt[4]{\frac{1}{ax}+1}}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{13x^2\sqrt[4]{\frac{1}{ax}+1}}{12a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (-287*(1 + 1/(a*x))^(1/4))/(24*a^3*(1 - 1/(a*x))^(1/4)) + (61*(1 + 1/(a*x))
^(1/4)*x)/(24*a^2*(1 - 1/(a*x))^(1/4)) + (13*(1 + 1/(a*x))^(1/4)*x^2)/(12*a
*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) +
 (55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) + (55*ArcTanh
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^4\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}+\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a\text{Subst}\left(\int \frac{165}{16a^4x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{55\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{55\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.98 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.07

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{8e^{\frac{9}{2}\coth^{-1}(ax)}\left(-\frac{27653}{195} - \frac{899079}{512}e^{-8\coth^{-1}(ax)} - \frac{3309759e^{-6\coth^{-1}(ax)}}{2560} + \frac{8521937e^{-4\coth^{-1}(ax)}}{7680} + \frac{69571361e^{-2\coth^{-1}(ax)}}{99840}\right.}{}$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

[Out] (-8*E^((9*ArcCoth[a*x])/2)*(-27653/195 - 899079/(512*E^(8*ArcCoth[a*x])) - 3309759/(2560*E^(6*ArcCoth[a*x])) + 8521937/(7680*E^(4*ArcCoth[a*x])) + 695 71361/(99840*E^(2*ArcCoth[a*x])) - (653*E^(2*ArcCoth[a*x]))/390 + (133407*H ypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/512 + (899079*Hypergeome tric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(512*E^(8*ArcCoth[a*x])) + (60267 *Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(64*E^(6*ArcCoth[a*x])) - (382227*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(256*E^(4*A rcCoth[a*x])) - (40827*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/ (64*E^(2*ArcCoth[a*x])) + (E^(2*ArcCoth[a*x])*(1117 + 1906*E^(2*ArcCoth[a*x]) + 821*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 25/4 }, E^(2*ArcCoth[a*x])])/3094 + (4*E^(2*ArcCoth[a*x])*(27 + 50*E^(2*ArcCoth[a*x]) + 23*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 13/4}, {1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(2*ArcCoth[a*x])*Hypergeometric PFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (16*E^(4*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1 , 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(6*ArcCoth[a*x])*HypergeometricPF Q[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547))/(9 *a^3)

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)

[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{330\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) - 2}{48\,(a^4x-a^3)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="fricas")

[Out] -1/48*(330*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*(a*x - 1)*lo g(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*(8*a^4*x^4 + 34*a^3*x^3 + 87*a^2*x^2 - 226*a*x - 287)*((a* x - 1)/(a*x + 1))^(3/4))/(a^4*x - a^3)

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**2,x)
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\left(\frac{4\left(\frac{425\,(ax-1)}{ax+1}-\frac{462\,(ax-1)^2}{(ax+1)^2}+\frac{165\,(ax-1)^3}{(ax+1)^3}-96\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\log\left(\left(\frac{a}{a}\right.\right.}{a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="maxima")
[Out] -1/48*a*(4*(425*(a*x - 1)/(a*x + 1) - 462*(a*x - 1)^2/(a*x + 1)^2 + 165*(a*
x - 1)^3/(a*x + 1)^3 - 96)/(a^4*((a*x - 1)/(a*x + 1))^(13/4) - 3*a^4*((a*x
- 1)/(a*x + 1))^(9/4) + 3*a^4*((a*x - 1)/(a*x + 1))^(5/4) - a^4*((a*x - 1)/
(a*x + 1))^(1/4)) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{384}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 + 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 + 384/(a^4*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(174*(a*x - 1)*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1) - 69*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^2 - 137*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{55\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{\frac{77\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}-\frac{55\,{\left(a\,x-1\right)}^3}{4\,{\left(a\,x+1\right)}^3}-\frac{425\,\left(a\,x-1\right)}{12\,\left(a\,x+1\right)}+8}{a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (55*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (55*atan(((a*x - 1)/(a*x
+ 1))^(1/4)))/(8*a^3) - ((77*(a*x - 1)^2)/(2*(a*x + 1)^2) - (55*(a*x - 1)^3
)/(4*(a*x + 1)^3) - (425*(a*x - 1))/(12*(a*x + 1)) + 8)/(a^3*((a*x - 1)/(a*
x + 1))^(1/4) - 3*a^3*((a*x - 1)/(a*x + 1))^(5/4) + 3*a^3*((a*x - 1)/(a*x +
 1))^(9/4) - a^3*((a*x - 1)/(a*x + 1))^(13/4))
```

3.80 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 176

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -25/2*(1+1/a/x)^(1/4)/a^2/(1-1/a/x)^(1/4)+5/4*(1+1/a/x)^(5/4)*x/a/(1-1/a/x)
^(1/4)+1/2*(1+1/a/x)^(9/4)*x^2/(1-1/a/x)^(1/4)+25/4*arctan((1+1/a/x)^(1/4)/
(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6306, 98, 96, 95, 218, 212, 209}

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx=\frac{25\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}-\frac{25\sqrt[4]{\frac{1}{ax}+1}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5x\left(\frac{1}{ax}+1\right)^{5/4}}{4a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (-25*(1 + 1/(a*x))^(1/4))/(2*a^2*(1 - 1/(a*x))^(1/4)) + (5*(1 + 1/(a*x))^(5
/4)*x)/(4*a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x
))^(1/4)) + (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
```

```
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4a}\\
&= \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{25\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}
\end{aligned}
$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{25\text{Subst}\left(\int\frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx,x,\frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{25\text{Subst}\left(\int\frac{1}{-1+x^4}dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\text{Subst}\left(\int\frac{1}{1-x^2}dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{Subst}\left(\int\frac{1}{1+x^2}dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(25-45e^{2\coth^{-1}(ax)}+16e^{4\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+25\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+25\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x,x]
```

[Out] ((-2*E^(ArcCoth[a*x]/2)*(25 - 45*E^(2*ArcCoth[a*x]) + 16*E^(4*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x]))^2 + 25*ArcTan[E^(ArcCoth[a*x]/2)] + 25*ArcTanh [E^(ArcCoth[a*x]/2)])/(4*a^2)

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)

[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx =$$
$$-\frac{50\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(2}{8\,(a^3x-a^2)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="fricas")

[Out] -1/8*(50*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1 /4) - 1) - 2*(2*a^3*x^3 + 11*a^2*x^2 - 34*a*x - 43)*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*x - a^2)

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x,x)

[Out] Integral(x/((a*x - 1)/(a*x + 1))**(5/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}\,a\left(\frac{4\left(\frac{45\,(ax-1)}{ax+1}-\frac{25\,(ax-1)^2}{(ax+1)^2}-16\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\,\log\left(\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(45*(a*x - 1)/(a*x + 1) - 25*(a*x - 1)^2/(a*x + 1)^2 - 16)/(a^3*((
a*x - 1)/(a*x + 1))^(9/4) - 2*a^3*((a*x - 1)/(a*x + 1))^(5/4) + a^3*((a*x -
 1)/(a*x + 1))^(1/4)) - 50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4)
- 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx =$$
$$-\frac{1}{8}\,a\left(\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{64}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{9\,(ax-}{}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 + 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 +
64/(a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(9*(a*x - 1)*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) - 13*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*((a*x - 1)/(a*x + 1
) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx=\frac{25\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{25\,(a\,x-1)^2}{2\,(a\,x+1)^2}-\frac{45\,(a\,x-1)}{2\,(a\,x+1)}+8}{a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-2\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}$$

```
[In] int(x/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (25*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (25*atan(((a*x - 1)/(a*x
+ 1))^(1/4)))/(4*a^2) - ((25*(a*x - 1)^2)/(2*(a*x + 1)^2) - (45*(a*x - 1))/
(2*(a*x + 1)) + 8)/(a^2*((a*x - 1)/(a*x + 1))^(1/4) - 2*a^2*((a*x - 1)/(a*x
 + 1))^(5/4) + a^2*((a*x - 1)/(a*x + 1))^(9/4))
```

3.81 $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx=-\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] -10*(1+1/a/x)^(1/4)/a/(1-1/a/x)^(1/4)+(1+1/a/x)^(5/4)*x/(1-1/a/x)^(1/4)+5*a
rctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a+5*arctanh((1+1/a/x)^(1/4)/(1-1/a/x
)^(1/4))/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6305, 96, 95, 218, 212, 209}

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{5\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{10\sqrt[4]{\frac{1}{ax}+1}}{a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-10*(1 + 1/(a*x))^(1/4))/(a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(5/4)*x)
/(1 - 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/
a + (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{5/4} x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4} x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4} x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{10\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-5+4e^{2\coth^{-1}(ax)}\right)}{-1+e^{2\coth^{-1}(ax)}} + 5\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 5\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2),x]
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-5 + 4*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
])) + 5*ArcTan[E^(ArcCoth[a*x]/2)] + 5*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4),x)
[Out] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx =$$
$$-\frac{10\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(a^2x^2 % truncated on page
}{2\,(a^2x-a)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] -1/2*(10*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 5*(a*x - 1)*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1) - 2*(a^2*x^2 - 8*a*x - 9)*((a*x - 1)/(a*x + 1))^(3/4))/(a^2*x - a)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4),x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(-5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx =$$
$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
[Out] -1/2*a*(4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)/(a*x + 1))^(5/4) - a^
2*((a*x - 1)/(a*x + 1))^(1/4)) + 10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2
 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 5*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} - \frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2} + \frac{5\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2} + \frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1} - \left(\frac{ax}{ax}\right.\right.}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 + 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*
(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{\frac{10\,(a\,x-1)}{a\,x+1}-8}{a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - a\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}} - \frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a} + \frac{5\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

```
[In] int(1/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] ((10*(a*x - 1))/(a*x + 1) - 8)/(a*((a*x - 1)/(a*x + 1))^(1/4) - a*((a*x - 1
)/(a*x + 1))^(5/4)) - (5*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a + (5*atanh(((
a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.82 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 320

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

```
[Out] -8*(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-
arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6306, 100, 21, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx=\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)+2\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + (4a)\text{Subst}\left(\int \frac{-\frac{1}{4a}+\frac{x}{4a^2}}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a}\\
&\quad - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - 4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$- 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$+ 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$+ 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$- \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$- \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$- \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$- \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$+ \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.09

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = 8e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x,x]
[Out] 8*E^(ArcCoth[a*x]/2)*(-1 + Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x]
)])
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{\sqrt{2}(-(i-1)\,ax+i-1)\log\left((i+1)\,\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\sqrt{2}((i+1)\,ax-i-1)\log\left(-(i-1)\,\sqrt{2}+2\left(\frac{ax}{ax}\right.\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(-(I - 1)*a*x + I - 1)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x
 + 1))^(1/4)) + sqrt(2)*((I + 1)*a*x - I - 1)*log(-(I - 1)*sqrt(2) + 2*((a*
x - 1)/(a*x + 1))^(1/4)) + sqrt(2)*(-(I + 1)*a*x + I + 1)*log((I - 1)*sqrt(
2) + 2*((a*x - 1)/(a*x + 1))^(1/4)) + sqrt(2)*((I - 1)*a*x - I + 1)*log(-(I
 + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(a*x - 1)*arctan(((a*x -
 1)/(a*x + 1))^(1/4)) + 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) -
2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) - 16*(a*x + 1)*((a*x - 1)/
(a*x + 1))^(3/4))/(a*x - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{a}{a}\right.\right.}{a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.79

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.37

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{8}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}} -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right.\right.$$

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(1
/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 -
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1
i) - 8/((a*x - 1)/(a*x + 1))^(1/4)
```

3.83 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 299

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}+\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

```
[Out] -5*a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-4*a*(1+1/a/x)^(5/4)/(1-1/a/x)^(1/4)-5/
2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctan
(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/4*a*ln(1-(1-1/a/x)^(1
/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+5/4*a*
ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2
))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6306, 49, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}} - \frac{5a\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}} - \frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - 5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}} + \frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (4*a*(1 + 1/(a*x))^(5/4))/(1
 - 1/(a*x))^(1/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a
*x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1
/(a*x))^(1/4)])/Sqrt[2] - (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
- (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) + (5*a*Lo
g[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}+5\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-(10a)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)
\end{aligned}$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-(10a)\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$
$$+(5a)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$-(5a)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$
$$-\frac{1}{2}(5a)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$-\frac{1}{2}(5a)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$-\frac{(5a)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$
$$-\frac{(5a)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}+\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$-\frac{(5a)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$+\frac{(5a)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$+\frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$-\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}+\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\frac{10e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-\frac{8e^{\frac{5}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-\frac{5\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{5\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] a*((-10*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - (8*E^((5*ArcCoth[a*x
])/2))/(1 + E^(2*ArcCoth[a*x])) - (5*ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)]
)/Sqrt[2] + (5*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (5*Log[1 -
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]) + (5*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^2}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.81

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{5\left(-a^4\right)^{\frac{1}{4}}\left(ax^2-x\right)\log\left(125\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+125\left(-a^4\right)^{\frac{3}{4}}\right)-5\left(-a^4\right)^{\frac{1}{4}}\left(i\,ax^2-i\,x\right)\log\left(125\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+125\right.}{}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")

[Out] -1/2*(5*(-a^4)^(1/4)*(a*x^2 - x)*log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 125*(-a^4)^(3/4)) - 5*(-a^4)^(1/4)*(I*a*x^2 - I*x)*log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 125*I*(-a^4)^(3/4)) - 5*(-a^4)^(1/4)*(-I*a*x^2 + I*x)*log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) - 125*I*(-a^4)^(3/4)) - 5*(-a^4)^(1/4)*(a*x^2 - x)*log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) - 125*(-a^4)^(3/4)) + 2*(9*a^2*x^2 + 8*a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^2 - x)

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**2,x)

[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(5/4)), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")

[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/(((a*x - 1)/(a*x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.73

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((a*x - 1)*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.36

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$= 5\,(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - 5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \frac{8\,a+\frac{10\,a\,(a\,x-1)}{a\,x+1}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+\left(\frac{a\,x-1}{a\,x+1}\right.}$$

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] 5*(-1)^(1/4)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 5*(-1)^(1/4)
*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (8*a + (10*a*(a*x - 1))/(
a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/4) + ((a*x - 1)/(a*x + 1))^(5/4))
```

3.84 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 351

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

```
[Out] -25/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-5/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)
^(5/4)-2*a^2*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-25/8*a^2*arctan(-1+(1-1/a/x)^(
1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
```

```
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 79, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}} - \frac{25a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}} - \frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4} - \frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} + \frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-25*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 - (5*a^2*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(5/4))/2 - (2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1
/4) + (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) - (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}(25a)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}\\
&\quad - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{8}(25a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int\frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{8}\left(25a^2\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{8}\left(25a^2\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x,\right.$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$
$$-\frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$
$$-\frac{(25a^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$
$$+\frac{(25a^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$
$$+\frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}-\frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$
$$-\frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(-128e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2} - \frac{104e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - 50\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 50\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 25\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)} + e^{\coth^{-1}(ax)}\right) + 25\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)} + e^{\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (a^2*(-128*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a
*x]))^2 - (104*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 50*Sqrt[2]*Ar
cTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)] + 50*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(Arc
Coth[a*x]/2)] - 25*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a
*x]] + 25*Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^3}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.74

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{25\,(-a^8)^{\frac{1}{4}}\,(ax^3-x^2)\log\left(15625\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 15625\,(-a^8)^{\frac{3}{4}}\right) - 25\,(-a^8)^{\frac{1}{4}}\,(i\,ax^3 - i\,x^2)\log\left(15625\,a^6\left(\frac{a}{a}\right.\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(25*(-a^8)^(1/4)*(a*x^3 - x^2)*log(15625*a^6*((a*x - 1)/(a*x + 1))^(1/
4) + 15625*(-a^8)^(3/4)) - 25*(-a^8)^(1/4)*(I*a*x^3 - I*x^2)*log(15625*a^6*
```

```
((a*x - 1)/(a*x + 1))^(1/4) + 15625*I*(-a^8)^(3/4)) - 25*(-a^8)^(1/4)*(-I*a
*x^3 + I*x^2)*log(15625*a^6*((a*x - 1)/(a*x + 1))^(1/4) - 15625*I*(-a^8)^(3
/4)) - 25*(-a^8)^(1/4)*(a*x^3 - x^2)*log(15625*a^6*((a*x - 1)/(a*x + 1))^(1
/4) - 15625*(-a^8)^(3/4)) + 2*(43*a^3*x^3 + 34*a^2*x^2 - 11*a*x - 2)*((a*x
- 1)/(a*x + 1))^(3/4))/(a*x^3 - x^2)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**3,x)
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(25\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
[Out] -1/16*(25*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1))*a + 8*(45*(a*x - 1)*a/(a*x + 1) + 25*(a*x - 1)^2*
a/(a*x + 1)^2 + 16*a)/(((a*x - 1)/(a*x + 1))^(9/4) + 2*((a*x - 1)/(a*x + 1)
)^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 128*a/((a*x - 1)/(a*x + 1))^(1/4) +
8*(9*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 13*a*((a*x - 1)/(a
*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.43

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{25\,{(-1)}^{1/4}\,a^2\,\mathrm{atanh}\left({(-1)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4} - \frac{25\,{(-1)}^{1/4}\,a^2\,\mathrm{atan}\left({(-1)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4} - \frac{8\,a^2+\frac{25\,a^2\,{(a\,x-1)}^2}{2\,{(a\,x+1)}^2}+\frac{45\,a^2\,(a\,x-1)}{2\,(a\,x+1)}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}$$

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] (25*(-1)^(1/4)*a^2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (25*(
-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (8*a^2 + (2
5*a^2*(a*x - 1)^2)/(2*(a*x + 1)^2) + (45*a^2*(a*x - 1))/(2*(a*x + 1)))/(((a
*x - 1)/(a*x + 1))^(1/4) + 2*((a*x - 1)/(a*x + 1))^(5/4) + ((a*x - 1)/(a*x
+ 1))^(9/4))
```

3.85 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 385

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} -\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4}+\frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\frac{55a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\ldots$$

```
[Out] -55/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-11/4*a^3*(1-1/a/x)^(3/4)*(1+1/a/x
)^(5/4)-2*a^3*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-1/3*a^3*(1-1/a/x)^(3/4)*(1+1/
a/x)^(9/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(
1/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-55
/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a
/x)^(1/2))*2^(1/2)+55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6306, 91, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}dx = \frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}} - \frac{55a^3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4} - \frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4} - \frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{55a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}} +$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (-55*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 - (11*a^3*(1 - 1/(a*x))
^(3/4)*(1 + 1/(a*x))^(5/4))/4 - (2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(
1/4) - (a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(9/4))/3 + (55*a^3*ArcTan[1 -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
- (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*
x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (55*a^3*Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \left(2a^3\right)\text{Subst}\left(\int \frac{\left(\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{8}\left(55a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{16}\left(55a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} - \frac{1}{4}\left(55a^3\right)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} - \frac{1}{4}\left(55a^3\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{8}\left(55a^3\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{8}\left(55a^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} - \frac{1}{16}(55a^3)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{16}(55a$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} - \frac{55a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} + \frac{55a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right.}{16\sqrt{2}}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{55a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{8\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.27

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= a^3\left(-\frac{e^{\frac{1}{2}\coth^{-1}(ax)}\left(165+462e^{2\coth^{-1}(ax)}+425e^{4\coth^{-1}(ax)}+96e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{55}{32}\text{RootSum}\left[1+\#1^4\&, \frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] a^3*(-1/12*(E^(ArcCoth[a*x]/2)*(165 + 462*E^(2*ArcCoth[a*x]) + 425*E^(4*Arc
Coth[a*x]) + 96*E^(6*ArcCoth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 - (55*RootS
um[1 + #1^4 & , (ArcCoth[a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ])/3
2)
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^{4}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{165\left(-a^{12}\right)^{\frac{1}{4}}\left(ax^4-x^3\right)\log\left(166375\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+166375\left(-a^{12}\right)^{\frac{3}{4}}\right)+165\left(-a^{12}\right)^{\frac{1}{4}}\left(-i\,ax^4+i\,x^3\right)\log\left(16\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(165*(-a^12)^(1/4)*(a*x^4 - x^3)*log(166375*a^9*((a*x - 1)/(a*x + 1))
^(1/4) + 166375*(-a^12)^(3/4)) + 165*(-a^12)^(1/4)*(-I*a*x^4 + I*x^3)*log(1
66375*a^9*((a*x - 1)/(a*x + 1))^(1/4) + 166375*I*(-a^12)^(3/4)) + 165*(-a^1
2)^(1/4)*(I*a*x^4 - I*x^3)*log(166375*a^9*((a*x - 1)/(a*x + 1))^(1/4) - 166
375*I*(-a^12)^(3/4)) - 165*(-a^12)^(1/4)*(a*x^4 - x^3)*log(166375*a^9*((a*x
 - 1)/(a*x + 1))^(1/4) - 166375*(-a^12)^(3/4)) + 2*(287*a^4*x^4 + 226*a^3*x
^3 - 87*a^2*x^2 - 34*a*x - 8)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^4 - x^3)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**4,x)
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.75

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(165\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
[Out] -1/96*(165*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(425*(a*x - 1)*a^2/(a*x + 1) + 462*(a*x
- 1)^2*a^2/(a*x + 1)^2 + 165*(a*x - 1)^3*a^2/(a*x + 1)^3 + 96*a^2)/(((a*x -
 1)/(a*x + 1))^(13/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + 3*((a*x - 1)/(a*x +
 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) - 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 768*a^2/((a*x - 1)/(a*x
+ 1))^(1/4) + 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) +
69*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 137*a^2*((a*x
- 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{8\,a^3+\frac{77\,a^3\,(a\,x-1)^2}{2\,(a\,x+1)^2}+\frac{55\,a^3\,(a\,x-1)^3}{4\,(a\,x+1)^3}+\frac{425\,a^3\,(a\,x-1)}{12\,(a\,x+1)}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}+\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}$$

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] (55*(-1)^(1/4)*a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (55*(
-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (8*a^3 + (7
7*a^3*(a*x - 1)^2)/(2*(a*x + 1)^2) + (55*a^3*(a*x - 1)^3)/(4*(a*x + 1)^3) +
 (425*a^3*(a*x - 1))/(12*(a*x + 1)))/(((a*x - 1)/(a*x + 1))^(1/4) + 3*((a*x
 - 1)/(a*x + 1))^(5/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + ((a*x - 1)/(a*x +
1))^(13/4))
```

3.86 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4}-\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] 611/1920*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4-269/960*(1-1/a/x)^(1/4)*(1+1
/a/x)^(3/4)*x^2/a^3+11/48*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2-9/40*(1-1
/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5+3
1/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-31/128*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{31\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{611x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{1920a^4}-\frac{269x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{960a^3}+\frac{11x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{48a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{40a}$$

```
[In] Int[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (611*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(1920*a^4) - (269*(1 - 1/(a
*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4)*x^3)/(48*a^2) - (9*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^6\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{1}{5}\text{Subst}\left(\int\frac{-\frac{9}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{1}{20}\text{Subst}\left(\int\frac{-\frac{55}{4a^2}+\frac{27x}{2a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{1}{60}\text{Subst}\left(\int\frac{-\frac{269}{8a^3}+\frac{55x}{2a^4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{1}{120}\text{Subst}\left(\int\frac{-\frac{611}{16a^4}+\frac{269x}{8a^5}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4}-\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{1}{120}\text{Subst}\left(\int-\frac{465}{32a^5x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{31\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{256a^5}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{31\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^5}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{31\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} + \frac{31\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+}}{\sqrt[4]{1}}\right.}{128a^5}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} - \frac{31\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{24576e^{\frac{19}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5} - \frac{62976e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{64640e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{34000e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{9620e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 930\arctan\left(\right.}{3840a^5}$$

```
[In] Integrate[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((24576*E^((19*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (62976*E^((1
5*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^((11*ArcCoth[a*x
])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (34000*E^((7*ArcCoth[a*x])/2))/(-1 + E
^(2*ArcCoth[a*x]))^2 + (9620*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x
])) - 930*ArcTan[E^(-1/2*ArcCoth[a*x])] + 465*Log[1 - E^(-1/2*ArcCoth[a*x])
] - 465*Log[1 + E^(-1/2*ArcCoth[a*x])])/(3840*a^5)
```

Maple [F]

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

```
[In] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\,(384\,a^5x^5-48\,a^4x^4+8\,a^3x^3-98\,a^2x^2+73\,ax+611)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-465\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{3840\,a^5}$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
[Out] 1/3840*(2*(384*a^5*x^5 - 48*a^4*x^4 + 8*a^3*x^3 - 98*a^2*x^2 + 73*a*x + 611
)*((a*x - 1)/(a*x + 1))^(1/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 4
65*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 465*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1))/a^5
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \int x^4\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(1/4),x)
[Out] Integral(x**4*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{4\left(2405\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\,\mathrm{arcta}}{}\right.$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
[Out] -1/3840*a*(4*(2405*((a*x - 1)/(a*x + 1))^(17/4) - 1120*((a*x - 1)/(a*x + 1)
)^(13/4) + 5090*((a*x - 1)/(a*x + 1))^(9/4) - 696*((a*x - 1)/(a*x + 1))^(5/
4) + 465*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
- 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^6
/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 465*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{696(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}\right.}{}\right.$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 - 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(696*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 5090*(a*x -
1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1120*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1)^3 - 2405*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1)^4 - 465*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$

$$=\frac{\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{509\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{6}+\frac{481\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{192}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}$$
$$-\frac{31\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{31\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((31*((a*x - 1)/(a*x + 1))^(1/4))/64 - (29*((a*x - 1)/(a*x + 1))^(5/4))/40
+ (509*((a*x - 1)/(a*x + 1))^(9/4))/96 - (7*((a*x - 1)/(a*x + 1))^(13/4))/6
 + (481*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*x
 + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)^
4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31*atan
(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (31*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(128*a^5)
```

3.87 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] -83/192*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+29/96*(1-1/a/x)^(1/4)*(1+1/a/
x)^(3/4)*x^2/a^2-7/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(
1/4)*(1+1/a/x)^(3/4)*x^4-11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+
11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{6306, 101, 156, 12, 95, 304, 209, 212\}$

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{11\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{11\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} - \frac{83x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{192a^3} + \frac{29x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{96a^2} + \frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} - \frac{7x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a}$$

```
[In] Int[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (-83*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(192*a^3) + (29*(1 - 1/(a*x
))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(96*a^2) - (7*(1 - 1/(a*x))^(1/4)*(1 + 1/
(a*x))^(3/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4
- (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^5\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}$$

$$+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4+\frac{1}{12}\text{Subst}\left(\int\frac{-\frac{29}{4a^2}+\frac{7x}{a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}$$

$$+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{1}{24}\text{Subst}\left(\int\frac{-\frac{83}{8a^3}+\frac{29x}{4a^4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}$$

$$+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4+\frac{1}{24}\text{Subst}\left(\int-\frac{33}{16a^4x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}$$

$$+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{11\text{Subst}\left(\int\frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{128a^4}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}$$

$$+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{11\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32a^4}$$

$$=-\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4+\frac{11\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{11\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

$$=-\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$

$$=\frac{\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{3200e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{2512e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{980e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+66\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-33}{384a^4}$$

```
[In] Integrate[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (3200*E^((11*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^((7*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^2 - (980*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*Ar
cCoth[a*x])) + 66*ArcTan[E^(-1/2*ArcCoth[a*x])] - 33*Log[1 - E^(-1/2*ArcCot
h[a*x])] + 33*Log[1 + E^(-1/2*ArcCoth[a*x])])/(384*a^4)
```

Maple [F]

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

[In] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)

[Out] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(48\,a^4x^4-8\,a^3x^3+2\,a^2x^2-25\,ax-83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-33\,\mathrm{lo}}{384\,a^4}$$

[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")

[Out] 1/384*(2*(48*a^4*x^4 - 8*a^3*x^3 + 2*a^2*x^2 - 25*a*x - 83)*((a*x - 1)/(a*x + 1))^(1/4) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/4),x)

[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(1/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}\,a\left(\frac{4\left(245\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-107\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+279\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\,\log\left(\right.}{}\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(245*((a*x - 1)/(a*x + 1))^(13/4) - 107*((a*x - 1)/(a*x + 1))^(
9/4) + 279*((a*x - 1)/(a*x + 1))^(5/4) - 33*((a*x - 1)/(a*x + 1))^(1/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{279\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{}\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^5 - 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5 +
 4*(279*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 107*(a*x - 1)^2*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 245*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(1/4)/(a*x + 1)^3 - 33*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*
x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx=\frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{93\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{107\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{245\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}+\frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (11*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - ((11*((a*x - 1)/(a*x + 1)
)^(1/4))/32 - (93*((a*x - 1)/(a*x + 1))^(5/4))/32 + (107*((a*x - 1)/(a*x +
1))^(9/4))/96 - (245*((a*x - 1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x -
 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a
*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (11*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(64*a^4)
```

3.88 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 11/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2-5/12*(1-1/a/x)^(1/4)*(1+1/a/x)^
(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3+3/8*arctan((1+1/a/x)^(1
/4)/(1-1/a/x)^(1/4))/a^3-3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{6306, 101, 156, 12, 95, 304, 209, 212\}$

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{3\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{3\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{11x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2} + \frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} - \frac{5x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

```
[In] Int[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) - (5*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
- (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^4\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{5}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}+\frac{5x}{2a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{6}\text{Subst}\left(\int -\frac{9}{8a^3x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.31 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.17

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(-22034705 - 26688365e^{2\coth^{-1}(ax)} - 3731255e^{4\coth^{-1}(ax)} + 3122405e^{6\coth^{-1}(ax)} + 220347\right.}{}$$

```
[In] Integrate[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -1/221760*(-22034705 - 26688365*E^(2*ArcCoth[a*x]) - 3731255*E^(4*ArcCoth[a
*x]) + 3122405*E^(6*ArcCoth[a*x]) + 22034705*Hypergeometric2F1[3/4, 1, 7/4,
 E^(2*ArcCoth[a*x])] + 17244920*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1
, 7/4, E^(2*ArcCoth[a*x])] - 9077530*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3
/4, 1, 7/4, E^(2*ArcCoth[a*x])] - 7043960*E^(6*ArcCoth[a*x])*Hypergeometric
2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 446985*E^(8*ArcCoth[a*x])*Hypergeome
tric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(685 + 10
90*E^(2*ArcCoth[a*x]) + 437*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2,
2, 2}, {1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(21 + 38
*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2, 2,
2, 2}, {1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*Hyper
geometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])]
+ 8192*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1,
 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ
[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5*
ArcCoth[a*x])/2))
```

Maple [F]

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

[In] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)

[Out] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.57

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^2 \, dx = \frac{2\,(8\,a^3x^3 - 2\,a^2x^2 + ax + 11)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{48\,a^3}$$

[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")

[Out] 1/48*(2*(8*a^3*x^3 - 2*a^2*x^2 + a*x + 11)*((a*x - 1)/(a*x + 1))^(1/4) - 18 *arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^2 \, dx = \int x^2 \sqrt[4]{\frac{ax-1}{ax+1}}\, dx$$

[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/4),x)

[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(1/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^2 \, dx = -\frac{1}{48}\,a\left(\frac{4\left(29\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}} - 6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} + 9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1} - \frac{3(ax-1)^2a^4}{(ax+1)^2} + \frac{(ax-1)^3a^4}{(ax+1)^3} - a^4} + \frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4} + \frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^4} - \frac{9\,\log}{}\right.$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(29*((a*x - 1)/(a*x + 1))^(9/4) - 6*((a*x - 1)/(a*x + 1))^(5/4)
+ 9*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2
*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 - 9*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{a^4(}\right.$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 29*(a*x - 1)^2*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} - \frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3} - \frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((3*((a*x - 1)/(a*x + 1))^(1/4))/4 - ((a*x - 1)/(a*x + 1))^(5/4)/2 + (29*((
a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^
3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*x -
 1)/(a*x + 1))^(1/4)))/(8*a^3) - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*
a^3)
```

3.89 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=-\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2-\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -1/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4
)*x^2-1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6306, 98, 96, 95, 304, 209, 212}

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

```
[In] Int[x/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -1/4*((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/a + ((1 - 1/(a*x))^(5/4)*(
1 + 1/(a*x))^(3/4)*x^2)/2 - ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^3\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - \frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{-\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-5+e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - \arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

```
[In] Integrate[x/E^(ArcCoth[a*x]/2),x]
[Out] ((-2*E^((3*ArcCoth[a*x])/2)*(-5 + E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a
*x]))^2 - ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

```
[In] int(x*((a*x-1)/(a*x+1))^(1/4),x)
[Out] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2-ax-3)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
[Out] 1/8*(2*(2*a^2*x^2 - a*x - 3)*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x -
 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/4),x)
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}\,a\left(\frac{4\left(5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
[Out] -1/8*a*(4*(5*((a*x - 1)/(a*x + 1))^(5/4) - ((a*x - 1)/(a*x + 1))^(1/4))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 +
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} + \frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3} - \frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3} + \frac{4\left(\frac{5(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1} - \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1)/a^3 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*(5*(a*
x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))
/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2} - \frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}} + \frac{\operatorname{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

```
[In] int(x*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] atan(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2) - (((a*x - 1)/(a*x + 1))^(1/4)/2
- (5*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 -
 (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2)
```

3.90 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
-arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6305, 96, 95, 304, 209, 212}

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

```
[In] Int[E^(-1/2*ArcCoth[a*x]),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{\text{Subst}\left(\int\frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{2\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x-\frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx=-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)}{3a}$$

```
[In] Integrate[E^(-1/2*ArcCoth[a*x]),x]
[Out] (-8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, E^(2*ArcCoth[a*x]
)])/(3*a)
```

Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

[In] int(((a*x-1)/(a*x+1))^(1/4),x)

[Out] int(((a*x-1)/(a*x+1))^(1/4),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} dx = \frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2a}$$

[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")

[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} dx = \int \sqrt[4]{\frac{ax-1}{ax+1}} dx$$

[In] integrate(((a*x-1)/(a*x+1))**(1/4),x)

[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} dx = -\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2} + \frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} + \frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2} - \frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
- 1)/(a*x + 1))^(1/4))/a - atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

3.91 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

```
[Out] -2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)
/(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6306, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)-2\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$- 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$- 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \sqrt{2}\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1 + \frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1 + \frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{3}e^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{8}, 1, \frac{11}{8}, e^{4\coth^{-1}(ax)}\right)$$

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x),x]
[Out] (8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/8, 1, 11/8, E^(4*ArcCoth[a*x]
)])/3
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x}\,dx$$

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -(1/2*I + 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
 + (1/2*I - 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)) - (1/2*I - 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4)) + (1/2*I + 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4)) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx =$$
$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\right.}{a}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx =$$
$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\right.}{}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(-1-\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(-1+1\mathrm{i})$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
```

3.92 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 268

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

```
[Out] -a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4))*2^(1/2)-1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6306, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}} - a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}} + \frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

```
[Out] -(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) - (a*ArcTan[1 - (Sqrt[2]*(1 -
1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (a*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(2*Sqrt[2]) + (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + a\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$+ \frac{1}{2}a\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$- \frac{a\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$
$$- \frac{a\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$
$$+ \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$
$$- \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.12

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{8}{3}ae^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)$$

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^2),x]
[Out] (-8*a*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a
*x])])/3
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^2}\,dx$$

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{(-a^4)^{\frac{1}{4}}\,x\log\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+(-a^4)^{\frac{1}{4}}\right)+i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+i\,(-a^4)^{\frac{1}{4}}\right)-i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\right.}{2\,x}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-a^4)^(1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) + (-a^4)^(1/4)) + I*(
-a^4)^(1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^4)^(1/4)) - I*(-a^4
)^(1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^4)^(1/4)) - (-a^4)^(1/4
)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) - (-a^4)^(1/4)) - 2*(a*x + 1)*((a*x -
 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,\mathrm{lo}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}$$
$$-(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}-(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^2,x)
```

```
[Out] - (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (-1)^(1/4)
*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (2*a*((a*x - 1)/(a*x
+ 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.93 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 319

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3} dx = \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

```
[Out] 1/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(
3/4)-1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-1/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6306, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}} - \frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} - \frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(5/4)*
(1 + 1/(a*x))^(3/4))/2 + (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}a\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\text{Subst}\left(\int\frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}a^2\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{4}a^2\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$-\frac{1}{8}a^2\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{8}a^2\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$- \frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+ \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

$$+ \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) - 2\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^3),x]
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCot
h[a*x])] - 2*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^3}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+(-a^8)^{\frac{1}{4}}\right)+i\,(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+i\,(-a^8)^{\frac{1}{4}}\right)-i\,(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^2\left(\right.\right.}{8\,x^2}$$

[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")

[Out] -1/8*((-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) + (-a^8)^(1/4)) + I*(-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^8)^(1/4)) - I*(-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^8)^(1/4)) - (-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) - (-a^8)^(1/4)) - 2*(3*a^2*x^2 + a*x - 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**3,x)

[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")

[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^

```
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*a*((a*x - 1)/(a*x + 1))^(5/4) + a*((a*x
 - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1) + a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{5\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}+\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4} + \frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^3,x)
```

```
[Out] ((a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (5*a^2*((a*x - 1)/(a*x + 1))^(5/4))/
2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*a^
2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4 + ((-1)^(1/4)*a^2*atan
h((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4
```

3.94 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 356

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx =& -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}\\ &-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x}\\ &-\frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\\ &-\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\\ &+\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\end{aligned}$$

```
[Out] -3/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-1/12*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)
^(3/4)+1/3*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)/x+3/16*a^3*arctan(-1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/4)
*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+
```

```
1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/32*a^3*ln(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6306, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{3a^3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4} - \frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} - \frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}} + \frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}$$

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (-3*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/8 - (a^3*(1 - 1/(a*x))^(5/
4)*(1 + 1/(a*x))^(3/4))/12 + (a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/
(3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(8*Sqrt[2]) - (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)
] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (3*a
^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}\left(-1+\frac{x}{2a}\right)}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x}$$
$$- \frac{1}{8}(3a^2)\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}$$
$$+ \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{1}{16}(3a^2)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}$$
$$+ \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{4}(3a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}$$
$$+ \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{4}(3a^3)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x}$$
$$+ \frac{1}{8}(3a^3)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{8}(3a^3)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x}$$

$$+ \frac{1}{16}(3a^3)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{16}(3a^3)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x,\right.$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} + \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} + \frac{(3a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{(3a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} + \frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(29+6e^{2\coth^{-1}(ax)}+9e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 9\mathrm{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^((3*ArcCoth[a*x])/2)*(29 + 6*E^(2*ArcCoth[a*x]) + 9*E^(4*ArcCot
h[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x]
 + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]))/96
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^4}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{9\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(3a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3\left(-a^{12}\right)^{\frac{1}{4}}\right)+9i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(3a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3i\left(-a^{12}\right)^{\frac{1}{4}}\right)-9i\left(-a^{12}\right)^{\frac{1}{4}}}{}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(9*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 3*(-a^12)
^(1/4)) + 9*I*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 3*I
```

```
*(-a^12)^(1/4)) - 9*I*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1))^(1/
4) - 3*I*(-a^12)^(1/4)) - 9*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1
))^(1/4) - 3*(-a^12)^(1/4)) - 2*(11*a^3*x^3 + a^2*x^2 - 2*a*x + 8)*((a*x -
1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**4,x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.78

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(29*a^2*((a*x - 1)/(a*x + 1))
^(9/4) + 6*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 9*a^2*((a*x - 1)/(a*x + 1))^(1
/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
+ 1)^3 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(1/4)/(a*x + 1) + 29*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/
(a*x + 1)^2 + 9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{\frac{3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} - \frac{(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,3\mathrm{i}}{8} - \frac{(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,3\mathrm{i}}{8}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^4,x)
```

```
[Out] - ((3*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (a^3*((a*x - 1)/(a*x + 1))^(5/4)
)/2 + (29*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)^2
 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^3
*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8 - ((-1)^(1/4)*a^3*atanh
((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8
```

3.95 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4}-\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3}+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2}-\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] 557/640*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4-157/320*(1-1/a/x)^(3/4)*(1+1/
a/x)^(1/4)*x^2/a^3+5/16*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2-11/40*(1-1/
a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5-23
7/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-237/128*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{237\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} +\frac{557x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{640a^4} -\frac{157x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{320a^3}+\frac{5x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{16a^2} +\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{11x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{40a}$$

```
[In] Int[x^4/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (557*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(640*a^4) - (157*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(3/4)*(1 +
1/(a*x))^(1/4)*x^3)/(16*a^2) - (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^6\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{11}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{75}{4a^2}+\frac{33x}{2a^3}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{60}\text{Subst}\left(\int \frac{-\frac{471}{8a^3}+\frac{75x}{2a^4}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{1}{120}\text{Subst}\left(\int \frac{-\frac{1671}{16a^4}+\frac{471x}{8a^5}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{120}\text{Subst}\left(\int -\frac{3555}{32a^5x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{237\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{256a^5}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{237\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^5}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{237\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} - \frac{237\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{128a^5}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} - \frac{237\,\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{8192e^{\frac{17}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5} - \frac{22016e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{23936e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{14032e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{5500e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 2370\arctan$$

$$1280a^5$$

```
[In] Integrate[x^4/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((8192*E^((17*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (22016*E^((13
*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((9*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (14032*E^((5*ArcCoth[a*x])/2))/(-1 + E^(
2*ArcCoth[a*x]))^2 + (5500*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) +
2370*ArcTan[E^(-1/2*ArcCoth[a*x])] + 1185*Log[1 - E^(-1/2*ArcCoth[a*x])] -
1185*Log[1 + E^(-1/2*ArcCoth[a*x])])/(1280*a^5)
```

Maple [F]

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

```
[In] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\left(128\,a^5x^5-48\,a^4x^4+24\,a^3x^3-114\,a^2x^2+243\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1185\log\left(\right.}{1280\,a^5}$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 - 48*a^4*x^4 + 24*a^3*x^3 - 114*a^2*x^2 + 243*a*x +
557)*((a*x - 1)/(a*x + 1))^(3/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))
 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1185*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x**4*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{1280}\,a\left(\frac{4\left(1375\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+395\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,\mathrm{arc}}{}\right.$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(1375*((a*x - 1)/(a*x + 1))^(19/4) - 1992*((a*x - 1)/(a*x + 1)
)^(15/4) + 3710*((a*x - 1)/(a*x + 1))^(11/4) - 1440*((a*x - 1)/(a*x + 1))^(
7/4) + 395*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*
x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a
^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 1
185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{1}{1280}a\left(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6} - \frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6} + \frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6} + \frac{4\left(\frac{1440\,(ax-1)}{ax+}\right.}{}\right.$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 + 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1440*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 1992*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1375*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3
/4)/(a*x + 1)^4 - 395*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{64} - \frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2} + \frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32} - \frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40} + \frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5 + \frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5} - \frac{5\,a^5\,(a\,x-1)}{a\,x+1}}$$
$$+ \frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5} - \frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((79*((a*x - 1)/(a*x + 1))^(3/4))/64 - (9*((a*x - 1)/(a*x + 1))^(7/4))/2 +
(371*((a*x - 1)/(a*x + 1))^(11/4))/32 - (249*((a*x - 1)/(a*x + 1))^(15/4))/
40 + (275*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*
x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)
^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (237*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (237*atanh(((a*x - 1)/(a*x + 1
))^(1/4)))/(128*a^5)
```

3.96 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4+\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] -63/64*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+15/32*(1-1/a/x)^(3/4)*(1+1/a/x
)^(1/4)*x^2/a^2-3/8*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(3/
4)*(1+1/a/x)^(1/4)*x^4+123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+1
23/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{6306, 101, 156, 12, 95, 218, 212, 209\}$

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{123\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{123\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} - \frac{63x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{64a^3} + \frac{15x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{32a^2} + \frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{3x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}$$

```
[In] Int[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-63*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(64*a^3) + (15*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(32*a^2) - (3*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
(123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^5\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}+\frac{9x}{a^3}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{189}{8a^3}+\frac{45x}{4a^4}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 + \frac{1}{24}\text{Subst}\left(\int -\frac{369}{16a^4x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{123\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)}{128a^4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{123\text{Subst}\left(\int \frac{1}{-1+x^4}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32a^4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}$$
$$+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4+\frac{123\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{64a^4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}$$
$$+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4+\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{\frac{512e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{1152e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1008e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{532e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-246\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-123}{128a^4}$$

```
[In] Integrate[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((512*E^((13*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (1152*E^((9*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((5*ArcCoth[a*x])/2))
/(-1 + E^(2*ArcCoth[a*x]))^2 - (532*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[
a*x])) - 246*ArcTan[E^(-1/2*ArcCoth[a*x])] - 123*Log[1 - E^(-1/2*ArcCoth[a*
x])] + 123*Log[1 + E^(-1/2*ArcCoth[a*x])])/(128*a^4)
```

Maple [F]

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
[Out] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^3 \, dx = \frac{2 (16 a^4 x^4 - 8 a^3 x^3 + 6 a^2 x^2 - 33 a x - 63) \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 246 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 123 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 1}{128 a^4}$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
[Out] 1/128*(2*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 33*a*x - 63)*((a*x - 1)/(a*x
 + 1))^(3/4) - 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^3 \, dx = \int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/4),x)
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^3 \, dx =$$

$$-\frac{1}{128} a \left(\frac{4 \left(133 \left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}} - 147 \left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}} + 183 \left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}} - 41 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4 (ax-1) a^5}{ax+1} - \frac{6 (ax-1)^2 a^5}{(ax+1)^2} + \frac{4 (ax-1)^3 a^5}{(ax+1)^3} - \frac{(ax-1)^4 a^5}{(ax+1)^4} - a^5} + \frac{246 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} - \frac{123}{} \right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/128*a*(4*(133*((a*x - 1)/(a*x + 1))^(15/4) - 147*((a*x - 1)/(a*x + 1))^(
11/4) + 183*((a*x - 1)/(a*x + 1))^(7/4) - 41*((a*x - 1)/(a*x + 1))^(3/4))/(
4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a
^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 123
*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{1}{128}\,a\left(\frac{246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{123\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{183\,(ax-1)\left(\frac{ax-1}{ax+1}\right.}{ax+1}\right.}{}\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^5 + 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a
^5 - 4*(183*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 147*(a*x - 1)
^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 133*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1)^3 - 41*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)
/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx=\frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{41\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{32}-\frac{183\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{32}+\frac{147\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{133\,\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}$$

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (123*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (123*atan(((a*x - 1)/(a
*x + 1))^(1/4)))/(64*a^4) - ((41*((a*x - 1)/(a*x + 1))^(3/4))/32 - (183*((a
*x - 1)/(a*x + 1))^(7/4))/32 + (147*((a*x - 1)/(a*x + 1))^(11/4))/32 - (133
*((a*x - 1)/(a*x + 1))^(15/4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 -
 (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(
a*x - 1))/(a*x + 1))
```

3.97 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 23/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2-7/12*(1-1/a/x)^(3/4)*(1+1/a/x)^
(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3-17/8*arctan((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^3-17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

= {6306, 101, 156, 12, 95, 218, 212, 209}

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{17\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{17\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{23x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2} + \frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{7x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

```
[In] Int[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (23*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) - (7*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 - (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^4\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{23}{4a^2}+\frac{7x}{2a^3}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{6}\text{Subst}\left(\int -\frac{51}{8a^3x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{17\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{17\text{Subst}\left(\int \frac{1}{-1+x^4}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{17\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{17\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

Mathematica [A] (verified)

Time = 5.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{128e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{240e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{180e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 102\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right) + 51\log\left(1-e^{-\frac{1}{2}\coth^{-1}(a}\right.}{48a^3}$$

```
[In] Integrate[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((128*E^((9*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (240*E^((5*ArcC
oth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (180*E^(ArcCoth[a*x]/2))/(-1 +
E^(2*ArcCoth[a*x])) + 102*ArcTan[E^(-1/2*ArcCoth[a*x])] + 51*Log[1 - E^(-1/
2*ArcCoth[a*x])] - 51*Log[1 + E^(-1/2*ArcCoth[a*x])])/(48*a^3)
```

Maple [F]

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\left(8a^3x^3-6a^2x^2+9ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 51\log\left(\left(\frac{ax-1}{ax+1}\right)\right.}{48a^3}$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
[Out] 1/48*(2*(8*a^3*x^3 - 6*a^2*x^2 + 9*a*x + 23)*((a*x - 1)/(a*x + 1))^(3/4) +
102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 51*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/4),x)
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}a\left(\frac{4\left(45\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\right.$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
[Out] -1/48*a*(4*(45*((a*x - 1)/(a*x + 1))^(11/4) - 30*((a*x - 1)/(a*x + 1))^(7/4
) + 17*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1
)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 -
 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$$

$$=\frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\right.}{a}\right.$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 + 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 45*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 17*((a*x - 1)/(a*x + 1))^(3/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} + \frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3} - \frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((17*((a*x - 1)/(a*x + 1))^(3/4))/12 - (5*((a*x - 1)/(a*x + 1))^(7/4))/2 +
(15*((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2
 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*atan(
((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (17*atanh(((a*x - 1)/(a*x + 1))^(1/
4)))/(8*a^3)
```

3.98 $\int e^{-\frac{3}{2}\coth^{-1}(ax)} x \, dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x \, dx = -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 + \frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -3/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4
)*x^2+9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 218, 212, 209}

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x \, dx = \frac{9\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{3x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

```
[In] Int[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(7/4)
*(1 + 1/(a*x))^(1/4)*x^2)/2 + (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^
2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^3\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2+\frac{3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a}$$

$$+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2-\frac{9\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a}$$

$$+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2-\frac{9\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2$$

$$+\frac{9\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 + \frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-7+3e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + 9\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

```
[In] Integrate[x/E^((3*ArcCoth[a*x])/2),x]
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-7 + 3*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
]))^2 + 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^
2)
```

Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

```
[In] int(x*((a*x-1)/(a*x+1))^(3/4),x)
[Out] int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{2\left(2a^2x^2-3ax-5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 3*a*x - 5)*((a*x - 1)/(a*x + 1))^(3/4) - 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx =$$
$$-\frac{1}{8}a\left(\frac{4\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right.}{a^3}\right.$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(7*((a*x - 1)/(a*x + 1))^(7/4) - 3*((a*x - 1)/(a*x + 1))^(3/4))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan(
((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx =$$
$$-\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{7(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-3\left(\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*
(7*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3*((a*x - 1)/(a*x + 1)
)^(3/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2} - \frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}}$$

```
[In] int(x*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (9*atan(((a*x - 1)/(a*x +
1))^(1/4)))/(4*a^2) - ((3*((a*x - 1)/(a*x + 1))^(3/4))/2 - (7*((a*x - 1)/(a
*x + 1))^(7/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))
/(a*x + 1))
```

3.99 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x-\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
/a-3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6305, 96, 95, 218, 212, 209}

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=-\frac{3\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

```
[In] Int[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a - (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x + \frac{6\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x - \frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 3\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 3\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

```
[In] Integrate[E^((-3*ArcCoth[a*x])/2),x]
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) - 3*ArcTan[E^(ArcCoth[a*x
]/2)] - 3*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

[In] int(((a*x-1)/(a*x+1))^(3/4),x)

[Out] int(((a*x-1)/(a*x+1))^(3/4),x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{2\,a}$$

[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")

[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) + 6*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

[In] integrate(((a*x-1)/(a*x+1))**(3/4),x)

[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1} - a^2} - \frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} + \frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^2} - \frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{a^2}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$
$$=\frac{1}{2}a\left(\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 + 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/a - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.100 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

```
[Out] 2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6306, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)+2\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)$$

$$= -\left(4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$- 4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$+ 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$
=2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)
-\frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}
-\frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}
-\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)
-\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)
$$

$$
=2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)
-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}
-\sqrt{2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)
+\sqrt{2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)
$$

$$= \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.10

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = 8e^{\frac{1}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)$$

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x),x]
[Out] 8*E^(ArcCoth[a*x]/2)*Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x])]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -(1/2*I - 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
 + (1/2*I + 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)) - (1/2*I + 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4)) + (1/2*I - 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4)) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\cdots\right.\right.}{a}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\cdots\right.\right.}{\cdots}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)$$
$$+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(-1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(-1-\mathrm{i})$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x,x)
```

```
[Out] - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(1
/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 -
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1
i)
```

3.101 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 269

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

```
[Out] -a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4))*2^(1/2)+3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6306, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{3a\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}} - a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}} - \frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] -(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - (3*a*ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*Log[1 + Sqrt[1 -
1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1
/4)])/(2*Sqrt[2]) - (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\\
&= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - (3a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&\quad + (3a)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)
\end{aligned}
$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}(3a)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{2}(3a)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{(3a)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}+\frac{(3a)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}-\frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}+\frac{(3a)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{(3a)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} + \frac{3\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} - \frac{3\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] a*((-2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + (3*ArcTan[1 - Sqrt[2]
*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/
Sqrt[2] + (3*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[
2]) - (3*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}\,dx$$

[In] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)

[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{3\left(-a^4\right)^{\frac{1}{4}} x\log\left(27\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 27\left(-a^4\right)^{\frac{3}{4}}\right) - 3i\left(-a^4\right)^{\frac{1}{4}} x\log\left(27\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 27i\left(-a^4\right)^{\frac{3}{4}}\right) + 3i\left(-a^4\right)^{\frac{1}{4}} x\,\mathrm{l}}{2\,x}$$

[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")

[Out] 1/2*(3*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 27*(-a^4)^(3/4)) - 3*I*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 27*I*(-a^4)^(3/4)) + 3*I*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) - 27*I*(-a^4)^(3/4)) - 3*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) - 27*(-a^4)^(3/4)) - 2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}\,dx$$

[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**2,x)

[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=3\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-3\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^2,x)
```

```
[Out] 3*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 3*(-1)^(1/4)*
a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (2*a*((a*x - 1)/(a*x + 1)
)^(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.102 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 319

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

```
[Out] 3/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(
1/4)-9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}} - \frac{9a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1} + \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} - \frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} + \frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (3*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4
)*(1 + 1/(a*x))^(1/4))/2 + (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{4}(3a)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\\
&\quad+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{8}(9a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\\
&\quad+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)
\end{aligned}$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{2}(9a^2)\,\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{4}(9a^2)\,\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{4}(9a^2)\,\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{8}(9a^2)\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{8}(9a^2)\,\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$-\frac{(9a^2)\,\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$-\frac{(9a^2)\,\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{(9a^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{(9a^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}-\frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{24e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - 18\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 18\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 9\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right) + 9\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (a^2*((32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (24*E^(ArcCoth[a
*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 18*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] + 18*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] - 9*Sqrt[2]*L
og[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] + 9*Sqrt[2]*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{9\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(729\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+729\left(-a^8\right)^{\frac{3}{4}}\right)-9i\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(729\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+729i\left(-a^8\right)^{\frac{3}{4}}\right)+9i}{}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(9*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(a*x + 1))^(1/4) + 729*(-a^
8)^(3/4)) - 9*I*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(a*x + 1))^(1/4) +
729*I*(-a^8)^(3/4)) + 9*I*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(a*x + 1)
)^(1/4) - 729*I*(-a^8)^(3/4)) - 9*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(
a*x + 1))^(1/4) - 729*(-a^8)^(3/4)) - 2*(5*a^2*x^2 + 3*a*x - 2)*((a*x - 1)/
(a*x + 1))^(3/4))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$
$$-\frac{1}{16}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a - 8*(7*a*((a*x - 1)/(a*x + 1))^(7/4) + 3*a*((a*x
 - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(7*(a*x - 1)*a*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) + 3*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) +
1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{3\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}+\frac{7\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}-\frac{9\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4}+\frac{9\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^3,x)
```

```
[Out] ((3*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (7*a^2*((a*x - 1)/(a*x + 1))^(7/4)
)/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - (9*(-1)^(1/4
)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 + (9*(-1)^(1/4)*a^2*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

3.103 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 356

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} - \frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}$$

```
[Out] -17/24*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-1/4*a^3*(1-1/a/x)^(7/4)*(1+1/a/x
)^(1/4)+1/3*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4)/x+17/16*a^3*arctan(-1+(1-1/
a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^(1
/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)
```

```
/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-17/32*a^3*ln(1+(1
-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6306, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{17a^3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} -\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1} +\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}} -\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}$$

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (-17*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/24 - (a^3*(1 - 1/(a*x))^(
7/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))
/(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/
4)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(
a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) -
(17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}\left(-1+\frac{3x}{2a}\right)}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$=-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}$$
$$-\frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}$$
$$+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}-\frac{1}{16}\left(17a^2\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}$$
$$+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}+\frac{1}{4}\left(17a^3\right)\text{Subst}\left(\int\frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{1-\frac{1}{ax}}\right)$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}$$
$$+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}+\frac{1}{4}\left(17a^3\right)\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}$$
$$-\frac{1}{8}\left(17a^3\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{8}\left(17a^3\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}$$
$$+\frac{1}{16}\left(17a^3\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\frac{1}{16}\left(17a^3\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\right.$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}+\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}-\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}+\frac{(17a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\frac{(17a^3)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$=-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}-\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}-\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(-\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(45+30e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 51\text{RootSum}\left[1 + \#1^4\&, \frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
[Out] (a^3*((-8*E^(ArcCoth[a*x]/2)*(45 + 30*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[
a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*x]
+ 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1 & ]))/96
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\,dx$$

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{51\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(4913\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4913\left(-a^{12}\right)^{\frac{3}{4}}\right)-51i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(4913\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4913i\left(-a^{12}\right)^{\frac{3}{4}}\right.}{}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
[Out] 1/48*(51*(-a^12)^(1/4)*x^3*log(4913*a^9*((a*x - 1)/(a*x + 1))^(1/4) + 4913*
(-a^12)^(3/4)) - 51*I*(-a^12)^(1/4)*x^3*log(4913*a^9*((a*x - 1)/(a*x + 1))^
```

```
(1/4) + 4913*I*(-a^12)^(3/4)) + 51*I*(-a^12)^(1/4)*x^3*log(4913*a^9*((a*x -
 1)/(a*x + 1))^(1/4) - 4913*I*(-a^12)^(3/4)) - 51*(-a^12)^(1/4)*x^3*log(491
3*a^9*((a*x - 1)/(a*x + 1))^(1/4) - 4913*(-a^12)^(3/4)) - 2*(23*a^3*x^3 + 9
*a^2*x^2 - 6*a*x + 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$=\frac{1}{96}\left(51\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(51*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a^2 - 8*(45*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 30*
a^2*((a*x - 1)/(a*x + 1))^(7/4) + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a
*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1
))*a
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) - 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1) + 45*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^2 + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{\frac{17\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{5\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} - \frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^4,x)
```

```
[Out] (17*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - ((17*a
^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (5*a^3*((a*x - 1)/(a*x + 1))^(7/4))/2
+ (15*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2 + (
a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - (17*(-1)^(1/4)*a^3*
atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

3.104 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 287

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] 26111/1920*(1-1/a/x)^(1/4)/a^5/(1+1/a/x)^(1/4)+5533/1920*(1-1/a/x)^(1/4)*x/
a^4/(1+1/a/x)^(1/4)-1189/960*(1-1/a/x)^(1/4)*x^2/a^3/(1+1/a/x)^(1/4)+181/24
0*(1-1/a/x)^(1/4)*x^3/a^2/(1+1/a/x)^(1/4)-21/40*(1-1/a/x)^(1/4)*x^4/a/(1+1/
a/x)^(1/4)+1/5*(1-1/a/x)^(1/4)*x^5/(1+1/a/x)^(1/4)+1003/128*arctan((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^5-1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/
4))/a^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6306, 100, 156, 160, 12, 95, 304, 209, 212}

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=\frac{1003\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{\frac{1}{ax}+1}}+\frac{5533x\sqrt[4]{1-\frac{1}{ax}}}{1920a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{1189x^2\sqrt[4]{1-\frac{1}{ax}}}{960a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{181x^3\sqrt[4]{1-\frac{1}{ax}}}{240a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}-\frac{21x^4\sqrt[4]{1-\frac{1}{ax}}}{40a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[In] Int[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (26111*(1 - 1/(a*x))^(1/4))/(1920*a^5*(1 + 1/(a*x))^(1/4)) + (5533*(1 - 1/(
a*x))^(1/4)*x)/(1920*a^4*(1 + 1/(a*x))^(1/4)) - (1189*(1 - 1/(a*x))^(1/4)*x
^2)/(960*a^3*(1 + 1/(a*x))^(1/4)) + (181*(1 - 1/(a*x))^(1/4)*x^3)/(240*a^2*
(1 + 1/(a*x))^(1/4)) - (21*(1 - 1/(a*x))^(1/4)*x^4)/(40*a*(1 + 1/(a*x))^(1/
4)) + ((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + (1003*ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (1003*ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^6\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}-\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{\frac{1189}{8a^3}-\frac{543x}{4a^4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{120}\text{Subst}\left(\int \frac{\frac{5533}{16a^4}-\frac{1189x}{4a^5}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1}{120}\text{Subst}\left(\int\frac{\frac{15045}{32a^5}-\frac{5533x}{16a^6}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1}{60}a\text{Subst}\left(\int\frac{15045}{64a^6x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1003\text{Subst}\left(\int\frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{256a^5}$$

$$=\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1003\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^5}$$

$$= \frac{26111 \sqrt[4]{1-\frac{1}{ax}}}{1920a^5 \sqrt[4]{1+\frac{1}{ax}}} + \frac{5533 \sqrt[4]{1-\frac{1}{ax}}x}{1920a^4 \sqrt[4]{1+\frac{1}{ax}}} - \frac{1189 \sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3 \sqrt[4]{1+\frac{1}{ax}}} + \frac{181 \sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2 \sqrt[4]{1+\frac{1}{ax}}} - \frac{21 \sqrt[4]{1-\frac{1}{ax}}x^4}{40a \sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5 \sqrt[4]{1+\frac{1}{ax}}} - \frac{1003\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} + \frac{1003\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

$$= \frac{26111 \sqrt[4]{1-\frac{1}{ax}}}{1920a^5 \sqrt[4]{1+\frac{1}{ax}}} + \frac{5533 \sqrt[4]{1-\frac{1}{ax}}x}{1920a^4 \sqrt[4]{1+\frac{1}{ax}}} - \frac{1189 \sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3 \sqrt[4]{1+\frac{1}{ax}}} + \frac{181 \sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2 \sqrt[4]{1+\frac{1}{ax}}} - \frac{21 \sqrt[4]{1-\frac{1}{ax}}x^4}{40a \sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5 \sqrt[4]{1+\frac{1}{ax}}} + \frac{1003 \arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} - \frac{1003\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{8e^{-\frac{1}{2}\coth^{-1}(ax)} - \frac{32e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^5} - \frac{122e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^4} - \frac{233e^{-\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{-2\coth^{-1}(ax)}\right)^3} - \frac{1661e^{-\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{-2\coth^{-1}(ax)}\right)^2} - \frac{41}{192(}}{a^5}$$

```
[In] Integrate[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (8/E^(ArcCoth[a*x]/2) - 32/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))
^5) - 122/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^4) - 233/(6*E^(A
```

```
rcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^3) - 1661/(48*E^(ArcCoth[a*x]/2)*
(-1 + E^(-2*ArcCoth[a*x]))^2) - 4117/(192*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*Ar
cCoth[a*x]))) - (1003*ArcTan[E^(-1/2*ArcCoth[a*x])])/128 + (1003*Log[1 - E^
(-1/2*ArcCoth[a*x])])/256 - (1003*Log[1 + E^(-1/2*ArcCoth[a*x])])/256)/a^5
```

Maple [F]

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

```
[In] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
[Out] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\left(384a^5x^5 - 1008a^4x^4 + 1448a^3x^3 - 2378a^2x^2 + 5533ax + 26111\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{3840a^5}$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] 1/3840*(2*(384*a^5*x^5 - 1008*a^4*x^4 + 1448*a^3*x^3 - 2378*a^2*x^2 + 5533*
a*x + 26111)*((a*x - 1)/(a*x + 1))^(1/4) - 30090*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \text{Timed out}$$

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(5/4),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.97

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{4\left(20585\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-49120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+61130\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-33816\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{300}{}\right.$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(20585*((a*x - 1)/(a*x + 1))^(17/4) - 49120*((a*x - 1)/(a*x +
1))^(13/4) + 61130*((a*x - 1)/(a*x + 1))^(9/4) - 33816*((a*x - 1)/(a*x + 1)
)^(5/4) + 7365*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10
*(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)
^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 30090*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^6 - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6 - 30720*((a*x - 1)/(a*x
 + 1))^(1/4)/a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{30720\,(}{a^6}\right.$$

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 - 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 - 30720*((a*x - 1)/(a*x + 1))^(1/4)/a^6 - 4*(33816*(a*x - 1)*((a*
x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1))
^(1/4)/(a*x + 1)^2 + 49120*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1
)^3 - 20585*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^4 - 7365*((a*
x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$
$$=\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^5}+\frac{\frac{491\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{1409\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{6113\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{307\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{6}+\frac{4117\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{192}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,1003\mathrm{i}}{128\,a^5}$$

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*1003i)/(128*a^5) + (8*((a*x - 1)/(a*x
 + 1))^(1/4))/a^5 + ((491*((a*x - 1)/(a*x + 1))^(1/4))/64 - (1409*((a*x - 1
)/(a*x + 1))^(5/4))/40 + (6113*((a*x - 1)/(a*x + 1))^(9/4))/96 - (307*((a*x
 - 1)/(a*x + 1))^(13/4))/6 + (4117*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5
+ (10*a^5*(a*x - 1)^2)/(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*
a^5*(a*x - 1)^4)/(a*x + 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x
- 1))/(a*x + 1)) - (1003*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5)
```

3.105 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx=-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}}-\frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] -2467/192*(1-1/a/x)^(1/4)/a^4/(1+1/a/x)^(1/4)-521/192*(1-1/a/x)^(1/4)*x/a^3
/(1+1/a/x)^(1/4)+113/96*(1-1/a/x)^(1/4)*x^2/a^2/(1+1/a/x)^(1/4)-17/24*(1-1/
a/x)^(1/4)*x^3/a/(1+1/a/x)^(1/4)+1/4*(1-1/a/x)^(1/4)*x^4/(1+1/a/x)^(1/4)-47
5/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6306, 100, 156, 160, 12, 95, 304, 209, 212}

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{475\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{521x\sqrt[4]{1-\frac{1}{ax}}}{192a^3\sqrt[4]{\frac{1}{ax}+1}} +\frac{113x^2\sqrt[4]{1-\frac{1}{ax}}}{96a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}-\frac{17x^3\sqrt[4]{1-\frac{1}{ax}}}{24a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[In] Int[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-2467*(1 - 1/(a*x))^(1/4))/(192*a^4*(1 + 1/(a*x))^(1/4)) - (521*(1 - 1/(a*
x))^(1/4)*x)/(192*a^3*(1 + 1/(a*x))^(1/4)) + (113*(1 - 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 + 1/(a*x))^(1/4)) - (17*(1 - 1/(a*x))^(1/4)*x^3)/(24*a*(1 + 1/(a
*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^5\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}-\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{24}\text{Subst}\left(\int \frac{\frac{1425}{16a^4}-\frac{521x}{8a^5}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}a\text{Subst}\left(\int \frac{1425}{32a^5x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{128a^4}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32a^4}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{475\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} - \frac{475\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4} + \frac{475\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

Mathematica [A] (verified)

Time = 5.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{-3072e^{-\frac{1}{2}\coth^{-1}(ax)} + \frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} - \frac{5248e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{7376e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - \frac{6292e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 2850\,\text{arc}}{384a^4}$$

```
[In] Integrate[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-3072/E^(ArcCoth[a*x]/2) + (1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x]))^4 - (5248*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 +
(7376*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (6292*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(-1/2*ArcCoth[a*x]
)] - 1425*Log[1 - E^(-1/2*ArcCoth[a*x])] + 1425*Log[1 + E^(-1/2*ArcCoth[a*x
])])/(384*a^4)
```

Maple [F]

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

```
[In] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.44

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(48\,a^4x^4-136\,a^3x^3+226\,a^2x^2-521\,ax-2467\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{384\,a^4}$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] 1/384*(2*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*((a*x -
1)/(a*x + 1))^(1/4) + 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 1425*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^4
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(5/4),x)
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}\,a\left(\frac{4\left(1573\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-2875\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2343\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
[Out] -1/384*a*(4*(1573*((a*x - 1)/(a*x + 1))^(13/4) - 2875*((a*x - 1)/(a*x + 1))
^(9/4) + 2343*((a*x - 1)/(a*x + 1))^(5/4) - 657*((a*x - 1)/(a*x + 1))^(1/4)
)/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^
3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 2850*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5
+ 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5 + 3072*((a*x - 1)/(a*x + 1)
)^(1/4)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{384}a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}\right.$$

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))
/a^5 - 3072*((a*x - 1)/(a*x + 1))^(1/4)/a^5 + 4*(2343*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1) - 2875*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(
a*x + 1)^2 + 1573*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 657
*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx=\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{219\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{781\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{2875\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{1573\,\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}-\frac{8\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{a^4}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)\,475{}\mathrm{i}}{64\,a^4}$$

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (475*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (8*((a*x - 1)/(a*x + 1))
^(1/4))/a^4 - ((219*((a*x - 1)/(a*x + 1))^(1/4))/32 - (781*((a*x - 1)/(a*x
+ 1))^(5/4))/32 + (2875*((a*x - 1)/(a*x + 1))^(9/4))/96 - (1573*((a*x - 1)/
(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x
 - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a
*x + 1)) - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*475i)/(64*a^4)
```

3.106 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 213

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}}+\frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 287/24*(1-1/a/x)^(1/4)/a^3/(1+1/a/x)^(1/4)+61/24*(1-1/a/x)^(1/4)*x/a^2/(1+1
/a/x)^(1/4)-13/12*(1-1/a/x)^(1/4)*x^2/a/(1+1/a/x)^(1/4)+1/3*(1-1/a/x)^(1/4)
*x^3/(1+1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3-55/8*
arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used

$= \{6306, 100, 156, 160, 12, 95, 304, 209, 212\}$

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{55\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{55\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{\frac{1}{ax}+1}} + \frac{61x\sqrt[4]{1-\frac{1}{ax}}}{24a^2\sqrt[4]{\frac{1}{ax}+1}} + \frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}} - \frac{13x^2\sqrt[4]{1-\frac{1}{ax}}}{12a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[In] Int[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (287*(1 - 1/(a*x))^(1/4))/(24*a^3*(1 + 1/(a*x))^(1/4)) + (61*(1 - 1/(a*x))^
(1/4)*x)/(24*a^2*(1 + 1/(a*x))^(1/4)) - (13*(1 - 1/(a*x))^(1/4)*x^2)/(12*a*
(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) +
(55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) - (55*ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^4\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}-\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a\text{Subst}\left(\int \frac{165}{16a^4x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{55\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{55\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{55\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{384e^{-\frac{1}{2}\coth^{-1}(ax)} + \frac{128e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{400e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{548e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 330\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right) + 165}{48a^3}$$

```
[In] Integrate[x^2/E^((5*ArcCoth[a*x])/2),x]
[Out] (384/E^(ArcCoth[a*x]/2) + (128*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[
a*x]))^3 - (400*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (548*
```

```
E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) - 330*ArcTan[E^(-1/2*ArcC
oth[a*x])] + 165*Log[1 - E^(-1/2*ArcCoth[a*x])] - 165*Log[1 + E^(-1/2*ArcCo
th[a*x])])/(48*a^3)
```

Maple [F]

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

```
[In] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
[Out] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.48

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\,(8\,a^3x^3-26\,a^2x^2+61\,ax+287)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,\log\left(\right.}{48\,a^3}$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] 1/48*(2*(8*a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*((a*x - 1)/(a*x + 1))^(1/4)
 - 330*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*log(((a*x - 1)/(a*x + 1))^
(1/4) + 1) + 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(5/4),x)
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}a\left(\frac{4\left(137\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-174\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}\right.$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(137*((a*x - 1)/(a*x + 1))^(9/4) - 174*((a*x - 1)/(a*x + 1))^(5/
4) + 69*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 330*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4
 - 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4 - 384*((a*x - 1)/(a*x + 1))
^(1/4)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{384\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^4}-\frac{4}{}\right.$$

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 - 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4 - 4*(174*(a*x - 1)*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1) - 137*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x +
1)^2 - 69*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{23\left(\frac{ax-1}{ax+1}\right)^{1/4}}{4}-\frac{29\left(\frac{ax-1}{ax+1}\right)^{5/4}}{2}+\frac{137\left(\frac{ax-1}{ax+1}\right)^{9/4}}{12}}{a^3+\frac{3a^3(ax-1)^2}{(ax+1)^2}-\frac{a^3(ax-1)^3}{(ax+1)^3}-\frac{3a^3(ax-1)}{ax+1}}+\frac{8\left(\frac{ax-1}{ax+1}\right)^{1/4}}{a^3} - \frac{55\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{8\,a^3}+\frac{\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\,1\mathrm{i}\right)\,55\mathrm{i}}{8\,a^3}$$

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] ((23*((a*x - 1)/(a*x + 1))^(1/4))/4 - (29*((a*x - 1)/(a*x + 1))^(5/4))/2 +
(137*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^
2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (atan(((
a*x - 1)/(a*x + 1))^(1/4)*1i)*55i)/(8*a^3) + (8*((a*x - 1)/(a*x + 1))^(1/4)
)/a^3 - (55*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3)
```

3.107 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 176

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx=-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -25/2*(1-1/a/x)^(1/4)/a^2/(1+1/a/x)^(1/4)-5/4*(1-1/a/x)^(5/4)*x/a/(1+1/a/x)
^(1/4)+1/2*(1-1/a/x)^(9/4)*x^2/(1+1/a/x)^(1/4)-25/4*arctan((1+1/a/x)^(1/4)/
(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 304, 209, 212}

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{25\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}-\frac{5x\left(1-\frac{1}{ax}\right)^{5/4}}{4a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[In] Int[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-25*(1 - 1/(a*x))^(1/4))/(2*a^2*(1 + 1/(a*x))^(1/4)) - (5*(1 - 1/(a*x))^(5
/4)*x)/(4*a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x
))^(1/4)) - (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
```

```
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^3\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4a}\\
&= -\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}
\end{aligned}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} + \frac{25\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - \frac{25\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(32-90e^{2\coth^{-1}(ax)}+50e^{4\coth^{-1}(ax)}+25e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)^2\arctan\left(e^{\frac{1}{2}\coth^{-1}(\right.\right.}{4a^2\left(-1+e^{2\coth^{-1}(ax)}\right)^2}$$

```
[In] Integrate[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] -1/4*(32 - 90*E^(2*ArcCoth[a*x]) + 50*E^(4*ArcCoth[a*x]) + 25*E^(ArcCoth[a*
x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTan[E^(ArcCoth[a*x]/2)] - 25*E^(ArcCot
h[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTanh[E^(ArcCoth[a*x]/2)])/(a^2*E^(
ArcCoth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2)
```

Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

```
[In] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.54

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{2\left(2a^2x^2-9ax-43\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 9*a*x - 43)*((a*x - 1)/(a*x + 1))^(1/4) + 50*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 25*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

```
[In] integrate(x*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx =$$
$$-\frac{1}{8}\,a\left(\frac{4\left(13\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^3}\right.$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(13*((a*x - 1)/(a*x + 1))^(5/4) - 9*((a*x - 1)/(a*x + 1))^(1/4))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 50*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^3 + 25*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3 + 64*((a*x - 1)/(a*x + 1
))^(1/4)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}\,a\left(\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}+\frac{4\left(\frac{13\,(ax-}{}\right.}{}\right.$$

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 - 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 6
4*((a*x - 1)/(a*x + 1))^(1/4)/a^3 + 4*(13*(a*x - 1)*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^3*((a*x - 1)/(a*x + 1) -
 1)^2))
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{8\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^2} - \frac{\frac{9\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2} - \frac{13\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,25\mathrm{i}}{4\,a^2}$$

```
[In] int(x*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (25*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - ((9*((a*x - 1)/(a*x + 1))^
(1/4))/2 - (13*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*
x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (8*((a*x - 1)/(a*x + 1))^(1/4))/a
^2 - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*25i)/(4*a^2)
```

3.108 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] 10*(1-1/a/x)^(1/4)/a/(1+1/a/x)^(1/4)+(1-1/a/x)^(5/4)*x/(1+1/a/x)^(1/4)+5*ar
ctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a-5*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)
^(1/4))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6305, 96, 95, 304, 209, 212}

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{5\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{5\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}} + \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[In] Int[E^((-5*ArcCoth[a*x])/2),x]
```

```
[Out] (10*(1 - 1/(a*x))^(1/4))/(a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(5/4)*x)/
(1 + 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
 - (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/4} x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4} x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4} x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{10\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$
$$+ \frac{5\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.24

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4}, 2, \frac{3}{4}, e^{2\coth^{-1}(ax)}\right)}{a}$$

```
[In] Integrate[E^((-5*ArcCoth[a*x])/2),x]
[Out] (8*Hypergeometric2F1[-1/4, 2, 3/4, E^(2*ArcCoth[a*x])])/(a*E^(ArcCoth[a*x]/
2))
```

Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

```
[In] int(((a*x-1)/(a*x+1))^(5/4),x)
[Out] int(((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+9)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] 1/2*(2*(a*x + 9)*((a*x - 1)/(a*x + 1))^(1/4) - 10*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4),x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2} + \frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} + \frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2} - \frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2} - \frac{16\,\left(\frac{ax-1}{ax+1}\right.}{a^2}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 10*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x + 1))^(1/4)
 + 1)/a^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2 - 16*((a*x - 1)/(a*x
 + 1))^(1/4)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 - 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 16
*((a*x - 1)/(a*x + 1))^(1/4)/a^2 + 4*((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x
 - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a}-\frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,5\mathrm{i}}{a}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (atan(((a*x
 - 1)/(a*x + 1))^(1/4)*1i)*5i)/a + (8*((a*x - 1)/(a*x + 1))^(1/4))/a - (5*a
tan(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.109 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 320

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

```
[Out] -8*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+
arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {6306, 100, 21, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right) - 2\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}} - \frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}} + \frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

```
[Out] (-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - (4a)\text{Subst}\left(\int \frac{\frac{1}{4a}+\frac{x}{4a^2}}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$
$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$
$$+ \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$+ \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$- \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$
$$+ \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$- \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \sqrt{2}\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1 + \frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1 + \frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.09

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -8e^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{8}, 1, \frac{7}{8}, e^{4\coth^{-1}(ax)}\right)$$

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x),x]
[Out] (-8*Hypergeometric2F1[-1/8, 1, 7/8, E^(4*ArcCoth[a*x])])/E^(ArcCoth[a*x]/2)
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] (1/2*I + 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
- (1/2*I - 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)) + (1/2*I - 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)) - (1/2*I + 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4)) - 8*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1
/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$=\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.}{a}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$=\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax-}\right.\right.}{}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.37

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - 8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right.\right.$$

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
 - 8*((a*x - 1)/(a*x + 1))^(1/4)
```

3.110 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 299

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

```
[Out] 4*a*(1-1/a/x)^(5/4)/(1+1/a/x)^(1/4)+5*a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-5/2
*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctan(
1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+5/4*a*ln(1-(1-1/a/x)^(1/
4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-5/4*a*l
n(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 49, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}} - \frac{5a\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}} + \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}} + 5a\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}} + \frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}} - \frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*a*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/Sqrt[2] + (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) - (5*a*Log
[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (5a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - (5a)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}(5a)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{2}(5a)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{(5a)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{(5a)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}+5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{(5a)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{(5a)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}+5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.10

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = 8ae^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},-e^{2\coth^{-1}(ax)}\right)$$

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
[Out] (8*a*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcCoth[a*x])])/E^(ArcCoth[a*x]/
2)
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.62

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$
$$-\frac{5\left(-a^4\right)^{\frac{1}{4}}x\log\left(5a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+5\left(-a^4\right)^{\frac{1}{4}}\right)+5i\left(-a^4\right)^{\frac{1}{4}}x\log\left(5a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+5i\left(-a^4\right)^{\frac{1}{4}}\right)-5i\left(-a^4\right)^{\frac{1}{4}}x\log\left(\right.}{2x}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
[Out] -1/2*(5*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) + 5*(-a^4)^(1/4)
) + 5*I*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) + 5*I*(-a^4)^(1/
4)) - 5*I*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) - 5*I*(-a^4)^(
1/4)) - 5*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) - 5*(-a^4)^(1/
4)) - 2*(9*a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = 8\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} + 5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right) + \frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1} + (-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,5\mathrm{i}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^2,x)
```

```
[Out] 8*a*((a*x - 1)/(a*x + 1))^(1/4) + (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(
a*x + 1))^(1/4))*5i + 5*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^
(1/4)*1i) + (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.111 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 351

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \cdots$$

```
[Out] -2*a^2*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)-25/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(
3/4)-5/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+25/8*a^2*arctan(-1+(1-1/a/x)^(
1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules

used = {6306, 79, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}} + \frac{25a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}} - \frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}} - \frac{5}{2}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4} - \frac{25}{4}a^2\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}} - \frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} + \frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right.}{8\sqrt{2}}$$

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - (25*a^2*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4))/4 - (5*a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/2 - (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) + (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - (5a)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}(25a)\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$- \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}(25a)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int\frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{4}\left(25a^2\right)\text{Subst}\Bigg($$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}\left(25a^2\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{8}\left(25a^2\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right.}{8\sqrt{2}}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.29

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{-\frac{1}{2}\coth^{-1}(ax)}\left(3 + e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + 2e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (-8*a^2*(3 + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[
a*x])] + 2*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[
a*x])]))/(3*E^(ArcCoth[a*x]/2))
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{25\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(25\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+25\left(-a^8\right)^{\frac{1}{4}}\right)+25i\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(25\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+25i\left(-a^8\right)^{\frac{1}{4}}\right)-25i\left(-a\right.}{}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(25*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 25*(-a^8)
^(1/4)) + 25*I*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 25
*I*(-a^8)^(1/4)) - 25*I*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x + 1))^(
1/4) - 25*I*(-a^8)^(1/4)) - 25*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 25*(-a^8)^(1/4)) - 2*(43*a^2*x^2 + 9*a*x - 2)*((a*x - 1)/(a*x
 + 1))^(1/4))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*a*((a*x - 1)/(a*x + 1))^(5/4) + 9*a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a
*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 9*a*((a*x - 1)/(a*
x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -8\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - \frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{13\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1} - \frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,25\mathrm{i}}{4} - \frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)}{4}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^3,x)
```

```
[Out] - 8*a^2*((a*x - 1)/(a*x + 1))^(1/4) - ((9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/
2 + (13*a^2*((a*x - 1)/(a*x + 1))^(5/4))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(
a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x
+ 1))^(1/4))*25i)/4 - (25*(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x +
1))^(1/4)*1i))/4
```

3.112 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 385

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4} \, dx = \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{55a^3}{}$$

```
[Out] 2*a^3*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)+55/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3
/4)+11/4*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+1/3*a^3*(1-1/a/x)^(9/4)*(1+1/a
/x)^(3/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1
/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+55/
32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/
x)^(1/2))*2^(1/2)-55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6306, 91, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}} - \frac{55a^3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4} + \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}} + \frac{55a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}} - \frac{\sqrt{2}\ldots}{\ldots}\right.}{16\sqrt{2}}$$

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + (55*a^3*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4))/8 + (11*a^3*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/4 + (a^3*(1 - 1/(a*x))^(9/4)*(1 + 1/(a*x))^(3/4))/3 + (55*a^3*ArcTan[1 -
(Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*A
rcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) +
 (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (55*a^3*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}}-\left(2a^3\right)\text{Subst}\left(\int \frac{\left(-\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}\left(55a^2\right)\text{Subst}\left(\int\frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{16}\left(55a^2\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{1}{4}\left(55a^3\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{2-x^4}}\,dx,\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{1}{4}\left(55a^3\right)\text{Subst}\left(\int\frac{1}{1+x^4}\,dx, x,\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{1}{8}\left(55a^3\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x,\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{1}{16}\left(55a^3\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x\ \cdots}\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{55a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1\ \cdots}}{\sqrt[4]{1+\ \cdots}}\right.}{16\sqrt{2}\ \cdots}$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\ \cdots\right.}{8\sqrt{2}\ \cdots}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.27

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= a^3\left(\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(96+425e^{2\coth^{-1}(ax)}+462e^{4\coth^{-1}(ax)}+165e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] a^3*((96 + 425*E^(2*ArcCoth[a*x]) + 462*E^(4*ArcCoth[a*x]) + 165*E^(6*ArcCo
th[a*x]))/(12*E^(ArcCoth[a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootSum[
1 + #1^4 & , (ArcCoth[a*x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ])/3
2)
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}dx$$

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}dx = -\frac{165\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(55\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+55\left(-a^{12}\right)^{\frac{1}{4}}\right)+165i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(55\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+55i\left(-a^{12}\right)^{\frac{1}{4}}\right)-}{}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(165*(-a^12)^(1/4)*x^3*log(55*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 55*(-
a^12)^(1/4)) + 165*I*(-a^12)^(1/4)*x^3*log(55*a^3*((a*x - 1)/(a*x + 1))^(1/
4) + 55*I*(-a^12)^(1/4)) - 165*I*(-a^12)^(1/4)*x^3*log(55*a^3*((a*x - 1)/(a
*x + 1))^(1/4) - 55*I*(-a^12)^(1/4)) - 165*(-a^12)^(1/4)*x^3*log(55*a^3*((a
*x - 1)/(a*x + 1))^(1/4) - 55*(-a^12)^(1/4)) - 2*(287*a^3*x^3 + 61*a^2*x^2
- 26*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(137*a^2*((a*x - 1)/(a*x + 1))^(9/4) + 174*a^2*((a*x - 1)/(
a*x + 1))^(5/4) + 69*a^2*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)/(a*x + 1
) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) +
137*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 69*a^2*((a*x
- 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{23\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1} + 8\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4} + \frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,55{}\mathrm{i}}{8} + \frac{55\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)}{8}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^4,x)
```

```
[Out] ((23*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (29*a^3*((a*x - 1)/(a*x + 1))^(5/
4))/2 + (137*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1
)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 8*a^3*((a*x
- 1)/(a*x + 1))^(1/4) + ((-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1
))^(1/4))*55i)/8 + (55*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))
^(1/4)*1i))/8
```

3.113 $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 285

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^2+\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3-\frac{19\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19}{81}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)-\frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+}}{\sqrt[6]{\frac{-1+}{x}}}\right.$$

```
[Out] 11/27*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+7/18*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*
x^2+1/3*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x^3+19/81*arctanh((1+1/x)^(1/6)/((-1
+x)/x)^(1/6))-19/324*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1/x)^(1/6)/((-1
+x)/x)^(1/6))+19/324*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)^(1/6)/((-1
+x)/x)^(1/6))-19/162*arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2
))*3^(1/2)+19/162*arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6306, 101, 156, 12, 95, 216, 648, 632, 210, 642, 212}

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$$
$$= -\frac{19\arctan\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19}{81}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)$$
$$+\frac{1}{3}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^3+\frac{7}{18}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{11}{27}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x$$
$$-\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)$$

```
[In] Int[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] (11*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/27 + (7*(1 + x^(-1))^(1/6)*((-
1 + x)/x)^(5/6)*x^2)/18 + ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x^3)/3 - (
19*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])/(54*Sqr
t[3]) + (19*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]]
)/(54*Sqrt[3]) + (19*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/81 - (
19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/324 + (19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (
1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/324
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{3}+2x}{\sqrt[6]{1-x}x^3(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2\\
&\quad + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{22}{9}-\frac{7x}{3}}{\sqrt[6]{1-x}x^2(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2\\
&\quad + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{19}{27\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2\\
&\quad + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19}{162}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$=\frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2+\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3-\frac{19}{27}\text{Subst}\left(\int\frac{1}{-1+x^6}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$=\frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2+\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3+\frac{19}{81}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{19}{81}\text{Subst}\left(\int\frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{19}{81}\text{Subs}$$

$$=\frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2+\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3+\frac{19}{81}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)-\frac{19}{324}\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{19}{324}\text{Subst}\left(\int\frac{1+}{1+x}\right.$$

$$=\frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2+\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3+\frac{19}{81}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)-\frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)+\frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}+\right.$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19}{81}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}+\cdots\right.$$

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.66

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \frac{1}{324}\left(\frac{864e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^3} + \frac{1368e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2} + \frac{732e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}} + 38\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right) + 38\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right) - 38\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) + 38\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right) - 19\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right) + 19\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] ((864*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^3 + (1368*E^(ArcCoth[x]/3))
/(-1 + E^(2*ArcCoth[x]))^2 + (732*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))
 + 38*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] + 38*Sqrt[3]*ArcTan
[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 38*Log[1 - E^(ArcCoth[x]/3)] + 38*Log[
1 + E^(ArcCoth[x]/3)] - 19*Log[1 - E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)]
 + 19*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/324
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.14 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.47

method	result
trager	$\frac{(1+x)(18x^2+21x+22)\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}}{54} - \frac{19\ln\left(3\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}x+9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+3\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}+9\,\mathrm{RootOf}\left(9\_\right.\right.}{}$
risch	Expression too large to display

```
[In] int(1/((x-1)/(1+x))^(1/6)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(1+x)*(18*x^2+21*x+22)*(-(1-x)/(1+x))^(5/6)-19/162*ln(3*(-(1-x)/(1+x))
^(5/6)*x+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5
/6)+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x-1
8*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x-3*(-(1-x)/(1+x))^(2/3)-18*Ro
otOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(
1+x))^(1/3)*x+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x)
)^(1/3)*x-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(
1/3)-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-
3*(-(1-x)/(1+x))^(1/6)+3*RootOf(9*_Z^2+3*_Z+1)+2)+19/54*RootOf(9*_Z^2+3*_Z+
1)*ln(3*(-(1-x)/(1+x))^(5/6)*x-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3
)*x+3*(-(1-x)/(1+x))^(5/6)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*
(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(2/3)-6*(-(1-x)/(1+x))^(1/2)*x+18*R
ootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-6*(-(1-x)/(1+x))^(1/2)+18*RootO
f(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/(1
+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(1/6)-6*RootOf(9*_Z^2+
3*_Z+1)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.61

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \frac{1}{54}\left(18x^3+39x^2+43x+22\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{19}{162}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right) - \frac{19}{162}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right) + \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) + \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="fricas")
[Out] 1/54*(18*x^3 + 39*x^2 + 43*x + 22)*((x - 1)/(x + 1))^(5/6) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 19/324*log(((x
 - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x - 1)/(
x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x + 1))
^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \int \frac{x^2}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**2,x)
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/6), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.77

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}} - 8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}} + 61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} + \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) + \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="maxima")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/27*(19*((
x - 1)/(x + 1))^(17/6) - 8*((x - 1)/(x + 1))^(11/6) + 61*((x - 1)/(x + 1))^
(5/6))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1
) + 19/324*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/
324*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log
(((x - 1)/(x + 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.75

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) + \frac{\frac{8(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1} - \frac{19(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2} - 61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}-1\right)^3} + \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) + \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{19}{162}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="giac")
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/27*(8*(x
- 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) - 19*(x - 1)^2*((x - 1)/(x + 1))^(5/6)
/(x + 1)^2 - 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) - 1)^3 + 19/324*l
og(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x
- 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x
 + 1))^(1/6) + 1) - 19/162*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.59

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)\,19\mathrm{i}}{81} - \frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27} - \frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27} + \frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1} - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19\sqrt{3}}{162}-\frac{19}{162}\mathrm{i}\right) - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)$$

```
[In] int(x^2/((x - 1)/(x + 1))^(1/6),x)
[Out] - (atan(((x - 1)/(x + 1))^(1/6)*1i)*19i)/81 - ((61*((x - 1)/(x + 1))^(5/6))
/27 - (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/(
```

```
(3*(x - 1))/(x + 1) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) -
atan((((x - 1)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348
907 - 2476099/14348907)))*((19*3^(1/2))/162 - 19i/162) - atan((((x - 1)/(x
+ 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/1434
8907)))*((19*3^(1/2))/162 + 19i/162)
```

3.114 $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 258

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2-\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{9}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)-\frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

```
[Out] 1/6*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+1/2*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)*x^2
+1/9*arctanh((1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/36*ln(1+(1+1/x)^(1/3)/((-1+x
)/x)^(1/3)-(1+1/x)^(1/6)/((-1+x)/x)^(1/6))+1/36*ln(1+(1+1/x)^(1/3)/((-1+x)/
x)^(1/3)+(1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/18*arctan(1/3*(1-2*(1+1/x)^(1/6)
/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/18*arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1
+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6306, 98, 96, 95, 216, 648, 632, 210, 642, 212}

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)+\frac{1}{2}\left(\frac{1}{x}+1\right)^{7/6}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{1}{6}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)$$

```
[In] Int[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/6 + ((1 + x^(-1))^(7/6)*((-1 + x)
/x)^(5/6)*x^2)/2 - ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/S
qrt[3]]/(6*Sqrt[3]) + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6)
)/Sqrt[3]]/(6*Sqrt[3]) + ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/9 -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/36 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-
1))^(1/6)/((-1 + x)/x)^(1/6)]/36
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x \\
&\quad + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x \\
&\quad + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)
\end{aligned}$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{9}\text{Subst}\Bigg(\int$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{36}\text{Subst}\Bigg(\int \frac{1+2x}{1+x+x^2}$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{36}\log\Bigg(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} + \frac{\sqrt[6]{1}}{\sqrt[6]{-}}$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{36}\log\Bigg(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} + \frac{\sqrt[6]{1}}{\sqrt[6]{-}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = \frac{1}{36}\left(\frac{72e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{84e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)+2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)-\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((72*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^2 + (84*E^(ArcCoth[x]/3))/(-
1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]]
 + 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E^(ArcCot
h[x]/3)] + 2*Log[1 + E^(ArcCoth[x]/3)] - Log[1 - E^(ArcCoth[x]/3) + E^((2*A
rcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/36
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.40 (sec) , antiderivative size = 1158, normalized size of antiderivative = 4.49

method	result	size
trager	Expression too large to display	1158
risch	Expression too large to display	1702

```
[In] int(1/((x-1)/(1+x))^(1/6)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)*(4+3*x)*(-(1-x)/(1+x))^(5/6)+1/18*ln(3*(-(1-x)/(1+x))^(5/6)*x+9*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*RootOf
(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_
Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*
x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+9*
RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+9*RootO
f(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1
+x))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)-1/6*ln(3*(-(1-x)/(1+x))^(5/6)*x+9*Roo
tOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^
```

```
2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*
_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+
18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+9*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+9*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x
))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)*RootOf(9*_Z^2-3*_Z+1)+1/6*RootOf(9*_Z^2
-3*_Z+1)*ln(3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))
^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3
)+6*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+
6*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+6*(-(1
-x)/(1+x))^(1/2)*x-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x
)/(1+x))^(1/2)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x
))^(1/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^
(1/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*RootOf(9*_Z^2-3*_Z+1)-
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = \frac{1}{6}\left(3\,x^2+7\,x+4\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{1}{18}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right) - \frac{1}{18}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right) + \frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) + \frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 7*x + 4)*((x - 1)/(x + 1))^(5/6) - 1/18*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/18*sqrt(3)*arctan(2/3*sqrt(
3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/36*log(((x - 1)/(x + 1))^(1/3
) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x -
 1)/(x + 1))^(1/6) + 1) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(
((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\int \frac{x}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x,x)
[Out] Integral(x/((x - 1)/(x + 1))**(1/6), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.75

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=&-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)\\&-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)\\&+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\&-\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\&+\frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="maxima")
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/3*(((x - 1)/(
x + 1))^(11/6) - 7*((x - 1)/(x + 1))^(5/6))/(2*(x - 1)/(x + 1) - (x - 1)^2/
(x + 1)^2 - 1) + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6)
 + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1
/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(((x - 1)/(x + 1))^(1/6) - 1
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = -\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1} - 7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}-1\right)^{2}} + \frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) + \frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{18}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/3*((x - 1)*((
x - 1)/(x + 1))^(5/6)/(x + 1) - 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1)
 - 1)^2 + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) -
 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1/18*log
(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.55

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = \frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3} - \frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{(x-1)^2}{(x+1)^2} - \frac{2(x-1)}{x+1} + 1} - \frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)1\mathrm{i}}{9} - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18} - \frac{1}{18}\mathrm{i}\right) - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18} + \frac{1}{18}\mathrm{i}\right)$$

```
[In] int(x/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] ((7*((x - 1)/(x + 1))^(5/6))/3 - ((x - 1)/(x + 1))^(11/6)/3)/((x - 1)^2/(x
+ 1)^2 - (2*(x - 1))/(x + 1) + 1) - (atan(((x - 1)/(x + 1))^(1/6)*1i)*1i)/9
 - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243 - 1/243)))*(3^(
1/2)/18 - 1i/18) - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243
 + 1/243)))*(3^(1/2)/18 + 1i/18)
```

3.115 $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 223

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x-\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)-\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

```
[Out] (1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+2/3*arctanh((1+1/x)^(1/6)/((-1+x)/x)^(1/6)
)-1/6*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1/x)^(1/6)/((-1+x)/x)^(1/6))+1
/6*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/3*
arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/3*arctan
(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6305, 96, 95, 216, 648, 632, 210, 642, 212}

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)}{\sqrt{3}} +\frac{2}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)+\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x -\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)$$

```
[In] Int[E^(ArcCoth[x]/3),x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x - ArcTan[(1 - (2*(1 + x^(-1))^(1/6)
)/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))
/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + (2*ArcTanh[(1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/3 - Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^
(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)
^(1/3) + (1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x - 2\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)\\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)\\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)
\end{aligned}
$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} + \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) - \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x+\frac{\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}$$
$$+\frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$
$$-\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)+\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.16

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=2e^{\frac{1}{3}\coth^{-1}(x)}\left(\frac{1}{-1+e^{2\coth^{-1}(x)}}+\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{2\coth^{-1}(x)}\right)\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3),x]
[Out] 2*E^(ArcCoth[x]/3)*((-1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6, 1
, 7/6, E^(2*ArcCoth[x])])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.15 (sec) , antiderivative size = 1151, normalized size of antiderivative = 5.16

method	result	size
trager	Expression too large to display	1151
risch	Expression too large to display	1761

```
[In] int(1/((x-1)/(1+x))^(1/6),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)*(-(1-x)/(1+x))^(5/6)+RootOf(9*_Z^2-3*_Z+1)*ln(3*(-(1-x)/(1+x))^(5/6)*
x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)-9*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+6*(-(1-x)/(1+x))^(2/3)*x-18*RootO
f(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+6*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+6*(-(1-x)/(1+x))^(1/2)*x-18*RootOf(9*_Z^2
-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)-18*RootOf(9*_Z^2-3*_
Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*
(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(1/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-
x)/(1+x))^(1/6)-3*RootOf(9*_Z^2-3*_Z+1)-1)+1/3*ln(3*(-(1-x)/(1+x))^(5/6)*x+
9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*Roo
tOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z
^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/
3)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x
+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+9*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)
/(1+x))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)-ln(3*(-(1-x)/(1+x))^(5/6)*x+9*Root
Of(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2
-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_
Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+1
8*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+9*Roo
tOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+9*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x)
)^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)*RootOf(9*_Z^2-3*_Z+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.72

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="fricas")
```

```
[Out] (x + 1)*((x - 1)/(x + 1))^(5/6) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x +
 1))^(1/6) - 1/3*sqrt(3)) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x +
 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6)
 + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1))^(1
/6) - 1)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=\int\frac{1}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/6), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = &-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\
&-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\
&-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
&-\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
&+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)
\end{aligned}
$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1
))^(1/6) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1} + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(abs(((x - 1)/(x
 + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)2\mathrm{i}}{3} - \frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}-1} - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right) - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

```
[In] int(1/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] - (atan(((x - 1)/(x + 1))^(1/6)*1i)*2i)/3 - (2*((x - 1)/(x + 1))^(5/6))/((x
 - 1)/(x + 1) - 1) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i - 32))
*(3^(1/2)/3 - 1i/3) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i + 32)
)*(3^(1/2)/3 + 1i/3)
```

3.116 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 402

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right) - \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+2\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{2}\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)$$

```
[Out] 2*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^
(1/6)-3^(1/2))+arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+2*arctanh((
1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/2*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1
/x)^(1/6)/((-1+x)/x)^(1/6))+1/2*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)
^(1/6)/((-1+x)/x)^(1/6))-arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^
```

```
(1/2))*3^(1/2)+arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(
1/2)+1/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/
x)^(1/6))*3^(1/2)-1/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*
3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6306, 132, 65, 338, 301, 648, 632, 210, 642, 209, 95, 216, 212}

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = & -\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right) \\ & -\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\arctan\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right) \\ & +2\arctan\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+2\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right) \\ & -\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right) \\ & +\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right) \\ & -\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)\end{aligned}$$

```
[In] Int[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])
+ Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]] -
 ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + ArcTan[Sqrt[
3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + 2*ArcTan[((-1 + x)/x)^(1/
6)/(1 + x^(-1))^(1/6)] + 2*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)] -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/2 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-1
))^(1/6)/((-1 + x)/x)^(1/6)]/2 + (Sqrt[3]*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1/
6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - (Sqrt[
3]*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(
1/3)/(1 + x^(-1))^(1/3)])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right) \\
&= 6\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right) - 6\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) \\
&= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + 2\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) \\
&\quad + 2\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + 6\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)
\end{aligned}$$

$$= 2\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$+ \frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$+ \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$+ \frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$+ 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$+ 2\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$+ 2\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= 2\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

$$-\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)+\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

$$+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$-3\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$-3\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$+\frac{1}{2}\sqrt{3}\text{Subst}\left(\int\frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$-\frac{1}{2}\sqrt{3}\text{Subst}\left(\int\frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= -\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + \sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)$$

$$+ 2\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

$$-\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)+\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

$$+\frac{1}{2}\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)$$

$$-\frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)$$

$$-\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$-\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, \sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= -\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + \sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)$$

$$- \arctan\left(\sqrt{3} - \frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \arctan\left(\sqrt{3} + \frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$+ 2\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

$$- \frac{1}{2}\log\left(1 + \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{2}\log\left(1 + \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} + \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)$$

$$+ \frac{1}{2}\sqrt{3}\log\left(1 - \frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}} + \frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)$$

$$- \frac{1}{2}\sqrt{3}\log\left(1 + \frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}} + \frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \frac{12}{7}e^{\frac{7}{3}\coth^{-1}(x)}\,\text{Hypergeometric2F1}\left(\frac{7}{12},1,\frac{19}{12},e^{4\coth^{-1}(x)}\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3)/x,x]
[Out] (12*E^((7*ArcCoth[x])/3)*Hypergeometric2F1[7/12, 1, 19/12, E^(4*ArcCoth[x])
])/7
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 22.19 (sec) , antiderivative size = 2088, normalized size of antiderivative = 5.19

method	result	size
trager	Expression too large to display	2088

```
[In] int(1/((x-1)/(1+x))^(1/6)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -3*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*ln(-18*RootOf(3*_Z*R
ootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(3*_
Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1
+x))^(5/6)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)
/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(3
*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/
(1+x))^(2/3)+6*(-(1-x)/(1+x))^(1/2)*x+6*(-(1-x)/(1+x))^(1/2)-3*(-(1-x)/(1+x
))^(1/3)*x-6*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)-3*(-(1-x)/
(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/6)+1)-3*ln(-(18*R
ootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))
^(5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(3*_Z*RootOf(_Z^2
+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^2+1)*
(-(1-x)/(1+x))^(2/3)+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))
^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+1
8*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/(1+x)
)^(1/2)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+6*RootOf(3*_Z*RootOf(_Z^2+1)+
9*_Z^2-1)*x+3*(-(1-x)/(1+x))^(1/6)*x+RootOf(_Z^2+1)*x+3*(-(1-x)/(1+x))^(1/6
))/x)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)-RootOf(_Z^2+1)*ln((9*RootOf(3*_Z
*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x-18*(-(1-x)/(1+x))^(1/2)*Ro
otOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*x-3*(-(1-x)/(1+x))^(5/6)*
x+6*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z
^2-1)*(-(1-x)/(1+x))^(2/3)-18*(-(1-x)/(1+x))^(1/2)*RootOf(3*_Z*RootOf(_Z^2+
1)+9*_Z^2-1)*RootOf(_Z^2+1)-3*(-(1-x)/(1+x))^(5/6)+6*RootOf(_Z^2+1)*(-(1-x)
/(1+x))^(2/3)-18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*
x+9*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/6
)*x+6*(-(1-x)/(1+x))^(1/2)*x-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-18*Roo
tOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(3*_Z*RootOf
(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/6)+6*(-(1-x)/(1+x))^(1/
2)-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^
2-1)*x-RootOf(_Z^2+1)*x)/x)-ln(-(18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-
(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x
))^(2/3)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*(
-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(3*_Z*Ro
otOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)*x+3*Ro
otOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)
```

```
*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/(1+x))^(1/2)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x)
)^(1/3)+6*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*x+3*(-(1-x)/(1+x))^(1/6)*x+R
ootOf(_Z^2+1)*x+3*(-(1-x)/(1+x))^(1/6))/x)*RootOf(_Z^2+1)+ln(-9*RootOf(3*_Z
*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-9*RootOf(3*
_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)-18*(-(1-x)/
(1+x))^(1/2)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*x-3*(-(1-x
)/(1+x))^(5/6)*x-18*(-(1-x)/(1+x))^(1/2)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-
1)*RootOf(_Z^2+1)-18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-
(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(5/6)-3*(-(1-x)/(1+x))^(2/3)*x-18*Roo
tOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*Roo
tOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/6)*x-3*(
-(1-x)/(1+x))^(2/3)-9*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(
-(1-x)/(1+x))^(1/6)+3*(-(1-x)/(1+x))^(1/3)*x-3*RootOf(3*_Z*RootOf(_Z^2+1)+9
*_Z^2-1)*RootOf(_Z^2+1)+3*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+3*(
-(1-x)/(1+x))^(1/6)+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = &-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)\\ &-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)\\ &+\frac{1}{2}\sqrt{2\sqrt{-3}+2}\log\left(\sqrt{2\sqrt{-3}+2}\left(\sqrt{-3}-1\right)+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &-\frac{1}{2}\sqrt{2\sqrt{-3}+2}\log\left(-\sqrt{2\sqrt{-3}+2}\left(\sqrt{-3}-1\right)+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &-\frac{1}{2}\sqrt{-2\sqrt{-3}+2}\log\left(\left(\sqrt{-3}+1\right)\sqrt{-2\sqrt{-3}+2}+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &+\frac{1}{2}\sqrt{-2\sqrt{-3}+2}\log\left(-\left(\sqrt{-3}+1\right)\sqrt{-2\sqrt{-3}+2}+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &+2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &-\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &+\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - sqrt(3
)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/2*sqrt(2*sq
rt(-3) + 2)*log(sqrt(2*sqrt(-3) + 2)*(sqrt(-3) - 1) + 4*((x - 1)/(x + 1))^(
1/6)) - 1/2*sqrt(2*sqrt(-3) + 2)*log(-sqrt(2*sqrt(-3) + 2)*(sqrt(-3) - 1) +
 4*((x - 1)/(x + 1))^(1/6)) - 1/2*sqrt(-2*sqrt(-3) + 2)*log((sqrt(-3) + 1)*
sqrt(-2*sqrt(-3) + 2) + 4*((x - 1)/(x + 1))^(1/6)) + 1/2*sqrt(-2*sqrt(-3) +
 2)*log(-(sqrt(-3) + 1)*sqrt(-2*sqrt(-3) + 2) + 4*((x - 1)/(x + 1))^(1/6))
+ 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(1/3) + ((x
 - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x +
 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(((x - 1)/(x + 1))^
(1/6) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((x - 1)/(x + 1))^(1/6)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.65

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) + \arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + 2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) + \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/2*sqrt(3)*log(sqrt(3)*(
(x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*sqrt(3)*log(-sq
rt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + arctan(sqrt(
3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(1/3) +
 ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/
(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(abs(((x - 1)/(
x + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.42

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right) - \mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)\,2\mathrm{i} - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{-743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(\sqrt{3}-\mathrm{i}\right) - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)$$

```
[In] int(1/(x*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/6)) - atan(((x - 1)/(x + 1))^(1/6)*1i)*2i - ata
n((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(1/2)*743008370688i - 7430083
70688))*(3^(1/2) - 1i) - atan((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(
1/2)*743008370688i + 743008370688))*(3^(1/2) + 1i) - atan((1486016741376*((
x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i - 743008370688))*(3^(1/2)*1i
+ 1) - atan((1486016741376*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i
+ 743008370688))*(3^(1/2)*1i - 1)
```

3.117 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 233

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}} - \frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}$$

```
[Out] (1+1/x)^(1/6)*((-1+x)/x)^(5/6)+2/3*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+1
/3*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+1/3*arctan(2*((-1+x)/x)
^(1/6)/(1+1/x)^(1/6)+3^(1/2))+1/6*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+
x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-1/6*ln(1+((-1+x)/x)^(1/3)/(1+1/x
)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6306, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\frac{1}{3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{3}\arctan\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}$$

```
[In] Int[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6) - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6)]/3 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x
^(-1))^(1/6)]/3 + (2*ArcTan[((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/3 + Log
[1 - (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(
1 + x^(-1))^(1/3)]/(2*Sqrt[3]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 +
x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(2*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)\\
&= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)\\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)\\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)
\end{aligned}$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$+ \frac{2}{3}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$+ \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$+ \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$+ \frac{2}{3}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}$$

$$- \frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}$$

$$- \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$- \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$
$$-\frac{1}{3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$
$$+\frac{2}{3}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}$$
$$-\frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -2e^{\frac{1}{3}\coth^{-1}(x)}\left(-\frac{1}{1+e^{2\coth^{-1}(x)}}+\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{2\coth^{-1}(x)}\right)\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] -2*E^(ArcCoth[x]/3)*(-(1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
1, 7/6, -E^(2*ArcCoth[x])])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 16.72 (sec) , antiderivative size = 1487, normalized size of antiderivative = 6.38

method	result	size
trager	Expression too large to display	1487
risch	Expression too large to display	2991

```
[In] int(1/((x-1)/(1+x))^(1/6)/x^2,x,method=_RETURNVERBOSE)
[Out] (1+x)*(-(1-x)/(1+x))^(5/6)/x-9*RootOf(81*_Z^4-9*_Z^2+1)^3*ln((-27*RootOf(81
*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1
-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/
(1+x))^(2/3)*x+18*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*Root
Of(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*RootOf
(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-
x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x
)/(1+x))^(1/2)*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf
(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^3*x
-2*(-(1-x)/(1+x))^(1/2)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*R
ootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-RootOf(81*_Z^4-9*_Z^2+1)*x)/
x)+RootOf(81*_Z^4-9*_Z^2+1)*ln((-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x
))^(2/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x)
)^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*x*RootOf(81*
_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)
/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+
x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_
Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x)/(1+x))^(1/2)*x+6*RootOf(81*_
Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(
1+x))^(1/6)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)+6*Root
Of(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1
-x)/(1+x))^(1/6)-RootOf(81*_Z^4-9*_Z^2+1)*x)/x)+RootOf(81*_Z^4-9*_Z^2+1)*ln
((54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+54*RootOf(81*_Z^4-9*
_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z
^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))
^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^
(5/6)-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(81*_Z^4-9*_
Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^
(1/3)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(81*_Z^4-9*
_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^3*x+6*RootOf(81
*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(
1+x))^(1/6)-(-(1-x)/(1+x))^(1/6)*x-RootOf(81*_Z^4-9*_Z^2+1)*x-(-(1-x)/(1+x)
)^(1/6))/x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$$

$$= \frac{x\sqrt{2i\sqrt{3}+2}\log\left(\sqrt{2i\sqrt{3}+2}(i\sqrt{3}-1)+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-x\sqrt{2i\sqrt{3}+2}\log\left(\sqrt{2i\sqrt{3}+2}(-i\sqrt{3}+1)+4\left(\right.\right.}{}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(x*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) + 2)*(I*sqrt(3) - 1) + 4*
((x - 1)/(x + 1))^(1/6)) - x*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) + 2
)*(-I*sqrt(3) + 1) + 4*((x - 1)/(x + 1))^(1/6)) - x*sqrt(-2*I*sqrt(3) + 2)*
log((I*sqrt(3) + 1)*sqrt(-2*I*sqrt(3) + 2) + 4*((x - 1)/(x + 1))^(1/6)) + x
*sqrt(-2*I*sqrt(3) + 2)*log((-I*sqrt(3) - 1)*sqrt(-2*I*sqrt(3) + 2) + 4*((x
 - 1)/(x + 1))^(1/6)) + 4*x*arctan(((x - 1)/(x + 1))^(1/6)) + 6*(x + 1)*((x
 - 1)/(x + 1))^(5/6))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**2,x)
```

```
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = & -\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) \\ & +\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) \\ & +\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & +\frac{1}{3}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{3}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{3} + \frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}+1} - \mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) - \mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

```
[In] int(1/(x^2*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] (2*atan(((x - 1)/(x + 1))^(1/6)))/3 + (2*((x - 1)/(x + 1))^(5/6))/((x - 1)/
(x + 1) + 1) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i - 32))*((3^(1
/2)*1i)/3 + 1/3) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i + 32))*((
3^(1/2)*1i)/3 - 1/3)
```

3.118 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 260

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$-\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{\log\left(1-\right.}{}$$

```
[Out] 1/6*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+1/2*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)+1/9*a
rctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1+1/x
)^(1/6)-3^(1/2))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+1/36
*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6)
)*3^(1/2)-1/36*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)
/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules

used = {6306, 81, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{18}\arctan\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right) + \frac{1}{9}\arctan\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{2}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6} + \frac{1}{6}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}} - \frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}$$

```
[In] Int[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/6 + ((1 + x^(-1))^(7/6)*((-1 + x)/x
)^(5/6))/2 - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18
 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18 + ArcTan[
((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)]/9 + Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3
]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)
^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$+\frac{1}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{9}\text{Subs}$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{36}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{36}\text{Subst}\left(\int \frac{}{1+\sqrt{}}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{12\sqrt{3}} - \frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\right.}{12\sqrt{3}}$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$-\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{9}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.48

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$$
$$= \frac{1}{54}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(1+7e^{2\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^2} - 6\arctan\left(e^{\frac{1}{3}\coth^{-1}(x)}\right) + \text{RootSum}\left[1-\#1^2 + \#1^4\&, \frac{2\coth^{-1}(x) - 6\log\left(e^{\frac{1}{3}\coth^{-1}(x)} - \#1\right) - \coth^{-1}(x)\#1^2 + 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)} - \#1\right)\#1^2}{-\#1 + 2\#1^3}\&\right]\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(1 + 7*E^(2*ArcCoth[x])))/(1 + E^(2*ArcCoth[x]))^2 -
6*ArcTan[E^(ArcCoth[x]/3)] + RootSum[1 - #1^2 + #1^4 & , (2*ArcCoth[x] - 6*
Log[E^(ArcCoth[x]/3) - #1] - ArcCoth[x]*#1^2 + 3*Log[E^(ArcCoth[x]/3) - #1]
*#1^2)/(-#1 + 2*#1^3) & ])/54
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 15.33 (sec) , antiderivative size = 1262, normalized size of antiderivative = 4.85

method	result	size
trager	Expression too large to display	1262
risch	Expression too large to display	3478

```
[In] int(1/((x-1)/(1+x))^(1/6)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)*(4*x+3)/x^2*(-(1-x)/(1+x))^(5/6)-1/18*RootOf(_Z^2+1)*ln(-(9*RootO
f(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x-18*(-(1-x)/(1+x))^(1
/2)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*x-3*(-(1-x)/(1+x))^
(5/6)*x+6*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(3*_Z*RootOf(_Z^2+1
)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)-18*(-(1-x)/(1+x))^(1/2)*RootOf(3*_Z*RootOf
(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)-3*(-(1-x)/(1+x))^(5/6)+6*RootOf(_Z^2+1)*(
-(1-x)/(1+x))^(2/3)-18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^
(1/3)*x+9*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x)
)^(1/6)*x+6*(-(1-x)/(1+x))^(1/2)*x-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-
18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(3*_Z*
RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/6)+6*(-(1-x)/(1+x
```

```
))^(1/2)-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*RootOf(3*_Z*RootOf(_Z^2+1)
+9*_Z^2-1)*x-RootOf(_Z^2+1)*x)/x)-1/18*ln((18*RootOf(3*_Z*RootOf(_Z^2+1)+9*
_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+3*RootOf(_Z^2+1)*(-
(1-x)/(1+x))^(2/3)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))
^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*Root
Of(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1
/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)
+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/(1+x))^(1/2)+3*RootOf(_Z^2+1)*(-(
1-x)/(1+x))^(1/3)+6*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*x+3*(-(1-x)/(1+x))
^(1/6)*x+RootOf(_Z^2+1)*x+3*(-(1-x)/(1+x))^(1/6))/x)*RootOf(_Z^2+1)-1/6*ln(
(18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(
1+x))^(5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(3*_Z*RootOf
(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^
2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(
1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3
)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/
(1+x))^(1/2)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+6*RootOf(3*_Z*RootOf(_Z^
2+1)+9*_Z^2-1)*x+3*(-(1-x)/(1+x))^(1/6)*x+RootOf(_Z^2+1)*x+3*(-(1-x)/(1+x))
^(1/6))/x)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.02

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\sqrt{2}x^2\sqrt{i\sqrt{3}+1}\log\left(\left(i\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{i\sqrt{3}+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-\sqrt{2}x^2\sqrt{i\sqrt{3}+1}\log\left(\left(-i\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{\cdots}\right.}{}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="fricas")
```

```
[Out] 1/36*(sqrt(2)*x^2*sqrt(I*sqrt(3) + 1)*log((I*sqrt(3)*sqrt(2) - sqrt(2))*sqr
t(I*sqrt(3) + 1) + 4*((x - 1)/(x + 1))^(1/6)) - sqrt(2)*x^2*sqrt(I*sqrt(3)
+ 1)*log((-I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(I*sqrt(3) + 1) + 4*((x - 1)/(x
 + 1))^(1/6)) - sqrt(2)*x^2*sqrt(-I*sqrt(3) + 1)*log((I*sqrt(3)*sqrt(2) + s
qrt(2))*sqrt(-I*sqrt(3) + 1) + 4*((x - 1)/(x + 1))^(1/6)) + sqrt(2)*x^2*sqr
t(-I*sqrt(3) + 1)*log((-I*sqrt(3)*sqrt(2) - sqrt(2))*sqrt(-I*sqrt(3) + 1) +
 4*((x - 1)/(x + 1))^(1/6)) + 4*x^2*arctan(((x - 1)/(x + 1))^(1/6)) + 6*(4*
x^2 + 7*x + 3)*((x - 1)/(x + 1))^(5/6))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**3,x)
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.68

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = &-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}+\frac{1}{18}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\&+\frac{1}{18}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="maxima")
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*(((x - 1)/(x + 1))^(11/6) + 7*((x - 1)/(x + 1))^(5/6))/
(2*(x - 1)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) + 1/18*arctan(sqrt(3) + 2*((x
 - 1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 1/9*arctan(((x - 1)/(x + 1))^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1} + 7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}+1\right)^2} + \frac{1}{18}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{18}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="giac")
```

```
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*((x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 7*((x - 1)/(
x + 1))^(5/6))/((x - 1)/(x + 1) + 1)^2 + 1/18*arctan(sqrt(3) + 2*((x - 1)/(
x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/9*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.52

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{9} + \frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3} + \frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{2(x-1)}{x+1} + \frac{(x-1)^2}{(x+1)^2} + 1} - \operatorname{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(-\frac{1}{243} + \frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{1}{18} + \frac{\sqrt{3}\,1\mathrm{i}}{18}\right) - \operatorname{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(\frac{1}{243} + \frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(-\frac{1}{18} + \frac{\sqrt{3}\,1\mathrm{i}}{18}\right)$$

```
[In] int(1/(x^3*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] atan(((x - 1)/(x + 1))^(1/6))/9 + ((7*((x - 1)/(x + 1))^(5/6))/3 + ((x - 1)
/(x + 1))^(11/6)/3)/((2*(x - 1))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - atan(
(2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 - 1/243)))*((3^(1/2)*1i)
/18 + 1/18) - atan((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 + 1/2
43)))*((3^(1/2)*1i)/18 - 1/18)
```

3.119 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 287

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x}$$

$$-\frac{19}{162}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{162}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{81}\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19\log}{}$$

```
[Out] 19/54*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+1/18*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)+1/
3*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)/x+19/81*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1
/6))+19/162*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+19/162*arctan(
2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+19/324*ln(1+((-1+x)/x)^(1/3)/(1+1
/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-19/324*ln(1+((-1+
x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6306, 92, 81, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = -\frac{19}{162}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{19}{162}\arctan\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right)+\frac{19}{81}\arctan\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{18}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}+\frac{19}{54}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}-\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}$$

```
[In] Int[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] (19*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/54 + ((1 + x^(-1))^(7/6)*((-1 +
x)/x)^(5/6))/18 + ((1 + x^(-1))^(7/6)*((-1 + x)/x)^(5/6))/(3*x) - (19*ArcTa
n[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[Sq
rt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[((-1 +
 x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/81 + (19*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt
[3]) - (19*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 +
 x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-1-\frac{x}{3}\right)\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{54}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6}\\
&\quad + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{19}{81}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{19}{81}\text{Su}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x}$$

$$+ \frac{19}{81}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{19}{324}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{19}{324}\text{Subst}\left(\int \frac{}{1+}\right.$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{19\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{108\sqrt{3}} - \frac{19\log\left(1+\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right.}{108\sqrt{3}}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}+\frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x}$$

$$-\frac{19}{162}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{162}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{81}\arctan\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.46

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{1}{486}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(19+8e^{2\coth^{-1}(x)}+61e^{4\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^3} - 114\arctan\left(e^{\frac{1}{3}\coth^{-1}(x)}\right) - 19\text{RootSum}\left[1-\#1^2+\#1^4\&, \frac{-2\coth^{-1}(x)+6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+\coth^{-1}(x)\#1^2-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

```
[In] Integrate[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(19 + 8*E^(2*ArcCoth[x]) + 61*E^(4*ArcCoth[x])))/(1 +
 E^(2*ArcCoth[x]))^3 - 114*ArcTan[E^(ArcCoth[x]/3)] - 19*RootSum[1 - #1^2 +
 #1^4 & , (-2*ArcCoth[x] + 6*Log[E^(ArcCoth[x]/3) - #1] + ArcCoth[x]*#1^2 -
 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-#1 + 2*#1^3) & ])/486
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.23 (sec) , antiderivative size = 1498, normalized size of antiderivative = 5.22

method	result	size
trager	Expression too large to display	1498
risch	Expression too large to display	3004

```
[In] int(1/((x-1)/(1+x))^(1/6)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(1+x)*(22*x^2+21*x+18)/x^3*(-(1-x)/(1+x))^(5/6)+19/54*RootOf(81*_Z^4-9
*_Z^2+1)*ln((54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+54*RootOf
(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootOf(8
1*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(
1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1
-x)/(1+x))^(5/6)-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(
81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1
-x)/(1+x))^(1/3)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+9*RootOf
(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^3*x+
6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(81*_Z^4-9*_Z^2+1)^
2*(-(1-x)/(1+x))^(1/6)-(-(1-x)/(1+x))^(1/6)*x-RootOf(81*_Z^4-9*_Z^2+1)*x-(-
(1-x)/(1+x))^(1/6))/x)+19/6*RootOf(81*_Z^4-9*_Z^2+1)^3*ln((27*RootOf(81*_Z^
4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/
(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x
))^(2/3)*x+18*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(8
1*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)+3*RootOf(81*
_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(
1+x))^(1/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x)/(1
+x))^(1/2)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(81*
_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-18*RootOf(81*_Z^4-9*_Z^2+1)^3*x-2*(
-(1-x)/(1+x))^(1/2)-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*RootO
f(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)+RootOf(81*_Z^4-9*_Z^2+1)*x)/x)-1
9/54*RootOf(81*_Z^4-9*_Z^2+1)*ln((27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+
x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x
))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*x*RootOf(81
*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x
)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1
+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(81*
_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x)/(1+x))^(1/2)*x-6*RootOf(81*
_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/
(1+x))^(1/6)*x-18*RootOf(81*_Z^4-9*_Z^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)-6*Roo
tOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(
1-x)/(1+x))^(1/6)+RootOf(81*_Z^4-9*_Z^2+1)*x)/x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$$

$$= \frac{\sqrt{2}x^3\sqrt{361i\sqrt{3}+361}\log\left(\left(i\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{361i\sqrt{3}+361}+76\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-\sqrt{2}x^3\sqrt{361i\sqrt{3}+361}\log\left(\left(\right.\right.}{}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="fricas")
```

```
[Out] 1/324*(sqrt(2)*x^3*sqrt(361*I*sqrt(3) + 361)*log((I*sqrt(3)*sqrt(2) - sqrt(
2))*sqrt(361*I*sqrt(3) + 361) + 76*((x - 1)/(x + 1))^(1/6)) - sqrt(2)*x^3*s
qrt(361*I*sqrt(3) + 361)*log((-I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(361*I*sqrt
(3) + 361) + 76*((x - 1)/(x + 1))^(1/6)) - sqrt(2)*x^3*sqrt(-361*I*sqrt(3)
+ 361)*log((I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(-361*I*sqrt(3) + 361) + 76*((
x - 1)/(x + 1))^(1/6)) + sqrt(2)*x^3*sqrt(-361*I*sqrt(3) + 361)*log((-I*sqr
t(3)*sqrt(2) - sqrt(2))*sqrt(-361*I*sqrt(3) + 361) + 76*((x - 1)/(x + 1))^(
1/6)) + 76*x^3*arctan(((x - 1)/(x + 1))^(1/6)) + 6*(22*x^3 + 43*x^2 + 39*x
+ 18)*((x - 1)/(x + 1))^(5/6))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**4,x)
```

```
[Out] Integral(1/(x**4*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = &-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}+8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1}+\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}+1\right)}\\&+\frac{19}{162}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\&+\frac{19}{162}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="maxima")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
 + 1))^(1/3) + 1) + 1/27*(19*((x - 1)/(x + 1))^(17/6) + 8*((x - 1)/(x + 1))
^(11/6) + 61*((x - 1)/(x + 1))^(5/6))/(3*(x - 1)/(x + 1) + 3*(x - 1)^2/(x +
 1)^2 + (x - 1)^3/(x + 1)^3 + 1) + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x +
1))^(1/6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = -\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{\frac{8\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1} + \frac{19\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2} + 61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}+1\right)^3} + \frac{19}{162}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{19}{162}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="giac")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
 + 1))^(1/3) + 1) + 1/27*(8*(x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 19*(x
 - 1)^2*((x - 1)/(x + 1))^(5/6)/(x + 1)^2 + 61*((x - 1)/(x + 1))^(5/6))/((x
 - 1)/(x + 1) + 1)^3 + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*arctan(((x - 1
)/(x + 1))^(1/6))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{19\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{81} + \frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27} + \frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27} + \frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1} + \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} + 1} - \mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(-\frac{2476099}{14348907} + \frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19}{162} + \frac{\sqrt{3}\,19\mathrm{i}}{162}\right) - \mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(\frac{2476099}{14348907} + \frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)$$

```
[In] int(1/(x^4*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] (19*atan(((x - 1)/(x + 1))^(1/6)))/81 + ((61*((x - 1)/(x + 1))^(5/6))/27 +
(8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/((3*(x
- 1))/(x + 1) + (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) - atan((
4952198*((x - 1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 - 2
476099/14348907)))*((3^(1/2)*19i)/162 + 19/162) - atan((4952198*((x - 1)/(x
 + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/14348907)))*
((3^(1/2)*19i)/162 - 19/162)
```

3.120 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 157

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x+\frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3-\frac{22\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{27\sqrt{3}}-\frac{11}{27}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{11\log(x)}{81}$$

```
[Out] 14/27*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x+4/9*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x
^2+1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x^3-11/27*ln((1+1/x)^(1/3)-((-1+x)/x)
^(1/3))-11/81*ln(x)-22/81*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(
1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used

$= \{6306, 101, 156, 12, 93\}$

$$\int e^{\frac{2}{3}\coth^{-1}(x)} x^2\,dx = -\frac{22\arctan\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}} + \frac{1}{\sqrt{3}}\right)}{27\sqrt{3}} + \frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3} x^3 + \frac{4}{9}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3} x^2 + \frac{14}{27}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3} x - \frac{11}{27}\log\left(\sqrt[3]{\frac{1}{x}+1} - \sqrt[3]{\frac{x-1}{x}}\right) - \frac{11\log(x)}{81}$$

```
[In] Int[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] (14*(1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/27 + (4*(1 + x^(-1))^(1/3)*((-
1 + x)/x)^(2/3)*x^2)/9 + ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x^3)/3 - (2
2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/
(27*Sqrt[3]) - (11*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/27 - (11*L
og[x])/81
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{8}{3}+2x}{\sqrt[3]{1-x}x^3(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2\\
&\quad + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{28}{9}-\frac{8x}{3}}{\sqrt[3]{1-x}x^2(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2\\
&\quad + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{44}{27\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2\\
&\quad + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{22}{81}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3\\
&\quad - \frac{22\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{27\sqrt{3}} - \frac{11}{27}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{-\frac{1-x}{x}}\right) - \frac{11\log(x)}{81}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx=\frac{1}{81}\left(\frac{216e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^3}+\frac{360e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{210e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+22\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-22\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-22\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-22\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)+11\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+11\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] ((216*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^3 + (360*E^((2*ArcCoth[
x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (210*E^((2*ArcCoth[x])/3))/(-1 + E^(2*A
rcCoth[x])) + 22*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 22*Sqr
t[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 22*Log[1 - E^(ArcCoth[x]/3)
] - 22*Log[1 + E^(ArcCoth[x]/3)] + 11*Log[1 - E^(ArcCoth[x]/3) + E^((2*ArcC
oth[x])/3)] + 11*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/81
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.60

method	result
trager	$\frac{(1+x)(9x^2+12x+14)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{27}-\frac{22\ln\left(-9\operatorname{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\operatorname{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}+9\right.}{}$
risch	$\frac{(9x^2+12x+14)(x-1)}{27\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(22\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{-2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)}{}\right.\right.}{}$

```
[In] int(1/((x-1)/(1+x))^(1/3)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(1+x)*(9*x^2+12*x+14)*(-(1-x)/(1+x))^(2/3)-22/81*ln(-9*RootOf(9*_Z^2-3
*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+
9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-36*RootOf(9*_Z^2-3*_Z+1)^2*x
+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+12*R
ootOf(9*_Z^2-3*_Z+1)*x-3*(-(1-x)/(1+x))^(1/3)-6*RootOf(9*_Z^2-3*_Z+1)-x+1)+
22/27*RootOf(9*_Z^2-3*_Z+1)*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)
*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x+18
*RootOf(9*_Z^2-3*_Z+1)^2*x-3*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x-
15*RootOf(9*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-3*_Z+1)+2
*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \frac{1}{27}\left(9\,x^3+21\,x^2+26\,x+14\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \frac{22}{81}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right) + \frac{11}{81}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{22}{81}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^3 + 21*x^2 + 26*x + 14)*((x - 1)/(x + 1))^(2/3) - 22/81*sqrt(3)*a
rctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 11/81*log(((x -
1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(((x - 1)/(x +
1))^(1/3) - 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \int \frac{x^2}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \frac{2\left(11\left(\frac{x-1}{x+1}\right)^{\frac{8}{3}} - 10\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}} + 35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} + \frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{22}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/27*(
11*((x - 1)/(x + 1))^(8/3) - 10*((x - 1)/(x + 1))^(5/3) + 35*((x - 1)/(x +
1))^(2/3))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3
 - 1) + 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) -
22/81*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) + \frac{2\left(\frac{10(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1} - \frac{11(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{(x+1)^2} - 35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{x-1}{x+1}-1\right)^3} + \frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{22}{81}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/27*(
10*(x - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 11*(x - 1)^2*((x - 1)/(x + 1))
^(2/3)/(x + 1)^2 - 35*((x - 1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)^3 + 11
/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(
abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\frac{22\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-\frac{484}{729}\right)}{81}-\frac{\frac{70\left(\frac{x-1}{x+1}\right)^{2/3}}{27}-\frac{20\left(\frac{x-1}{x+1}\right)^{5/3}}{27}+\frac{22\left(\frac{x-1}{x+1}\right)^{8/3}}{27}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1} \\ -\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(-\frac{11}{81}+\frac{\sqrt{3}\,11i}{81}\right)^2\right)\left(-\frac{11}{81}+\frac{\sqrt{3}\,11i}{81}\right)+\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(\frac{11}{81}+\frac{\sqrt{3}\,11i}{81}\right)^2\right)$$

```
[In] int(x^2/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] log((484*((x - 1)/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 + 11/81)^2)*((3
^(1/2)*11i)/81 + 11/81) - ((70*((x - 1)/(x + 1))^(2/3))/27 - (20*((x - 1)/(
x + 1))^(5/3))/27 + (22*((x - 1)/(x + 1))^(8/3))/27)/((3*(x - 1))/(x + 1) -
 (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) - log((484*((x - 1)/(x
+ 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 - 11/81)^2)*((3^(1/2)*11i)/81 - 11/8
1) - (22*log((484*((x - 1)/(x + 1))^(1/3))/729 - 484/729))/81
```

3.121 $\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2-\frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{\log(x)}{9}$$

```
[Out] 1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x+1/2*(1+1/x)^(4/3)*((-1+x)/x)^(2/3)*x^2
-1/3*ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-1/9*ln(x)-2/9*arctan(1/3*3^(1/2)+2/
3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{6306, 98, 96, 93\}$

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\frac{2\arctan\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\left(\frac{1}{x}+1\right)^{4/3}\left(\frac{x-1}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x - \frac{1}{3}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right) - \frac{\log(x)}{9}$$

```
[In] Int[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/3 + ((1 + x^(-1))^(4/3)*((-1 + x)
/x)^(2/3)*x^2)/2 - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1
 + x^(-1))^(1/3))])/(3*Sqrt[3]) - Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/
3)]/3 - Log[x]/9
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x \\
&\quad + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 \\
&\quad - \frac{2\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(\sqrt[3]{1+\frac{1}{x}} - \sqrt[3]{-\frac{1-x}{x}}\right) - \frac{\log(x)}{9}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\begin{aligned}
\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \frac{1}{9}\Bigg(&\frac{18e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2} + \frac{24e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}} \\
&+ 2\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right) \\
&- 2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) - 2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right) \\
&+ \log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right) + \log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\Bigg)
\end{aligned}$$

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x,x]
[Out] ((18*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (24*E^((2*ArcCoth[x]
)/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/
```

```
Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E
^(ArcCoth[x]/3)] - 2*Log[1 + E^(ArcCoth[x]/3)] + Log[1 - E^(ArcCoth[x]/3) +
 E^((2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/
9
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.10

method	result
risch	$\frac{(5+3x)(x-1)}{6\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3\,\mathrm{R}}{}\right.}{}\right.}{}$
trager	$\frac{(1+x)(5+3x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6}+\frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+18\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootOf}\left(9\_Z^2-3\right.\right.}{}$

```
[In] int(1/((x-1)/(1+x))^(1/3)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(5+3*x)*(x-1)/((x-1)/(1+x))^(1/3)+(-2/9*ln(-(4*RootOf(_Z^2-_Z+1)^2*x^2+
4*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(_Z
^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x-4*RootOf(_Z^2-_Z+1)*x^2-3*RootOf(_Z^2-_Z+1)*
(x^3+x^2-x-1)^(1/3)-2*RootOf(_Z^2-_Z+1)*x+3*(x^3+x^2-x-1)^(1/3)*x+x^2+2*Roo
tOf(_Z^2-_Z+1)+3*(x^3+x^2-x-1)^(1/3)-1)/(1+x))+2/9*RootOf(_Z^2-_Z+1)*ln((2*
RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^
2-x-1)^(2/3)-5*RootOf(_Z^2-_Z+1)*x^2-6*RootOf(_Z^2-_Z+1)*x-3*(x^3+x^2-x-1)^
(2/3)+3*(x^3+x^2-x-1)^(1/3)*x+2*x^2-RootOf(_Z^2-_Z+1)+3*(x^3+x^2-x-1)^(1/3)
+4*x+2)/(1+x)))/((x-1)/(1+x))^(1/3)*((1+x)^2*(x-1))^(1/3)/(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\frac{1}{6}\left(3x^2+8x+5\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{9}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 8*x + 5)*((x - 1)/(x + 1))^(2/3) - 2/9*sqrt(3)*arctan(2/3*sqrt
(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/9*log(((x - 1)/(x + 1))^(2/3
) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\int\frac{x}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/3*(((x
 - 1)/(x + 1))^(5/3) - 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^
(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^2} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/3*((x
- 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/
(x + 1) - 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3)
+ 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1} - \frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-\frac{4}{9}\right)}{9} - \ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right.$$

```
[In] int(x/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] ((8*((x - 1)/(x + 1))^(2/3))/3 - (2*((x - 1)/(x + 1))^(5/3))/3)/((x - 1)^2/
(x + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))/9
 - 4/9))/9 - log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*1i)/9 - 1/9)^2
)*((3^(1/2)*1i)/9 - 1/9) + log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*
1i)/9 + 1/9)^2)*((3^(1/2)*1i)/9 + 1/9)
```

3.122 $\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{\log(x)}{3}$$

```
[Out] (1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x-ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-1/3*ln(
x)-2/3*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6305, 96, 93}

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=-\frac{2\arctan\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{3}$$

```
[In] Int[E^((2*ArcCoth[x])/3),x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x
)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[(1 + x^(-1))^(1/3)
 - ((-1 + x)/x)^(1/3)] - Log[x]/3
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x - \frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x - \frac{2\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}\\
&\quad - \log\left(\sqrt[3]{1+\frac{1}{x}} - \sqrt[3]{-\frac{1-x}{x}}\right) - \frac{\log(x)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\frac{1}{3}\left(\frac{6e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)\right)$$

```
[In] Integrate[E^((2*ArcCoth[x])/3),x]
```

```
[Out] ((6*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(1 + 2
*E^((2*ArcCoth[x])/3))/Sqrt[3]] - 2*Log[1 - E^((2*ArcCoth[x])/3)] + Log[1 +
 E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 503, normalized size of antiderivative = 5.24

method	result
trager	$(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-\frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-36\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_\right.\right.}{}$
risch	$\frac{x-1}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}+3\,\mathrm{RootOf}\left(\_\right.}{}\right.\right.}{}$

```
[In] int(1/((x-1)/(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)*(-(1-x)/(1+x))^(2/3)-2/3*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2
/3)*x-36*RootOf(9*_Z^2-3*_Z+1)^2*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(
2/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3)*
x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+12*RootOf(9*_Z^2-3*_Z+1)*x-3
*(-(1-x)/(1+x))^(2/3)+6*RootOf(9*_Z^2-3*_Z+1)-x-1)+2/3*ln(-9*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2*x-9*RootOf(9*_Z^2
-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(9*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1
/3)*x+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x+1)-2*ln(-9*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2*x-9*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(9*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x
```

```
))^(1/3)*x+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x+1)*RootOf(9*_Z^
2-3*_Z+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = (x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="fricas")
[Out] (x + 1)*((x - 1)/(x + 1))^(2/3) - 2/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/3) + 1/3*sqrt(3)) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(
x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = \int \frac{1}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3),x)
[Out] Integral(((x - 1)/(x + 1))**(-1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1} - 1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-4\right)}{3} - \ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}-1}$$

```
[In] int(1/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 + 1/3)^2)*((3^(1/2)*1i)/3
 + 1/3) - log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 - 1/3)^2)*((3^(
1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) - 4))/3 - (2*((x - 1)/
(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)
```

3.123 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 155

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{3}{2}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right) - \frac{3}{2}\log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{2}\log\left(1+\frac{1}{x}\right) - \frac{\log(x)}{2}$$

```
[Out] -3/2*ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-3/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(
1/3))-1/2*ln(1+1/x)-1/2*ln(x)+arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1
/x)^(1/3)*3^(1/2))*3^(1/2)-arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^
(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6306, 132, 62, 93}

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right) -\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right) -\frac{3}{2}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)-\frac{\log(x)}{2}$$

```
[In] Int[E^((2*ArcCoth[x])/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(
1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x
^(-1))^(1/3))] - (3*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/2 - (3*Lo
g[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 - Log[x
]/2
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right) \\
&= -\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) \\
&\quad - \frac{3}{2}\log\left(\sqrt[3]{1+\frac{1}{x}} - \sqrt[3]{\frac{-1+x}{x}}\right) - \frac{3}{2}\log\left(1 + \frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{2}\log\left(1 + \frac{1}{x}\right) - \frac{\log(x)}{2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \frac{3}{2}e^{\frac{8}{3}\coth^{-1}(x)}\,\text{Hypergeometric2F1}\left(\frac{2}{3}, 1, \frac{5}{3}, e^{4\coth^{-1}(x)}\right)$$

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x,x]
[Out] (3*E^((8*ArcCoth[x])/3)*Hypergeometric2F1[2/3, 1, 5/3, E^(4*ArcCoth[x])])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.62

method	result	size
trager	Expression too large to display	1026

```
[In] int(1/((x-1)/(1+x))^(1/3)/x,x,method=_RETURNVERBOSE)
[Out] -3*ln((945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*RootOf(9*_Z^
2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(2/3)*x^2-504*(-(1-x)/(
1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+72*RootOf(9*_Z^2-3*_Z+1)^2*x^2+945*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-336*(-(1-x)/(1+x))^(2/3)*x-147*(-(
1-x)/(1+x))^(1/3)*x^2-180*RootOf(9*_Z^2-3*_Z+1)^2*x-465*RootOf(9*_Z^2-3*_Z+
1)*x^2-168*(-(1-x)/(1+x))^(2/3)+504*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1
/3)+72*RootOf(9*_Z^2-3*_Z+1)^2+1026*RootOf(9*_Z^2-3*_Z+1)*x+323*x^2+147*(-(
1-x)/(1+x))^(1/3)-465*RootOf(9*_Z^2-3*_Z+1)-34*x+323)/x)*RootOf(9*_Z^2-3*_Z
+1)-ln(-(945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*RootOf(9*_
Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(2/3)*x^2-441*(-(1-x)
/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2-1152*RootOf(9*_Z^2-3*_Z+1)^2*x^2+94
5*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-294*(-(1-x)/(1+x))^(2/3)*x-168
*(-(1-x)/(1+x))^(1/3)*x^2+2880*RootOf(9*_Z^2-3*_Z+1)^2*x-120*RootOf(9*_Z^2-
3*_Z+1)*x^2-147*(-(1-x)/(1+x))^(2/3)+441*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x
))^(1/3)-1152*RootOf(9*_Z^2-3*_Z+1)^2-1884*RootOf(9*_Z^2-3*_Z+1)*x+187*x^2+
168*(-(1-x)/(1+x))^(1/3)-120*RootOf(9*_Z^2-3*_Z+1)+306*x+187)/x)+ln((945*(-
(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*RootOf(9*_Z^2-3*_Z+1)*(-(
1-x)/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(2/3)*x^2-504*(-(1-x)/(1+x))^(1/3)*R
ootOf(9*_Z^2-3*_Z+1)*x^2+72*RootOf(9*_Z^2-3*_Z+1)^2*x^2+945*RootOf(9*_Z^2-3
*_Z+1)*(-(1-x)/(1+x))^(2/3)-336*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(
1/3)*x^2-180*RootOf(9*_Z^2-3*_Z+1)^2*x-465*RootOf(9*_Z^2-3*_Z+1)*x^2-168*(-
(1-x)/(1+x))^(2/3)+504*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+72*RootOf
(9*_Z^2-3*_Z+1)^2+1026*RootOf(9*_Z^2-3*_Z+1)*x+323*x^2+147*(-(1-x)/(1+x))^(
1/3)-465*RootOf(9*_Z^2-3*_Z+1)-34*x+323)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right)+\frac{1}{2}\log\left(\frac{(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+x+1}{x+1}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(2/3) + 1/3*sqrt(3)) - log(((x
 - 1)/(x + 1))^(2/3) - 1) + 1/2*log(((x + 1)*((x - 1)/(x + 1))^(2/3) + (x -
 1)*((x - 1)/(x + 1))^(1/3) + x + 1)/(x + 1))
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx=\int \frac{1}{x\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx=&-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)\\&+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)\\&+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1))
^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3) - (
(x - 1)/(x + 1))^(1/3) + 1) - log(((x - 1)/(x + 1))^(1/3) + 1) - log(((x -
1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+1\right)\right) + \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}{x+1} + 1\right) - \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - 1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(2/3) + 1)) + 1/2*log(((x -
 1)/(x + 1))^(2/3) + (x - 1)*((x - 1)/(x + 1))^(1/3)/(x + 1) + 1) - log(abs
(((x - 1)/(x + 1))^(2/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3} - 1296\right) - \ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3} + 648 - \sqrt{3}\,648\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3} + 648 + \sqrt{3}\,648\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

```
[In] int(1/(x*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] log(3^(1/2)*648i + 1296*((x - 1)/(x + 1))^(2/3) + 648)*((3^(1/2)*1i)/2 + 1/
2) - log(1296*((x - 1)/(x + 1))^(2/3) - 3^(1/2)*648i + 648)*((3^(1/2)*1i)/2
 - 1/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 1296)
```

3.124 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 99

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2} dx = \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}} - \log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{3}\log\left(1+\frac{1}{x}\right)$$

```
[Out] (1+1/x)^(1/3)*((-1+x)/x)^(2/3)-ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3))-1/3*ln(
1+1/x)+2/3*arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6306, 52, 62}

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{\sqrt{3}}+\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3} - \log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)$$

```
[In] Int[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3) - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/
x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[1 + ((-1 + x)/x)^(1/
3)/(1 + x^(-1))^(1/3)] - Log[1 + x^(-1)]/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}\\
&\quad - \log\left(1+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{3}\log\left(1+\frac{1}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx &= \frac{2e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - \frac{2\arctan\left(\frac{-1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)}{\sqrt{3}}\\
&\quad - \frac{2}{3}\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) + \frac{1}{3}\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)
\end{aligned}
$$

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (2*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x])) - (2*ArcTan[(-1 + 2*E^((2*A
rcCoth[x])/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 + E^((2*ArcCoth[x])/3)])/3 + Lo
g[1 - E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.22

method	result
trager	$\frac{(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{x} - \frac{2\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-3\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int(1/((x-1)/(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)/x*(-(1-x)/(1+x))^(2/3)-2/3*ln((9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))
^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3
)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3)+9
*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-36*RootOf(9*_Z^2-3*_Z+1)^2+6*Ro
otOf(9*_Z^2-3*_Z+1)*x+12*RootOf(9*_Z^2-3*_Z+1)-x-1)/x)+2/3*ln(-(9*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^
(2/3)+3*(-(1-x)/(1+x))^(1/3)*x-18*RootOf(9*_Z^2-3*_Z+1)^2-3*RootOf(9*_Z^2-3
*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-3*_Z+1)-x+1)/x)-2*ln(-(9*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/
(1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x-18*RootOf(9*_Z^2-3*_Z+1)^2-3*RootOf(9
*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-3*_Z+1)-x+1)/x)*Root
Of(9*_Z^2-3*_Z+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-2x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3(x+1)(}{3x}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3))
+ x*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 2*x*log(((
x - 1)/(x + 1))^(1/3) + 1) + 3*(x + 1)*((x - 1)/(x + 1))^(2/3))/x
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**2,x)
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$$
$$= \frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}+1} - \ln\left(9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$
$$+\ln\left(9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) - \frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}+4\right)}{3}$$

```
[In] int(1/(x^2*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/3 + 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(1/2)*1i)/3
 + 1/3) - log(9*((3^(1/2)*1i)/3 - 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(
1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) + 4))/3 + (2*((x - 1)/
(x + 1))^(2/3))/((x - 1)/(x + 1) + 1)
```

3.125 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 130

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{9}\log\left(1+\frac{1}{x}\right)$$

```
[Out] 1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)+1/2*(1+1/x)^(4/3)*((-1+x)/x)^(2/3)-1/3*l
n(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3))-1/9*ln(1+1/x)+2/9*arctan(-1/3*3^(1/2)+2
/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6306, 81, 52, 62\}$

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{3\sqrt{3}} + \frac{1}{2}\left(\frac{x-1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3} + \frac{1}{3}\left(\frac{x-1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1} - \frac{1}{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right) - \frac{1}{9}\log\left(\frac{1}{x}+1\right)$$

```
[In] Int[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3))/3 + ((1 + x^(-1))^(4/3)*((-1 + x)/x
)^(2/3))/2 - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(
-1))^(1/3))])/(3*Sqrt[3]) - Log[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/
3 - Log[1 + x^(-1)]/9
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}\\
&\quad + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}\\
&\quad - \frac{2\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{9}\log\left(1+\frac{1}{x}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{2}{27}\left(\frac{27e^{\frac{2}{3}\coth^{-1}(x)}}{\left(1+e^{2\coth^{-1}(x)}\right)^2} - \frac{36e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - 2\coth^{-1}(x) + 3\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) - \text{RootSum}\left[1-\#1^2 + \#1^4\&, \frac{\coth^{-1}(x) - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)} - \#1\right) + \coth^{-1}(x)\#1^2 - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)} - \#1\right)\#1^2}{-2+\#1^2}\&\right]\right)$$

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] (-2*((27*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x]))^2 - (36*E^((2*ArcCoth
[x])/3))/(1 + E^(2*ArcCoth[x])) - 2*ArcCoth[x] + 3*Log[1 + E^((2*ArcCoth[x]
)/3)] - RootSum[1 - #1^2 + #1^4 & , (ArcCoth[x] - 3*Log[E^(ArcCoth[x]/3) -
#1] + ArcCoth[x]*#1^2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-2 + #1^2) & ])
)/27
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.89

method	result
risch	$\frac{(x-1)(5x+3)}{6x^2\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x^2-8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}}{}\right.}{}\right.}{}$
trager	$\frac{(1+x)(5x+3)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6x^2}+\frac{2\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\ln\left(\frac{-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)}{}\right.}{}$

```
[In] int(1/((x-1)/(1+x))^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x-1)*(5*x+3)/x^2/((x-1)/(1+x))^(1/3)+(-2/9*ln((8*RootOf(_Z^2-3*_Z+9)^2
*x^2-8*RootOf(_Z^2-3*_Z+9)^2*x+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-4
5*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x-30*RootOf(_Z^2-3*_Z+9)*x^2-16*R
ootOf(_Z^2-3*_Z+9)^2-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-54*RootOf(_
Z^2-3*_Z+9)*x-216*(x^3+x^2-x-1)^(2/3)-81*(x^3+x^2-x-1)^(1/3)*x-27*x^2-24*Ro
otOf(_Z^2-3*_Z+9)-81*(x^3+x^2-x-1)^(1/3)-36*x-9)/x/(1+x))+2/27*RootOf(_Z^2-
3*_Z+9)*ln((2*RootOf(_Z^2-3*_Z+9)^2*x^2-2*RootOf(_Z^2-3*_Z+9)^2*x-27*RootOf
(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-72*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3
)*x+27*RootOf(_Z^2-3*_Z+9)*x^2-4*RootOf(_Z^2-3*_Z+9)^2-72*RootOf(_Z^2-3*_Z+
9)*(x^3+x^2-x-1)^(1/3)-6*RootOf(_Z^2-3*_Z+9)*x-135*(x^3+x^2-x-1)^(2/3)+81*(
x^3+x^2-x-1)^(1/3)*x+36*x^2-33*RootOf(_Z^2-3*_Z+9)+81*(x^3+x^2-x-1)^(1/3)+2
16*x+180)/x/(1+x)))/((x-1)/(1+x))^(1/3)*((1+x)^2*(x-1))^(1/3)/(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$$
$$= \frac{4\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-4\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(5\,x^2}{18\,x^2}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3
)) + 2*x^2*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 4*x
^2*log(((x - 1)/(x + 1))^(1/3) + 1) + 3*(5*x^2 + 8*x + 3)*((x - 1)/(x + 1))
^(2/3))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}$$
$$+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*(((x
- 1)/(x + 1))^(5/3) + 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) + (x -
1)^2/(x + 1)^2 + 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(
1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}+1\right)^2} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*((x -
 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) + 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/(
x + 1) + 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) +
 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}+\frac{4}{9}\right)}{9} - \ln\left(9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(9\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

```
[In] int(1/(x^3*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] ((8*((x - 1)/(x + 1))^(2/3))/3 + (2*((x - 1)/(x + 1))^(5/3))/3)/((2*(x - 1)
)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))/9
 + 4/9))/9 - log(9*((3^(1/2)*1i)/9 - 1/9)^2 + (4*((x - 1)/(x + 1))^(1/3))/9
)*((3^(1/2)*1i)/9 - 1/9) + log(9*((3^(1/2)*1i)/9 + 1/9)^2 + (4*((x - 1)/(x
+ 1))^(1/3))/9)*((3^(1/2)*1i)/9 + 1/9)
```

3.126 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 429

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2}+\frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3-\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}+\frac{11\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}-\frac{11\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}+\frac{11\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}$$

```
[Out] 37/96*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a^2+3/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(
1/8)*x^2/a+1/3*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x^3+11/64*arctan((1+1/a/x)^(
```

```
1/8)/(1-1/a/x)^(1/8))/a^3+11/64*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^
3-11/128*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)+11/1
28*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)-11/256*ln(
1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/
a^3*2^(1/2)+11/256*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(
1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6306, 101, 156, 12, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = -\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}+\frac{11\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}-\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}+\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}+\frac{37x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{96a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{3x^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

```
[In] Int[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] (37*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(96*a^2) + (3*(1 - 1/(a*x))^
(7/8)*(1 + 1/(a*x))^(1/8)*x^2)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x^3)/3 - (11*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1
/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 -
```

```
1/(a*x))^(1/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(
a*x))^(1/8)])/(64*a^3) + (11*ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8
)])/(64*a^3) - (11*Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8
) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3) + (11*Log[1
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^4\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{9}{4a}+\frac{2x}{a^2}}{x^3\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{37}{16a^2}-\frac{9x}{4a^3}}{x^2\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{33}{64a^3x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\mathrm{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{128a^3}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\mathrm{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^3}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{32a^3} + \frac{11\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{32a^3}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} + \frac{11\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} + \frac{11\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} + \frac{11\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a}$$

$$+ \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}$$

$$+ \frac{11\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} + \frac{11\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{128a^3}$$

$$+ \frac{11\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{128a^3}$$

$$- \frac{11\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}$$

$$- \frac{11\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} + \frac{11\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} - \frac{11\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3} + \frac{11\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3} + \frac{11\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3} - \frac{11\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3} + \frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3} + \frac{11\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} + \frac{11\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3} - \frac{11\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3} + \frac{11\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.39

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$$

$$= \frac{-4\left(-\frac{1024e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{1600e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - \frac{840e^{\frac{1}{4}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 66\arctan\left(e^{\frac{1}{4}\coth^{-1}(ax)}\right) + 33\log\left(1-e^{\frac{1}{4}\coth^{-1}(ax)}\right.\right.}{1536a^3}$$

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] (-4*((-1024*E^(ArcCoth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^3 - (1600*E^(ArcC
oth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^2 - (840*E^(ArcCoth[a*x]/4))/(-1 + E
^(2*ArcCoth[a*x])) - 66*ArcTan[E^(ArcCoth[a*x]/4)] + 33*Log[1 - E^(ArcCoth[
a*x]/4)] - 33*Log[1 + E^(ArcCoth[a*x]/4)]) - 33*RootSum[1 + #1^4 & , (ArcCo
th[a*x] - 4*Log[E^(ArcCoth[a*x]/4) - #1])/#1^3 & ])/(1536*a^3)
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)

[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.62

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = -\frac{33\,a^3\left(-\frac{1}{a^{12}}\right)^{\frac{1}{4}}\log\left(a^9\left(-\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-33i\,a^3\left(-\frac{1}{a^{12}}\right)^{\frac{1}{4}}\log\left(i\,a^9\left(-\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+33i\,a^3\left(-\frac{1}{a^{12}}\right.}{}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="fricas")

[Out] -1/384*(33*a^3*(-1/a^12)^(1/4)*log(a^9*(-1/a^12)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8)) - 33*I*a^3*(-1/a^12)^(1/4)*log(I*a^9*(-1/a^12)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8)) + 33*I*a^3*(-1/a^12)^(1/4)*log(-I*a^9*(-1/a^12)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8)) - 33*a^3*(-1/a^12)^(1/4)*log(-a^9*(-1/a^12)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8)) - 4*(32*a^3*x^3 + 68*a^2*x^2 + 73*a*x + 37)*((a*x - 1)/(a*x + 1))^(7/8) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/8)) - 33*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) + 33*log(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^3

Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**2,x)

[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/8), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.79

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{768}a\left(\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="maxima")
```

```
[Out] -1/768*a*(16*(33*((a*x - 1)/(a*x + 1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(1
5/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x
 - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*l
og(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1))/a^4 + 132*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x
 - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a
^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.72

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{768}a\left(\frac{66\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}+\frac{66\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}-\frac{33\sqrt{2}\,\mathrm{lo}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="giac")
```

```
[Out] -1/768*a*(66*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/8)))/a^4 + 66*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/8)))/a^4 - 33*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 33*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 132*arctan(((a*x - 1)
/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*l
og(-((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 16*(33*((a*x - 1)/(a*x + 1))^(23
/8) - 10*((a*x - 1)/(a*x + 1))^(15/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(a
^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.53

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx=\frac{\frac{35\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{16}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{24}+\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{23/8}}{16}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,11\mathrm{i}}{64\,a^3}-\frac{11\,\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{64\,a^3}+\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{11}{128}+\frac{11}{128}\mathrm{i}\right)}{a^3}+\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{11}{128}-\frac{11}{128}\mathrm{i}\right)}{a^3}$$

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] ((35*((a*x - 1)/(a*x + 1))^(7/8))/16 - (5*((a*x - 1)/(a*x + 1))^(15/8))/24
+ (11*((a*x - 1)/(a*x + 1))^(23/8))/16)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1
)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (atan(
((a*x - 1)/(a*x + 1))^(1/8)*1i)*11i)/(64*a^3) - (11*atan(((a*x - 1)/(a*x +
1))^(1/8)))/(64*a^3) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1
/2 - 1i/2))*(11/128 - 11i/128))/a^3 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x
 + 1))^(1/8)*(1/2 + 1i/2))*(11/128 + 11i/128))/a^3
```

3.127 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 392

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} + \frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2}$$

```
[Out] 1/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a+1/2*(1-1/a/x)^(7/8)*(1+1/a/x)^(9/8)
*x^2+1/16*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^2+1/16*arctanh((1+1/a/x
```

```
)^(1/8)/(1-1/a/x)^(1/8))/a^2-1/32*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x
)^(1/8))/a^2*2^(1/2)+1/32*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))
/a^2*2^(1/2)-1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1
/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)+1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+
(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6306, 98, 96, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{16\sqrt{2}a^2}+\frac{\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

```
[In] Int[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1
 + 1/(a*x))^(9/8)*x^2)/2 - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/
(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(
1 - 1/(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a
*x))^(1/8)]/(16*a^2) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/(16
```

```
*a^2) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(32*Sqrt[2]*a^2) + Log[1 + (Sqrt[2]*(1 +
1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/(32*Sqrt[2]*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^3\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{32a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$=\frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2+\frac{\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{8a^2}+\frac{\text{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{8a^2}$$

$$=\frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2+\frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}$$

$$=\frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{32a^2}+\frac{\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{32a^2}-\frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2}-\frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} + \frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \frac{\frac{2}{\left(-1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{-1+e^{\frac{1}{4}\coth^{-1}(ax)}}-\frac{2}{\left(1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{1+e^{\frac{1}{4}\coth^{-1}(ax)}}+\frac{8e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)^2}-\frac{12e^{\frac{1}{4}\coth^{-1}(ax)}}{1+e^{\frac{1}{2}\coth^{-1}(ax)}}+}{}$$

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] (2/(-1 + E^(ArcCoth[a*x]/4))^2 + 6/(-1 + E^(ArcCoth[a*x]/4)) - 2/(1 + E^(Ar
cCoth[a*x]/4))^2 + 6/(1 + E^(ArcCoth[a*x]/4)) + (8*E^(ArcCoth[a*x]/4))/(1 +
 E^(ArcCoth[a*x]/2))^2 - (12*E^(ArcCoth[a*x]/4))/(1 + E^(ArcCoth[a*x]/2)) +
 (32*E^(ArcCoth[a*x]/4))/(1 + E^ArcCoth[a*x])^2 - (40*E^(ArcCoth[a*x]/4))/(
1 + E^ArcCoth[a*x]) + 4*ArcTan[E^(ArcCoth[a*x]/4)] - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^(ArcCoth[a*x]/4)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/4
)] + 4*ArcTanh[E^(ArcCoth[a*x]/4)] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x
]/4) + E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/4) + E
^(ArcCoth[a*x]/2)])/(64*a^2)
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = -\frac{a^2\left(-\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(a^6\left(-\frac{1}{a^8}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-i\,a^2\left(-\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(i\,a^6\left(-\frac{1}{a^8}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+i\,a^2\left(-\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(-i\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="fricas")
```

```
[Out] -1/32*(a^2*(-1/a^8)^(1/4)*log(a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(a*x + 1))^(1
/8)) - I*a^2*(-1/a^8)^(1/4)*log(I*a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(a*x + 1)
```

```
)^(1/8)) + I*a^2*(-1/a^8)^(1/4)*log(-I*a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(a*x
 + 1))^(1/8)) - a^2*(-1/a^8)^(1/4)*log(-a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(a*
x + 1))^(1/8)) - 4*(4*a^2*x^2 + 9*a*x + 5)*((a*x - 1)/(a*x + 1))^(7/8) + 2*
arctan(((a*x - 1)/(a*x + 1))^(1/8)) - log(((a*x - 1)/(a*x + 1))^(1/8) + 1)
+ log(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^2
```

Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x,x)
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/8), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.78

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \frac{1}{64}a\left(\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\right.\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="maxima")
[Out] 1/64*a*(16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x - 1)/(a*x + 1))^(7/8))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - (2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*lo
g(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1))/a^3 - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^3 + 2*log(((a*x - 1
)/(a*x + 1))^(1/8) + 1)/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.73

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{1}{64}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}-\frac{\sqrt{2}\log\left(\sqrt{\right.}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="giac")
```

```
[Out] -1/64*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/8)))/a^3 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8)))/a^3 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^3 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 + 4*arctan(((a*x - 1)/(a*x + 1))
^(1/8))/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 2*log(-((a*x - 1
)/(a*x + 1))^(1/8) + 1)/a^3 + 16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x -
1)/(a*x + 1))^(7/8))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.48

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^2}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{16\,a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}+\frac{1}{32}\mathrm{i}\right)}{a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)}{a^2}$$

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] ((9*((a*x - 1)/(a*x + 1))^(7/8))/4 - ((a*x - 1)/(a*x + 1))^(15/8)/4)/(a^2 +
 (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (atan(((a*x
 - 1)/(a*x + 1))^(1/8)*1i)*1i)/(16*a^2) - atan(((a*x - 1)/(a*x + 1))^(1/8))
/(16*a^2) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))
*(1/32 - 1i/32))/a^2 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1
/2 + 1i/2))*(1/32 + 1i/32))/a^2
```

3.128 $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 352

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx=\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}+\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}$$

```
[Out] (1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x+1/2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8
))/a+1/2*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a-1/4*arctan(1-(1+1/a/x)^
(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/4*arctan(1+(1+1/a/x)^(1/8)*2^(1/
2)/(1-1/a/x)^(1/8))/a*2^(1/2)-1/8*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1
```

```
/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/8*ln(1+(1+1/a/x)^(1/4)/(1-
1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {6305, 96, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{2\sqrt{2}a}+\frac{\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}a}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}a}$$

```
[In] Int[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x
))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*(1 + 1/(
a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)]/(2*a) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/(2*a) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1
 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*Sqrt[2]*a) + Log[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/(4*Sqrt[2]*a)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$=\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x-\frac{\text{Subst}\left(\int\frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$=\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x-\frac{2\text{Subst}\left(\int\frac{1}{-1+x^8}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$=\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x+\frac{\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}+\frac{\text{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$=\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x+\frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}$$
$$+\frac{\operatorname{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4a}+\frac{\operatorname{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4a}$$
$$-\frac{\operatorname{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}-\frac{\operatorname{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}$$
$$-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}+\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}$$
$$+\frac{\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}-\frac{\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}+\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.16

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \frac{2e^{\frac{1}{4}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

```
[In] Integrate[E^(ArcCoth[a*x]/4),x]
[Out] (2*E^(ArcCoth[a*x]/4)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[1/8,
 1, 9/8, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8),x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\frac{a\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-i\,a\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(i\,a^3\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+i\,a\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-i\,a^3\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="fricas")
```

```
[Out] -1/4*(a*(-1/a^4)^(1/4)*log(a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8)
) - I*a*(-1/a^4)^(1/4)*log(I*a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/
8)) + I*a*(-1/a^4)^(1/4)*log(-I*a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1))^
(1/8)) - a*(-1/a^4)^(1/4)*log(-a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1))^(
1/8)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(7/8) + 2*arctan(((a*x - 1)/(a*x
+ 1))^(1/8)) - log(((a*x - 1)/(a*x + 1))^(1/8) + 1) + log(((a*x - 1)/(a*x +
 1))^(1/8) - 1))/a
```

Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/8), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.75

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\frac{1}{8}\,a\left(\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right.}{a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="maxima")
```

```
[Out] -1/8*a*(16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)*a^2/(a*x + 1) - a^2) + (2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sq
rt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
 + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
+ 1))^(1/4) + 1))/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^2 - 2*log((
(a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1
)/a^2)
```

Giac [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx=\int\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(-1/8), x)
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.42

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a}-\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2\,a}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{a}$$

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(7/8))/(a - (a*(a*x - 1))/(a*x + 1)) - (atan(((a*x
 - 1)/(a*x + 1))^(1/8)*1i)*1i)/(2*a) - atan(((a*x - 1)/(a*x + 1))^(1/8))/(2
*a) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4
- 1i/4))/a - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 + 1i/2)
)*(1/4 + 1i/4))/a
```

3.129 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 919

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\ldots} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\ldots}\right.$$

```
[Out] 2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))+2*arctanh((1+1/a/x)^(1/8)/(1-1/a/
x)^(1/8))-1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/
(1-1/a/x)^(1/8))*2^(1/2)+1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)
^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^(1/2)-arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1
-1/a/x)^(1/8))*2^(1/2)+arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^
(1/2)-arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1
/2))^(1/2))*(2-2^(1/2))^(1/2)+arctan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+
2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/
4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^
(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^
(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-arctan((-2*(1-1/a/x)^(1/8)/
(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+arc
tan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
)*(2+2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8
)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(
1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+
2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {6306, 132, 65, 338, 305, 1136, 1183, 648, 632, 210, 642, 95, 220, 218, 212, 209, 217, 1179, 1176, 631}

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2+\sqrt{2}}\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right) + 2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}+1\right)$$

```
[In] Int[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] -(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]]) - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 +
Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]]
+ Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + S
qrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]] -
 Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + Sq
rt[2]*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + 2*Arc
Tan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] + 2*ArcTanh[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)] + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1
+ 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1
/8)])/2 - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4
) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 + (Sqrt
[2 + Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + S
qrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8
))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2] +
 Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 95</u>

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

<u>Rule 132</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
```

```
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= 8\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right) - 8\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$= 4\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+ 8\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+ 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+ \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$- \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} \\
&\quad - \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) \\
&\quad + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)
\end{aligned}
$$

$$= 2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$- \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$+ \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$- \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$- \sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$- \sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+ \sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+ \sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$+\frac{1}{2}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int\frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int\frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\left(-2+\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\left(-2+\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+\frac{1}{2}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int\frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int\frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+\frac{1}{2}\left(2+\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+\frac{1}{2}\left(2+\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+\frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$-\left(2-\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,-\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\left(2-\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\left(2+\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,-\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\left(2+\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= -\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$-\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+2\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{\ldots}\sqrt{\ldots}\left(\frac{\sqrt[4]{1-\frac{1}{\ldots}}}{\ldots}\quad\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{\ldots}}}{\ldots}\right.$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.03

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \frac{16}{9}e^{\frac{9}{4}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{9}{16},1,\frac{25}{16},e^{4\coth^{-1}(ax)}\right)$$

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] (16*E^((9*ArcCoth[a*x])/4)*Hypergeometric2F1[9/16, 1, 25/16, E^(4*ArcCoth[a
*x])])/9
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.45

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i+1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i-1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i-1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i+1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+(-1)^{\frac{1}{8}}\log\left((-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$-i(-1)^{\frac{1}{8}}\log\left(i(-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+i(-1)^{\frac{1}{8}}\log\left(-i(-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$-(-1)^{\frac{1}{8}}\log\left(-(-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)$$
$$+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="fricas")
```

```
[Out] -(1/2*I - 1/2)*sqrt(2)*(-1)^(1/8)*log((I + 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x
- 1)/(a*x + 1))^(1/8)) + (1/2*I + 1/2)*sqrt(2)*(-1)^(1/8)*log(-(I - 1)*sqrt
(2)*(-1)^(7/8) + 2*((a*x - 1)/(a*x + 1))^(1/8)) - (1/2*I + 1/2)*sqrt(2)*(-1
)^(1/8)*log((I - 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x - 1)/(a*x + 1))^(1/8)) + (
1/2*I - 1/2)*sqrt(2)*(-1)^(1/8)*log(-(I + 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x -
```

```
 1)/(a*x + 1))^(1/8)) - (1/2*I - 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x
 - 1)/(a*x + 1))^(1/8)) + (1/2*I + 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((
a*x - 1)/(a*x + 1))^(1/8)) - (1/2*I + 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*
((a*x - 1)/(a*x + 1))^(1/8)) + (1/2*I - 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) +
 2*((a*x - 1)/(a*x + 1))^(1/8)) + (-1)^(1/8)*log((-1)^(7/8) + ((a*x - 1)/(a
*x + 1))^(1/8)) - I*(-1)^(1/8)*log(I*(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/
8)) + I*(-1)^(1/8)*log(-I*(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/8)) - (-1)^
(1/8)*log(-(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/8)) - 2*arctan(((a*x - 1)/
(a*x + 1))^(1/8)) + log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - log(((a*x - 1)/(
a*x + 1))^(1/8) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x,x)
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="maxima")
[Out] integrate(1/(x*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Giac [A] (verification not implemented)

none

Time = 1.04 (sec) , antiderivative size = 661, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
8)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/8)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + (
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a
 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a + 2*log(-((a*x - 1)/(a*x + 1))^
(1/8) + 1)/a - 4*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))
/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4)) - 4*arctan(-(sqrt(sqrt(2) + 2)
 - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4
)) - 4*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqr
t(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) - 4*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a
*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) + 2*l
og(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) - 2*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*
x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) +
 2*log(sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1
))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4)) - 2*log(-sqrt(-sqrt(2) + 2)*((a*x - 1
)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4
)))
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right) +\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(-1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(-1-\mathrm{i})+\mathrm{ata}$$

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (((a*x - 1)/(a*
x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8
)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2
*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i + (2^(1/2) + 2)^(1/2)*1i) -
2*atan(((a*x - 1)/(a*x + 1))^(1/8)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x
+ 1))^(1/8)*(1/2 - 1i/2))*(1 - 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x +
 1))^(1/8)*(1/2 + 1i/2))*(1 + 1i) - atan(((a*x - 1)/(a*x + 1))^(1/8)*1i)*2i
 - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (((a*x - 1)/
(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (2^(1/2)*((a*x - 1)/(a*x + 1))^(
1/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)
/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i - (2^(1/2) + 2)^(1/2)*1i)
```

```
 - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2) - (((a*x - 1
)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)
*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i + (2 - 2^(1/2))^(1/
2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2) + (((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x - 1)/(a*x
 + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) - (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i - (2 - 2^(1/2
))^(1/2)*1i)
```

3.130 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 676

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\sqrt{2+\sqrt{2}}\,a\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2-\sqrt{2}}\,a\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}}\,a\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2-\sqrt{2}}\,a\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

```
[Out] a*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x
)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*a*arcta
n((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*
(2-2^(1/2))^(1/2)+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8
)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)
^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(
2-2^(1/2))^(1/2)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*a*arctan((2*(1-1/a/x)^(1
/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/
2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules

used = {6306, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$-\frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+\frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$+\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)$$

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)$$

$$+\frac{1}{8}\sqrt{2+\sqrt{2}}a\log\left(\sqrt[4]{1-\frac{1}{ax}} \quad \sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}} \quad +1\right)$$

```
[In] Int[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) - (Sqrt[2 + Sqrt[2]]*a*ArcTan[(Sq
rt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/4 - (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x
))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqrt[2]]*a
*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/S
qrt[2 + Sqrt[2]]])/4 + (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*
(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 -
 Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 - Sqrt[2]]*a*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 + (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1
/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))
/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)
/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x)
)^(1/8)])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
```

```
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$- \frac{a\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}-\frac{a\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{a\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{a\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{a\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{a\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{a\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$
\begin{aligned}
&= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} \\
&\quad + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \frac{1}{8}\left(\sqrt{2-\sqrt{2}}a\right)\text{Subst}\left(\int\frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad - \frac{1}{8}\left(\sqrt{2-\sqrt{2}}a\right)\text{Subst}\left(\int\frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \frac{1}{8}\left(\sqrt{2+\sqrt{2}}a\right)\text{Subst}\left(\int\frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad - \frac{1}{8}\left(\sqrt{2+\sqrt{2}}a\right)\text{Subst}\left(\int\frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
&\quad + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)
\end{aligned}
$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$+\frac{1}{8}\sqrt{2+\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{8}\sqrt{2+\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,-\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\left(\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,-\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$-\frac{1}{2}\left(\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}a\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$
\begin{aligned}
&= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)\\
&\quad - \frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)\\
&\quad + \frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)\\
&\quad + \frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)\\
&\quad + \frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)\\
&\quad - \frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)\\
&\quad + \frac{1}{8}\sqrt{2+\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.07

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -2ae^{\frac{1}{4}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}} + \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^2,x]
[Out] -2*a*E^(ArcCoth[a*x]/4)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometric2F1
[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])])
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^2}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \frac{-(i-1)\sqrt{2}(-a^8)^{\frac{1}{8}}\,x\log\left(2\,a^7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+(i+1)\sqrt{2}(-a^8)^{\frac{7}{8}}\right)+(i+1)\sqrt{2}(-a^8)^{\frac{1}{8}}\,x\log\left(2\,a^7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-(\cdots\right.}{\cdots}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="fricas")
[Out] 1/8*(-(I - 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1))^(1/8)
+ (I + 1)*sqrt(2)*(-a^8)^(7/8)) + (I + 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*
((a*x - 1)/(a*x + 1))^(1/8) - (I - 1)*sqrt(2)*(-a^8)^(7/8)) - (I + 1)*sqrt(
2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1))^(1/8) + (I - 1)*sqrt(2)*(
-a^8)^(7/8)) + (I - 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1
))^(1/8) - (I + 1)*sqrt(2)*(-a^8)^(7/8)) + 2*(-a^8)^(1/8)*x*log(a^7*((a*x -
 1)/(a*x + 1))^(1/8) + (-a^8)^(7/8)) - 2*I*(-a^8)^(1/8)*x*log(a^7*((a*x - 1
)/(a*x + 1))^(1/8) + I*(-a^8)^(7/8)) + 2*I*(-a^8)^(1/8)*x*log(a^7*((a*x - 1
```

```
)/(a*x + 1))^(1/8) - I*(-a^8)^(7/8)) - 2*(-a^8)^(1/8)*x*log(a^7*((a*x - 1)/
(a*x + 1))^(1/8) - (-a^8)^(7/8)) + 8*(a*x + 1)*((a*x - 1)/(a*x + 1))^(7/8))
/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**2,x)
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="maxima")
[Out] integrate(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.64

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{8}\left(2\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="giac")
[Out] 1/8*(2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1
))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2)
+ 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(sqrt(2)
+ 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(
2) + 2)) + 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a*x - 1)/(
```

```
a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) +
2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(sq
rt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1) - sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*((a*x - 1
)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(-sqrt(2) + 2)*
log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))
^(1/4) + 1) + 16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)/(a*x + 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.24

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \frac{(-1)^{1/8}\,a\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2} + \frac{(-1)^{1/8}\,a\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}1\mathrm{i}\right)\,1\mathrm{i}}{2} + \frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{\frac{a\,x-1}{a\,x+1}+1} + (-1)^{1/8}\sqrt{2}\,a\,\mathrm{atan}\left((-1)^{1/8}\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+(-1)^{1/8}\sqrt{2}\,a\,\mathrm{atan}\left((-1)^{1/8}\sqrt{2}\left(\right.\right.$$

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] ((-1)^(1/8)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/2 + ((-1)^(1/8)
*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/2 + (2*a*((a*x - 1)/
(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1) + (-1)^(1/8)*2^(1/2)*a*atan((-1
)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4 - 1i/4) + (-
1)^(1/8)*2^(1/2)*a*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2
 + 1i/2))*(1/4 + 1i/4)
```

3.131 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 731

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}$$
$$-\frac{1}{32}\sqrt{2+\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{32}\sqrt{2-\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

```
[Out] 1/8*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)+1/2*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(
9/8)-1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))
/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)/(1
+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/64*
a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/
(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-
1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)/(1+1/a/
x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/64*a^2*l
n(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/
a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)
+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules

used = {6306, 81, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{32}\sqrt{2+\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$-\frac{1}{32}\sqrt{2-\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\frac{1}{32}\sqrt{2+\sqrt{2}}a^2\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+\frac{1}{32}\sqrt{2-\sqrt{2}}a^2\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$+\frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{}}{\sqrt[8]{\frac{1}{ax}}}\right.$$

```
[In] Int[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8))/8 + (a^2*(1 - 1/(a*x))^(7/8)*
(1 + 1/(a*x))^(9/8))/2 - (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sqrt[2 - Sqrt[2]] -
 (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]])/32 - (Sqr
t[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 +
 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sq
rt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/32 + (Sqrt[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(
a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 - Sqrt[2
]]*a^2*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]
*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 - Sqrt[2]]*a^2*Log
[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a
*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 + (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 -
1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8)
)/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 - 1/(a*x))^(
1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(
a*x))^(1/8)])/64
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{8}a\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}a\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{a^2\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{a^2\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}$$

$$+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}+\frac{a^2\text{Subst}\left(\int\frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$+\frac{a^2\text{Subst}\left(\int\frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$-\frac{a^2\text{Subst}\left(\int\frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{16\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$-\frac{a^2\text{Subst}\left(\int\frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{16\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}$$

$$+\frac{1}{32}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a^2\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx,x,\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)+\frac{1}{32}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{\right.\right.}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}$$

$$+\frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)-\frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}$$
$$-\frac{1}{32}\sqrt{2+\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{32}\sqrt{2-\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-}}{\sqrt[8]{1+}}}{\sqrt{2-\sqrt{2}}}\right.$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.10

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{a^2e^{\frac{1}{4}\coth^{-1}(ax)}\left(-1-9e^{2\coth^{-1}(ax)}+\left(1+e^{2\coth^{-1}(ax)}\right)^2\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)\right)}{4\left(1+e^{2\coth^{-1}(ax)}\right)^2}$$

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^3,x]
[Out] -1/4*(a^2*E^(ArcCoth[a*x]/4)*(-1 - 9*E^(2*ArcCoth[a*x]) + (1 + E^(2*ArcCoth
[a*x]))^2*Hypergeometric2F1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])]))/(1 + E^(2*A
rcCoth[a*x]))^2
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}x^3}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{-(i-1)\sqrt{2}(-a^{16})^{\frac{1}{8}}x^2\log\left(2a^{14}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+(i+1)\sqrt{2}(-a^{16})^{\frac{7}{8}}\right)+(i+1)\sqrt{2}(-a^{16})^{\frac{1}{8}}x^2\log\left(2a^{14}\left(\frac{ax-1}{ax+1}\right.\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="fricas")
```

```
[Out] 1/64*(-(I - 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14*((a*x - 1)/(a*x + 1))^(
1/8) + (I + 1)*sqrt(2)*(-a^16)^(7/8)) + (I + 1)*sqrt(2)*(-a^16)^(1/8)*x^2*l
og(2*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (I - 1)*sqrt(2)*(-a^16)^(7/8)) - (I
 + 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14*((a*x - 1)/(a*x + 1))^(1/8) + (I
 - 1)*sqrt(2)*(-a^16)^(7/8)) + (I - 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14
*((a*x - 1)/(a*x + 1))^(1/8) - (I + 1)*sqrt(2)*(-a^16)^(7/8)) + 2*(-a^16)^(
1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) + (-a^16)^(7/8)) - 2*I*(-a^16
)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) + I*(-a^16)^(7/8)) + 2*I*(
-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) - I*(-a^16)^(7/8)) -
2*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) - (-a^16)^(7/8)) +
 8*(5*a^2*x^2 + 9*a*x + 4)*((a*x - 1)/(a*x + 1))^(7/8))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$$
$$=\frac{1}{64}\left(2\,a\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(2*a*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x
+ 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqr
t(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(s
qrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqr
t(sqrt(2) + 2)) + 2*a*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a
*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - a*sqrt(sqrt(2) + 2)*log(sqrt
(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1
) + a*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)
+ ((a*x - 1)/(a*x + 1))^(1/4) + 1) - a*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + a*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) + 16*(a*((a*x - 1)/(a*x + 1))^(15/8) + 9*a*((a
*x - 1)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.29

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx=\frac{\frac{9\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}+\frac{a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}$$
$$+\frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{16}+\frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{16}$$
$$+(-1)^{1/8}\,\sqrt{2}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\,\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)+(-1)^{1/8}\,\sqrt{2}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\,\sqrt{2}\right.$$

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] ((9*a^2*((a*x - 1)/(a*x + 1))^(7/8))/4 + (a^2*((a*x - 1)/(a*x + 1))^(15/8))
/4)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/8)*a
```

```
^2*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/16 + ((-1)^(1/8)*a^2*atan(
(-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/16 + (-1)^(1/8)*2^(1/2)*a^2*
atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/32 - 1
i/32) + (-1)^(1/8)*2^(1/2)*a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1)
)^(1/8)*(1/2 + 1i/2))*(1/32 + 1i/32)
```

3.132 $\int e^{4\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(-a*x+1)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6261, 91, 81, 66}

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=-4x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

```
[In] Int[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)} x^m \, dx \\
&= \int \frac{x^m(1+ax)^2}{(1-ax)^2} \, dx \\
&= \frac{4x^{1+m}}{1-ax} - \frac{\int \frac{x^m\left(a^2(3+4m)+a^3x\right)}{1-ax} \, dx}{a^2} \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - (4(1+m)) \int \frac{x^m}{1-ax} \, dx \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - 4x^{1+m} \, \text{Hypergeometric2F1}(1, 1+m, 2+m, ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^m\,dx = \frac{x^{1+m}(-5-4m+ax-4(1+m)(-1+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{(1+m)(-1+ax)}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^m,x]
[Out] (x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.47

method	result
meijerg	$-\frac{(-a)^{-m}\left(\frac{x^m(-a)^m\left(a^2m\,x^2+amx+2ax-m^2-3m-2\right)}{(1+m)m(-ax+1)}+x^m(-a)^m(2+m)\,\text{LerchPhi}(ax,1,m)\right)}{a}+\frac{2(-a)^{-m}\left(-\frac{x^m(-a)^m(ax-m-1)}{m(-ax+1)}-x\right.}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^m,x,method=_RETURNVERBOSE)
[Out] -(-a)^(-m)/a*(x^m*(-a)^m*(a^2*m*x^2+a*m*x+2*a*x-m^2-3*m-2)/(1+m)/m/(-a*x+1)
+x^m*(-a)^m*(2+m)*LerchPhi(a*x,1,m))+2*(-a)^(-m)/a*(-x^m*(-a)^m*(a*x-m-1)/m
/(-a*x+1)-x^m*(-a)^m*(1+m)*LerchPhi(a*x,1,m))-(-a)^(-m)/a*(1/(1+m)*x^m*(-a)
^m*(-m-1)/(-a*x+1)+x^m*(-a)^m*m*LerchPhi(a*x,1,m))
```

Fricas [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="fricas")
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m(ax+1)^2}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**m,x)
[Out] Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

Maxima [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="maxima")
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

Giac [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="giac")
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{4\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

```
[In] int((x^m*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] int((x^m*(a*x + 1)^2)/(a*x - 1)^2, x)
```

3.133 $\int e^{3\coth^{-1}(ax)} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 151

$$\int e^{3\coth^{-1}(ax)} x^m \, dx = -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} + \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-x^m*hyp
ergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2, -1/
2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+4*x^m*hypergeom([3/2, -1/2*m],[1-1/2*
m],1/a^2/x^2)/a/m
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6307, 6874, 371, 864, 822}

$$\int e^{3\coth^{-1}(ax)} x^m \, dx = -\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1} + \frac{4x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am} + \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m) +
 (4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (4*x^m*Hypergeometric2F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}-\frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)\,dx, x, \frac{1}{x}\right)\right)
\end{aligned}$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + \frac{\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right) - \frac{\left(4\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-1-m}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} + \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}x^m\,dx$$
$$= \frac{x^{1+m}\left(3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)-2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1}\right.}{m(1}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^
2]*AppellF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - 2*(1 + m)*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, -(a*x)
, a*x] + m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]*Hypergeometric2
F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]))/(m*(1 + m)*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*Sqrt[-a^(-2) + x^2])
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m,x)
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.134 $\int e^{2\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{1+m}$$

```
[Out] x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6261, 81, 66}

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(
1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}x^m\,dx\\
&= -\int \frac{x^m(1+ax)}{1-ax}\,dx\\
&= \frac{x^{1+m}}{1+m} - 2\int \frac{x^m}{1-ax}\,dx\\
&= \frac{x^{1+m}}{1+m} - \frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{1+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = \frac{x^{1+m}(1-2\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{1+m}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m,x]
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

method	result
meijerg	$-\frac{(-a)^{-m}\left(-\frac{x^m(-a)^m(amx+m+1)}{(1+m)m}+x^m(-a)^m \operatorname{LerchPhi}(ax,1,m)\right)}{a}+\frac{(-a)^{-m}\left(-\frac{x^m(-a)^m(-m-1)}{(1+m)m}-x^m(-a)^m \operatorname{LerchPhi}(ax,1,m\right.}{a}$

```
[In] int(1/(a*x-1)*(a*x+1)*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(-m)/a*(-x^m*(-a)^m*(a*m*x+m+1)/(1+m)/m+x^m*(-a)^m*LerchPhi(a*x,1,m))
+(-a)^(-m)/a*(-1/(1+m)*x^m*(-a)^m*(-m-1)/m-x^m*(-a)^m*LerchPhi(a*x,1,m))
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\int\frac{(ax+1)x^m}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(26) = 52.

Time = 1.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^m\,dx=&-\frac{amx^{m+2}\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma\left(m+3\right)}\\&-\frac{2ax^{m+2}\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma\left(m+3\right)}\\&-\frac{mx^{m+1}\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma\left(m+2\right)}-\frac{x^{m+1}\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma\left(m+2\right)}\end{aligned}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m,x)
```

```
[Out] -a*m*x**(m + 2)*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - 2*a*x**
(m + 2)*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - m*x**(m + 1)*le
rchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - x**(m + 1)*lerchphi(a*x,
1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)x^m}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="maxima")
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)x^m}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="giac")
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^m*(a*x + 1))/(a*x - 1),x)
[Out] int((x^m*(a*x + 1))/(a*x - 1), x)
```

3.135 $\int e^{\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int e^{\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m}+\frac{x^m\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+x^m*hyperg
eom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6307, 822, 371}

$$\int e^{\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[In] Int[E^ArcCoth[a*x]*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) + (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right)-\frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m}\\
&\quad+\frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int e^{\coth^{-1}(ax)}x^m\,dx = x^{1+m}\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{-\frac{1}{a^2}+x^2}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)}{m\sqrt{-1+ax}\sqrt{\frac{1+ax}{a^2}}\sqrt{1-a^2x^2}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{1+m}\right)$$

```
[In] Integrate[E^ArcCoth[a*x]*x^m,x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[-a^(-2) + x^2]*AppellF1[m, -1/2, 1
/2, 1 + m, -(a*x), a*x])/(m*Sqrt[-1 + a*x]*Sqrt[(1 + a*x)/a^2]*Sqrt[1 - a^2
*x^2])) + Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 +
m))
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}} dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)} x^m \, dx = \int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}} \, dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)} x^m \, dx = \int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}} \, dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***m,x)
```

```
[Out] Integral(x**m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="giac")
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.136 $\int e^{-\coth^{-1}(ax)} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int e^{-\coth^{-1}(ax)} x^m \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am}$$

```
[Out] x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-x^m*hyperg
eom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6307, 822, 371}

$$\int e^{-\coth^{-1}(ax)} x^m \, dx = \frac{x^{m+1} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-m-1), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{m+1} - \frac{x^m \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am}$$

```
[In] Int[x^m/E^ArcCoth[a*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) - (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)+\frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m}\\
&\quad-\frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.53

$$
\int e^{-\coth^{-1}(ax)}x^m\,dx = x^{1+m}\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right)}{m\sqrt{1-ax}\sqrt{-\frac{1}{a^2}+x^2}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{1+m}\right)
$$

```
[In] Integrate[x^m/E^ArcCoth[a*x],x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[m, -1/2,
1/2, 1 + m, a*x, -(a*x)])/(m*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])) + Hypergeo
metric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 + m))
```

Maple [F]

$$\int x^m \sqrt{\frac{ax-1}{ax+1}}dx$$

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)} x^m\,dx = \int x^m \sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)} x^m\,dx = \int x^m \sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)} x^m\,dx = \int x^m \sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^m\,dx = \int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^m\,dx = \int x^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.137 $\int e^{-2\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{1+m}$$

```
[Out] x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6261, 81, 66}

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{m+1}$$

```
[In] Int[x^m/E^(2*ArcCoth[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]
)/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}x^m\,dx\\ &= -\int \frac{x^m(1-ax)}{1+ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-2\int \frac{x^m}{1+ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{1+m}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}(1-2\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{1+m}$$

```
[In] Integrate[x^m/E^(2*ArcCoth[a*x]),x]
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

method	result
meijerg	$a^{-m-1}\left(\frac{x^m a^m (amx-m-1)}{(1+m)m}+x^m a^m \operatorname{LerchPhi}(-ax,1,m)\right)-a^{-m-1}\left(\frac{x^m a^m}{m}+\frac{x^m a^m (-m-1)\operatorname{LerchPhi}(-ax,}{1+m}\right.$

```
[In] int(x^m*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] a^(-m-1)*(x^m*a^m*(a*m*x-m-1)/(1+m)/m+x^m*a^m*LerchPhi(-a*x,1,m))-a^(-m-1)*
(x^m*a^m/m+1/(1+m)*x^m*a^m*(-m-1)*LerchPhi(-a*x,1,m))
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\int \frac{(ax-1)x^m}{ax+1}\,dx$$

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\frac{amx^{m+2}\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}+\frac{2ax^{m+2}\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}-\frac{mx^{m+1}\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}-\frac{x^{m+1}\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}$$

```
[In] integrate(x**m*(a*x-1)/(a*x+1),x)
[Out] a*m*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(m
 + 3) + 2*a*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)
/gamma(m + 3) - m*x**(m + 1)*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(
m + 1)/gamma(m + 2) - x**(m + 1)*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*ga
mma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax-1)x^m}{ax+1}\,dx$$

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax-1)x^m}{ax+1}\,dx$$

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x^m*(a*x - 1))/(a*x + 1),x)
[Out] int((x^m*(a*x - 1))/(a*x + 1), x)
```

3.138 $\int e^{-3\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 150

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+x^m*hyp
ergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2, -1/
2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-4*x^m*hypergeom([3/2, -1/2*m],[1-1/2*
m],1/a^2/x^2)/a/m
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6307, 6874, 371, 864, 822}

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = -\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1} + \frac{4x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1} + \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am} - \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[In] Int[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m) +
 (4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (4*x^m*Hypergeometric2F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}+\frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\right)$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right) + \frac{\left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{1}{a^2x^2}\right)}{1+m} - \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2}, -\frac{m}{2}, 1-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{am}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.28

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx$$
$$= \frac{x^{1+m}\left(-3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right)+2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,\text{A}}{m(1+m)\sqrt{1}}$$

```
[In] Integrate[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*(-3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[
m, -1/2, 1/2, 1 + m, a*x, -(a*x)] + 2*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-
1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, a*x, -(a*x)] + m*Sqrt[1 - a*x]*
Sqrt[-a^(-2) + x^2]*Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x
^2)]))/(m*(1 + m)*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])
```

Maple [F]

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\text{Timed out}$$

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.139 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,5/4,-5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}} \, dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m} \text{AppellF1}\left(-1-m, \frac{5}{4}, -\frac{5}{4}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx = \int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx$$

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^m,x]
[Out] Integrate[E^((5*ArcCoth[a*x])/2)*x^m, x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}} dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}} \, dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="fricas")
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a^2*x^2 - 2
*a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**m,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Giac [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}\,dx$$

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(5/4),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.140 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,3/4,-3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m, \frac{3}{4}, -\frac{3}{4}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)} x^m\,dx = \int e^{\frac{3}{2}\coth^{-1}(ax)} x^m\,dx$$

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^m,x]
[Out] Integrate[E^((3*ArcCoth[a*x])/2)*x^m, x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)} x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**m,x)
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Giac [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}\,dx$$

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(3/4),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.141 $\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(-1-m, \frac{1}{4}, -\frac{1}{4}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/4,-1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx = \frac{x^{m+1} \operatorname{AppellF1}\left(-m-1, \frac{1}{4}, -\frac{1}{4}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[E^(ArcCoth[a*x]/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m, \frac{1}{4}, -\frac{1}{4}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^m,x]
[Out] Integrate[E^(ArcCoth[a*x]/2)*x^m, x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**m,x)
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Giac [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}\,dx$$

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/4),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-1/4,1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[x^m/E^(ArcCoth[a*x]/2),x]
[Out] (x^(1 + m)*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

```
[In] Integrate[x^m/E^(ArcCoth[a*x]/2),x]
[Out] Integrate[x^m/E^(ArcCoth[a*x]/2), x]
```

Maple [F]

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/4),x)
[Out] Integral(x**m*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Giac [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,dx$$

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(1/4),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.143 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-3/4,3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[x^m/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m, -\frac{3}{4}, \frac{3}{4}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

```
[In] Integrate[x^m/E^((3*ArcCoth[a*x])/2),x]
[Out] Integrate[x^m/E^((3*ArcCoth[a*x])/2), x]
```

Maple [F]

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/4),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Giac [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}\,dx$$

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(3/4),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.144 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-5/4,5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[x^m/E^((5*ArcCoth[a*x])/2),x]
[Out] (x^(1 + m)*AppellF1[-1 - m, -5/4, 5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

```
[In] Integrate[x^m/E^((5*ArcCoth[a*x])/2),x]
[Out] Integrate[x^m/E^((5*ArcCoth[a*x])/2), x]
```

Maple [F]

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

```
[In] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] integral((a*x - 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(5/4),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

Giac [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}\,dx$$

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(5/4),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.145 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/3,-1/3,-m,1/x,-1/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6308, 138}

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

```
[In] Int[E^((2*ArcCoth[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m, \frac{1}{3}, -\frac{1}{3}, -m, \frac{1}{x}, -\frac{1}{x}\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)} x^m\,dx = \int e^{\frac{2}{3}\coth^{-1}(x)} x^m\,dx$$

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^m,x]
[Out] Integrate[E^((2*ArcCoth[x])/3)*x^m, x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/((x-1)/(1+x))^(1/3)*x^m,x)
[Out] int(1/((x-1)/(1+x))^(1/3)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)} x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="fricas")
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(2/3)/(x - 1), x)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**m,x)
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\left(\frac{x-1}{x+1}\right)^{1/3}}\,dx$$

```
[In] int(x^m/((x - 1)/(x + 1))^(1/3),x)
[Out] int(x^m/((x - 1)/(x + 1))^(1/3), x)
```

3.146 $\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx = \frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/6,-1/6,-m,1/x,-1/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6308, 138}

$$\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx = \frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

```
[In] Int[E^(ArcCoth[x]/3)*x^m,x]
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m, \frac{1}{6}, -\frac{1}{6}, -m, \frac{1}{x}, -\frac{1}{x}\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx$$

```
[In] Integrate[E^(ArcCoth[x]/3)*x^m,x]
[Out] Integrate[E^(ArcCoth[x]/3)*x^m, x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{6}}}dx$$

```
[In] int(1/((x-1)/(1+x))^(1/6)*x^m,x)
[Out] int(1/((x-1)/(1+x))^(1/6)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="fricas")
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(5/6)/(x - 1), x)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**m,x)
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/6), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

Giac [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{1/6}}\,dx$$

```
[In] int(x^m/((x - 1)/(x + 1))^(1/6),x)
[Out] int(x^m/((x - 1)/(x + 1))^(1/6), x)
```

3.147 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{1}{8},-\frac{1}{8},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/8,-1/8,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{1}{8},-\frac{1}{8},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[E^(ArcCoth[a*x]/4)*x^m,x]
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(-1-m, \frac{1}{8}, -\frac{1}{8}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$$

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^m,x]
[Out] Integrate[E^(ArcCoth[a*x]/4)*x^m, x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(7/8)/(a*x - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**m,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Giac [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}}\,dx$$

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/8),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

3.148 $\int e^{n \coth^{-1}(ax)} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int e^{n \coth^{-1}(ax)} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(-1-m, \frac{n}{2}, -\frac{n}{2}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/2*n,-1/2*n,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6308, 138}

$$\int e^{n \coth^{-1}(ax)} x^m \, dx = \frac{x^{m+1} \operatorname{AppellF1}\left(-m-1, \frac{n}{2}, -\frac{n}{2}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

```
[In] Int[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, n/2, -1/2*n, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
```

```
; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int x^{-2-m}\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2} dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m} \text{AppellF1}\left(-1-m, \frac{n}{2}, -\frac{n}{2}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F]

$$\int e^{n \coth^{-1}(ax)} x^m \, dx = \int e^{n \coth^{-1}(ax)} x^m \, dx$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^m,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*x^m, x]
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} x^m dx$$

```
[In] int(exp(n*arccoth(a*x))*x^m,x)
[Out] int(exp(n*arccoth(a*x))*x^m,x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)} x^m \, dx = \int x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="fricas")
[Out] integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*x**m,x)
[Out] Integral(x**m*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="giac")
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

```
[In] int(x^m*exp(n*acoth(a*x)),x)
[Out] int(x^m*exp(n*acoth(a*x)), x)
```

3.149 $\int e^{n\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 174

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3+\frac{2(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}$$

```
[Out] 1/6*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/a+1/3*(1-1/a/x)^(1-1/2*n)
*(1+1/a/x)^(1+1/2*n)*x^3+2/3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*
n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^3/(2-n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6306, 105, 156, 12, 133}

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \frac{2(n^2+2)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{nx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{6a}$$

```
[In] Int[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/(6*a) + ((1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 + (2*(2 + n^2)*(1 - 1/(a*x
))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (a - x^(-1))/(a + x^(-1))])/(3*a^3*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{(2+n^2)\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{a^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 - \frac{(2+n^2)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{6a^2}$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{2(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(2+n^2)\,\text{Hypergeometric2F1}\left(1, 1+\frac{n}{2}, 2+\frac{n}{2}, e^{2\coth^{-1}(ax)}\right) + (2+n)\left(an^2x+2a^3\right.\right.}{6a^3(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F1[1, 1
+ n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^2*x + 2*a^3*x^3 + n*(-1
+ a^2*x^2) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)])))/(6*a^3*(2 + n))
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} x^2 dx$$

```
[In] int(exp(n*arccoth(a*x))*x^2,x)
[Out] int(exp(n*arccoth(a*x))*x^2,x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)} x^2 \, dx = \int x^2 \left(\frac{ax + 1}{ax - 1}\right)^{\frac{1}{2}\,n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="fricas")
[Out] integral(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(ax)} x^2 \, dx = \int x^2 e^{n \operatorname{acoth}(ax)} \, dx$$

```
[In] integrate(exp(n*acoth(a*x))*x**2,x)
[Out] Integral(x**2*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} x^2 \, dx = \int x^2 \left(\frac{ax + 1}{ax - 1}\right)^{\frac{1}{2}\,n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="giac")
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \int x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

```
[In] int(x^2*exp(n*acoth(a*x)),x)
[Out] int(x^2*exp(n*acoth(a*x)), x)
```

3.150 $\int e^{n\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 122

$$\int e^{n\coth^{-1}(ax)}x\,dx = \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}$$

```
[Out] 1/2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2+2*n*(1-1/a/x)^(1-1/2*n)*(1+
1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^2/(2-
n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6306, 98, 133}

$$\int e^{n\coth^{-1}(ax)}x\,dx = \frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)} + \frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] ((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a*
x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2-\frac{n\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\\
&\quad+\frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}x\,dx$$
$$=\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+a^2x^2\right.\right.}{2a^2(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + a^2*x^2 + n*Hypergeomet
ric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a^2*(2 + n))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}xdx$$

```
[In] int(exp(n*arccoth(a*x))*x,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int xe^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*x,x)
```

```
[Out] Integral(x*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int x \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int x \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="giac")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int x \, e^{n \operatorname{acoth}(a x)} \, dx$$

```
[In] int(x*exp(n*acoth(a*x)),x)
[Out] int(x*exp(n*acoth(a*x)), x)
```

3.151 $\int e^{n \coth^{-1}(ax)} dx$

Optimal result

Integrand size = 8, antiderivative size = 78

$$\int e^{n \coth^{-1}(ax)} dx = \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] 4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(a-1/x)/(a+1/x))/a/(2-n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6305, 133}

$$\int e^{n \coth^{-1}(ax)} dx = \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}} \text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
```

```
|| !SumSimplerQ[p, 1]) && !ILtQ[m, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}\,dx$$
$$= \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(ax+\text{Hypergeometr}\right.\right.}{a(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Maple [F]

$$\int e^{n\,\text{arccoth}(ax)}dx$$

```
[In] int(exp(n*arccoth(a*x)),x)
[Out] int(exp(n*arccoth(a*x)),x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(ax)}\,dx = \int e^{n \operatorname{acoth}(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x)),x)
[Out] Integral(exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n \coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

```
[In] int(exp(n*acoth(a*x)),x)
[Out] int(exp(n*acoth(a*x)), x)
```

3.152 $\int \frac{e^{n \coth^{-1}(ax)}}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{e^{n \coth^{-1}(ax)}}{x} dx = -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2} \text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}$$

```
[Out] -2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/n/((1
-1/a/x)^(1/2*n))+2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2*(a-1/
x)/a)/n/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6306, 132, 71, 133}

$$\int \frac{e^{n \coth^{-1}(ax)}}{x} dx = \frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2} \text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n} - \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

```
[In] Int[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/
(a + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-1/
2*n, -1/2*n, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} \\
&\quad + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}n\,\text{Hypergeome}\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, -E^(2*ArcCoth[a*x])] + E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] - (2 + n)*(Hypergeometric2F1[1, n/2, 1 +
n/2, -E^(2*ArcCoth[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth
[a*x])])))/(n*(2 + n))
```

Maple [F]

$$\int \frac{e^{n\,\operatorname{arccoth}(ax)}}{x}dx$$

```
[In] int(exp(n*arccoth(a*x))/x,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{e^{n \operatorname{acoth}(ax)}}{x}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/x,x)
[Out] Integral(exp(n*acoth(a*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{e^{n \operatorname{acoth}(a\,x)}}{x}\,dx$$

```
[In] int(exp(n*acoth(a*x))/x,x)
[Out] int(exp(n*acoth(a*x))/x, x)
```

3.153 $\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx = \frac{2^{1+\frac{n}{2}} a \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[Out] 2^(1+1/2*n)*a*(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2
*(a-1/x)/a)/(2-n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6306, 71}

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx = \frac{a 2^{\frac{n}{2}+1} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[In] Int[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2
 - n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2} dx, x, \frac{1}{x}\right)$$
$$= \frac{2^{1+\frac{n}{2}} a \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2} dx = -\frac{4ae^{(2+n)\coth^{-1}(ax)} \text{Hypergeometric2F1}\left(2, 1+\frac{n}{2}, 2+\frac{n}{2}, -e^{2\coth^{-1}(ax)}\right)}{2+n}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^2,x]
[Out] (-4*a*E^((2 + n)*ArcCoth[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2
*ArcCoth[a*x])])/(2 + n)
```

Maple [F]

$$\int \frac{e^{n\,\operatorname{arccoth}(ax)}}{x^2} dx$$

```
[In] int(exp(n*arccoth(a*x))/x^2,x)
[Out] int(exp(n*arccoth(a*x))/x^2,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{n\operatorname{acoth}(ax)}}{x^2}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/x**2,x)
[Out] Integral(exp(n*acoth(a*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx = \int \frac{\mathrm{e}^{n \,\mathrm{acoth}(a\,x)}}{x^2} \, dx$$

```
[In] int(exp(n*acoth(a*x))/x^2,x)
[Out] int(exp(n*acoth(a*x))/x^2, x)
```

3.154 $\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[Out] 1/2*a^2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+2^(1/2*n)*a^2*n*(1-1/a/x)^(
1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6306, 81, 71}

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \frac{a^2 2^{n/2} n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n} + \frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n*
(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a - x^
(-1))/(2*a)])/(2 - n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2} dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} - \frac{1}{2}(an)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2} dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3} dx = -\frac{a^2e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\frac{1}{a^2x^2}\right.\right.}{2(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^3,x]
```

[Out] -1/2*(a^2*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-1 + 1/(a^2*x^2) + n/(a*x) + n*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^3}dx$$

[In] int(exp(n*arccoth(a*x))/x^3,x)

[Out] int(exp(n*arccoth(a*x))/x^3,x)

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3} dx$$

[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="fricas")

[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \int \frac{e^{n \operatorname{acoth}(ax)}}{x^3} dx$$

[In] integrate(exp(n*acoth(a*x))/x**3,x)

[Out] Integral(exp(n*acoth(a*x))/x**3, x)

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3} dx$$

[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="maxima")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3} dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} dx = \int \frac{e^{n \operatorname{acoth}(a x)}}{x^3} dx$$

```
[In] int(exp(n*acoth(a*x))/x^3,x)
[Out] int(exp(n*acoth(a*x))/x^3, x)
```

3.155 $\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 167

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx &= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}\\ &\quad+\frac{2^{n/2}a^3(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}\end{aligned}$$

```
[Out] 1/6*a^3*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+1/3*a^2*(1-1/a/x)^(1-1/2*
n)*(1+1/a/x)^(1+1/2*n)/x+1/3*2^(1/2*n)*a^3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*hype
rgeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6306, 92, 81, 71}

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx =& \frac{a^3 2^{n/2}(n^2+2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}\\ &+\frac{1}{6}a^3n\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}\end{aligned}$$

```
[In] Int[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] (a^3*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/6 + (a^2*(1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) + (2^(n/2)*a^3*(2 + n^2)*(
1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a - x^(
-1))/(2*a)])/(3*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}+\frac{1}{3}a^2\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{nx}{a}\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}\\
&\quad -\frac{1}{6}\left(a^2\left(2+n^2\right)\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} + \frac{2^{n/2}a^3(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = -\frac{a^3e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n(2+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-\right.\right.}{6(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/6*(a^3*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometr
ic2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-((1 - 1/(a^2*x
^2))*(n + 2/(a*x))) + (2 + n^2)/(a*x) + (2 + n^2)*Hypergeometric2F1[1, n/2,
 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

Maple [F]

$$\int \frac{e^{n\,\text{arccoth}(ax)}}{x^4}dx$$

```
[In] int(exp(n*arccoth(a*x))/x^4,x)
[Out] int(exp(n*arccoth(a*x))/x^4,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4} \, dx = \int \frac{e^{n \operatorname{acoth}{(ax)}}}{x^4} \, dx$$

```
[In] integrate(exp(n*acoth(a*x))/x**4,x)
[Out] Integral(exp(n*acoth(a*x))/x**4, x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^4} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^4} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4} \, dx = \int \frac{\mathrm{e}^{n \operatorname{acoth}(a\,x)}}{x^4} \, dx$$

```
[In] int(exp(n*acoth(a*x))/x^4,x)
[Out] int(exp(n*acoth(a*x))/x^4, x)
```

3.156 $\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx$

Optimal result

Integrand size = 12, antiderivative size = 183

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx = \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a(6+n^2)+\frac{2n}{x}\right)+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{2^{-2+\frac{n}{2}}a^4n(8+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}$$

```
[Out] 1/24*a^3*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(a*(n^2+6)+2*n/x)+1/4*a^2*
(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)/x^2+1/3*2^(-2+1/2*n)*a^4*n*(n^2+8)*
(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2
-n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6306, 102, 152, 71}

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx = \frac{a^4 2^{\frac{n}{2}-2}n(n^2+8)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)} + \frac{1}{24}a^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a(n^2+6)+\frac{2n}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} + \frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{4x^2}$$

```
[In] Int[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] (a^3*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*n)
/x))/24 + (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) +
 (2^(-2 + n/2)*a^4*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1
- n/2, -1/2*n, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x^3\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2}+\frac{1}{4}a^2\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-2-\frac{nx}{a}\right)\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right)+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} - \frac{1}{24}\left(a^3n\left(8+n^2\right)\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right)+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{2^{-2+\frac{n}{2}}a^4n(8+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

$$\int\frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$$
$$= -\frac{1}{24}a^4e^{n\coth^{-1}(ax)}\left(-6-n^2+\frac{6}{a^4x^4}+\frac{2n}{a^3x^3}+\frac{n^2}{a^2x^2}+\frac{6n}{ax}+\frac{n^3}{ax} - \frac{e^{2\coth^{-1}(ax)}n^2(8+n^2)\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)}{2+n} + n\left(8+n^2\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2},1+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] -1/24*(a^4*E^(n*ArcCoth[a*x])*(-6 - n^2 + 6/(a^4*x^4) + (2*n)/(a^3*x^3) + n
^2/(a^2*x^2) + (6*n)/(a*x) + n^3/(a*x) - (E^(2*ArcCoth[a*x])*n^2*(8 + n^2)*
Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])])/(2 + n) + n*(8
 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])]))
```

Maple [F]

$$\int \frac{e^{n\ \mathrm{arccoth}(ax)}}{x^5}dx$$

```
[In] int(exp(n*arccoth(a*x))/x^5,x)
[Out] int(exp(n*arccoth(a*x))/x^5,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^5}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^5}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/x**5,x)
[Out] Integral(exp(n*acoth(a*x))/x**5, x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^5}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^5} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} \, dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^5} \, dx$$

```
[In] int(exp(n*acoth(a*x))/x^5,x)
[Out] int(exp(n*acoth(a*x))/x^5, x)
```

3.157 $\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 143

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p}{1+p} + \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2-p)*(-a*c*x+c)^p*hypergeom([-p, 1/2-p],[1-p],2/(a+1/x
)/x)*(1+1/a/x)^(1/2)/a/p/(p+1)/(1-1/a/x)^(1/2)+x*(-a*c*x+c)^p*(1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2)/(p+1)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 134}

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}} + \frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}(c-acx)^p}{p+1}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^p)/(1 + p) + (((a - x^(-
1))/(a + x^(-1)))^(1/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric2
F1[1/2 - p, -p, 1 - p, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-p} x^{-p}(c-acx)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)\right)$$

$$
\begin{aligned}
&= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p}{1+p} \\
&\quad - \frac{\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\mathrm{Subst}\left(\int\frac{x^{-1-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a(1+p)} \\
&= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p}{1+p} \\
&\quad + \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(1+p)\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$
\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx
$$

$$
= \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left(p(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^p+\sqrt{\frac{-1+ax}{1+ax}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{1+a\ldots}\right.\right.}{ap(1+p)\sqrt{1-\frac{1}{ax}}}
$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*(p*(-1 + a*x)*((-1 + a*x)/(1 + a*x))^p + S
qrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p, 2/(1 + a*x)
]))/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^p)
```

Maple [F]

$$
\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx
$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] integral((a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-c(ax-1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(c-a\,c\,x)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.158 $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 -\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 +\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{7c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] 17/15*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3-3/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1
/5*a^4*c^4*(1-1/a^2/x^2)^(3/2)*x^5+7/8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-7
/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6313, 1821, 821, 272, 43, 65, 214}

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{7c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}-\frac{7}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}} +\frac{17}{15}a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} +\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (-7*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (17*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2
)*x^3)/15 - (3*a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (a^4*c^4*(1 - 1/(a^
2*x^2))^(3/2)*x^5)/5 + (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(a^4c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left(\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{5}\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(\frac{15}{a}-\frac{17x}{a^2}+\frac{5x^2}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5-\frac{1}{20}\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(\frac{68}{a^2}-\frac{35x}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{4}\left(7ac^4\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{8}\left(7ac^4\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 - \frac{(7c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a}$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 + \frac{1}{8}(7ac^4)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 + \frac{7c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4)+105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$\frac{(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136)(ax-1)c^4}{120a\sqrt{\frac{ax-1}{ax+1}}}+\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)c^4\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-90(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-105\sqrt{a^2x^2-1}\sqrt{a^2}\,ax+120((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+10\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(24*a^4*x^4-90*a^3*x^3+112*a^2*x^2-15*a*x-136)*(a*x-1)/a*c^4/((a*x-1)
/(a*x+1))^(1/2)+7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4
/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(24\,a^5c^4x^5-66\,a^4c^4x^4+22\,a^3c^4x^3+97\,a^2c^4x^2-15}{120\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^4*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^4*x^5 - 66*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 9
7*a^2*c^4*x^2 - 151*a*c^4*x - 136*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = c^4\left(\int\left(-\frac{4ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a*
*2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4/sqrt(a*x/(a*x + 1) -
1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(112) = 224.

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^4\left(\frac{ax}{ax}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^4*((a*x - 1)/(a*x + 1))^(9/2) + 790*
c^4*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 490
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a
*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2
/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
- a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{7\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)} - \frac{1}{120}\sqrt{a^2x^2-1}\left(\left(\frac{15\,c^4}{\mathrm{sgn}\,(ax+1)}-2\left(\frac{56\,ac^4}{\mathrm{sgn}\,(ax+1)}+3\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\,(ax+1)}-\frac{15\,a^2c^4}{\mathrm{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{13}{a\mathrm{sgn}\,(}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -7/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1/
120*sqrt(a^2*x^2 - 1)*((15*c^4/sgn(a*x + 1) - 2*(56*a*c^4/sgn(a*x + 1) + 3*
(4*a^3*c^4*x/sgn(a*x + 1) - 15*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 136*c^4/(a*s
gn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{\frac{49\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}} + \frac{7\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((49*c^4*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^4*((a*x - 1)/(a*x + 1))^(1/2
))/4 - (224*c^4*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^4*((a*x - 1)/(a*x +
 1))^(7/2))/6 + (7*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*c^4*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.159 $\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{5c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] 2/3*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3-1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4+5/8
*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-5/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6313, 1821, 821, 272, 43, 65, 214}

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{5c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}-\frac{5}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{2}{3}a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (-5*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (2*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)
*x^3)/3 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (5*c^3*ArcTanh[Sqrt[1 -
 1/(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
```

```
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left((a^3c^3)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= (a^3c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{4}(a^3c^3)\,\text{Subst}\left(\int \frac{\left(\frac{8}{a}-\frac{5x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{4}(5ac^3)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}(5ac^3)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{(5c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a}$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}(5ac^3)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{5c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{c^3\left(-a\sqrt{1-\frac{1}{a^2x^2}}x(16+9ax-16a^2x^2+6a^3x^3)+15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
[Out] (c^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3)) +
15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

method	result	size
risch	$-\frac{(6a^3x^3-16a^2x^2+9ax+16)(ax-1)c^3}{24a\sqrt{\frac{ax-1}{ax+1}}}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	120
default	$-\frac{(ax-1)c^3\left(6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-16((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{24a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	141

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/24*(6*a^3*x^3-16*a^2*x^2+9*a*x+16)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2)
+5/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-
1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^3x^4-10\,a^3c^3x^3-7\,a^2c^3x^2+25\,ac^3x+16\,c^3)\sqrt{\frac{a}{a}}}{24\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (6*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 7*a^2*c^3*x^2 + 25*a*c^3*x + 16*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int\frac{3ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**3,x)

[Out] -c**3*(Integral(3*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(89) = 178.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.10

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{1}{24}\left(\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(15\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+73\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4}{(ax+1)}}\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")

[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + 2*(15*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 73*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 55*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*a

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)} - \frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\left(\frac{3\,a^2c^3x}{\mathrm{sgn}\,(ax+1)}-\frac{8\,ac^3}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{9\,c^3}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{16\,c^3}{a\mathrm{sgn}\,(ax+1)}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -5/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1/
24*sqrt(a^2*x^2 - 1)*((2*(3*a^2*c^3*x/sgn(a*x + 1) - 8*a*c^3/sgn(a*x + 1))*
x + 9*c^3/sgn(a*x + 1))*x + 16*c^3/(a*sgn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a} - \frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{55\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{73\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{12}+\frac{5\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a-\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{6\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}$$

```
[In] int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a) - ((5*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/4 - (55*c^3*((a*x - 1)/(a*x + 1))^(3/2))/12 + (73*c^3*((a*x -
1)/(a*x + 1))^(5/2))/12 + (5*c^3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a - (4*a*
(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2 - (4*a*(a*x - 1)^3)/(a
*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4)
```

3.160 $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] 1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3+1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-
1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6310, 6313, 821, 272, 43, 65, 214}

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -1/2*(a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x
^3)/3 + (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (a^2c^2)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx\\
&= -\left((a^2c^2)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + (ac^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{1}{2}(ac^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}\\
&= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\\
&\quad + \frac{1}{2}(ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$
\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-2-3ax+2a^2x^2)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}
$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - 3*a*x + 2*a^2*x^2) + 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{(2a^2x^2-3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	112
default	$-\frac{(ax-1)c^2\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	121

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*(2*a^2*x^2-3*a*x-2)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x/
(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^2x^3-a^2c^2x^2-5\,ac^2x-2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] 1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (2*a^3*c^2*x^3 - a^2*c^2*x^2 - 5*a*c^2*x - 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{2ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.32

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$$
$$=\frac{1}{6}\,a\left(\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 8*c^2*((a*
x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2
/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$$
$$=\frac{1}{6}\,\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\mathrm{sgn}\,(ax+1)}-\frac{3\,c^2}{\mathrm{sgn}\,(ax+1)}\right)x-\frac{2\,c^2}{a\mathrm{sgn}\,(ax+1)}\right)-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) - 3*c^2/sgn(a*x + 1))*x - 2*
c^2/(a*sgn(a*x + 1))) - 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(ab
s(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{\frac{8\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - c^2*((a*x - 1)/(a*x + 1))^(1/2) +
c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (c^2*atanh(((a*x - 1)
/(a*x + 1))^(1/2)))/a
```

3.161 $\int e^{\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] 1/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6310, 6313, 272, 43, 65, 214}

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] -1/2*(a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(
2*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((ac)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\
&= (ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}(ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}
\end{aligned}
$$

$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}(ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = \frac{c\left(-a^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] (c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]
))/(2*a)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(39) = 78.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result	size
risch	$-\frac{x(ax-1)c}{2\sqrt{\frac{ax-1}{ax+1}}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	92
default	$-\frac{(ax-1)c\left(x\sqrt{a^2x^2-1}\,\sqrt{a^2}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	93

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(a*x-1)*c/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-
1)^(1/2))/(a^2)^(1/2)*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = \frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+acx)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - c*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (a^2*c*x^2 + a*c*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int\frac{ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a
*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(39) = 78.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.81

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}\,a\left(\frac{2\left(c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(c*((a*x - 1)/(a*x + 1))^(3/2) + c*sqrt((a*x - 1)/(a*x + 1)))/(2*(
a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + c*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = -\frac{\sqrt{a^2x^2-1}cx}{2\,\mathrm{sgn}\,(ax+1)} - \frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 - 1)*c*x/sgn(a*x + 1) - 1/2*c*log(abs(-x*abs(a) + sqrt(a^
2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = \frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}$$

```
[In] int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2) +
 c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x -
1)^2)/(a*x + 1)^2)
```

3.162 $\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6310, 6313, 866, 1819, 272, 65, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$
\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}x + (1-ax)\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}
$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x),x]
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*x + (1 - a*x)*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x])/(a*c*(-1 + a*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(47) = 94.

Time = 0.42 (sec) , antiderivative size = 247, normalized size of antiderivative = 4.84

method	result
default	$-\frac{\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x-((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-2\sqrt{}}{a\sqrt{a^2}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -1/a*(ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3*x^2+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-2*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)/
(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
[Out] -((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - 2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{1}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
[Out] -Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c} - \frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{1}{(acx-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

```
[In] int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] 2/(a*c*((a*x - 1)/(a*x + 1))^(1/2)) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
)/(a*c)
```

3.163 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

```
[Out] -1/3*a^2*(1-1/a^2/x^2)^(3/2)/c^2/(a-1/x)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6310, 6313, 665}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

```
[Out] -1/3*(a^2*(1 - 1/(a^2*x^2))^(3/2))/(c^2*(a - x^(-1))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(1+ax)}{3c^2(-1+ax)^2}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x))/(c^2*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{ax+1}{3(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}\, a}$	36
default	$-\frac{ax+1}{3(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}\, a}$	36
trager	$-\frac{(ax+1)^{2}\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{2}(ax-1)^{2}}$	40

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(a^2x^2+2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*c
^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
[Out] Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{3\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/3/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)
^3*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{3\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

```
[In] int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] -1/(3*a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

3.164 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3} dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

```
[Out] 1/5*a^3*(1-1/a^2/x^2)^(3/2)/c^3/(a-1/x)^4-4/15*a^2*(1-1/a^2/x^2)^(3/2)/c^3/
(a-1/x)^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6310, 6313, 807, 665}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3} dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2))/(5*c^3*(a - x^(-1))^4) - (4*a^2*(1 - 1/(a^2*x
^2))^(3/2))/(15*c^3*(a - x^(-1))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{5a^2c^3}\\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-4-3ax+a^2x^2)}{15c^3(-1+ax)^3}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

```
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(-4 - 3*a*x + a^2*x^2))/(c^3*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(ax-4)(ax+1)}{15(ax-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}\,a}$	41
default	$-\frac{(ax-4)(ax+1)}{15(ax-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}\,a}$	41
trager	$-\frac{(ax+1)(a^2x^2-3ax-4)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax-1)^3}$	51

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^3x^3-2\,a^2x^2-7\,ax-4)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(a^3*x^3 - 2*a^2*x^2 - 7*a*x - 4)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3*
x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{1}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{5\,(ax-1)}{ax+1}-3}{30\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(a*x - 1)/(a*x + 1) - 3)/(a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(15\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 5*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
5*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{3\,(a\,x+1)}-\frac{1}{5}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] -((a*x - 1)/(3*(a*x + 1)) - 1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.165 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4} \, dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

```
[Out] -1/7*a^4*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^5+12/35*a^3*(1-1/a^2/x^2)^(3/2)/c^
4/(a-1/x)^4-23/105*a^2*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4} \, dx = -\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -1/7*(a^4*(1 - 1/(a^2*x^2))^(3/2))/(c^4*(a - x^(-1))^5) + (12*a^3*(1 - 1/(a
^2*x^2))^(3/2))/(35*c^4*(a - x^(-1))^4) - (23*a^2*(1 - 1/(a^2*x^2))^(3/2))/
(105*c^4*(a - x^(-1))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

0]

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{3x}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{7a^2c^4}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{35a^2c^4}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(23+13ax-8a^2x^2+2a^3x^3)}{105c^4(-1+ax)^4}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -1/105*(Sqrt[1 - 1/(a^2*x^2)]*x*(23 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^4
*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
default	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
trager	$-\frac{(ax+1)(2a^3x^3-8a^2x^2+13ax+23)\sqrt{-\frac{-ax+1}{ax+1}}}{105a\,c^4(ax-1)^4}$	60

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/
a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+36\,ax+23)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + 36*a*x + 23)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{42\,(ax-1)}{ax+1} - \frac{35\,(ax-1)^2}{(ax+1)^2} - 15}{420\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/420*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15)/(a*c^4*((a
*x - 1)/(a*x + 1))^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4 x^4 + 35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3 x^3 + 21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2 x^2 - 7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x + \right.}{105\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7 ac^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 35*(a + sqrt(a^2 - 1/x^2))^3*x^3
 + 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 7*(a + sqrt(a^2 - 1/x^2))*x + 1)/(((a
 + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{3\,(a\,x+1)^2} - \frac{2\,(a\,x-1)}{5\,(a\,x+1)} + \frac{1}{7}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -((a*x - 1)^2/(3*(a*x + 1)^2) - (2*(a*x - 1))/(5*(a*x + 1)) + 1/7)/(4*a*c^4
*((a*x - 1)/(a*x + 1))^(7/2))
```

3.166 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

```
[Out] 1/9*a^5*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^6-8/21*a^4*(1-1/a^2/x^2)^(3/2)/c^5/
(a-1/x)^5+47/105*a^3*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^4-58/315*a^2*(1-1/a^2/
x^2)^(3/2)/c^5/(a-1/x)^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx = -\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3} + \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(3/2))/(9*c^5*(a - x^(-1))^6) - (8*a^4*(1 - 1/(a^2*x
^2))^(3/2))/(21*c^5*(a - x^(-1))^5) + (47*a^3*(1 - 1/(a^2*x^2))^(3/2))/(105
```

```
*c^5*(a - x^(-1))^4) - (58*a^2*(1 - 1/(a^2*x^2))^(3/2))/(315*c^5*(a - x^(-1
))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{7x}{a^3}+\frac{2x^2}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{a^4\text{Subst}\left(\int \frac{\left(\frac{18}{a^6}-\frac{20x}{a^7}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{29\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{3a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{21a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{105a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-58-25ax+21a^2x^2-10a^3x^3+2a^4x^4)}{315c^5(-1+ax)^5}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

```
[Out] -1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(-58 - 25*a*x + 21*a^2*x^2 - 10*a^3*x^3 + 2
*a^4*x^4))/(c^5*(-1 + a*x)^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
default	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2-25ax-58)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^5(ax-1)^5}$	68

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a*x
+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3-4\,a^2x^2-83\,ax-58)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^5x^5-5\,a^5c^5x^4+10\,a^4c^5x^3-10\,a^3c^5x^2+5\,a^2c^5x-ac^5)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 - 4*a^2*x^2 - 83*a*x - 58)*sqrt(
(a*x - 1)/(a*x + 1))/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a^3
*c^5*x^2 + 5*a^2*c^5*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{1}{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
[Out] -Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 5*a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 10*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 5*a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{105\,(ax-1)^3}{(ax+1)^3}-35}{2520\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
[Out] -1/2520*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 105*(a*x -
 1)^3/(a*x + 1)^3 - 35)/(a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+189\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+84\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-36\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2\right.}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9ac^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
[Out] 4/315*(315*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 189*(a + sqrt(a^2 - 1/x^2))^4*x^
4 + 84*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 9
*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{3\,(a\,x-1)^2}{5\,(a\,x+1)^2}-\frac{(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{3\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{9}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

```
[In] int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((3*(a*x - 1)^2)/(5*(a*x + 1)^2) - (a*x - 1)^3/(3*(a*x + 1)^3) - (3*(a*x -
1))/(7*(a*x + 1)) + 1/9)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

3.167 $\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

```
[Out] 2*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(p+1)/a/c/(p+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6265, 21, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (2*(c - a*c*x)^p)/(a*p) - (c - a*c*x)^(1 + p)/(a*c*(1 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^p\,dx\\
&= -\int \frac{(1+ax)(c-acx)^p}{1-ax}\,dx\\
&= -\left(c\int (1+ax)(c-acx)^{-1+p}\,dx\right)\\
&= -\left(c\int \left(2(c-acx)^{-1+p}-\frac{(c-acx)^p}{c}\right)dx\right)\\
&= \frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^p(2+p+apx)}{ap(1+p)}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
[Out] ((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{(pax+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
risch	$\frac{(pax+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
norman	$\frac{x\,\mathrm{e}^{p\ln(-acx+c)}}{p+1}+\frac{(2+p)\mathrm{e}^{p\ln(-acx+c)}}{ap(p+1)}$	46
parallelrisch	$\frac{x(-acx+c)^{p}ap+(-acx+c)^{p}p+2(-acx+c)^{p}}{ap(p+1)}$	49

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
[Out] (a*p*x+p+2)*(-a*c*x+c)^p/a/p/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{p}\,dx=\frac{(apx+p+2)(-acx+c)^{p}}{ap^{2}+ap}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] (a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(29) = 58.

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{p}\,dx=\begin{cases}-c^{p}x & \text{for}\ a=0\\ -\frac{ax\log\left(x-\frac{1}{a}\right)}{a^{2}cx-ac}+\frac{\log\left(x-\frac{1}{a}\right)}{a^{2}cx-ac}+\frac{2}{a^{2}cx-ac} & \text{for}\ p=-1\\ x+\frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for}\ p=0\\ \frac{apx(-acx+c)^{p}}{ap^{2}+ap}+\frac{p(-acx+c)^{p}}{ap^{2}+ap}+\frac{2(-acx+c)^{p}}{ap^{2}+ap} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**p,x)
[Out] Piecewise((-c**p*x, Eq(a, 0)), (-a*x*log(x - 1/a)/(a**2*c*x - a*c) + log(x
- 1/a)/(a**2*c*x - a*c) + 2/(a**2*c*x - a*c), Eq(p, -1)), (x + 2*log(x - 1/
a)/a, Eq(p, 0)), (a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) + p*(-a*c*x + c)**p/
(a*p**2 + a*p) + 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(ac^ppx+c^p)(-ax+1)^p}{(p^2+p)a} + \frac{(-ax+1)^pc^p}{ap}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] (a*c^p*p*x + c^p)*(-a*x + 1)^p/((p^2 + p)*a) + (-a*x + 1)^p*c^p/(a*p)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(ax+1)(-acx+c)^p}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(c-a\,c\,x)^p\,(p+a\,p\,x+2)}{a\,p\,(p+1)}$$

```
[In] int(((c - a*c*x)^p*(a*x + 1))/(a*x - 1),x)
[Out] ((c - a*c*x)^p*(p + a*p*x + 2))/(a*p*(p + 1))
```

3.168 $\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[Out] 2/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (2*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^5\,dx\\ &= -\left(c^5\int (1-ax)^4(1+ax)\,dx\right)\\ &= -\left(c^5\int \left(2(1-ax)^4-(1-ax)^5\right)\,dx\right)\\ &= \frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(-1+ax)^5(7+5ax)}{30a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
[Out] -1/30*(c^5*(-1 + a*x)^5*(7 + 5*a*x))/a
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result
gosper	$-\frac{(5a^5x^5-18a^4x^4+15a^3x^3+20a^2x^2-45ax+30)c^5x}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{3}{5}a^4x^5-\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3+\frac{3}{2}a\,x^2-x\right)$
norman	$-c^5x+\frac{3}{2}a\,c^5x^2-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6-\frac{2}{3}c^5a^2x^3$
risch	$-c^5x+\frac{3}{2}a\,c^5x^2-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6-\frac{2}{3}c^5a^2x^3$
parallelrisch	$-c^5x+\frac{3}{2}a\,c^5x^2-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6-\frac{2}{3}c^5a^2x^3$
meijerg	$-\frac{c^5\left(\frac{ax(70a^5x^5+84a^4x^4+105a^3x^3+140a^2x^2+210ax+420)}{420}+\ln(-ax+1)\right)}{a}-\frac{4c^5\left(-\frac{ax(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60)}{60}-\ln(-\right.}{a}$

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*c^5*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5c^5x^6}{6}+\frac{3a^4c^5x^5}{5}-\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}+\frac{3ac^5x^2}{2}-c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + 3*a**4*c**5*x**5/5 - a**3*c**5*x**4/2 - 2*a**2*c**5*x**
3/3 + 3*a*c**5*x**2/2 - c**5*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{1}{6}\,a^5c^5x^6 + \frac{3}{5}\,a^4c^5x^5 - \frac{1}{2}\,a^3c^5x^4 - \frac{2}{3}\,a^2c^5x^3 + \frac{3}{2}\,ac^5x^2 - c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{1}{6}\,a^5c^5x^6 + \frac{3}{5}\,a^4c^5x^5 - \frac{1}{2}\,a^3c^5x^4 - \frac{2}{3}\,a^2c^5x^3 + \frac{3}{2}\,ac^5x^2 - c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{a^5\,c^5\,x^6}{6} + \frac{3\,a^4\,c^5\,x^5}{5} - \frac{a^3\,c^5\,x^4}{2} - \frac{2\,a^2\,c^5\,x^3}{3} + \frac{3\,a\,c^5\,x^2}{2} - c^5\,x$$

```
[In] int(((c - a*c*x)^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (3*a*c^5*x^2)/2 - c^5*x - (2*a^2*c^5*x^3)/3 - (a^3*c^5*x^4)/2 + (3*a^4*c^5*
x^5)/5 - (a^5*c^5*x^6)/6
```

3.169 $\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

```
[Out] 1/2*c^4*(-a*x+1)^4/a-1/5*c^4*(-a*x+1)^5/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(1 - a*x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^4\,dx\\ &= -\left(c^4\int (1-ax)^3(1+ax)\,dx\right)\\ &= -\left(c^4\int \left(2(1-ax)^3-(1-ax)^4\right)\,dx\right)\\ &= \frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{10}c^4x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
[Out] (c^4*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{x(2a^4x^4-5a^3x^3+10ax-10)c^4}{10}$
default	$c^4\left(\frac{1}{5}a^4x^5-\frac{1}{2}a^3x^4+a\,x^2-x\right)$
norman	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
risch	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
parallelrisch	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
meijerg	$-\frac{c^4\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln(-ax+1)\right)}{a}-\frac{3c^4\left(\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)\right)}{a}-\frac{2c^4\left(-\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/10*x*(2*a^4*x^4-5*a^3*x^3+10*a*x-10)*c^4

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{1}{2}\,a^3c^4x^4+ac^4x^2-c^4x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="fricas")

[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{a^4c^4x^5}{5}-\frac{a^3c^4x^4}{2}+ac^4x^2-c^4x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**4,x)

[Out] a**4*c**4*x**5/5 - a**3*c**4*x**4/2 + a*c**4*x**2 - c**4*x

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="giac")
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{a^4\,c^4\,x^5}{5} - \frac{a^3\,c^4\,x^4}{2} + a\,c^4\,x^2 - c^4\,x$$

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(a*x - 1),x)
[Out] a*c^4*x^2 - c^4*x - (a^3*c^4*x^4)/2 + (a^4*c^4*x^5)/5
```

3.170 $\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

```
[Out] 2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x)^4)/(4*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^3\,dx\\ &= -\left(c^3\int(1-ax)^2(1+ax)\,dx\right)\\ &= -\left(c^3\int\left(2(1-ax)^2-(1-ax)^3\right)\,dx\right)\\ &= \frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{12}c^3x\left(12-6ax-4a^2x^2+3a^3x^3\right)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
[Out] -1/12*(c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$-\frac{(3a^3x^3-4a^2x^2-6ax+12)c^3x}{12}$
default	$c^3\left(-\frac{1}{4}a^3x^4+\frac{1}{3}a^2x^3+\frac{1}{2}a\,x^2-x\right)$
norman	$-c^3x+\frac{1}{2}a\,c^3x^2+\frac{1}{3}a^2c^3x^3-\frac{1}{4}a^3c^3x^4$
risch	$-c^3x+\frac{1}{2}a\,c^3x^2+\frac{1}{3}a^2c^3x^3-\frac{1}{4}a^3c^3x^4$
parallelrisch	$-c^3x+\frac{1}{2}a\,c^3x^2+\frac{1}{3}a^2c^3x^3-\frac{1}{4}a^3c^3x^4$
meijerg	$-\frac{c^3\left(\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)\right)}{a}-\frac{2c^3\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}+\frac{2c^3(-ax-\ln(-ax+1))}{a}+$

[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/12*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*c^3*x

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="fricas")

[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3c^3x^4}{4}+\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}-c^3x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**3,x)

[Out] -a**3*c**3*x**4/4 + a**2*c**3*x**3/3 + a*c**3*x**2/2 - c**3*x

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3\,c^3\,x^4}{4}+\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}-c^3\,x$$

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(a*x - 1),x)
[Out] (a*c^3*x^2)/2 - c^3*x + (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

3.171 $\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=-c^2x+\frac{1}{3}a^2c^2x^3$$

```
[Out] -c^2*x+1/3*a^2*c^2*x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 41}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}a^2c^2x^3-c^2x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x) + (a^2*c^2*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^2\,dx\\ &= -\left(c^2\int (1-ax)(1+ax)\,dx\right)\\ &= -\left(c^2\int \left(1-a^2x^2\right)\,dx\right)\\ &= -c^2x+\frac{1}{3}a^2c^2x^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx = -c^2\left(x-\frac{a^2x^3}{3}\right)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
[Out] -(c^2*(x - (a^2*x^3)/3))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	siz
gosper	$\frac{x(a^2x^2-3)c^2}{3}$	16
default	$c^2\left(\frac{1}{3}a^2x^3-x\right)$	17
norman	$-c^2x+\frac{1}{3}a^2c^2x^3$	19
risch	$-c^2x+\frac{1}{3}a^2c^2x^3$	19
parallelrisch	$-c^2x+\frac{1}{3}a^2c^2x^3$	19
meijerg	$-\frac{c^2\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c^2\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c^2(-ax-\ln(-ax+1))}{a}+\frac{c^2\ln(-ax+1)}{a}$	99

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x*(a^2*x^2-3)*c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}-c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**2,x)
[Out] a**2*c**2*x**3/3 - c**2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{3}\,a^2c^2x^3 - c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{3}\,a^2c^2x^3 - c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="giac")
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\,x\,(a^2\,x^2-3)}{3}$$

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(a*x - 1),x)
[Out] (c^2*x*(a^2*x^2 - 3))/3
```

3.172 $\int e^{2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-cx-\frac{1}{2}acx^2$$

```
[Out] -c*x-1/2*a*c*x^2
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2326}

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{c(1-a^2x^2)\,e^{2\coth^{-1}(ax)}}{2a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral}=\frac{ce^{2\coth^{-1}(ax)}(1-a^2x^2)}{2a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\coth^{-1}(ax)}(1-a^2x^2)}{2a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{cx(ax+2)}{2}$	10
default	$c\left(-\frac{1}{2}a\,x^{2}-x\right)$	13
norman	$-cx-\frac{1}{2}ac\,x^{2}$	13
risch	$-cx-\frac{1}{2}ac\,x^{2}$	13
parallelrisch	$-cx-\frac{1}{2}ac\,x^{2}$	13
meijerg	$-\frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c\ln(-ax+1)}{a}$	38

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -1/2*c*x*(a*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="fricas")
[Out] -1/2*a*c*x^2 - c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{acx^2}{2}-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
[Out] -a*c*x**2/2 - c*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="maxima")
[Out] -1/2*a*c*x^2 - c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="giac")
[Out] -1/2*a*c*x^2 - c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,x\,(a\,x+2)}{2}$$

```
[In] int(((c - a*c*x)*(a*x + 1))/(a*x - 1),x)
[Out] -(c*x*(a*x + 2))/2
```

3.173 $\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[Out] -2/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -2/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{c-acx}\,dx\\ &= -\frac{\int \frac{1+ax}{(1-ax)^2}\,dx}{c}\\ &= -\frac{\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{c}\\ &= -\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
[Out] -((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{2}{a(ax-1)}-\frac{\ln(ax-1)}{a}}{c}$	29
norman	$\frac{2x}{c(ax-1)}-\frac{\ln(ax-1)}{ac}$	29
risch	$\frac{2}{ac(ax-1)}-\frac{\ln(ax-1)}{ac}$	31
parallelrisch	$\frac{-a\ln(ax-1)x+2ax+\ln(ax-1)}{c(ax-1)a}$	36

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] 1/c*(2/a/(a*x-1)-1/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] -((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{a^2cx-ac} - \frac{\log(ax-1)}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
[Out] 2/(a**2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{a^2cx-ac} - \frac{\log(ax-1)}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
[Out] 2/(a^2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(|ax-1|)}{ac} + \frac{2}{(ax-1)ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2}{a\,(c-a\,c\,x)} - \frac{\ln(a\,x-1)}{a\,c}$$

```
[In] int((a*x + 1)/((c - a*c*x)*(a*x - 1)),x)
[Out] - 2/(a*(c - a*c*x)) - log(a*x - 1)/(a*c)
```

3.174 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2(1-ax)^2}$$

```
[Out] -x/c^2/(-a*x+1)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 34}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2(1-ax)^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(x/(c^2*(1 - a*x)^2))
```

<u>Rule 34</u>

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

<u>Rule 6264</u>

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^2}\,dx$$
$$= -\frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c^2}$$
$$= -\frac{x}{c^2(1-ax)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(1+ax)^2}{4ac^2(1-ax)^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -1/4*(1 + a*x)^2/(a*c^2*(1 - a*x)^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{x}{c^2(ax-1)^2}$	14
norman	$-\frac{x}{c^2(ax-1)^2}$	14
risch	$-\frac{x}{c^2(ax-1)^2}$	14
parallelrisch	$-\frac{x}{c^2(ax-1)^2}$	14
default	$\frac{-\frac{1}{(ax-1)^2 a}-\frac{1}{a(ax-1)}}{c^2}$	30

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -x/c^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2ac^2x+c^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**2,x)
[Out] -x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{(acx-c)^2a}-\frac{1}{(acx-c)ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2\,(a\,x-1)^2}$$

```
[In] int((a*x + 1)/((c - a*c*x)^2*(a*x - 1)),x)
[Out] -x/(c^2*(a*x - 1)^2)
```

3.175 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

```
[Out] -2/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -2/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{(c-acx)^3}\,dx \\
&= -\frac{\int \frac{1+ax}{(1-ax)^4}\,dx}{c^3} \\
&= -\frac{\int \left(\frac{2}{(-1+ax)^4}+\frac{1}{(-1+ax)^3}\right)dx}{c^3} \\
&= -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1+3ax}{6ac^3(-1+ax)^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
[Out] (1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{\frac{x}{2}+\frac{1}{6a}}{(ax-1)^3 c^3}$	21
gosper	$\frac{3ax+1}{6a\,c^3(ax-1)^3}$	22
default	$\frac{\frac{1}{2(ax-1)^2 a}+\frac{2}{3a(ax-1)^3}}{c^3}$	30
parallelrisch	$\frac{a^2x^3-3a\,x^2+6x}{6c^3(ax-1)^3}$	30
norman	$\frac{\frac{x}{c}-\frac{a\,x^2}{2c}+\frac{a^2x^3}{6c}}{(ax-1)^3 c^2}$	38

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] (1/2*x+1/6/a)/(a*x-1)^3/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{-3ax-1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**3,x)
[Out] -(-3*a*x - 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a*
c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] 1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{x}{2}+\frac{1}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}$$

```
[In] int((a*x + 1)/((c - a*c*x)^3*(a*x - 1)),x)
[Out] -(x/2 + 1/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^3*x)
```

3.176 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

```
[Out] -1/2/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{1}{3ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/2*1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^4}\,dx \\
&= -\frac{\int \frac{1+ax}{(1-ax)^5}\,dx}{c^4} \\
&= -\frac{\int \left(-\frac{2}{(-1+ax)^5}-\frac{1}{(-1+ax)^4}\right)dx}{c^4} \\
&= -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1+2ax}{6ac^4(-1+ax)^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
[Out] -1/6*(1 + 2*a*x)/(a*c^4*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{-\frac{x}{3}-\frac{1}{6a}}{c^4(ax-1)^4}$	21
gosper	$-\frac{2ax+1}{6a\,c^4(ax-1)^4}$	22
default	$\frac{-\frac{1}{2a(ax-1)^4}-\frac{1}{3a(ax-1)^3}}{c^4}$	30
parallelrisch	$\frac{a^3x^4-4a^2x^3+6a\,x^2-6x}{6c^4(ax-1)^4}$	38
norman	$\frac{\frac{a\,x^2}{c}-\frac{x}{c}-\frac{2a^2x^3}{3c}+\frac{a^3x^4}{6c}}{(ax-1)^4c^3}$	49

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*x-1/6/a)/c^4/(a*x-1)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2ax-1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (-2*a*x - 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 24
*a**2*c**4*x + 6*a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
[Out] -1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{x}{3}+\frac{1}{6\,a}}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

```
[In] int((a*x + 1)/((c - a*c*x)^4*(a*x - 1)),x)
[Out] -(x/3 + 1/(6*a))/(c^4 + 6*a^2*c^4*x^2 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c
^4*x)
```

3.177 $\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 202

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)\left(1-\frac{1}{ax}\right)^{3/2}x}$$

```
[Out] (1+1/a/x)^(3/2)*x*(-a*c*x+c)^p/(p+1)/(1-1/a/x)^(1/2)-3*((a-1/x)/(a+1/x))^(3
/2-p)*(-a*c*x+c)^p*hypergeom([1-p, 3/2-p],[2-p],2/(a+1/x)/x)*(1+1/a/x)^(1/2
)/a^2/p/(-p^2+1)/(1-1/a/x)^(3/2)/x+3*(-a*c*x+c)^p*(1+1/a/x)^(1/2)/a/p/(p+1)
/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6311, 6316, 96, 134}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$=-\frac{3\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}(c-acx)^p\,\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)x\left(1-\frac{1}{ax}\right)^{3/2}}$$
$$+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^p}{(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{3\sqrt{\frac{1}{ax}+1}(c-acx)^p}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p)/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]) + ((1 +
 1/(a*x))^(3/2)*x*(c - a*c*x)^p)/((1 + p)*Sqrt[1 - 1/(a*x)]) - (3*((a - x^(
-1))/(a + x^(-1)))^(3/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric
2F1[1 - p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a^2*p*(1 - p^2)*(1 - 1/(a*
x))^(3/2)*x)
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-p} x^{-p}(c-acx)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{3/2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-1-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\sqrt{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{a(1+p)}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int \frac{x^{-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2p(1+p)}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,\text{Hypergeometric2F1}\left(1-p, \frac{3}{2}-p, 2-p, \frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)\left(1-\frac{1}{ax}\right)^{3/2}x}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left((-1+p)\left(\frac{-1+ax}{1+ax}\right)^p(1+ax)(3+p+apx)+3\sqrt{\frac{-1+ax}{1+ax}}\,\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}\right.\right.}{a^2(-1+p)p(1+p)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((-1 + p)*((-1 + a*x)/(1 + a*x))^p*(1 + a*x)*(3 + p + a*p*x)
 + 3*Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1 - p, 3/2 - p, 2 - p, 2/
(1 + a*x)]))/(a^2*(-1 + p)*p*(1 + p)*Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1
 + a*x))^p)
```

Maple [F]

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-c(ax-1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(c-a\,c\,x)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.178 $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] -1/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1/5*a^4*c^4*(1-1/a^2/x^2)^(5/2)*x^5-3/
8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+3/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6310, 6313, 821, 272, 43, 65, 214}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=-\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{3}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}-\frac{1}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 + (a^4*c^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/5 - (3*c^4*ArcTanh[Sqrt[1 - 1/
(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = (a^4c^4)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left((a^4c^4)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 + (a^3c^4)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 + \frac{1}{2}(a^3c^4)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 - \frac{1}{8}(3ac^4)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a}$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 - \frac{1}{8}(3ac^4)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 - \frac{3c^4\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8)(ax-1)c^4}{40a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^4\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+45\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-40((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-4\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(8*a^4*x^4-10*a^3*x^3-16*a^2*x^2+25*a*x+8)*(a*x-1)/a*c^4/((a*x-1)/(a*x
+1))^(1/2)-3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/(a*x
+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=$$
$$-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(8\,a^5c^4x^5-2\,a^4c^4x^4-26\,a^3c^4x^3+9\,a^2c^4x^2+33\,ac^4}{40\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^4*x^5 - 2*a^4*c^4*x^4 - 26*a^3*c^4*x^3 + 9*a^2
*c^4*x^2 + 33*a*c^4*x + 8*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = c^4\left(\int\left(-\frac{4ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2*x**2/(a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x) + Integral(-4*a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Inte
gral(a**4*x**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(89) = 178.

Time = 0.21 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.47

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^4\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^4*
((a*x - 1)/(a*x + 1))^(7/2) - 128*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^4*
((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x - 1
)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
+ 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2)
)*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\text{sgn}\,(ax+1)} + \frac{1}{40}\sqrt{a^2x^2-1}\left(\left(\frac{25\,c^4}{\text{sgn}\,(ax+1)}-2\left(\frac{8\,ac^4}{\text{sgn}\,(ax+1)}-\left(\frac{4\,a^3c^4x}{\text{sgn}\,(ax+1)}-\frac{5\,a^2c^4}{\text{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{8\,c}{a\text{sgn}\,(a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 3/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1/4
0*sqrt(a^2*x^2 - 1)*((25*c^4/sgn(a*x + 1) - 2*(8*a*c^4/sgn(a*x + 1) - (4*a^
3*c^4*x/sgn(a*x + 1) - 5*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 8*c^4/(a*sgn(a*x +
 1)))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.04

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{\frac{3c^4\sqrt{\frac{ax-1}{ax+1}}}{4}-\frac{7c^4\left(\frac{ax-1}{ax+1}\right)^{3/2}}{2}+\frac{32c^4\left(\frac{ax-1}{ax+1}\right)^{5/2}}{5}+\frac{7c^4\left(\frac{ax-1}{ax+1}\right)^{7/2}}{2}-\frac{3c^4\left(\frac{ax-1}{ax+1}\right)^{9/2}}{4}}{a-\frac{5a(ax-1)}{ax+1}+\frac{10a(ax-1)^2}{(ax+1)^2}-\frac{10a(ax-1)^3}{(ax+1)^3}+\frac{5a(ax-1)^4}{(ax+1)^4}-\frac{a(ax-1)^5}{(ax+1)^5}}$$
$$-\frac{3c^4\operatorname{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{4a}$$

```
[In] int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((3*c^4*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^4*((a*x - 1)/(a*x + 1))^(3/2)
)/2 + (32*c^4*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^4*((a*x - 1)/(a*x + 1))
^(7/2))/2 - (3*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^4*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.179 $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{3c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] -1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4-3/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
+3/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 272, 43, 65, 214}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{3c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a} + \frac{3}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (3*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 - (3*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left((a^3c^3)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= (a^3c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}(a^3c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{8}(3ac^3)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{(3c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a}$$

$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{8}(3ac^3)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{3c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{c^3\left(a^2\sqrt{1-\frac{1}{a^2x^2}}x^2(5-2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{8a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
[Out] (c^3*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(5 - 2*a^2*x^2) - 3*Log[a*(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(8*a)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

method	result	size
risch	$-\frac{x(2a^2x^2-5)(ax-1)c^3}{8\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	106
default	$-\frac{(ax-1)^2c^3\left(2x(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3x\sqrt{a^2x^2-1}\,\sqrt{a^2}+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	124

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(2*a^2*x^2-5)*(a*x-1)*c^3/((a*x-1)/(a*x+1))^(1/2)-3/8*ln(a^2*x/(a^2)
^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(
(a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4+2\,a^3c^3x^3-5\,a^2c^3x^2-5\,ac^3x\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) + (2*a^4*c^3*x^4 + 2*a^3*c^3*x^3 - 5*a^2*c^3*x^2 - 5*a*c^3*x
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int \frac{3ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int \frac{a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**3,x)
[Out] -c**3*(Integral(3*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2*x**2/(a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x) + Integral(a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(66) = 132$.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.83

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{8}\left(\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\cdots\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}\cdots}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1)/a^2 + 2*(3*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 11*c^3*((a*
x - 1)/(a*x + 1))^(5/2) - 11*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((
a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x +
1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*
a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{8}\left(\frac{2\,a^2c^3x^2}{\mathrm{sgn}\,(ax+1)}-\frac{5\,c^3}{\mathrm{sgn}\,(ax+1)}\right)\sqrt{a^2x^2-1}x+\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*c^3*x^2/sgn(a*x + 1) - 5*c^3/sgn(a*x + 1))*sqrt(a^2*x^2 - 1)*x
+ 3/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{\frac{3\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}-\frac{11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}+\frac{3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{6\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{4\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}+\frac{a\,{(a\,x-1)}^4}{{(a\,x+1)}^4}}-\frac{3\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((3*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(3/2
))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 + (3*c^3*((a*x - 1)/(a*x + 1)
)^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2
- (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (3*c^3*ata
nh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.180 $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] 1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3-1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+
1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6310, 6313, 864, 821, 272, 43, 65, 214}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)
/3 - (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\left(ac^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)$$

$$= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a}$$

$$= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-2+3ax+2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*a*x + 2*a^2*x^2) - 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{(2a^2x^2+3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	112
default	$\frac{(ax-1)^2c^2\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	130

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*(2*a^2*x^2+3*a*x-2)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)-1/2*ln(a^2*x/
(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^2x^3+5\,a^2c^2x^2+ac^2x-2\,c^2)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) - (2*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + a*c^2*x - 2*c^2)*sqrt((a*
x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{2ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2*x**2/(a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}\,a\left(\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 8*c^2*((a
*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^
2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
 a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\operatorname{sgn}(ax+1)}+\frac{3\,c^2}{\operatorname{sgn}(ax+1)}\right)x-\frac{2\,c^2}{a\operatorname{sgn}(ax+1)}\right) + \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) + 3*c^2/sgn(a*x + 1))*x - 2*
c^2/(a*sgn(a*x + 1))) + 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(ab
s(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}} - \frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (c^2*((a*x - 1)/(a*x + 1))^(1/2) + (8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 -
c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^2*atanh(((a*x - 1)
/(a*x + 1))^(1/2)))/a
```

3.181 $\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-2c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2
*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6313, 866, 1821, 821, 272, 65, 214}

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}-2cx\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left((ac)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$
$$= (ac)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(ac)\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(3ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{3c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
[Out] -1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{(ax+4)(ax-1)c}{2a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	99
default	$-\frac{(ax-1)^2c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+4\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+4a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	162

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+4)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)-3/2*ln(a^2*x/(a^2)^(1/2)+(
a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a
*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2+5\,acx+4\,c)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 + 5*a*c*x + 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int\frac{ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{1}{2}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(3/2) - 5*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 3*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{cx}{\operatorname{sgn}(ax+1)}+\frac{4\,c}{a\operatorname{sgn}(ax+1)}\right)+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 - 1)*(c*x/sgn(a*x + 1) + 4*c/(a*sgn(a*x + 1))) + 3/2*c*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] - (5*c*((a*x - 1)/(a*x + 1))^(1/2) - 3*c*((a*x - 1)/(a*x + 1))^(3/2))/(a -
(2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x
 - 1)/(a*x + 1))^(1/2)))/a
```

3.182 $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] 8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)-arctanh((1-1/a^2/x^2)^(1/2))/a/c+4/3/
a^2/c/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6310, 6313, 866, 1819, 12, 272, 65, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac} + \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2cx\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) + 4/(3*a^2*c*Sqrt[1 - 1/
(a^2*x^2)]*x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^4}{x\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3-\frac{4x}{a}+\frac{3x^2}{a^2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3ac}\\
&= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{3}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3ac}\\
&= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+2ax)}{(-1+ax)^2}-\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] ((4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x))/(-1 + a*x)^2 - (3*Log[a*(1 + Sqrt
[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(70) = 140.

Time = 0.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.31

method	result
default	$-\frac{3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-3\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*(3*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^4*x
^3+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^3*x^3-9*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3*x^2-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(3/2)*a*x-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+9*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x+((a*x-1)*(a*x+1))^(3/2)*(
a^2)^(1/2)+9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x-3*a*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1
))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-4\left(2\,a^2x^2+ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - 4*(2*a^2*x^2 + a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{1}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1)/(a^2*c) - 2*(3*(a*x - 1)/(a*x + 1) + 1)/(a^2*c*((a*x - 1)/
(a*x + 1))^(3/2)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\frac{2\,(a\,x-1)}{a\,x+1}+\frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

```
[In] int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] ((2*(a*x - 1))/(a*x + 1) + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2)) - (2*atan
h(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.183 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

```
[Out] -1/5*a^4*(1-1/a^2/x^2)^(5/2)/c^2/(a-1/x)^5
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6313, 665}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -1/5*(a^4*(1 - 1/(a^2*x^2))^(5/2))/(c^2*(a - x^(-1))^5)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2x^2}\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^2c^2}\\ &= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2}{5c^2(-1+ax)^3}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -1/5*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2)/(c^2*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{ax+1}{5(ax-1)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
default	$-\frac{ax+1}{5(ax-1)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
trager	$-\frac{(ax+1)\left(a^{2}x^{2}+2ax+1\right)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^{2}(ax-1)^{3}}$	51

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/5*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/5*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*x
^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
[Out] Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x
)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{5\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/5/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2\left(5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{5\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -2/5*(5*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 10*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{5\,a\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] -1/(5*a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.184 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

```
[Out] 1/7*a^5*(1-1/a^2/x^2)^(5/2)/c^3/(a-1/x)^6-6/35*a^4*(1-1/a^2/x^2)^(5/2)/c^3/
(a-1/x)^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6310, 6313, 807, 665}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2))/(7*c^3*(a - x^(-1))^6) - (6*a^4*(1 - 1/(a^2*x
^2))^(5/2))/(35*c^3*(a - x^(-1))^5)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{7a^2c^3}\\
&= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-6+ax)(1+ax)^2}{35c^3(-1+ax)^4}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-6 + a*x)*(1 + a*x)^2)/(c^3*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(ax-6)(ax+1)}{35(ax-1)^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	41
default	$-\frac{(ax-6)(ax+1)}{35(ax-1)^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	41
trager	$-\frac{(ax+1)\left(a^3x^3-4a^2x^2-11ax-6\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^3(ax-1)^4}$	59

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/35*(a*x-6)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^4x^4-3\,a^3x^3-15\,a^2x^2-17\,ax-6)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/35*(a^4*x^4 - 3*a^3*x^3 - 15*a^2*x^2 - 17*a*x - 6)*sqrt((a*x - 1)/(a*x +
 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{1}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
[Out] -Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**3
*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{7\,(ax-1)}{ax+1}-5}{70\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/70*(7*(a*x - 1)/(a*x + 1) - 5)/(a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(59) = 118.

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+14\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+\cdots\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] 2/35*(35*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 35*(a + sqrt(a^2 - 1/x^2))^4*x^4 +
 70*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 14*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 7*(a
 + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{5\,(a\,x+1)}-\frac{1}{7}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] -((a*x - 1)/(5*(a*x + 1)) - 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

3.185 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[Out] -47/315*(a+1/x)^5/a^6/c^4/(1-1/a^2/x^2)^(5/2)+16/63*(a+1/x)^6/a^7/c^4/(1-1/
a^2/x^2)^(7/2)-1/9*(a+1/x)^7/a^8/c^4/(1-1/a^2/x^2)^(9/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 866, 1649, 803, 665}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (-47*(a + x^(-1))^5)/(315*a^6*c^4*(1 - 1/(a^2*x^2))^(5/2)) + (16*(a + x^(-1
))^6)/(63*a^7*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (a + x^(-1))^7/(9*a^8*c^4*(1 -
 1/(a^2*x^2))^(9/2))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

0]

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^7}\,dx,x,\frac{1}{x}\right)}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^7}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx,x,\frac{1}{x}\right)}{a^4c^4}\\
&= -\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6(7a^2+9ax)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx,x,\frac{1}{x}\right)}{9a^4c^4}\\
&= \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{47\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{63a^2c^4}\\
&= -\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2\left(47-14ax+2a^2x^2\right)}{315c^4(-1+ax)^5}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
[Out] -1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(47 - 14*a*x + 2*a^2*x^2))/(c^4
*(-1 + a*x)^5)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{(2a^2x^2-14ax+47)(ax+1)}{315(ax-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
default	$-\frac{(2a^2x^2-14ax+47)(ax+1)}{315(ax-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2+80ax+47)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^4(ax-1)^5}$	68

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
[Out] -1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/
a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3+101\,a^2x^2+127\,ax+47)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 + 101*a^2*x^2 + 127*a*x + 47)*sq
rt((a*x - 1)/(a*x + 1))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*
a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
[Out] Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 5*a**4*
x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqrt(a*x/(a
```

```
*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{90\,(ax-1)}{ax+1}-\frac{63\,(ax-1)^2}{(ax+1)^2}-35}{1260\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 35)/(a*c^4*((
a*x - 1)/(a*x + 1))^(9/2))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.54

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(210\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6x^6+315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+441\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+126\left(a+\sqrt{a^2-\frac{1}{x^2}}\right.\cdots\right.}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9ac^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/315*(210*(a + sqrt(a^2 - 1/x^2))^6*x^6 + 315*(a + sqrt(a^2 - 1/x^2))^5*x
^5 + 441*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 126*(a + sqrt(a^2 - 1/x^2))^3*x^3
+ 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 9*(a + sqrt(a^2 - 1/x^2))*x + 1)/(((a
+ sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{5\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

```
[In] int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)^2/(5*(a*x + 1)^2) - (2*(a*x - 1))/(7*(a*x + 1)) + 1/9)/(4*a*c^4
*((a*x - 1)/(a*x + 1))^(9/2))
```

3.186 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}$$

```
[Out] -152/1155*(a+1/x)^5/a^6/c^5/(1-1/a^2/x^2)^(5/2)+79/231*(a+1/x)^6/a^7/c^5/(1
-1/a^2/x^2)^(7/2)-10/33*(a+1/x)^7/a^8/c^5/(1-1/a^2/x^2)^(9/2)+1/11*(a+1/x)^
8/a^9/c^5/(1-1/a^2/x^2)^(11/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 866, 1649, 803, 665}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

```
[Out] (-152*(a + x^(-1))^5)/(1155*a^6*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (79*(a + x^(
-1))^6)/(231*a^7*c^5*(1 - 1/(a^2*x^2))^(7/2)) - (10*(a + x^(-1))^7)/(33*a^8
```

```
*c^5*(1 - 1/(a^2*x^2))^(9/2)) + (a + x^(-1))^8/(11*a^9*c^5*(1 - 1/(a^2*x^2)
)^(11/2))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^8}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^8}{\left(1-\frac{x^2}{a^2}\right)^{13/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^7(8a^3+11a^2x+11ax^2)}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{11a^5c^5}$$

$$= -\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6(138a^3+99a^2x)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{99a^5c^5}$$

$$= \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{152\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{231a^2c^5}$$

$$= -\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(1+ax)^2\left(-152+61ax-16a^2x^2+2a^3x^3\right)}{1155c^5(-1+ax)^6}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
[Out] -1/1155*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(-152 + 61*a*x - 16*a^2*x^2 + 
2*a^3*x^3))/(c^5*(-1 + a*x)^6)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
default	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
trager	$-\frac{(ax+1)(2a^5x^5-12a^4x^4+31a^3x^3-46a^2x^2-243ax-152)\sqrt{-\frac{-ax+1}{ax+1}}}{1155a\,c^5(ax-1)^6}$	76

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
[Out] -1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a
*x+1))^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\left(2\,a^6x^6-10\,a^5x^5+19\,a^4x^4-15\,a^3x^3-289\,a^2x^2-395\,ax-152\right)\sqrt{\frac{ax-1}{ax+1}}}{1155\left(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
[Out] -1/1155*(2*a^6*x^6 - 10*a^5*x^5 + 19*a^4*x^4 - 15*a^3*x^3 - 289*a^2*x^2 - 3
95*a*x - 152)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15*a
^5*c^5*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{1}{\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{20a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\sqrt{\frac{a}{ax}}}}{c^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 6*a**5
*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 15*a**4*x**4*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 20*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) + 15*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - 6*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1)), x)/c**5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^2}{(ax+1)^2}+\frac{231\,(ax-1)^3}{(ax+1)^3}-105}{9240\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/9240*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2 + 231*(a*x -
 1)^3/(a*x + 1)^3 - 105)/(a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(1155\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^7x^7+2079\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6x^6+2541\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+825\left(a+\sqrt{a^2-\frac{1}{x^2}}\right.\right.}{1155\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 4/1155*(1155*(a + sqrt(a^2 - 1/x^2))^7*x^7 + 2079*(a + sqrt(a^2 - 1/x^2))^6
*x^6 + 2541*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 825*(a + sqrt(a^2 - 1/x^2))^4*x
^4 + 165*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 55*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 11*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^11*a*c^
5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{\frac{3\,(a\,x-1)^2}{7\,(a\,x+1)^2}-\frac{(a\,x-1)^3}{5\,(a\,x+1)^3}-\frac{a\,x-1}{3\,(a\,x+1)}+\frac{1}{11}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

```
[In] int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((3*(a*x - 1)^2)/(7*(a*x + 1)^2) - (a*x - 1)^3/(5*(a*x + 1)^3) - (a*x - 1)/
(3*(a*x + 1)) + 1/11)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

3.187 $\int e^{4 \coth^{-1}(ax)}(c-acx)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int e^{4 \coth^{-1}(ax)}(c-acx)^p \, dx = \frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

```
[Out] 4*c*(-a*c*x+c)^(-1+p)/a/(1-p)+4*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(p+1)/a/c/(p+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6265, 21, 45}

$$\int e^{4 \coth^{-1}(ax)}(c-acx)^p \, dx = \frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}(c-acx)^p\,dx\\
&= \int \frac{(1+ax)^2(c-acx)^p}{(1-ax)^2}\,dx\\
&= c^2\int (1+ax)^2(c-acx)^{-2+p}\,dx\\
&= c^2\int \left(4(c-acx)^{-2+p}-\frac{4(c-acx)^{-1+p}}{c}+\frac{(c-acx)^p}{c^2}\right)dx\\
&= \frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^p\left(\frac{4+3p}{p(1+p)}+\frac{ax}{1+p}+\frac{4}{(-1+p)(-1+ax)}\right)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 + a
*x))))/a
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

method	result
gosper	$\frac{\left(a^{2} p^{2} x^{2}-a^{2} x^{2} p+2 a \,p^{2} x+2 p a x-4 a x+p^{2}+3 p+4\right) \left(-a c x+c\right)^{p}}{\left(a x-1\right) a p \left(p^{2}-1\right)}$
risch	$\frac{\left(a^{2} p^{2} x^{2}-a^{2} x^{2} p+2 a \,p^{2} x+2 p a x-4 a x+p^{2}+3 p+4\right) \left(-a c x+c\right)^{p}}{a p \left(p+1\right) \left(-1+p\right) \left(a x-1\right)}$
norman	$\frac{\frac{a \,x^{2} {\mathrm e}^{p \ln \left(-a c x+c\right)}}{p+1}+\frac{\left(p^{2}+3 p+4\right) {\mathrm e}^{p \ln \left(-a c x+c\right)}}{a p \left(p^{2}-1\right)}+\frac{2 \left(2+p\right) x \,{\mathrm e}^{p \ln \left(-a c x+c\right)}}{p \left(p+1\right)}}{a x-1}$
parallelrisch	$\frac{x^{2} \left(-a c x+c\right)^{p} a^{2} p^{2}-x^{2} \left(-a c x+c\right)^{p} a^{2} p+2 x \left(-a c x+c\right)^{p} a \,p^{2}+2 x \left(-a c x+c\right)^{p} a p-4 \left(-a c x+c\right)^{p} x a+\left(-a c x+c\right)^{p} p^{2}+3 \left(-a c x+c\right)^{p} p+4(}{\left(a x-1\right) a p \left(p^{2}-1\right)}$

[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)

[Out] (a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)*(-a*c*x+c)^p/(a*x
-1)/a/p/(p^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = -\frac{((a^2p^2-a^2p)x^2+p^2+2\,(ap^2+ap-2\,a)x+3\,p+4)(-acx+c)^p}{ap^3-ap-(a^2p^3-a^2p)x}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")

[Out] -((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
+ c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(48) = 96.

Time = 0.58 (sec) , antiderivative size = 530, normalized size of antiderivative = 8.03

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = \begin{cases} c^p x \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{4ax}{a^3cx^2-2a^2cx+ac}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}-\frac{2}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{5}{a^2x-a} \\ -\frac{acx^2}{2}-3cx-\frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{p}{a^2p^3x} \end{cases}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a**
2*c*x + a*c) + 2*a*x*log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 4*a*x/
(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x +
 a*c) - 2/(a**3*c*x**2 - 2*a**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**2*x
- a) + 4*a*x*log(x - 1/a)/(a**2*x - a) - 4*log(x - 1/a)/(a**2*x - a) - 5/(a
**2*x - a), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(p, 1))
, (a**2*p**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) -
 a**2*p*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*
p**2*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p*x*(-
a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - 4*a*x*(-a*c*x + c)*
*p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*x + c)**p/(a**2*p**
3*x - a**2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*
x - a*p**3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*
p), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{((p^2-p)a^2c^px^2+2\,ac^p(p-1)x+2\,c^p)(-ax+1)^pa^2}{(p^3-p)a^4x-(p^3-p)a^3} + \frac{2\,(ac^p(p-1)x+c^p)(-ax+1)^pa}{(p^2-p)a^3x-(p^2-p)a^2} + \frac{(-ax+1)^pc^p}{a^2(p-1)x-a(p-1)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 - p)*a^2*c^p*x^2 + 2*a*c^p*(p - 1)*x + 2*c^p)*(-a*x + 1)^p*a^2/((p^3
- p)*a^4*x - (p^3 - p)*a^3) + 2*(a*c^p*(p - 1)*x + c^p)*(-a*x + 1)^p*a/((p^
2 - p)*a^3*x - (p^2 - p)*a^2) + (-a*x + 1)^p*c^p/(a^2*(p - 1)*x - a*(p - 1)
)
```

Giac [F]

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(ax+1)^2(-acx+c)^p}{(ax-1)^2}\,dx$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="giac")

[Out] integrate((a*x + 1)^2*(-a*c*x + c)^p/(a*x - 1)^2, x)

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{4\,(c-a\,c\,x)^p}{a\,(a\,x-1)\,(p-1)}+\frac{(c-a\,c\,x)^p\,(3\,p+a\,p\,x+4)}{a\,p\,(p+1)}$$

[In] int(((c - a*c*x)^p*(a*x + 1)^2)/(a*x - 1)^2,x)

[Out] (4*(c - a*c*x)^p)/(a*(a*x - 1)*(p - 1)) + ((c - a*c*x)^p*(3*p + a*p*x + 4))/(a*p*(p + 1))

3.188 $\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[Out] -c^5*(-a*x+1)^4/a+4/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}(c-acx)^5\,dx \\
&= c^5\int (1-ax)^3(1+ax)^2\,dx \\
&= c^5\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)\,dx \\
&= -\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{c^5(-1+ax)^4\left(11+14ax+5a^2x^2\right)}{30a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
[Out] -1/30*(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/a
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{x(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30)c^5}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{1}{5}a^4x^5+\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3-\frac{1}{2}a\,x^2+x\right)$
risch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}c^5a^2x^3-\frac{1}{2}a\,c^5x^2+c^5x$
parallelrisch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}c^5a^2x^3-\frac{1}{2}a\,c^5x^2+c^5x$
norman	$\frac{-c^5x+\frac{3}{2}a\,c^5x^2-\frac{7}{6}a^3c^5x^4+\frac{3}{10}a^4c^5x^5+\frac{11}{30}a^5c^5x^6-\frac{1}{6}a^6c^5x^7+\frac{1}{6}c^5a^2x^3}{ax-1}$
meijerg	$-\frac{c^5\left(\frac{ax\left(-20a^6x^6-28a^5x^5-42a^4x^4-70a^3x^3-140a^2x^2-420ax+840\right)}{-120ax+120}+7\ln(-ax+1)\right)}{a}-\frac{3c^5\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70\right.}{70(-ax+1)}\right.}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3/
3 - a*c**5*x**2/2 + c**5*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{\left(5\,c^5+\frac{24\,c^5}{ax-1}+\frac{30\,c^5}{(ax-1)^2}\right)(ax-1)^6}{30\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/30*(5*c^5 + 24*c^5/(a*x - 1) + 30*c^5/(a*x - 1)^2)*(a*x - 1)^6/a
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{a^5\,c^5\,x^6}{6}+\frac{a^4\,c^5\,x^5}{5}+\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}-\frac{a\,c^5\,x^2}{2}+c^5\,x$$

```
[In] int(((c - a*c*x)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^5*x - (a*c^5*x^2)/2 - (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 + (a^4*c^5*x^5)
/5 - (a^5*c^5*x^6)/6
```

3.189 $\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$$

```
[Out] c^4*x-2/3*a^2*c^4*x^3+1/5*a^4*c^4*x^5
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6264, 41, 200}

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}(c-acx)^4\,dx \\
&= c^4\int (1-ax)^2(1+ax)^2\,dx \\
&= c^4\int \left(1-a^2x^2\right)^2\,dx \\
&= c^4\int \left(1-2a^2x^2+a^4x^4\right)\,dx \\
&= c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=c^4\left(x-\frac{2a^2x^3}{3}+\frac{a^4x^5}{5}\right)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
[Out] c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result
default	$c^{4} \left(\frac{1}{5} a^{4} x^{5}-\frac{2}{3} a^{2} x^{3}+x \right)$
gosper	$\frac{x \left(3 a^{4} x^{4}-10 a^{2} x^{2}+15\right) c^{4}}{15}$
risch	$c^{4} x -\frac{2}{3} a^{2} c^{4} x^{3}+\frac{1}{5} a^{4} c^{4} x^{5}$
parallelrisch	$c^{4} x -\frac{2}{3} a^{2} c^{4} x^{3}+\frac{1}{5} a^{4} c^{4} x^{5}$
norman	$\frac{-c^{4} x +a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}-\frac{2}{3} a^{3} c^{4} x^{4}-\frac{1}{5} a^{4} c^{4} x^{5}+\frac{1}{5} a^{5} c^{4} x^{6}}{a x -1}$
meijerg	$-\frac{c^{4} \left(-\frac{a x \left(-14 a^{5} x^{5}-21 a^{4} x^{4}-35 a^{3} x^{3}-70 a^{2} x^{2}-210 a x +420\right)}{70 \left(-a x +1\right)}-6 \ln \left(-a x +1\right)\right)}{a}-\frac{2 c^{4} \left(\frac{a x \left(-3 a^{4} x^{4}-5 a^{3} x^{3}-10 a^{2} x^{2}-30 a x +60\right)}{-12 a x +12}+5 \right.}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
[Out] c^4*(1/5*a^4*x^5-2/3*a^2*x^3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{a^4c^4x^5}{5}-\frac{2a^2c^4x^3}{3}+c^4x$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**4,x)
[Out] a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{\left(3\,c^4+\frac{15\,c^4}{ax-1}+\frac{20\,c^4}{(ax-1)^2}\right)(ax-1)^5}{15\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
[Out] 1/15*(3*c^4 + 15*c^4/(a*x - 1) + 20*c^4/(a*x - 1)^2)*(a*x - 1)^5/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4\,x\,\left(3\,a^4\,x^4-10\,a^2\,x^2+15\right)}{15}$$

```
[In] int(((c - a*c*x)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^4*x*(3*a^4*x^4 - 10*a^2*x^2 + 15))/15
```

3.190 $\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}$$

```
[Out] 2/3*c^3*(a*x+1)^3/a-1/4*c^3*(a*x+1)^4/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int e^{4\text{arctanh}(ax)}(c-acx)^3\,dx \\ &= c^3\int (1-ax)(1+ax)^2\,dx \\ &= c^3\int \left(2(1+ax)^2-(1+ax)^3\right)\,dx \\ &= \frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{12}c^3x\left(-12-6ax+4a^2x^2+3a^3x^3\right)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
[Out] -1/12*(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
gosper	$-\frac{x(3a^3x^3+4a^2x^2-6ax-12)c^3}{12}$
default	$c^3\left(-\frac{1}{4}a^3x^4-\frac{1}{3}a^2x^3+\frac{1}{2}a\,x^2+x\right)$
risch	$-\frac{1}{4}a^3c^3x^4-\frac{1}{3}a^2c^3x^3+\frac{1}{2}a\,c^3x^2+c^3x$
parallelrisch	$-\frac{1}{4}a^3c^3x^4-\frac{1}{3}a^2c^3x^3+\frac{1}{2}a\,c^3x^2+c^3x$
norman	$\frac{-c^3x+\frac{1}{2}a\,c^3x^2+\frac{5}{6}a^2c^3x^3-\frac{1}{12}a^3c^3x^4-\frac{1}{4}a^4c^3x^5}{ax-1}$
meijerg	$-\frac{c^3\left(\frac{ax\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+5\ln(-ax+1)\right)}{a}-\frac{c^3\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}+\frac{2c^3}{}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3c^3x^4}{4}-\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}+c^3x$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**3,x)
[Out] -a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4 - \frac{1}{3}\,a^2c^3x^3 + \frac{1}{2}\,ac^3x^2 + c^3x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{\left(3\,c^3 + \frac{16\,c^3}{ax-1} + \frac{24\,c^3}{(ax-1)^2}\right)(ax-1)^4}{12\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/12*(3*c^3 + 16*c^3/(a*x - 1) + 24*c^3/(a*x - 1)^2)*(a*x - 1)^4/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3\,c^3\,x^4}{4} - \frac{a^2\,c^3\,x^3}{3} + \frac{a\,c^3\,x^2}{2} + c^3\,x$$

```
[In] int(((c - a*c*x)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c^3*x + (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

3.191 $\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 17

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2(1+ax)^3}{3a}$$

```
[Out] 1/3*c^2*(a*x+1)^3/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 32}

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2(ax+1)^3}{3a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int e^{4\text{arctanh}(ax)}(c-acx)^2\,dx$$
$$= c^2\int (1+ax)^2\,dx$$
$$= \frac{c^2(1+ax)^3}{3a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(x+ax^2+\frac{a^2x^3}{3}\right)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
[Out] c^2*(x + a*x^2 + (a^2*x^3)/3)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{c^2(ax+1)^3}{3a}$	16
gosper	$\frac{x(a^2x^2+3ax+3)c^2}{3}$	20
parallelrisch	$\frac{1}{3}a^2c^2x^3+a\,c^2x^2+c^2x$	26
risch	$\frac{a^2c^2x^3}{3}+a\,c^2x^2+c^2x+\frac{c^2}{3a}$	34
norman	$\frac{-\frac{c^2}{a}+\frac{2a^2c^2x^3}{3}+\frac{a^3c^2x^4}{3}}{ax-1}$	40
meijerg	$-\frac{c^2\left(-\frac{ax(-5a^3x^3-10a^2x^2-30ax+60)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}+\frac{2c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c^2x}{-ax+1}$	103

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*c^2*(a*x+1)^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{a^2c^2x^3}{3} + ac^2x^2 + c^2x$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**2,x)
[Out] a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.35

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{\left(c^2 + \frac{6\,c^2}{ax-1} + \frac{12\,c^2}{(ax-1)^2}\right)(ax-1)^3}{3\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
[Out] 1/3*(c^2 + 6*c^2/(a*x - 1) + 12*c^2/(a*x - 1)^2)*(a*x - 1)^3/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

```
[In] int(((c - a*c*x)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^2*x*(3*a*x + a^2*x^2 + 3))/3
```

3.192 $\int e^{4\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}$$

```
[Out] -3*c*x-1/2*a*c*x^2-4*c*ln(-a*x+1)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6302, 6264, 45}

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}-3cx$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}(c-acx)\,dx \\
&= c\int \frac{(1+ax)^2}{1-ax}\,dx \\
&= c\int \left(-3-ax+\frac{4}{1-ax}\right)dx \\
&= -3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$
\int e^{4\coth^{-1}(ax)}(c-acx)\,dx = c\left(-3x-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}\right)
$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
[Out] c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	siz
default	$c\left(-\frac{a\,x^2}{2}-3x-\frac{4\ln(ax-1)}{a}\right)$	24
risch	$-\frac{ac\,x^2}{2}-3cx-\frac{4c\ln(ax-1)}{a}$	25
parallelrisch	$-\frac{a^2c\,x^2+6acx+8c\ln(ax-1)}{2a}$	29
norman	$\frac{3cx-\frac{5}{2}ac\,x^2-\frac{1}{2}a^2c\,x^3}{ax-1}-\frac{4c\ln(ax-1)}{a}$	43
meijerg	$-\frac{c\left(\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)\right)}{a}+\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}+\frac{cx}{-ax+1}$	11

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] c*(-1/2*a*x^2-3*x-4/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx = -\frac{a^2cx^2 + 6\,acx + 8\,c\log(ax-1)}{2\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="fricas")
[Out] -1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx = -\frac{acx^2}{2} - 3cx - \frac{4c\log(ax-1)}{a}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c),x)
[Out] -a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx = -\frac{1}{2}\,acx^2 - 3\,cx - \frac{4\,c\log(ax-1)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="maxima")
[Out] -1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{(ax-1)^2\left(c+\frac{8\,c}{ax-1}\right)}{2\,a}+\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="giac")
[Out] -1/2*(a*x - 1)^2*(c + 8*c/(a*x - 1))/a + 4*c*log(abs(a*x - 1)/((a*x - 1)^2*
abs(a)))/a
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,(8\,\ln(a\,x-1)+6\,a\,x+a^2\,x^2)}{2\,a}$$

```
[In] int(((c - a*c*x)*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] -(c*(8*log(a*x - 1) + 6*a*x + a^2*x^2))/(2*a)
```

3.193 $\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2}{ac(1-ax)^2}-\frac{4}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[Out] 2/a/c/(-a*x+1)^2-4/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{c-acx}\,dx \\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^3}\,dx}{c} \\
&= \frac{\int \left(\frac{1}{1-ax}-\frac{4}{(-1+ax)^3}-\frac{4}{(-1+ax)^2}\right)dx}{c} \\
&= \frac{2}{ac(1-ax)^2}-\frac{4}{ac(1-ax)}-\frac{\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{-2+4ax-(-1+ax)^2\log(1-ax)}{ac(-1+ax)^2}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
[Out] (-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

method	result	size
norman	$\frac{2a\,x^2}{c(ax-1)^2}-\frac{\ln(ax-1)}{ac}$	32
risch	$\frac{4x-\frac{2}{a}}{c(ax-1)^2}-\frac{\ln(ax-1)}{ac}$	36
default	$\frac{\frac{2}{(ax-1)^2a}+\frac{4}{a(ax-1)}-\frac{\ln(ax-1)}{a}}{c}$	41
parallelrisch	$\frac{-a^2\ln(ax-1)x^2+2a^2x^2+2a\ln(ax-1)x-\ln(ax-1)}{(ax-1)^2ca}$	56

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] 2*a/c*x^2/(a*x-1)^2-1/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{4\,ax-(a^2x^2-2\,ax+1)\log{(ax-1)}-2}{a^3cx^2-2\,a^2cx+ac}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="fricas")
[Out] (4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x + a
*c)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{-4ax+2}{a^3cx^2-2a^2cx+ac}-\frac{\log{(ax-1)}}{ac}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c),x)
[Out] -(-4*a*x + 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\,(2\,ax-1)}{a^3cx^2-2\,a^2cx+ac}-\frac{\log{(ax-1)}}{ac}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="maxima")
[Out] 2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac} + \frac{2\left(\frac{2\,ac}{ax-1} + \frac{ac}{(ax-1)^2}\right)}{a^2c^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) + 2*(2*a*c/(a*x - 1) + a*c/(a*
x - 1)^2)/(a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{4\,x-\frac{2}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c} - \frac{\ln\left(a\,x-1\right)}{a\,c}$$

```
[In] int((a*x + 1)^2/((c - a*c*x)*(a*x - 1)^2),x)
```

```
[Out] (4*x - 2/a)/(c + a^2*c*x^2 - 2*a*c*x) - log(a*x - 1)/(a*c)
```

3.194 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

```
[Out] 1/6*(a*x+1)^3/a/c^2/(-a*x+1)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 37}

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int \frac{e^{4\text{arctanh}(ax)}}{(c-acx)^2}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{-a\,x^2-\frac{1}{3a}}{(ax-1)^3c^2}$	24
parallelrisch	$\frac{-a^2x^3-3x}{3(ax-1)^3c^2}$	25
gosper	$-\frac{3a^2x^2+1}{3(ax-1)^3a\,c^2}$	26
norman	$\frac{-\frac{x}{c}-\frac{a^2x^3}{3c}}{(ax-1)^3c}$	30
default	$\frac{-\frac{2}{(ax-1)^2a}-\frac{1}{a(ax-1)}-\frac{4}{3a(ax-1)^3}}{c^2}$	42

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] (-a*x^2-1/3/a)/(a*x-1)^3/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-3a^2x^2-1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**2,x)
[Out] (-3*a**2*x**2 - 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3
*a*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2}{(acx-c)^2a} - \frac{1}{(acx-c)ac} - \frac{4c}{3(acx-c)^3a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3a^2x^2+1}{3ac^2(ax-1)^3}$$

```
[In] int((a*x + 1)^2/((c - a*c*x)^2*(a*x - 1)^2),x)
[Out] -(3*a^2*x^2 + 1)/(3*a*c^2*(a*x - 1)^3)
```

3.195 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3} dx = \frac{1}{ac^3(1-ax)^4} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{2ac^3(1-ax)^2}$$

```
[Out] 1/a/c^3/(-a*x+1)^4-4/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3} dx = \frac{1}{2ac^3(1-ax)^2} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{ac^3(1-ax)^4}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{(c-acx)^3}\,dx \\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^5}\,dx}{c^3} \\
&= \frac{\int \left(-\frac{4}{(-1+ax)^5}-\frac{4}{(-1+ax)^4}-\frac{1}{(-1+ax)^3}\right)dx}{c^3} \\
&= \frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{1+2ax+3a^2x^2}{6ac^3(-1+ax)^4}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
[Out] (1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{\frac{a\,x^2}{2}+\frac{x}{3}+\frac{1}{6a}}{(ax-1)^4 c^3}$	27
gosper	$\frac{3a^2x^2+2ax+1}{6(ax-1)^4 a\,c^3}$	30
parallelrisch	$\frac{-a^3x^4+4a^2x^3-3a\,x^2+6x}{6(ax-1)^4 c^3}$	39
default	$\frac{\frac{1}{a(ax-1)^4}+\frac{1}{2(ax-1)^2 a}+\frac{4}{3a(ax-1)^3}}{c^3}$	41
norman	$\frac{\frac{x}{c}-\frac{a\,x^2}{2c}+\frac{2a^2x^3}{3c}-\frac{a^3x^4}{6c}}{(ax-1)^4 c^2}$	49

[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)

[Out] (1/2*a*x^2+1/3*x+1/6/a)/(a*x-1)^4/c^3

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")

[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{-3a^2x^2-2ax-1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**3,x)

[Out] -(-3*a**2*x**2 - 2*a*x - 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\frac{3}{(ax-1)^2a}+\frac{8}{(ax-1)^3a}+\frac{6}{(ax-1)^4a}}{6\,c^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3/((a*x - 1)^2*a) + 8/((a*x - 1)^3*a) + 6/((a*x - 1)^4*a))/c^3
```

Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2\,x^2+2\,a\,x+1}{6\,a\,c^3\,(a\,x-1)^4}$$

```
[In] int((a*x + 1)^2/((c - a*c*x)^3*(a*x - 1)^2),x)
```

```
[Out] (2*a*x + 3*a^2*x^2 + 1)/(6*a*c^3*(a*x - 1)^4)
```

3.196 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{4}{5ac^4(1-ax)^5} - \frac{1}{ac^4(1-ax)^4} + \frac{1}{3ac^4(1-ax)^3}$$

```
[Out] 4/5/a/c^4/(-a*x+1)^5-1/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 45}

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{1}{3ac^4(1-ax)^3} - \frac{1}{ac^4(1-ax)^4} + \frac{4}{5ac^4(1-ax)^5}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{(c-acx)^4}\,dx \\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^6}\,dx}{c^4} \\
&= \frac{\int \left(\frac{4}{(-1+ax)^6}+\frac{4}{(-1+ax)^5}+\frac{1}{(-1+ax)^4}\right)dx}{c^4} \\
&= \frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2+5ax+5a^2x^2}{15ac^4(-1+ax)^5}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
[Out] -1/15*(2 + 5*a*x + 5*a^2*x^2)/(a*c^4*(-1 + a*x)^5)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{-\frac{a\,x^{2}}{3}-\frac{x}{3}-\frac{2}{15a}}{\left(ax-1\right)^{5}c^{4}}$	27
gosper	$-\frac{5a^{2}x^{2}+5ax+2}{15\left(ax-1\right)^{5}a\,c^{4}}$	30
default	$\frac{-\frac{1}{a\left(ax-1\right)^{4}}-\frac{1}{3a\left(ax-1\right)^{3}}-\frac{4}{5a\left(ax-1\right)^{5}}}{c^{4}}$	42
parallelrisch	$\frac{-2a^{4}x^{5}+10a^{3}x^{4}-20a^{2}x^{3}+15a\,x^{2}-15x}{15\left(ax-1\right)^{5}c^{4}}$	47
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{x}{c}-\frac{4a^{2}x^{3}}{3c}+\frac{2a^{3}x^{4}}{3c}-\frac{2a^{4}x^{5}}{15c}}{\left(ax-1\right)^{5}c^{3}}$	60

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*a*x^2-1/3*x-2/15/a)/(a*x-1)^5/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{5\,a^{2}x^{2}+5\,ax+2}{15\,\left(a^{6}c^{4}x^{5}-5\,a^{5}c^{4}x^{4}+10\,a^{4}c^{4}x^{3}-10\,a^{3}c^{4}x^{2}+5\,a^{2}c^{4}x-ac^{4}\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx=\frac{-5a^{2}x^{2}-5ax-2}{15a^{6}c^{4}x^{5}-75a^{5}c^{4}x^{4}+150a^{4}c^{4}x^{3}-150a^{3}c^{4}x^{2}+75a^{2}c^{4}x-15ac^{4}}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**4,x)
```

```
[Out] (-5*a**2*x**2 - 5*a*x - 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**
4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2x^2+5\,ax+2}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{5}{(ax-1)^3a}+\frac{15}{(ax-1)^4a}+\frac{12}{(ax-1)^5a}}{15\,c^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
[Out] -1/15*(5/((a*x - 1)^3*a) + 15/((a*x - 1)^4*a) + 12/((a*x - 1)^5*a))/c^4
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2\,x^2+5\,a\,x+2}{15\,a\,c^4\,{\left(a\,x-1\right)}^5}$$

```
[In] int((a*x + 1)^2/((c - a*c*x)^4*(a*x - 1)^2),x)
[Out] -(5*a*x + 5*a^2*x^2 + 2)/(15*a*c^4*(a*x - 1)^5)
```

3.197 $\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

```
[Out] ((a-1/x)/(a+1/x))^(-1/2-p)*x*(-a*c*x+c)^p*hypergeom([-1-p, -1/2-p],[-p],2/(
a+1/x)/x)*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(p+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6311, 6316, 134}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,\text{Hypergeometric2F1}\left(-p-1,-p-\frac{1}{2},-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

```
[In] Int[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*x)
])/(1 + p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-p} x^{-p}(c-acx)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{1+ax}\right)}{1+p}$$

```
[In] Integrate[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(c - a*c*x)^p*Hy
pergeometric2F1[-1 - p, -1/2 - p, -p, 2/(1 + a*x)])/(1 + p)
```

Maple [F]

$$\int (-acx+c)^p \sqrt{\frac{ax-1}{ax+1}}dx$$

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \int \sqrt{\frac{ax-1}{ax+1}}(-c(ax-1))^p\,dx$$

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**p, x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(c-a\,c\,x)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.198 $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 127

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x-\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4-\frac{35c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] -35/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+20/3*c^3*x*(1-1/a^2/x^2)^(1/2)-27/
8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+4/3*a^2*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3
*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6310, 6313, 1821, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{35c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}-\frac{27}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{20}{3}c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4}{3}a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

```
[Out] (20*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (27*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8
 + (4*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]
*x^4)/4 - (35*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(\left(a^3c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{16}{a}-\frac{27x}{a^2}+\frac{16x^2}{a^3}-\frac{4x^3}{a^4}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{81}{a^2}-\frac{80x}{a^3}+\frac{12x^2}{a^4}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{24}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{160}{a^3}-\frac{105x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{\left(35c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{\left(35c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{8}\left(35ac^3\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$
= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{35c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$
\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(160-81ax+32a^2x^2-6a^3x^3)-105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}
$$

```
[In] Integrate[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(160 - 81*a*x + 32*a^2*x^2 - 6*a^3*x^3) - 1
05*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(6a^3x^3-32a^2x^2+81ax-160)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{24a}-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,ax+87\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-32((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-87\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-192\sqrt{a^2}\,\sqrt{}\right.}{24a\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*a^3*x^3-32*a^2*x^2+81*a*x-160)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/
2)-35/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x-1)*((a
*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^3x^4-26\,a^3c^3x^3+49\,a^2c^3x^2-79\,ac^3x-160\right.}{24\,a}$$

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (6*a^4*c^3*x^4 - 26*a^3*c^3*x^3 + 49*a^2*c^3*x^2 - 79
*a*c^3*x - 160*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int 3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(107) = 214.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{24}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(279\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-511\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+385\,c^3\left(\right.\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(279*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 511*
c^3*((a*x - 1)/(a*x + 1))^(5/2) + 385*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105
*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*
a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)
^4 - a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{35\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{8\,|a|} + \frac{1}{24}\sqrt{a^2x^2-1}\left(\frac{160\,c^3\mathrm{sgn}(ax+1)}{a}-\left(81\,c^3\mathrm{sgn}(ax+1)+2\left(3\,a^2c^3x\mathrm{sgn}(ax+1)-16\,ac^3\mathrm{sgn}(ax+1)\right)x\right)x\right.$$

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 35/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/24
*sqrt(a^2*x^2 - 1)*(160*c^3*sgn(a*x + 1)/a - (81*c^3*sgn(a*x + 1) + 2*(3*a^
2*c^3*x*sgn(a*x + 1) - 16*a*c^3*sgn(a*x + 1))*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{\frac{35\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{385\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{511\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{35\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((35*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (385*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/12 + (511*c^3*((a*x - 1)/(a*x + 1))^(5/2))/12 - (93*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x +
 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (35*
c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.199 $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -5/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+11/3*c^2*x*(1-1/a^2/x^2)^(1/2)-3/2*
a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6310, 6313, 1821, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{3}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{11}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (11*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (3*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2
+ (a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (5*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
```

```
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = (a^2c^2)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left((a^2c^2)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{3}(a^2c^2)\,\text{Subst}\left(\int \frac{\frac{9}{a}-\frac{11x}{a^2}+\frac{3x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{6}(a^2c^2)\,\text{Subst}\left(\int \frac{\frac{22}{a^2}-\frac{15x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{2}(5ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{5c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(22-9ax+2a^2x^2)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

```
[In] Integrate[(c - a*c*x)^2/E^ArcCoth[a*x],x]
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 - 9*a*x + 2*a^2*x^2) - 15*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(2a^2x^2-9ax+22)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}-\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+24a\ln\left(\right.\right.}{6\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(2*a^2*x^2-9*a*x+22)*(a*x+1)/a*c^2*((a*x-1)/(a*x+1))^(1/2)-5/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2*((a*x-1)/(a*x+1))^(1/2)/(a*
x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^2x^3-7\,a^2c^2x^2+13\,ac^2x+22\,c^2)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 7*a^2*c^2*x^2 + 13*a*c^2*x + 22*c^2)*sq
rt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) -
 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leave count of result is larger than twice the leaf count of optimal. 181 vs. $2(84) = 168$.

Time $= 0.20$ (sec) , antiderivative size $= 181$, normalized size of antiderivative $= 1.81$

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}a\left(\frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} + \frac{2\left(33c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 40c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 15c^2\sqrt{\frac{ax-1}{ax+1}}\right.}{\frac{3(ax-1)a^2}{ax+1} - \frac{3(ax-1)^2a^2}{(ax+1)^2} + \frac{(ax-1)^3a^2}{(ax+1)^3} - a^2}\right.$$

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 + 2*(33*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 40*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x -
1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1
)^3 - a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{5\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|} + \frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,ac^2x\operatorname{sgn}(ax+1)-9\,c^2\operatorname{sgn}(ax+1)\right)x+\frac{22\,c^2\operatorname{sgn}(ax+1)}{a}\right)$$

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/6*s
qrt(a^2*x^2 - 1)*((2*a*c^2*x*sgn(a*x + 1) - 9*c^2*sgn(a*x + 1))*x + 22*c^2*
sgn(a*x + 1)/a)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{5\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+11\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}} - \frac{5\,c^2\,\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (40*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + 11*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*
a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c^2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.200 $\int e^{-\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=2c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2
*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6310, 6313, 1821, 821, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+2cx\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] 2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left((ac)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$

$$= (ac)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(ac)\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$

$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(3ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{3c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

```
[In] Integrate[(c - a*c*x)/E^ArcCoth[a*x],x]
[Out] -1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result	si
risch	$-\frac{(ax-4)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$	99
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-4\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+4a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)\right)}{2\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	15

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-4)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-3/2*ln(a^2*x/(a^2)^(1/2)+(
a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(
1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-3\,acx-4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 - 3*a*c*x - 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}\,a\left(\frac{2\left(5\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

[Out] 1/2*a*(2*(5*c*((a*x - 1)/(a*x + 1))^(3/2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,|a|} - \frac{1}{2}\,\sqrt{a^2x^2-1}\left(cx\mathrm{sgn}(ax+1)-\frac{4\,c\mathrm{sgn}(ax+1)}{a}\right)$$

[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")

[Out] 3/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/2*sqrt(a^2*x^2 - 1)*(c*x*sgn(a*x + 1) - 4*c*sgn(a*x + 1)/a)

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{3\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}} - \frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

[In] int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] (3*c*((a*x - 1)/(a*x + 1))^(1/2) - 5*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a

3.201 $\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6310, 6313, 272, 65, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= -\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log\left(ax\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)}{ac}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
[Out] -(Log[a*x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]/(a*c))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(21) = 42.

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)}{\sqrt{(ax-1)(ax+1)}\,c\sqrt{a^2}}$	76

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2))/(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
[Out] -(log(sqrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
(a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a))
```

Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x),x)
[Out] -(2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.202 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

```
[Out] -(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6313, 665}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(-1+ax)}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(-1 + a*x)))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)a\,c^2}$	36
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)a\,c^2}$	36
trager	$-\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^2(ax-1)}$	38

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/(a*x-1)/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/(a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(acx-c)^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^2,x)
[Out] -1/(a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

3.203 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

```
[Out] 1/3*a*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^2-2/3*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6310, 6313, 807, 665}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)])/(3*c^3*(a - x^(-1))^2) - (2*Sqrt[1 - 1/(a^2*x^2)]
)/(3*c^3*(a - x^(-1)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3a^2c^3}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-2+ax)}{3c^3(-1+ax)^2}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

```
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(c^3*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)(ax+1)}{3(ax-1)^2c^3a}$	41
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)(ax+1)}{3(ax-1)^2c^3a}$	41
trager	$-\frac{(ax-2)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^3(ax-1)^2}$	43

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(a*x-2)*(a*x+1)/(a*x-1)^2/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^3*x^2 - 2*a^2*c^3
*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x**2 + 3*a*
x - 1), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{3\,(ax-1)}{ax+1}-1}{6\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/6*(3*(a*x - 1)/(a*x + 1) - 1)/(a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^3*a*
c^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{a\,x+1}-\frac{1}{3}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^3,x)
[Out] -((a*x - 1)/(a*x + 1) - 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.204 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} dx = -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

```
[Out] -1/5*a^2*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^3+8/15*a*(1-1/a^2/x^2)^(1/2)/c^4/(
a-1/x)^2-7/15*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} dx = -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

```
[Out] -1/5*(a^2*Sqrt[1 - 1/(a^2*x^2)])/(c^4*(a - x^(-1))^3) + (8*a*Sqrt[1 - 1/(a^
2*x^2)])/(15*c^4*(a - x^(-1))^2) - (7*Sqrt[1 - 1/(a^2*x^2)])/(15*c^4*(a - x
^(-1)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

0]

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{x}{a^3}}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^4}\\
&= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5a^2c^4}\\
&= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15a^2c^4}\\
&= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(7-6ax+2a^2x^2)}{15c^4(-1+ax)^3}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^4*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3c^4a}$	50
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3c^4a}$	50
trager	$-\frac{(2a^2x^2-6ax+7)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^4(ax-1)^3}$	52

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2-6*a*x+7)*(a*x+1)/(a*x-1)^3/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^3x^3-4\,a^2x^2+ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^4*
x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x**3 + 6*a**
2*x**2 - 4*a*x + 1), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{10\,(ax-1)}{ax+1} - \frac{15\,(ax-1)^2}{(ax+1)^2} - 3}{60\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^4*((a*x
 - 1)/(a*x + 1))^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2 - 5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x + 1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,(a\,x-1)}{3\,(a\,x+1)} + \frac{1}{5}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^4,x)
```

```
[Out] -((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^4*((a
*x - 1)/(a*x + 1))^(5/2))
```

3.205 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 128

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} dx = \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

```
[Out] 1/7*a^3*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^4-18/35*a^2*(1-1/a^2/x^2)^(1/2)/c^5
/(a-1/x)^3+23/35*a*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^2-12/35*(1-1/a^2/x^2)^(1
/2)/c^5/(a-1/x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} dx = -\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)} + \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)])/(7*c^5*(a - x^(-1))^4) - (18*a^2*Sqrt[1 - 1/(a^
2*x^2)])/(35*c^5*(a - x^(-1))^3) + (23*a*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a
- x^(-1))^2) - (12*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a - x^(-1)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}\\
&= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{3x}{a^3}}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^5}\\
&= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{18\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{7a^2c^5}\\
&= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{36\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35a^2c^5}\\
&= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35a^2c^5}\\
&= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$
\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-12+13ax-8a^2x^2+2a^3x^3)}{35c^5(-1+ax)^4}
$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^5
*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^3x^3-8a^2x^2+13ax-12)(ax+1)}{35(ax-1)^4c^5a}$	58
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^3x^3-8a^2x^2+13ax-12)(ax+1)}{35(ax-1)^4c^5a}$	58
trager	$-\frac{(2a^3x^3-8a^2x^2+13ax-12)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^5(ax-1)^4}$	60

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)*(a*x+1)/(a*x-
1)^4/c^5/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+ax-12)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
[Out] -1/35*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + a*x - 12)*sqrt((a*x - 1)/(a*x +
1))/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx}{c^5}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 5*a**4*x**4 + 10*a
**3*x**3 - 10*a**2*x**2 + 5*a*x - 1), x)/c**5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{21\,(ax-1)}{ax+1} - \frac{35\,(ax-1)^2}{(ax+1)^2} + \frac{35\,(ax-1)^3}{(ax+1)^3} - 5}{280\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/280*(21*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 + 35*(a*x - 1)^
3/(a*x + 1)^3 - 5)/(a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3 - 21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2 + 7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x - 1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^5}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 4/35*(35*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 7*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{(a\,x-1)^2}{(a\,x+1)^2} - \frac{(a\,x-1)^3}{(a\,x+1)^3} - \frac{3\,(a\,x-1)}{5\,(a\,x+1)} + \frac{1}{7}}{8\,a\,c^5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^5,x)
```

```
[Out] ((a*x - 1)^2/(a*x + 1)^2 - (a*x - 1)^3/(a*x + 1)^3 - (3*(a*x - 1))/(5*(a*x
+ 1)) + 1/7)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

3.206 $\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^{2+p}\,\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}$$

```
[Out] 1/2*(-a*c*x+c)^(2+p)*hypergeom([1, 2+p],[3+p],-1/2*a*x+1/2)/a/c^2/(2+p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6265, 21, 70}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^{p+2}\,\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

```
[In] Int[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a*
c^2*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}(c-acx)^p\,dx\\ &= -\int \frac{(1-ax)(c-acx)^p}{1+ax}\,dx\\ &= -\frac{\int \frac{(c-acx)^{1+p}}{1+ax}\,dx}{c}\\ &= \frac{(c-acx)^{2+p}\,\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx = -\frac{(-1+ax)(c-acx)^p\left(-1+\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{1}{2}(1-ax)\right)\right)}{a(1+p)}$$

```
[In] Integrate[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
[Out] -(((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 - a
*x)/2]))/(a*(1 + p)))
```

Maple [F]

$$\int \frac{(-acx+c)^p (ax-1)}{ax+1} dx$$

```
[In] int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
[Out] int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p \, dx = \int \frac{(ax-1)(-acx+c)^p}{ax+1} \, dx$$

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p \, dx = \int \frac{(-c(ax-1))^p (ax-1)}{ax+1} \, dx$$

```
[In] integrate((-a*c*x+c)**p*(a*x-1)/(a*x+1),x)
[Out] Integral((-c*(a*x - 1))**p*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p \, dx = \int \frac{(ax-1)(-acx+c)^p}{ax+1} \, dx$$

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(c-a\,c\,x)^p\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1), x)
```

3.207 $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+ax)}{a}$$

```
[Out] 16*c^4*x-4*c^4*(-a*x+1)^2/a-4/3*c^4*(-a*x+1)^3/a-1/2*c^4*(-a*x+1)^4/a-1/5*c
^4*(-a*x+1)^5/a-32*c^4*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=-\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}-\frac{4c^4(1-ax)^3}{3a}-\frac{4c^4(1-ax)^2}{a}-\frac{32c^4\log(ax+1)}{a}+16c^4x$$

```
[In] Int[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 16*c^4*x - (4*c^4*(1 - a*x)^2)/a - (4*c^4*(1 - a*x)^3)/(3*a) - (c^4*(1 - a*
x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a) - (32*c^4*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}(c-acx)^4\,dx\\ &= -\left(c^4\int\frac{(1-ax)^5}{1+ax}\,dx\right)\\ &= -\left(c^4\int\left(-16-8(1-ax)-4(1-ax)^2-2(1-ax)^3-(1-ax)^4+\frac{32}{1+ax}\right)dx\right)\\ &= 16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{c^4(-181+930ax-390a^2x^2+160a^3x^3-45a^4x^4+6a^5x^5-960\log(1+ax))}{30a}$$

```
[In] Integrate[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5 -
 960*Log[1 + a*x]))/(30*a)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

method	result
default	$c^{4} \left(\frac{a^{4} x^{5}}{5}-\frac{3 a^{3} x^{4}}{2}+\frac{16 a^{2} x^{3}}{3}-13 a\,x^{2}+31 x -\frac{32 \ln \left(a x +1\right)}{a}\right)$
norman	$31 c^{4} x -13 a\,c^{4} x^{2}+\frac{16 a^{2} c^{4} x^{3}}{3}-\frac{3 a^{3} c^{4} x^{4}}{2}+\frac{a^{4} c^{4} x^{5}}{5}-\frac{32 c^{4} \ln \left(a x +1\right)}{a}$
risch	$31 c^{4} x -13 a\,c^{4} x^{2}+\frac{16 a^{2} c^{4} x^{3}}{3}-\frac{3 a^{3} c^{4} x^{4}}{2}+\frac{a^{4} c^{4} x^{5}}{5}-\frac{32 c^{4} \ln \left(a x +1\right)}{a}$
parallelrisch	$-\frac{-6 a^{5} c^{4} x^{5}+45 a^{4} c^{4} x^{4}-160 a^{3} c^{4} x^{3}+390 a^{2} c^{4} x^{2}-930 a\,c^{4} x +960 c^{4} \ln \left(a x +1\right)}{30 a}$
meijerg	$\frac{c^{4} \left(\frac{a x \left(12 a^{4} x^{4}-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}-\ln \left(a x +1\right)\right)}{a}-\frac{5 c^{4} \left(-\frac{a x \left(-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}+\ln \left(a x +1\right)\right)}{a}+\frac{10 c^{4} \left(\frac{a x (}{}\right.}{}$

```
[In] int((-a*c*x+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(1/5*a^4*x^5-3/2*a^3*x^4+16/3*a^2*x^3-13*a*x^2+31*x-32*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{6\,a^{5}c^{4}x^{5} - 45\,a^{4}c^{4}x^{4} + 160\,a^{3}c^{4}x^{3} - 390\,a^{2}c^{4}x^{2} + 930\,ac^{4}x - 960\,c^{4}\log\left(ax+1\right)}{30\,a}$$

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{a^{4}c^{4}x^{5}}{5} - \frac{3a^{3}c^{4}x^{4}}{2} + \frac{16a^{2}c^{4}x^{3}}{3} - 13ac^{4}x^{2} + 31c^{4}x - \frac{32c^{4}\log\left(ax+1\right)}{a}$$

```
[In] integrate((-a*c*x+c)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**4*x**5/5 - 3*a**3*c**4*x**4/2 + 16*a**2*c**4*x**3/3 - 13*a*c**4*x**
2 + 31*c**4*x - 32*c**4*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}\,a^4c^4x^5 - \frac{3}{2}\,a^3c^4x^4 + \frac{16}{3}\,a^2c^4x^3 - 13\,ac^4x^2 + 31\,c^4x - \frac{32\,c^4\log\left(ax+1\right)}{a}$$

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 3/2*a^3*c^4*x^4 + 16/3*a^2*c^4*x^3 - 13*a*c^4*x^2 + 31*c^
4*x - 32*c^4*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{32\,c^4\log\left(|ax+1|\right)}{a} + \frac{6\,a^9c^4x^5 - 45\,a^8c^4x^4 + 160\,a^7c^4x^3 - 390\,a^6c^4x^2 + 930\,a^5c^4x}{30\,a^5}$$

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*c^4*log(abs(a*x + 1))/a + 1/30*(6*a^9*c^4*x^5 - 45*a^8*c^4*x^4 + 160*a^
7*c^4*x^3 - 390*a^6*c^4*x^2 + 930*a^5*c^4*x)/a^5
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = 31\,c^4\,x - 13\,a\,c^4\,x^2 + \frac{16\,a^2\,c^4\,x^3}{3} - \frac{3\,a^3\,c^4\,x^4}{2} + \frac{a^4\,c^4\,x^5}{5} - \frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - a*c*x)^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] 31*c^4*x - 13*a*c^4*x^2 + (16*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + (a^4*c^4
*x^5)/5 - (32*c^4*log(a*x + 1))/a
```

3.208 $\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = 8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}$$

```
[Out] 8*c^3*x-2*c^3*(-a*x+1)^2/a-2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a-16*c^3
*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}-\frac{2c^3(1-ax)^2}{a}-\frac{16c^3\log(ax+1)}{a}+8c^3x$$

```
[In] Int[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 8*c^3*x - (2*c^3*(1 - a*x)^2)/a - (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x
)^4)/(4*a) - (16*c^3*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}(c-acx)^3\,dx\\ &= -\left(c^3\int \frac{(1-ax)^4}{1+ax}\,dx\right)\\ &= -\left(c^3\int \left(-8-4(1-ax)-2(1-ax)^2-(1-ax)^3+\frac{16}{1+ax}\right)dx\right)\\ &= 8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx\\ &= -\frac{c^3(35-180ax+66a^2x^2-20a^3x^3+3a^4x^4+192\log(1+ax))}{12a}\end{aligned}$$

```
[In] Integrate[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
[Out] -1/12*(c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1
+ a*x]))/a
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

method	result
default	$c^{3}\left(-\frac{a^{3}x^{4}}{4}+\frac{5a^{2}x^{3}}{3}-\frac{11a\,x^{2}}{2}+15x-\frac{16\ln(ax+1)}{a}\right)$
norman	$15c^{3}x-\frac{11a\,c^{3}x^{2}}{2}+\frac{5a^{2}c^{3}x^{3}}{3}-\frac{a^{3}c^{3}x^{4}}{4}-\frac{16c^{3}\ln(ax+1)}{a}$
risch	$15c^{3}x-\frac{11a\,c^{3}x^{2}}{2}+\frac{5a^{2}c^{3}x^{3}}{3}-\frac{a^{3}c^{3}x^{4}}{4}-\frac{16c^{3}\ln(ax+1)}{a}$
parallelrisch	$-\frac{3a^{4}c^{3}x^{4}-20a^{3}c^{3}x^{3}+66a^{2}c^{3}x^{2}-180a\,c^{3}x+192c^{3}\ln(ax+1)}{12a}$
meijerg	$-\frac{c^{3}\left(-\frac{ax\left(-15a^{3}x^{3}+20a^{2}x^{2}-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}+\frac{4c^{3}\left(\frac{ax\left(4a^{2}x^{2}-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}-\frac{6c^{3}\left(-\frac{ax(-3ax+6)}{6}+\ln(ax\right.}{a}$

```
[In] int((-a*c*x+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*a^3*x^4+5/3*a^2*x^3-11/2*a*x^2+15*x-16*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3}\,dx = -\frac{3\,a^{4}c^{3}x^{4}-20\,a^{3}c^{3}x^{3}+66\,a^{2}c^{3}x^{2}-180\,ac^{3}x+192\,c^{3}\log\left(ax+1\right)}{12\,a}$$

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192*
c^3*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3}\,dx = -\frac{a^{3}c^{3}x^{4}}{4}+\frac{5a^{2}c^{3}x^{3}}{3}-\frac{11ac^{3}x^{2}}{2}+15c^{3}x-\frac{16c^{3}\log\left(ax+1\right)}{a}$$

```
[In] integrate((-a*c*x+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**3*c**3*x**4/4 + 5*a**2*c**3*x**3/3 - 11*a*c**3*x**2/2 + 15*c**3*x - 16*
c**3*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4 + \frac{5}{3}\,a^2c^3x^3 - \frac{11}{2}\,ac^3x^2 + 15\,c^3x - \frac{16\,c^3\log\left(ax+1\right)}{a}$$

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 + 5/3*a^2*c^3*x^3 - 11/2*a*c^3*x^2 + 15*c^3*x - 16*c^3*log
(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{16\,c^3\log\left(|ax+1|\right)}{a} - \frac{3\,a^7c^3x^4 - 20\,a^6c^3x^3 + 66\,a^5c^3x^2 - 180\,a^4c^3x}{12\,a^4}$$

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*c^3*log(abs(a*x + 1))/a - 1/12*(3*a^7*c^3*x^4 - 20*a^6*c^3*x^3 + 66*a^5
*c^3*x^2 - 180*a^4*c^3*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = 15\,c^3\,x - \frac{11\,a\,c^3\,x^2}{2} + \frac{5\,a^2\,c^3\,x^3}{3} - \frac{a^3\,c^3\,x^4}{4} - \frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - a*c*x)^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] 15*c^3*x - (11*a*c^3*x^2)/2 + (5*a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4 - (16*c^3
*log(a*x + 1))/a
```

3.209 $\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}$$

```
[Out] 4*c^2*x-c^2*(-a*x+1)^2/a-1/3*c^2*(-a*x+1)^3/a-8*c^2*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{c^2(1-ax)^3}{3a}-\frac{c^2(1-ax)^2}{a}-\frac{8c^2\log(ax+1)}{a}+4c^2x$$

```
[In] Int[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 4*c^2*x - (c^2*(1 - a*x)^2)/a - (c^2*(1 - a*x)^3)/(3*a) - (8*c^2*Log[1 + a*
x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}(c-acx)^2\,dx\\ &= -\left(c^2\int\frac{(1-ax)^3}{1+ax}\,dx\right)\\ &= -\left(c^2\int\left(-4-2(1-ax)-(1-ax)^2+\frac{8}{1+ax}\right)dx\right)\\ &= 4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2(-4+21ax-6a^2x^2+a^3x^3-24\log(1+ax))}{3a}$$

```
[In] Integrate[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
[Out] (c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

method	result	size
default	$c^{2} \left(\frac{a^{2} x^{3}}{3}-2 a \,x^{2}+7 x -\frac{8 \ln \left(a x +1\right)}{a}\right)$	34
norman	$7 c^{2} x -2 a \,c^{2} x^{2}+\frac{a^{2} c^{2} x^{3}}{3}-\frac{8 c^{2} \ln \left(a x +1\right)}{a}$	42
risch	$7 c^{2} x -2 a \,c^{2} x^{2}+\frac{a^{2} c^{2} x^{3}}{3}-\frac{8 c^{2} \ln \left(a x +1\right)}{a}$	42
parallelrisch	$-\frac{-a^{3} c^{2} x^{3}+6 a^{2} c^{2} x^{2}-21 a \,c^{2} x +24 c^{2} \ln \left(a x +1\right)}{3 a}$	47
meijerg	$\frac{c^{2} \left(\frac{a x \left(4 a^{2} x^{2}-6 a x +12\right)}{12}-\ln \left(a x +1\right)\right)}{a}-\frac{3 c^{2} \left(-\frac{a x \left(-3 a x +6\right)}{6}+\ln \left(a x +1\right)\right)}{a}+\frac{3 c^{2} \left(a x -\ln \left(a x +1\right)\right)}{a}-\frac{c^{2} \ln \left(a x +1\right)}{a}$	95

```
[In] int((-a*c*x+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^2*(1/3*a^2*x^3-2*a*x^2+7*x-8*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^{3} c^{2} x^{3}-6 \, a^{2} c^{2} x^{2}+21 \, a c^{2} x-24 \, c^{2} \log \left(a x+1\right)}{3 \, a}$$

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^{2}c^{2}x^{3}}{3}-2ac^{2}x^{2}+7c^{2}x-\frac{8c^{2}\log{\left(ax+1\right)}}{a}$$

```
[In] integrate((-a*c*x+c)**2*(a*x-1)/(a*x+1),x)
[Out] a**2*c**2*x**3/3 - 2*a*c**2*x**2 + 7*c**2*x - 8*c**2*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{3}\,a^2c^2x^3 - 2\,ac^2x^2 + 7\,c^2x - \frac{8\,c^2\log\left(ax+1\right)}{a}$$

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 - 2*a*c^2*x^2 + 7*c^2*x - 8*c^2*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{8\,c^2\log\left(|ax+1|\right)}{a} + \frac{a^5c^2x^3 - 6\,a^4c^2x^2 + 21\,a^3c^2x}{3\,a^3}$$

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -8*c^2*log(abs(a*x + 1))/a + 1/3*(a^5*c^2*x^3 - 6*a^4*c^2*x^2 + 21*a^3*c^2*
x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = 7\,c^2\,x - 2\,a\,c^2\,x^2 + \frac{a^2\,c^2\,x^3}{3} - \frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - a*c*x)^2*(a*x - 1))/(a*x + 1),x)
[Out] 7*c^2*x - 2*a*c^2*x^2 + (a^2*c^2*x^3)/3 - (8*c^2*log(a*x + 1))/a
```

3.210 $\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

```
[Out] 3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6302, 6264, 45}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}acx^2-\frac{4c\log(ax+1)}{a}+3cx$$

```
[In] Int[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}(c-acx)\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{1+ax}\,dx\right) \\
&= -\left(c\int \left(-3+ax+\frac{4}{1+ax}\right)\,dx\right) \\
&= 3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

```
[In] Integrate[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
default	$c\left(-\frac{a\,x^2}{2}+3x-\frac{4\ln(ax+1)}{a}\right)$	24
norman	$3cx-\frac{ac\,x^2}{2}-\frac{4c\ln(ax+1)}{a}$	25
risch	$3cx-\frac{ac\,x^2}{2}-\frac{4c\ln(ax+1)}{a}$	25
parallelrisch	$-\frac{a^2c\,x^2-6acx+8c\ln(ax+1)}{2a}$	29
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{2c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}$	55

```
[In] int((-a*c*x+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*a*x^2+3*x-4*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{a^2cx^2-6\,acx+8\,c\log{(ax+1)}}{2\,a}$$

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{acx^2}{2}+3cx-\frac{4c\log{(ax+1)}}{a}$$

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a*c*x**2/2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2+3\,cx-\frac{4\,c\log{(ax+1)}}{a}$$

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{4\,c\log\left(|ax+1|\right)}{a}-\frac{a^3cx^2-6\,a^2cx}{2\,a^2}$$

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -4*c*log(abs(a*x + 1))/a - 1/2*(a^3*c*x^2 - 6*a^2*c*x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,(8\,\ln\left(a\,x+1\right)-6\,a\,x+a^2\,x^2)}{2\,a}$$

```
[In] int(((c - a*c*x)*(a*x - 1))/(a*x + 1),x)
[Out] -(c*(8*log(a*x + 1) - 6*a*x + a^2*x^2))/(2*a)
```

3.211 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(1+ax)}{ac}$$

```
[Out] -ln(a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 31}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(ax+1)}{ac}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] -(Log[1 + a*x]/(a*c))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}}{c-acx}\,dx$$
$$= -\frac{\int \frac{1}{1+ax}\,dx}{c}$$
$$= -\frac{\log(1+ax)}{ac}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(1+ax)}{ac}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
[Out] -(Log[1 + a*x]/(a*c))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\ln(ax+1)}{ac}$	15
norman	$-\frac{\ln(ax+1)}{ac}$	15
risch	$-\frac{\ln(ax+1)}{ac}$	15
parallelrisch	$-\frac{\ln(ax+1)}{ac}$	15

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -ln(a*x+1)/a/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(ax+1)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] -log(a*x + 1)/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(acx+c)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x)
[Out] -log(a*c*x + c)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(ax+1)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
[Out] -log(a*x + 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(|ax+1|)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x + 1))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\ln(a\,x+1)}{a\,c}$$

```
[In] int((a*x - 1)/((c - a*c*x)*(a*x + 1)),x)
[Out] -log(a*x + 1)/(a*c)
```

3.212 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{ac^2}$$

```
[Out] -arctanh(a*x)/a/c^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6264, 35, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{ac^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] -(ArcTanh[a*x]/(a*c^2))
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^2}\,dx\\ &= -\frac{\int \frac{1}{(1-ax)(1+ax)}\,dx}{c^2}\\ &= -\frac{\int \frac{1}{1-a^2x^2}\,dx}{c^2}\\ &= -\frac{\text{arctanh}(ax)}{ac^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\text{arctanh}(ax)}{ac^2}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
[Out] -(ArcTanh[a*x]/(a*c^2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

method	result	size
parallelrisch	$\frac{-\ln(ax+1)+\ln(ax-1)}{2a\,c^2}$	24
default	$\frac{-\frac{\ln(ax+1)}{2a}+\frac{\ln(ax-1)}{2a}}{c^2}$	28
norman	$\frac{\ln(ax-1)}{2a\,c^2}-\frac{\ln(ax+1)}{2a\,c^2}$	30
risch	$-\frac{\ln(ax+1)}{2a\,c^2}+\frac{\ln(-ax+1)}{2a\,c^2}$	31

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(-ln(a*x+1)+ln(a*x-1))/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{\log{(ax+1)}-\log{(ax-1)}}{2\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\frac{\log{\left(x-\frac{1}{a}\right)}}{2}-\frac{\log{\left(x+\frac{1}{a}\right)}}{2}}{ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**2,x)
[Out] (log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log(ax+1)}{2\,ac^2} + \frac{\log(ax-1)}{2\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{2\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\mathrm{atanh}(a\,x)}{a\,c^2}$$

```
[In] int((a*x - 1)/((c - a*c*x)^2*(a*x + 1)),x)
[Out] -atanh(a*x)/(a*c^2)
```

3.213 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3} dx = -\frac{1}{2ac^3(1-ax)} - \frac{\operatorname{arctanh}(ax)}{2ac^3}$$

```
[Out] -1/2/a/c^3/(-a*x+1)-1/2*arctanh(a*x)/a/c^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6264, 46, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3} dx = -\frac{\operatorname{arctanh}(ax)}{2ac^3} - \frac{1}{2ac^3(1-ax)}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] -1/2*1/(a*c^3*(1 - a*x)) - ArcTanh[a*x]/(2*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2(-1+a^2x^2)}\right)dx}{c^3}\\
&= -\frac{1}{2ac^3(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{2c^3}\\
&= -\frac{1}{2ac^3(1-ax)}-\frac{\text{arctanh}(ax)}{2ac^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{1}{2a(1-ax)}+\frac{\text{arctanh}(ax)}{2a}}{c^3}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
[Out] -((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{-\frac{\ln(ax+1)}{4a}+\frac{1}{2a(ax-1)}+\frac{\ln(ax-1)}{4a}}{c^3}$	40
risch	$\frac{1}{2a(ax-1)c^3}+\frac{\ln(-ax+1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	46
parallelrisch	$\frac{a\ln(ax-1)x-a\ln(ax+1)x+2ax-\ln(ax-1)+\ln(ax+1)}{4c^3(ax-1)a}$	54
norman	$\frac{-\frac{x}{2c}+\frac{a\,x^2}{2c}}{c^2(ax-1)^2}+\frac{\ln(ax-1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	57

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(-1/4*ln(a*x+1)/a+1/2/a/(a*x-1)+1/4/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{(ax-1)\log(ax+1)-(ax-1)\log(ax-1)-2}{4\,(a^2c^3x-ac^3)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*c
^3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{1}{2a^2c^3x-2ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**3,x)
[Out] 1/(2*a**2*c**3*x - 2*a*c**3) - (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2\,(a^2c^3x-ac^3)} - \frac{\log(ax+1)}{4\,ac^3} + \frac{\log(ax-1)}{4\,ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] 1/2/(a^2*c^3*x - a*c^3) - 1/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c^
3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\log(|ax+1|)}{4\,ac^3} + \frac{\log(|ax-1|)}{4\,ac^3} + \frac{1}{2\,(ax-1)ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) + 1/2/((a*x
- 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)} - \frac{\mathrm{atanh}(a\,x)}{2\,a\,c^3}$$

```
[In] int((a*x - 1)/((c - a*c*x)^3*(a*x + 1)),x)
[Out] - 1/(2*a*(c^3 - a*c^3*x)) - atanh(a*x)/(2*a*c^3)
```

3.214 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^4}$$

```
[Out] -1/4/a/c^4/(-a*x+1)^2-1/4/a/c^4/(-a*x+1)-1/4*arctanh(a*x)/a/c^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6264, 46, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{\operatorname{arctanh}(ax)}{4ac^4}-\frac{1}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] -1/4*1/(a*c^4*(1 - a*x)^2) - 1/(4*a*c^4*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}}{(c-acx)^4}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^4} \\
&= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^4} \\
&= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^4} \\
&= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2+ax-(-1+ax)^2\operatorname{arctanh}(ax)}{4ac^4(-1+ax)^2}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{(ax-1)^2 c^4}+\frac{\ln(-ax+1)}{8a\,c^4}-\frac{\ln(ax+1)}{8a\,c^4}$	51
default	$\frac{-\frac{\ln(ax+1)}{8a}-\frac{1}{4(ax-1)^2 a}+\frac{1}{4a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^4}$	52
norman	$\frac{\frac{3x}{4c}-\frac{5a\,x^2}{4c}+\frac{a^2x^3}{2c}}{c^3(ax-1)^3}+\frac{\ln(ax-1)}{8a\,c^4}-\frac{\ln(ax+1)}{8a\,c^4}$	68
parallelrisch	$\frac{a^2\ln(ax-1)x^2-a^2\ln(ax+1)x^2+4a^2x^2-2a\ln(ax-1)x+2a\ln(ax+1)x-6ax+\ln(ax-1)-\ln(ax+1)}{8c^4(ax-1)^2 a}$	90

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*x-1/2/a)/(a*x-1)^2/c^4+1/8*ln(-a*x+1)/a/c^4-1/8*ln(a*x+1)/a/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+(a^2x^2-2\,ax+1)\log(ax-1)-4}{8\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{ax-2}{4a^3c^4x^2-8a^2c^4x+4ac^4}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{ax-2}{4\,(a^3c^4x^2-2\,a^2c^4x+ac^4)} - \frac{\log(ax+1)}{8\,ac^4} + \frac{\log(ax-1)}{8\,ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - 1/8*log(a*x + 1)/(a*c^4
) + 1/8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\log(|ax+1|)}{8\,ac^4} + \frac{\log(|ax-1|)}{8\,ac^4} + \frac{ax-2}{4\,(ax-1)^2ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
[Out] -1/8*log(abs(a*x + 1))/(a*c^4) + 1/8*log(abs(a*x - 1))/(a*c^4) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4} - \frac{\operatorname{atanh}(a\,x)}{4\,a\,c^4}$$

```
[In] int((a*x - 1)/((c - a*c*x)^4*(a*x + 1)),x)
[Out] (x/4 - 1/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) - atanh(a*x)/(4*a*c^4)
```

3.215 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}-\frac{\operatorname{arctanh}(ax)}{8ac^5}$$

```
[Out] -1/6/a/c^5/(-a*x+1)^3-1/8/a/c^5/(-a*x+1)^2-1/8/a/c^5/(-a*x+1)-1/8*arctanh(a
*x)/a/c^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6264, 46, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\operatorname{arctanh}(ax)}{8ac^5}-\frac{1}{8ac^5(1-ax)}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{6ac^5(1-ax)^3}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] -1/6*1/(a*c^5*(1 - a*x)^3) - 1/(8*a*c^5*(1 - a*x)^2) - 1/(8*a*c^5*(1 - a*x)
) - ArcTanh[a*x]/(8*a*c^5)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}}{(c-acx)^5}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^5} \\
&= -\frac{\int \left(\frac{1}{2(-1+ax)^4} - \frac{1}{4(-1+ax)^3} + \frac{1}{8(-1+ax)^2} - \frac{1}{8(-1+a^2x^2)}\right)dx}{c^5} \\
&= -\frac{1}{6ac^5(1-ax)^3} - \frac{1}{8ac^5(1-ax)^2} - \frac{1}{8ac^5(1-ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8c^5} \\
&= -\frac{1}{6ac^5(1-ax)^3} - \frac{1}{8ac^5(1-ax)^2} - \frac{1}{8ac^5(1-ax)} - \frac{\operatorname{arctanh}(ax)}{8ac^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{10-9ax+3a^2x^2-3(-1+ax)^3\operatorname{arctanh}(ax)}{24ac^5(-1+ax)^3}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
[Out] (10 - 9*a*x + 3*a^2*x^2 - 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x)
^3)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\frac{a\,x^2}{8}-\frac{3x}{8}+\frac{5}{12a}}{(ax-1)^3 c^5}-\frac{\ln(ax+1)}{16c^5 a}+\frac{\ln(-ax+1)}{16c^5 a}$
default	$\frac{-\frac{\ln(ax+1)}{16a}+\frac{1}{6a(ax-1)^3}-\frac{1}{8(ax-1)^2 a}+\frac{1}{8a(ax-1)}+\frac{\ln(ax-1)}{16a}}{c^5}$
norman	$\frac{-\frac{7x}{8c}+\frac{2a\,x^2}{c}-\frac{37a^2x^3}{24c}+\frac{5a^3x^4}{12c}}{c^4(ax-1)^4}+\frac{\ln(ax-1)}{16a\,c^5}-\frac{\ln(ax+1)}{16c^5 a}$
parallelrisch	$\frac{3a^3\ln(ax-1)x^3-3a^3\ln(ax+1)x^3+20a^3x^3-9a^2\ln(ax-1)x^2+9a^2\ln(ax+1)x^2-54a^2x^2+9a\ln(ax-1)x-9a\ln(ax+1)x+42ax-}{48c^5(ax-1)^3 a}$

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*a*x^2-3/8*x+5/12/a)/(a*x-1)^3/c^5-1/16/c^5/a*ln(a*x+1)+1/16/c^5/a*ln(-
a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$=\frac{6\,a^2x^2-18\,ax-3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log(ax+1)+3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log(ax-1)+2}{48\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1)
 + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 - 3*
a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx=-\frac{-3a^2x^2+9ax-10}{24a^4c^5x^3-72a^3c^5x^2+72a^2c^5x-24ac^5}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^5}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**5,x)
```

```
[Out] -(-3*a**2*x**2 + 9*a*x - 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a*
*2*c**5*x - 24*a*c**5) - (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{3\,a^2x^2 - 9\,ax + 10}{24\,(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5)} - \frac{\log(ax+1)}{16\,ac^5} + \frac{\log(ax-1)}{16\,ac^5}$$

[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="maxima")

[Out] 1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5) - 1/16*log(a*x + 1)/(a*c^5) + 1/16*log(a*x - 1)/(a*c^5)

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\log\left(\left|-\frac{2\,c}{acx-c} - 1\right|\right)}{16\,ac^5} + \frac{\frac{3\,a^2c^2}{acx-c} - \frac{3\,a^2c^3}{(acx-c)^2} + \frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="giac")

[Out] -1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) + 1/24*(3*a^2*c^2/(a*c*x - c) - 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)

Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{a\,x^2}{8} - \frac{3\,x}{8} + \frac{5}{12\,a}}{-a^3\,c^5\,x^3 + 3\,a^2\,c^5\,x^2 - 3\,a\,c^5\,x + c^5} - \frac{\mathrm{atanh}(a\,x)}{8\,a\,c^5}$$

[In] int((a*x - 1)/((c - a*c*x)^5*(a*x + 1)),x)

[Out] - ((a*x^2)/8 - (3*x)/8 + 5/(12*a))/(c^5 + 3*a^2*c^5*x^2 - a^3*c^5*x^3 - 3*a*c^5*x) - atanh(a*x)/(8*a*c^5)

3.216 $\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

```
[Out] ((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
-3/2-p],[-p],2/(a+1/x)/x)/(p+1)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6311, 6316, 134}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,\text{Hypergeometric2F1}\left(-p-\frac{3}{2},-p-1,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^p
*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sqrt
[1 + 1/(a*x)])
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-p} x^{-p}(c-acx)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p (c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2} x(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(1+ax)(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{1+ax}\right)}{a(1+p)\sqrt{1+\frac{1}{ax}}}$$

```
[In] Integrate[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(1 + a*x)*(c - a*c*x)^
p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*Sqrt[1 +
 1/(a*x)])
```

Maple [F]

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] integral((a*x - 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \int (c-a\,c\,x)^p\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.217 $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 152

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+30c^3\sqrt{1-\frac{1}{a^2x^2}}x-\frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4-\frac{315c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] -315/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/a^2/(1-1/a^2/x^2)^
(1/2)+30*c^3*x*(1-1/a^2/x^2)^(1/2)-67/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+2*a^2
*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6310, 6313, 1819, 1821, 821, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{315c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}-\frac{67}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 30*c^3*Sqrt[1 - 1/(a^2*
x^2)]*x - (67*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + 2*a^2*c^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - (315*c^3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)\\
&= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^6}{x^5\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \left(a^3c^3\right)\text{Subst}\left(\int \frac{-1+\frac{6x}{a}-\frac{16x^2}{a^2}+\frac{26x^3}{a^3}-\frac{31x^4}{a^4}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{24}{a}+\frac{67x}{a^2}-\frac{104x^2}{a^3}+\frac{124x^3}{a^4}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4\\
&\quad - \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{201}{a^2}+\frac{360x}{a^3}-\frac{372x^2}{a^4}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{24}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{720}{a^3}+\frac{945x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{(315c^3)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{(315c^3)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{8}\left(315ac^3\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{315c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{1}{8}c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}x(496+173ax-51a^2x^2+14a^3x^3-2a^4x^4)}{1+ax} - \frac{315\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

```
[In] Integrate[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*x*(496 + 173*a*x - 51*a^2*x^2 + 14*a^3*x^3 - 2
*a^4*x^4))/(1 + a*x) - (315*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/8
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(2a^3x^3-16a^2x^2+67ax-240)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{8a}-\frac{\left(\frac{315\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{8\sqrt{a^2}}-\frac{32\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(a}}{ax-1}$
default	$-\frac{\left(2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+69\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-16\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+\right.}{}$

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(2*a^3*x^3-16*a^2*x^2+67*a*x-240)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/2
)-(315/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-32/a^2/(x+1/a)
*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*c^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a
*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx =$$

$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4-14\,a^3c^3x^3+51\,a^2c^3x^2-173\,ac^3x-49\right.}{8\,a}$$

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 173
*a*c^3*x - 496*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\left(-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int \frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\left(-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int \frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
-4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{8}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{256\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2} - \frac{2\left(325\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} - 765\,c^3\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{4\,(ax-1)a^2}{ax+1} - \frac{6\,(ax-1)^2a^2}{(ax+1)^2}}\right.$$

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 256*c^3*sqrt((a*x - 1)/(a*x + 1))/a^2 - 2*(325
*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 765*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 64
3*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 187*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(
a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/
(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx=\int -(acx-c)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] undef

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.31

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{\frac{187\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{643\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}+\frac{765\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4}-\frac{325\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a-\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{6\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{315\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

[In] int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2),x)

[Out] ((187*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (643*c^3*((a*x - 1)/(a*x + 1))^(3/2))/4 + (765*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 - (325*c^3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) + (32*c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (315*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)

3.218 $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{35c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -35/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2/(1-1/a^2/x^2)^(
1/2)+35/3*c^2*x*(1-1/a^2/x^2)^(1/2)-5/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a
^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6310, 6313, 1819, 1821, 821, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{35c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{5}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{35}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[In] Int[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (35*c^2*Sqrt[1 - 1/(a^2
*x^2)]*x)/3 - (5*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*Sqrt[1 - 1/(
a^2*x^2)]*x^3)/3 - (35*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = (a^2c^2)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left((a^2c^2)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^5}{x^4\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + (a^2c^2)\,\text{Subst}\left(\int \frac{-1+\frac{5x}{a}-\frac{11x^2}{a^2}+\frac{15x^3}{a^3}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{3}(a^2c^2)\,\text{Subst}\left(\int \frac{-\frac{15}{a}+\frac{35x}{a^2}-\frac{45x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{6}(a^2c^2)\,\text{Subst}\left(\int \frac{-\frac{70}{a^2}+\frac{105x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{(35c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2$$

$$+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3+\frac{(35c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2$$

$$+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{2}(35ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2$$

$$+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{35c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{6}c^2\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}x(166+55ax-13a^2x^2+2a^3x^3)}{1+ax}-\frac{105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

```
[In] Integrate[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
[Out] (c^2*((Sqrt[1 - 1/(a^2*x^2)]*x*(166 + 55*a*x - 13*a^2*x^2 + 2*a^3*x^3))/(1
+ a*x) - (105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/6
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(2a^2x^2-15ax+70)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{\left(-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\left(15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-1)(a$

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*a^2*x^2-15*a*x+70)*(a*x+1)/a*c^2*((a*x-1)/(a*x+1))^(1/2)+(-35/2*ln(a
^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+16/a^2/(x+1/a)*(a^2*(x+1/a)
^2-2*a*(x+1/a))^(1/2))*c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.58

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{96\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2} + \frac{2\left(87\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 136\,c^2\left(\frac{ax}{ax}\right.\right.}{\frac{3\,(ax-1)a^2}{ax+1} - \frac{3\,(ax-1)^2a^2}{(ax+1)^2}}\right.$$

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 96*c^2*sqrt((a*x - 1)/(a*x + 1))/a^2 + 2*(87
*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 136*c^2*((a*x - 1)/(a*x + 1))^(3/2) + 57
*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*
a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2))
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.26

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{19\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{136\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{35\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (19*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (136*c^2*((a*x - 1)/(a*x + 1))^(3/2))
/3 + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (
3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (16*c^2*((a*x
 - 1)/(a*x + 1))^(1/2))/a - (35*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.219 $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{15c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -15/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+
4*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6313, 1819, 1821, 821, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{15c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+4cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 4*c*Sqrt[1 - 1/(a^2*x^2)]*
x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (15*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left((ac)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$

$$= (ac)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - (ac)\text{Subst}\left(\int \frac{-1+\frac{4x}{a}-\frac{7x^2}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}(ac)\text{Subst}\left(\int \frac{-\frac{8}{a}+\frac{15x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(15c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(15c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2$$
$$- \frac{1}{2}(15ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{15c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{1}{2}c\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}x(24+7ax-a^2x^2)}{1+ax}-\frac{15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

```
[In] Integrate[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*((Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 7*a*x - a^2*x^2))/(1 + a*x) - (15*Log[a*
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{(ax-8)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{\left(\frac{15\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}-\frac{8\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-16\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+16\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-}{\sqrt{a^2}}\right.\right.}{}$

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-8)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-(15/2*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-8/a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(
1/2))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-7\,acx-24\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] -1/2*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (a^2*c*x^2 - 7*a*c*x - 24*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int \left(-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int \frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] -c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.70

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}\,a\left(\frac{2\left(9\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-7\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{16\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(9*c*((a*x - 1)/(a*x + 1))^(3/2) - 7*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 15*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/a^2 + 16*c*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\int -(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{7\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-9\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{15\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}$$

```
[In] int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (7*c*((a*x - 1)/(a*x + 1))^(1/2) - 9*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2
*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (15*c*atanh(((a*x
- 1)/(a*x + 1))^(1/2)))/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

3.220 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 1819, 272, 65, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax} - \frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
[Out] ((2*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x]/a)/c
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(49) = 98.

Time = 0.41 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.68

method	result
default	$-\frac{\left(\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-\right.}{a\sqrt{a^2}\,c(ax-1)\sqrt{(ax-1)(ax+1)}}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -(ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))*a^2*x+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-2*(a^2)
```

```
^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+
1))^(3/2)/(a^2)^(1/2)/c/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sqr
t((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Integra
l(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2*sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{acx-c}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2
)))/(a*c)
```

3.221 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6313, 651}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
[Out] (a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\int\frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2x^2}\,dx}{a^2c^2}\\&=-\frac{\text{Subst}\left(\int\frac{1-\frac{x}{a}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{a^2c^2}\\&=\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int\frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(1+ax)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(1 + a*x))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
trager	$\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^{2}}$	25
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^{2}}$	35
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^{2}}$	35

[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/a/c^2*(-(-a*x+1)/(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")

[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)

[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x + 1), x))/c**2

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^2,x)
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(a*c^2)
```

3.222 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/a/c^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6313, 267}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] 1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$
$$= \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^3\left(-1+a^2x^2\right)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(c^3*(-1 + a^2*x^2))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

method	result	size
trager	$\frac{x\sqrt{-\frac{-ax+1}{ax+1}}}{c^3(ax-1)}$	30
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x}{(ax-1)^2c^3}$	33
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x}{(ax-1)^2c^3}$	33

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*x/(a*x-1)*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{x\sqrt{\frac{ax-1}{ax+1}}}{ac^3x-c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] x*sqrt((a*x - 1)/(a*x + 1))/(a*c^3*x - c^3)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*x**3 + 2*
a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 -
2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2}\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] 1/2*a*(sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + 1/(a^2*c^3*sqrt((a*x - 1)/(a*x
 + 1))))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{x\mathrm{sgn}(ax+1)}{\sqrt{a^2x^2-1}c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] x*sgn(a*x + 1)/(sqrt(a^2*x^2 - 1)*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\frac{a\,x-1}{a\,x+1}+1}{2\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^3,x)
[Out] ((a*x - 1)/(a*x + 1) + 1)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(1/2))
```

3.223 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}$$

```
[Out] 2/3/a/c^4/(1-1/a^2/x^2)^(1/2)-1/3/a^2/c^4/(a-1/x)/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6310, 6313, 869, 12, 267}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3a^2c^4x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] 2/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) - 1/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a -
 x^(-1))*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}\\
&= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2} + \frac{\text{Subst}\left(\int \frac{2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}\\
&= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2} + \frac{2\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}
\end{aligned}$$

$$= \frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(-1-2ax+2a^2x^2)}{3c^4(-1+ax)^2(1+ax)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^4*(-1 + a*x)^2*(1 +
 a*x))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^3x^3-3ax-1)}{3(ax-1)^3c^4a}$	45
trager	$\frac{(2a^2x^2-2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^4(ax-1)^2}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2-2ax-1)(ax+1)}{3(ax-1)^3c^4a}$	50

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
[Out] 1/3*((a*x-1)/(a*x+1))^(3/2)*(2*a^3*x^3-3*a*x-1)/(a*x-1)^3/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{(2\,a^2x^2-2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^4*x^2 - 2*a^2*
c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a
**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x
+ 1), x))/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{1}{12}\,a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4} + \frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^4-3\,a^3\,c^4\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^4,x)
[Out] (2*a*x - 2*a^2*x^2 + 1)/((3*a*c^4 - 3*a^3*c^4*x^2)*((a*x - 1)/(a*x + 1))^(1
/2))
```

3.224 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -4/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(3/2)+1/5*(a+1/x)^2/a^3/c^5/(1-1/a^2/x^2
)^(5/2)+1/5*(5*a+2/x)/a^2/c^5/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6310, 6313, 866, 1649, 651}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) + (a + x^(-1))^2/(5*a
^3*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (5*a + 2/x)/(5*a^2*c^5*Sqrt[1 - 1/(a^2*x^
2)])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}\\
&= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}\\
&= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}
\end{aligned}
$$

$$= \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)(2a^3+5a^2x+5ax^2)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5a^5c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{6a^3+15a^2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15a^5c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax-4a^2x^2+2a^3x^3)}{5c^5(-1+ax)^3(1+ax)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3))/(5*c^5*(-1 + a*
x)^3*(1 + a*x))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result	size
trager	$\frac{(2a^3x^3-4a^2x^2+ax+2)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^5(ax-1)^3}$	54
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^3x^3-4a^2x^2+ax+2)(ax+1)}{5(ax-1)^4c^5a}$	57
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^4x^4-2a^3x^3-3a^2x^2+3ax+2)}{5(ax-1)^4c^5a}$	61

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
[Out] 1/5/a/c^5*(2*a^3*x^3-4*a^2*x^2+a*x+2)/(a*x-1)^3*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{(2\,a^3x^3 - 4\,a^2x^2 + ax + 2)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^5*x^
3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*
a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x
 - 1), x))/c**5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{1}{40}\,a\left(\frac{5\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5} - \frac{\frac{5\,(ax-1)}{ax+1} - \frac{15\,(ax-1)^2}{(ax+1)^2} - 1}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
[Out] 1/40*a*(5*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^5) - (5*(a*x - 1)/(a*x + 1) - 15
*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(5/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^5}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2}{5\,a\,c^5\,(a\,x+1)^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^5,x)
```

```
[Out] (a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2)/(5*a*c^5*(a*x + 1)^3*((a*x - 1)/(a*x + 1)
)^(5/2))
```

3.225 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -46/35*(a+1/x)/a^2/c^6/(1-1/a^2/x^2)^(3/2)+24/35*(a+1/x)^2/a^3/c^6/(1-1/a^2
/x^2)^(5/2)-1/7*(a+1/x)^3/a^4/c^6/(1-1/a^2/x^2)^(7/2)+1/35*(35*a+13/x)/a^2/
c^6/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6310, 6313, 866, 1649, 651}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (-46*(a + x^(-1)))/(35*a^2*c^6*(1 - 1/(a^2*x^2))^(3/2)) + (24*(a + x^(-1))^
2)/(35*a^3*c^6*(1 - 1/(a^2*x^2))^(5/2)) - (a + x^(-1))^3/(7*a^4*c^6*(1 - 1/
(a^2*x^2))^(7/2)) + (35*a + 13/x)/(35*a^2*c^6*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^6 x^6}\,dx}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-\frac{x}{a}\right)^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2\left(3a^4+7a^3x+7a^2x^2+7ax^3\right)}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7a^6c^6}$$

$$= \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\left(33a^4+70a^3x+35a^2x^2\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{39a^4+105a^3x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{105a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4\right)}{35c^6(-1+ax)^4(1+ax)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^6*(-1 + a*x)^4*(1 + a*x))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

method	result	size
trager	$\frac{(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^6(ax-1)^4}$	63
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13)(ax+1)}{35(ax-1)^5c^6a}$	66
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-13)}{35(ax-1)^5c^6a}$	69

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x,method=_RETURNVERBOSE)
[Out] 1/35/a/c^6*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^4*(-(-a*x+1)/
(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^6x^4-4\,a^4c^6x^3+6\,a^3c^6x^2-4\,a^2c^6x+ac^6)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas")
[Out] 1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^6*x^4 - 4*a^4*c^6*x^3 + 6*a^3*c^6*x^2 - 4*a^2*c^6*x + a*c^6)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\,dx}{c^6}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**6,x)
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a
**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x) + Integ
ral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a**5
*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x))/c**6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^6} + \frac{\frac{28\,(ax-1)}{ax+1} - \frac{70\,(ax-1)^2}{(ax+1)^2} + \frac{140\,(ax-1)^3}{(ax+1)^3} - 5}{a^2c^6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima")
[Out] 1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^6) + (28*(a*x - 1)/(a*x + 1) -
 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^6*((a
*x - 1)/(a*x + 1))^(7/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^6}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^6\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^6,x)
[Out] (4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^6*(a*x + 1)^4*((
a*x - 1)/(a*x + 1))^(7/2))
```

3.226 $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 254

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3} - \frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] -32/99*(a-1/x)^3*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)+9088/
3465*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)/x^3-768/385*(1+1/
a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+128/231*(1+1/a/x)^(3/2)
*(-a*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^4*(1+1/a/x)^(3/2)*x*(-
a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6311, 6316, 96, 91, 79, 37\}$

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{9088\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{3465a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{32\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2x\left(a-\frac{1}{x}\right)^4\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{768\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{385a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{128\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{231a^2x\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (-32*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1/(
a*x))^(9/2)) + (9088*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(3465*a^4*(1 -
1/(a*x))^(9/2)*x^3) - (768*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(385*a^3*
(1 - 1/(a*x))^(9/2)*x^2) + (128*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(231
*a^2*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2)*x*(c -
a*c*x)^(9/2))/(11*a^4*(1 - 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^{13/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(16\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(64\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{33a^2\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(128\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\mathrm{Subst}\left(\int\frac{\left(-\frac{9}{a}+\frac{7x}{2a^2}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(4544\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\mathrm{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}dx,x,\frac{1}{x}\right)}{1155a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3} - \frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2c^4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(5419-977ax-1866a^2x^2+2710a^3x^3-1505a^4x^4+315a^5x^5)}{3465a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(5419 - 977*a*x - 1866*a^2*x^2 + 2
710*a^3*x^3 - 1505*a^4*x^4 + 315*a^5*x^5))/(3465*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\,c^{4}(ax+1)(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419)}{3465\sqrt{\frac{ax-1}{ax+1}}\,a}$	69
gosper	$\frac{2(ax+1)(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419)(-acx+c)^{\frac{9}{2}}}{3465a(ax-1)^{4}\sqrt{\frac{ax-1}{ax+1}}}$	72
risch	$-\frac{2c^{5}(ax-1)(315a^{5}x^{5}-1505a^{4}x^{4}+2710a^{3}x^{3}-1866a^{2}x^{2}-977ax+5419)}{3465\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	77

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^4*(a*x+1)*(315*a^4*x^4-
1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,(315\,a^{6}c^{4}x^{6}-1190\,a^{5}c^{4}x^{5}+1205\,a^{4}c^{4}x^{4}+844\,a^{3}c^{4}x^{3}-2843\,a^{2}c^{4}x^{2}+4442\,ac^{4}x+5419\,c^{4}}{3465\,(a^{2}x-a)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*a^6*c^4*x^6 - 1190*a^5*c^4*x^5 + 1205*a^4*c^4*x^4 + 844*a^3*c^4
*x^3 - 2843*a^2*c^4*x^2 + 4442*a*c^4*x + 5419*c^4)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(315\,a^5\sqrt{-c}c^4x^5 - 1505\,a^4\sqrt{-c}c^4x^4 + 2710\,a^3\sqrt{-c}c^4x^3 - 1866\,a^2\sqrt{-c}c^4x^2 - 977\,a\sqrt{-c}c^4x +\right.}{3465\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
[Out] 2/3465*(315*a^5*sqrt(-c)*c^4*x^5 - 1505*a^4*sqrt(-c)*c^4*x^4 + 2710*a^3*sqr
t(-c)*c^4*x^3 - 1866*a^2*sqrt(-c)*c^4*x^2 - 977*a*sqrt(-c)*c^4*x + 5419*sqr
t(-c)*c^4)*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(4096\,\sqrt{2}\sqrt{-c}c^3 - \frac{315\,(acx+c)^5\sqrt{-acx-c}-3080\,(acx+c)^4\sqrt{-acx-c}c+11880\,(acx+c)^3\sqrt{-acx-c}c^2-22176\,(acx+c)^2\sqrt{}}{c^2}\right.}{3465\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
[Out] 2/3465*(4096*sqrt(2)*sqrt(-c)*c^3 - (315*(a*c*x + c)^5*sqrt(-a*c*x - c) - 3
080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*c*x - c)
*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c)^(3/2)*
c^4)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(315\,a^4\,x^4-1820\,a^3\,x^3+4530\,a^2\,x^2-6396\,a\,x+5419\right)}{3465\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*c^4*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(4530*a^2*
x^2 - 6396*a*x - 1820*a^3*x^3 + 315*a^4*x^4 + 5419))/(3465*a*(a*x - 1))
```

3.227 $\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 197

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2}+\frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -8/21*(1+1/a/x)^(3/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)-568/315*(1+1/a/x)^
(3/2)*(-a*c*x+c)^(7/2)/a^3/(1-1/a/x)^(7/2)/x^2+48/35*(1+1/a/x)^(3/2)*(-a*c*
x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(a-1/x)^3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)
^(7/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{568\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{315a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(21*a*(1 - 1/(a*x))^(7/2)) - (56
8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(315*a^3*(1 - 1/(a*x))^(7/2)*x^2)
+ (48*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(35*a^2*(1 - 1/(a*x))^(7/2)*x)
 + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(7/2))/(9*a^3*(1 - 1
/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{3a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(8\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a}+\frac{7x}{2a^2}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(284\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{105a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + \frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-319+2ax+156a^2x^2-130a^3x^3+35a^4x^4)}{315a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-319 + 2*a*x + 156*a^2*x^2 - 130
*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\,c^3(ax+1)(35a^3x^3-165a^2x^2+321ax-319)}{315\sqrt{\frac{ax-1}{ax+1}}\,a}$	61
gosper	$\frac{2(ax+1)(35a^3x^3-165a^2x^2+321ax-319)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}$	64
risch	$\frac{2c^4(ax-1)(35a^4x^4-130a^3x^3+156a^2x^2+2ax-319)}{315\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}\,a}$	69

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^3*(a*x+1)*(35*a^3*x^3-1
65*a^2*x^2+321*a*x-319)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(35\,a^5c^3x^5-95\,a^4c^3x^4+26\,a^3c^3x^3+158\,a^2c^3x^2-317\,ac^3x-319\,c^3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^2x-a)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
[Out] -2/315*(35*a^5*c^3*x^5 - 95*a^4*c^3*x^4 + 26*a^3*c^3*x^3 + 158*a^2*c^3*x^2
- 317*a*c^3*x - 319*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
- a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(35\,a^4\sqrt{-c}c^3x^4-130\,a^3\sqrt{-c}c^3x^3+156\,a^2\sqrt{-c}c^3x^2+2\,a\sqrt{-c}c^3x-319\,\sqrt{-c}c^3\right)\sqrt{ax+1}}{315\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 130*a^3*sqrt(-c)*c^3*x^3 + 156*a^2*sqrt(-
c)*c^3*x^2 + 2*a*sqrt(-c)*c^3*x - 319*sqrt(-c)*c^3)*sqrt(a*x + 1)/a
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-35\,a^4\,x^4+60\,a^3\,x^3+34\,a^2\,x^2-124\,a\,x+193\right)}{315\,a}+\frac{1024\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

[In] int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(34*a^2*x^2 - 124*a*x + 60*a^3*x^3 - 35*a^4*x^4 + 193))/(315*a) + (1024*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))

3.228 $\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{64a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{105(c-acx)^{3/2}}+\frac{16a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{35\sqrt{c-acx}}+\frac{2}{7}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\sqrt{c-acx}$$

```
[Out] 64/105*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+16/35*a^2*c^3*(1-1/
a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)+2/7*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3*(-a
*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6311, 6316, 91, 79, 37}

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{142\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{105a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{36\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (-36*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (1
42*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(105*a^2*(1 - 1/(a*x))^(5/2)*x) +
 (2*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a}+\frac{7x}{2a^2}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(71\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}dx,x,\frac{1}{x}\right)}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{142\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{105a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2c^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(71+17ax-39a^2x^2+15a^3x^3)}{105a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(71 + 17*a*x - 39*a^2*x^2 + 15*a^3
*x^3))/(105*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\,c^2(ax+1)(15a^2x^2-54ax+71)}{105\sqrt{\frac{ax-1}{ax+1}}\,a}$	53
gosper	$\frac{2(ax+1)(15a^2x^2-54ax+71)(-acx+c)^{\frac{5}{2}}}{105a(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}$	56
risch	$-\frac{2c^3(ax-1)(15a^3x^3-39a^2x^2+17ax+71)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	61

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/105/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^2*(a*x+1)*(15*a^2*x^2-54
*a*x+71)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(15\,a^4c^2x^4-24\,a^3c^2x^3-22\,a^2c^2x^2+88\,ac^2x+71\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^2x-a\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/105*(15*a^4*c^2*x^4 - 24*a^3*c^2*x^3 - 22*a^2*c^2*x^2 + 88*a*c^2*x + 71*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(15\,a^3\sqrt{-c}c^2x^3-39\,a^2\sqrt{-c}c^2x^2+17\,a\sqrt{-c}c^2x+71\,\sqrt{-c}c^2\right)\sqrt{ax+1}}{105\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] 2/105*(15*a^3*sqrt(-c)*c^2*x^3 - 39*a^2*sqrt(-c)*c^2*x^2 + 17*a*sqrt(-c)*c^
2*x + 71*sqrt(-c)*c^2)*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(64\sqrt{2}\sqrt{-c}c-\frac{15\,(acx+c)^3\sqrt{-acx-c}-84\,(acx+c)^2\sqrt{-acx-c}c-140\,(-acx-c)^{\frac{3}{2}}c^2}{c^2}\right)c^2}{105\,a|c|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(64*sqrt(2)*sqrt(-c)*c - (15*(a*c*x + c)^3*sqrt(-a*c*x - c) - 84*(a*c
*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)/c^2)*c^2/(a*abs(
c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,{(a\,x+1)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-54\,a\,x+71\right)}{105\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^
2 - 54*a*x + 71))/(105*a*(a*x - 1))
```

3.229 $\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{8a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{15(c-acx)^{3/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{5\sqrt{c-acx}}$$

```
[Out] 8/15*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+2/5*a^2*c^2*(1-1/a^2/
x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6311, 6316, 79, 37}

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-14*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (2
*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(c-acx)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\left(7\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}\\
&= -\frac{14\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{1+\frac{1}{ax}}(1+ax)(-7+3ax)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt[
1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\,c(ax+1)(3ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\,a}$	43
gosper	$\frac{2(ax+1)(3ax-7)(-acx+c)^{\frac{3}{2}}}{15a(ax-1)\sqrt{\frac{ax-1}{ax+1}}}$	48
risch	$\frac{2c^{2}(ax-1)(3a^{2}x^{2}-4ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	53

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c*(a*x+1)*(3*a*x-7)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(3\,a^{3}cx^{3}-a^{2}cx^{2}-11\,acx-7\,c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{2}x-a\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - a^2*c*x^2 - 11*a*c*x - 7*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int\frac{(-c(ax-1))^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(3\,a^2\sqrt{-c}cx^2-4\,a\sqrt{-c}cx-7\,\sqrt{-c}c\right)\sqrt{ax+1}}{15\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2/15*(3*a^2*sqrt(-c)*c*x^2 - 4*a*sqrt(-c)*c*x - 7*sqrt(-c)*c)*sqrt(a*x + 1
)/a
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,c\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-7\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] -(2*c*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 7)*((a*x - 1)/(a*x + 1))^(1/2)
)/(15*a*(a*x - 1))
```

3.230 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6309}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	35
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	42

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)

[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")

[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,c^2\left(\frac{2\sqrt{2}\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")

[Out] 2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x + 1))

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1
))
```

3.231 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] 2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)-2*arctanh(2^(1/2)*(1/x
)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2
)/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - a*c*x] - (2*Sqrt[2]*Sqrt
[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rule steps

$$\text{integral} = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int\frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{1-\frac{1}{ax}}x\left(\sqrt{a}\sqrt{1+\frac{1}{ax}} - \sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\sqrt{c-acx}}$$

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*x*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[c
 - a*c*x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\left(\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{-c(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax+1)}\,ca}$	83
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a\sqrt{c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$	115

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*2^(1/2)*arctan(1/2*(-
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/c/
a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.03

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \left[\frac{\sqrt{2}(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}, -\frac{2\left(\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\frac{\sqrt{2}(acx-c)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{\sqrt{c}}\right)}{a^2cx-ac}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] [(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a
*x + 1)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c), -2*(sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)
*(a*c*x - c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*
x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,c\left(\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{-c}\right)}{c}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-\sqrt{-acx-c}}{c}\right)}{a|c|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2*c*(sqrt(2)*(sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - sqrt(-c))/c - (sqrt(2)*sqr
t(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - sqrt(-a*c*x - c))/c)/(a
*abs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.232 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 128

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx = -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -1/2*(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a
^(1/2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^(
1/2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx = -\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}} - \frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - a*c*x)^(3/
2))) - (Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]
*Sqrt[1 + 1/(a*x)])])/(Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int\frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2\sqrt{a}c(-1+ax)\sqrt{c-acx}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(-
1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*S
qrt[a]*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\sqrt{-c(ax-1)}\left(\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx-\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c(ax+1)}\sqrt{c}\right)}{2\sqrt{\frac{ax-1}{ax+1}}(ax-1)\sqrt{-c(ax+1)}c^{\frac{5}{2}}a}$	118

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/c
^(5/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.20

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[-\frac{\sqrt{2}(a^2x^2 - 2\,ax + 1)\sqrt{-c}\log\left(-\frac{a^2cx^2 + 2\,acx - 2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}} - 3\,c}{a^2x^2 - 2\,ax + 1}\right) + 4\,\sqrt{-}}{4\,(a^3c^2x^2 - 2\,a^2c^2x + ac^2)} \right.$$

$$\left. -\frac{\sqrt{2}(a^2x^2 - 2\,ax + 1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right) + 2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3c^2x^2 - 2\,a^2c^2x + ac^2)} \right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] [-1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/2*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{2\sqrt{-acx-c}}{acx-c}}{2\,a|c|}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 2*sqrt(
-a*c*x - c)/(a*c*x - c))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.233 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 193

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] -1/4*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)+1/1
6*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/
2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/8*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/a
/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}} - \frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(8*(a - x^(-1))*(c - a*c*x)
^(5/2)) - (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(4*(a - x^(-1))
^2*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[
x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(8*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x
)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right) \int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \operatorname{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right) \operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{8\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}(3+ax)+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{16\sqrt{a}c^2(-1+ax)^2\sqrt{c-acx}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(3 + a*x) + Sqrt[2]*Sqrt
[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(16*Sqrt[a]*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2+2\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx+2ax\sqrt{c}\sqrt{-c(ax+1)}-\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}}{2\sqrt{c}}\right.\right.}{16\sqrt{\frac{ax-1}{ax+1}}(ax-1)^2c^{\frac{7}{2}}\sqrt{-c(ax+1)}\,a}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(
1/2))*a^2*c*x^2+2*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*
c*x+2*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*
2^(1/2)/c^(1/2))*c+6*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a
*x-1)^2/c^(7/2)/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.75

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)}{32\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}\right.$$

$$\left.-\frac{\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(a^2x^2+4\,ax+3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan
(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) -
2*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^
3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left((-acx-c)^{\frac{3}{2}}-2\sqrt{-acx-c}c\right)}{(acx-c)^{2}c}}{16\,a|c|}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 2*((-a
*c*x - c)^(3/2) - 2*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.234 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 250

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}} dx = -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -1/6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(7/2)+1/1
6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)-1/64*a
^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))
/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/32*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/a/x
)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}} dx = -\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}} + \frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{a^4x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] -1/6*(a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^2)/((a - x^(-1))^3*(c -
 a*c*x)^(7/2)) - (a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(32*(a - x
^(-1))*(c - a*c*x)^(7/2)) + (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^
3)/(16*(a - x^(-1))^2*(c - a*c*x)^(7/2)) - (a^(5/2)*(1 - 1/(a*x))^(7/2)*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(32*Sqrt[2]*(x^(-
1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right) \int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}} \, dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2} \text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^4} \, dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3} \, dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^3}{16\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2} \, dx, x, \frac{1}{x}\right)}{32\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^3}{16\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{64\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^3}{16\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}} \, dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{32\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^3}{16\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} - \frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2} \text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}(25+10ax-3a^2x^2)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(-1+ax)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{192\sqrt{a}c^3\sqrt{\frac{1}{x}}(-1+ax)^3\sqrt{c-acx}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(25 + 10*a*x - 3*a^2*x^2))
/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])]))/(192*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 + a*x)^3*Sqrt[c
 - a*c*x])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+9\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2+6a^2x^2\sqrt{-c(ax+1)}\sqrt{c}-9\sqrt{2}\arctan\left(\frac{\sqrt{-c}\right.}{}\right.}{192\sqrt{\frac{ax-1}{ax+1}}(ax-1)^3c^{\frac{9}{2}}\sqrt{-c(ax+1)}a}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*a^3*c*x^3+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
*a^2*c*x^2+6*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-20*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+3*2^(
1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-50*(-c*(a*x+1))^(1/2)
*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}}{a^2x^2-2ax+1}\right.}{384\,(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2}\right.$$

$$-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3a^3x^3-7a^2x^2-35ax-2}{192\,(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 - 7*a^
2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x
^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/192*(3*sqrt(2
)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt
(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3
- 7*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5
*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}} - \frac{2\left(3\,(acx+c)^2\sqrt{-acx-c}+16\,(-acx-c)^{\frac{3}{2}}c-12\,\sqrt{-acx-c}c^2\right)}{(acx-c)^3c^2}}{192\,a|c|}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*(
3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*x
 - c)*c^2)/((a*c*x - c)^3*c^2))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.235 $\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

```
[Out] 4/7*(-a*c*x+c)^(7/2)/a-2/9*(-a*c*x+c)^(9/2)/a/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^{7/2}\,dx\\
&= -\int \frac{(1+ax)(c-acx)^{7/2}}{1-ax}\,dx\\
&= -\left(c\int (1+ax)(c-acx)^{5/2}\,dx\right)\\
&= -\left(c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)\,dx\right)\\
&= \frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3(-1+ax)^3(11+7ax)\sqrt{c-acx}}{63a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
[Out] (-2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{7}{2}}(7ax+11)}{63a}$	21
pseudoelliptic	$-\frac{2\sqrt{-c(ax-1)}\,(ax-1)^{3}(ax+\frac{11}{7})c^{3}}{9a}$	31
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{9}{2}}}{9}+\frac{4c(-acx+c)^{\frac{7}{2}}}{7}}{ac}$	33
trager	$-\frac{2c^{3}(7a^{4}x^{4}-10a^{3}x^{3}-12a^{2}x^{2}+26ax-11)\sqrt{-acx+c}}{63a}$	48
risch	$\frac{2c^{4}(7a^{4}x^{4}-10a^{3}x^{3}-12a^{2}x^{2}+26ax-11)(ax-1)}{63a\sqrt{-c(ax-1)}}$	54

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(7\,a^{4}c^{3}x^{4}-10\,a^{3}c^{3}x^{3}-12\,a^{2}c^{3}x^{2}+26\,ac^{3}x-11\,c^{3})\sqrt{-acx+c}}{63\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
[Out] -2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c^
3)*sqrt(-a*c*x + c)/a
```

Sympy [A] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\begin{cases}-\frac{2\left(-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{ac} & \text{for } ac\neq 0\\ c^{\frac{7}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(7/2),x)
[Out] Piecewise((-2*(-2*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a*c), N
e(a*c, 0)), (c**(7/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)
/a, True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2\left(7(-acx+c)^{\frac{9}{2}}-18(-acx+c)^{\frac{7}{2}}c\right)}{63\,ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] -2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.12

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{2\left(90(acx-c)^3\sqrt{-acx+c}+378(acx-c)^2\sqrt{-acx+c}c-630(-acx+c)^{\frac{3}{2}}c^2+945\sqrt{-acx\right.}{}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
[Out] 2/315*(90*(a*c*x - c)^3*sqrt(-a*c*x + c) + 378*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c - 630*(-a*c*x + c)^(3/2)*c^2 + 945*sqrt(-a*c*x + c)*c^3 + 210*((-a*c*x
 + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c^2 - (35*(a*c*x - c)^4*sqrt(-a*c*x + c
) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 378*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sqrt(-a*c*x + c)*c^4)/c)/a
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4\,(c-a\,c\,x)^{7/2}}{7\,a}-\frac{2\,(c-a\,c\,x)^{9/2}}{9\,a\,c}$$

```
[In] int(((c - a*c*x)^(7/2)*(a*x + 1))/(a*x - 1),x)
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

3.236 $\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

```
[Out] 4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}(c-acx)^{5/2}\,dx\\ &= -\int \frac{(1+ax)(c-acx)^{5/2}}{1-ax}\,dx\\ &= -\left(c\int (1+ax)(c-acx)^{3/2}\,dx\right)\\ &= -\left(c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)\,dx\right)\\ &= \frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2(-1+ax)^2(9+5ax)\sqrt{c-acx}}{35a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
[Out] (2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{5}{2}}(5ax+9)}{35a}$	21
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+\frac{9}{5})(ax-1)^{2}c^{2}}{7a}$	31
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c(-acx+c)^{\frac{5}{2}}}{5}}{ac}$	33
trager	$\frac{2c^{2}(5a^{3}x^{3}-a^{2}x^{2}-13ax+9)\sqrt{-acx+c}}{35a}$	40
risch	$-\frac{2c^{3}(5a^{3}x^{3}-a^{2}x^{2}-13ax+9)(ax-1)}{35a\sqrt{-c(ax-1)}}$	46

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,(5\,a^{3}c^{2}x^{3}-a^{2}c^{2}x^{2}-13\,ac^{2}x+9\,c^{2})\sqrt{-acx+c}}{35\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\begin{cases}-\frac{2\left(-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac} & \text{for } ac\neq 0\\ c^{\frac{5}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*(-2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a*c), N
e(a*c, 0)), (c**(5/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)
/a, True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{2\left(5\,(-acx+c)^{\frac{7}{2}}-14\,(-acx+c)^{\frac{5}{2}}c\right)}{35\,ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.52

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=$$

$$-\frac{2\left(21\,(acx-c)^2\sqrt{-acx+c}-70\,(-acx+c)^{\frac{3}{2}}c-35\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c\right)c-\frac{3\left(5\,(acx-c)^3\sqrt{-acx\cdots}\right.}{}\right.}{105\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(21*(a*c*x - c)^2*sqrt(-a*c*x + c) - 70*(-a*c*x + c)^(3/2)*c - 35*((
-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c - 3*(5*(a*c*x - c)^3*sqrt(-a*c*
x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 +
35*sqrt(-a*c*x + c)*c^3)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}$$

```
[In] int(((c - a*c*x)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

3.237 $\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

```
[Out] 4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}(c-acx)^{3/2}\,dx\\ &= -\int \frac{(1+ax)(c-acx)^{3/2}}{1-ax}\,dx\\ &= -\left(c\int (1+ax)\sqrt{c-acx}\,dx\right)\\ &= -\left(c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)\,dx\right)\\ &= \frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c(-1+ax)(7+3ax)\sqrt{c-acx}}{15a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
[Out] (-2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{3}{2}}(3ax+7)}{15a}$	21
pseudoelliptic	$-\frac{2\sqrt{-c(ax-1)}\,(ax+\frac{7}{3})(ax-1)c}{5a}$	27
trager	$-\frac{2c(3a^2x^2+4ax-7)\sqrt{-acx+c}}{15a}$	30
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c(-acx+c)^{\frac{3}{2}}}{3}}{ac}$	33
risch	$\frac{2c^2(3a^2x^2+4ax-7)(ax-1)}{15a\sqrt{-c(ax-1)}}$	38

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,(3\,a^2cx^2+4\,acx-7\,c)\sqrt{-acx+c}}{15\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \begin{cases} -\frac{2\left(-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Piecewise((-2*(-2*c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a*c), N
e(a*c, 0)), (c**(3/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)
/a, True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(3\,(-acx+c)^{\frac{5}{2}}-10\,(-acx+c)^{\frac{3}{2}}c\right)}{15\,ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2\left(15\,\sqrt{-acx+c}c-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}c^2}{c}\right)}{15\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(-a*c*x + c)*c - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*
x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{4\,(c-a\,c\,x)^{3/2}}{3\,a}-\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a\,c}$$

```
[In] int(((c - a*c*x)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

3.238 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\sqrt{c-acx}\,dx\\ &= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+5)}{3a}$	21
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4c\sqrt{-acx+c}}{ac}$	33

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{-acx+c}(ax+5)}{3a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\begin{cases}-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0\\ \sqrt{c}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((-2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c
, 0)), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, Tr
ue)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c}{c}\right)}{3\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

3.239 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{4}{a\sqrt{c-acx}}-\frac{2\sqrt{c-acx}}{ac}$$

```
[Out] -4/a/(-a*c*x+c)^(1/2)-2*(-a*c*x+c)^(1/2)/a/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac}-\frac{4}{a\sqrt{c-acx}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
[Out] -4/(a*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x])/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-acx}}\,dx\\ &= -\int \frac{1+ax}{(1-ax)\sqrt{c-acx}}\,dx\\ &= -\left(c\int \frac{1+ax}{(c-acx)^{3/2}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)\,dx\right)\\ &= -\frac{4}{a\sqrt{c-acx}}-\frac{2\sqrt{c-acx}}{ac}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{-6+2ax}{a\sqrt{c-acx}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
[Out] (-6 + 2*a*x)/(a*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2ax-6}{a\sqrt{-acx+c}}$	20
pseudoelliptic	$\frac{2ax-6}{a\sqrt{-c(ax-1)}}$	21
trager	$-\frac{2(ax-3)\sqrt{-acx+c}}{ca(ax-1)}$	30
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ca}$	31
default	$\frac{-2\sqrt{-acx+c}-\frac{4c}{\sqrt{-acx+c}}}{ac}$	33
risch	$\frac{2ax-2}{a\sqrt{-c(ax-1)}}-\frac{4}{a\sqrt{-c(ax-1)}}$	37

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(a*x-3)/a/(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=-\frac{2\sqrt{-acx+c}(ax-3)}{a^2cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\begin{cases}-\frac{2\cdot\left(\frac{2c}{\sqrt{-acx+c}}+\sqrt{-acx+c}\right)}{ac} & \text{for } ac\neq 0\\ \frac{\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}}{\sqrt{c}} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(1/2),x)
[Out] Piecewise((-2*(2*c/sqrt(-a*c*x + c) + sqrt(-a*c*x + c))/(a*c), Ne(a*c, 0)),
 (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, True))/sqrt(c),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\left(\sqrt{-acx+c}+\frac{2\,c}{\sqrt{-acx+c}}\right)}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{4}{\sqrt{-acx+c}a} - \frac{2\sqrt{-acx+c}}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] -4/(sqrt(-a*c*x + c)*a) - 2*sqrt(-a*c*x + c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,a\,x-6}{a\,\sqrt{c-a\,c\,x}}$$

```
[In] int((a*x + 1)/((c - a*c*x)^(1/2)*(a*x - 1)),x)
[Out] (2*a*x - 6)/(a*(c - a*c*x)^(1/2))
```

3.240 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{4}{3a(c-acx)^{3/2}} + \frac{2}{ac\sqrt{c-acx}}$$

```
[Out] -4/3/a/(-a*c*x+c)^(3/2)+2/a/c/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2}{ac\sqrt{c-acx}} - \frac{4}{3a(c-acx)^{3/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] -4/(3*a*(c - a*c*x)^(3/2)) + 2/(a*c*Sqrt[c - a*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{3/2}}\,dx\\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{3/2}}\,dx\\
&= -\left(c\int \frac{1+ax}{(c-acx)^{5/2}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx\right)\\
&= -\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\frac{-2+6ax}{3ac(-1+ax)\sqrt{c-acx}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
[Out] (-2 + 6*a*x)/(3*a*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(3ax-1)}{3a(-acx+c)^{\frac{3}{2}}}$	21
default	$\frac{\frac{2}{\sqrt{-acx+c}}-\frac{4c}{3(-acx+c)^{\frac{3}{2}}}}{ac}$	31
trager	$-\frac{2(3ax-1)\sqrt{-acx+c}}{3c^{2}(ax-1)^{2}a}$	31
pseudoelliptic	$\frac{6ax-2}{3ac(ax-1)\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(-\frac{1}{\sqrt{-acx+c}}+\frac{2c}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(3*a*x-1)/a/(-a*c*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{2\sqrt{-acx+c}(3\,ax-1)}{3\,(a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2})}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\begin{cases} -\frac{2\cdot\left(\frac{2c}{3(-acx+c)^{\frac{3}{2}}}-\frac{1}{\sqrt{-acx+c}}\right)}{ac} & \text{for } ac\neq 0\\ \frac{\begin{cases} -x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}}{c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Piecewise((-2*(2*c/(3*(-a*c*x + c)**(3/2)) - 1/sqrt(-a*c*x + c))/(a*c), Ne(
a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, True))/
c**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\,(3\,acx-c)}{3\,(acx-c)\sqrt{-acx+c}ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] 2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{6\,a\,x-2}{3\,a\,(c-a\,c\,x)^{3/2}}$$

```
[In] int((a*x + 1)/((c - a*c*x)^(3/2)*(a*x - 1)),x)
[Out] -(6*a*x - 2)/(3*a*(c - a*c*x)^(3/2))
```

3.241 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}$$

```
[Out] -4/5/a/(-a*c*x+c)^(5/2)+2/3/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{2}{3ac(c-acx)^{3/2}}-\frac{4}{5a(c-acx)^{5/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -4/(5*a*(c - a*c*x)^(5/2)) + 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{5/2}}\,dx\\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{5/2}}\,dx\\
&= -\left(c\int \frac{1+ax}{(c-acx)^{7/2}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)dx\right)\\
&= -\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2(1+5ax)}{15ac^2(-1+ax)^2\sqrt{c-acx}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
[Out] (-2*(1 + 5*a*x))/(15*a*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(5ax+1)}{15(-acx+c)^{\frac{5}{2}}a}$	21
trager	$\frac{2(5ax+1)\sqrt{-acx+c}}{15c^3(ax-1)^3a}$	31
pseudoelliptic	$\frac{-\frac{2ax}{3}-\frac{2}{15}}{c^2(ax-1)^2\sqrt{-c(ax-1)}\,a}$	32
derivativedivides	$-\frac{2\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33
default	$\frac{\frac{2}{3(-acx+c)^{\frac{3}{2}}}-\frac{4c}{5(-acx+c)^{\frac{5}{2}}}}{ac}$	33

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/15*(5*a*x+1)/(-a*c*x+c)^(5/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{2\sqrt{-acx+c}(5\,ax+1)}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ac} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*(2*c/(5*(-a*c*x + c)**(5/2)) - 1/(3*(-a*c*x + c)**(3/2)))/(a*
c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a,
True))/c**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] -2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\,(5\,acx+c)}{15\,(acx-c)^2\sqrt{-acx+c}ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] -2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{10\,a\,x+2}{15\,a\,{\left(c-a\,c\,x\right)}^{5/2}}$$

```
[In] int((a*x + 1)/((c - a*c*x)^(5/2)*(a*x - 1)),x)
[Out] -(10*a*x + 2)/(15*a*(c - a*c*x)^(5/2))
```

3.242 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{4}{7a(c-acx)^{7/2}} + \frac{2}{5ac(c-acx)^{5/2}}$$

```
[Out] -4/7/a/(-a*c*x+c)^(7/2)+2/5/a/c/(-a*c*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{2}{5ac(c-acx)^{5/2}} - \frac{4}{7a(c-acx)^{7/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
[Out] -4/(7*a*(c - a*c*x)^(7/2)) + 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{7/2}}\,dx \\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{7/2}}\,dx \\
&= -\left(c\int \frac{1+ax}{(c-acx)^{9/2}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx\right) \\
&= -\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{6+14ax}{35ac^3(-1+ax)^3\sqrt{c-acx}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
[Out] (6 + 14*a*x)/(35*a*c^3*(-1 + a*x)^3*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(7ax+3)}{35a(-acx+c)^{\frac{7}{2}}}$	21
trager	$-\frac{2(7ax+3)\sqrt{-acx+c}}{35c^{4}(ax-1)^{4}a}$	31
pseudoelliptic	$\frac{\frac{2ax}{5}+\frac{6}{35}}{a\,c^{3}(ax-1)^{3}\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(-\frac{1}{5(-acx+c)^{\frac{5}{2}}}+\frac{2c}{7(-acx+c)^{\frac{7}{2}}}\right)}{ca}$	33
default	$\frac{\frac{2}{5(-acx+c)^{\frac{5}{2}}}-\frac{4c}{7(-acx+c)^{\frac{7}{2}}}}{ac}$	33

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/35*(7*a*x+3)/a/(-a*c*x+c)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\sqrt{-acx+c}(7\,ax+3)}{35\,(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-4\,a^{2}c^{4}x+ac^{4})}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
[Out] -2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4
*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{7(-acx+c)^{\frac{7}{2}}}-\frac{1}{5(-acx+c)^{\frac{5}{2}}}\right)}{ac} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Piecewise((-2*(2*c/(7*(-a*c*x + c)**(7/2)) - 1/(5*(-a*c*x + c)**(5/2))))/(a*
c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a,
True))/c**(7/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^3\sqrt{-acx+c}ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{14\,a\,x+6}{35\,a\,{\left(c-a\,c\,x\right)}^{7/2}}$$

```
[In] int((a*x + 1)/((c - a*c*x)^(7/2)*(a*x - 1)),x)
```

```
[Out] -(14*a*x + 6)/(35*a*(c - a*c*x)^(7/2))
```

3.243 $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 197

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] -8/33*(1+1/a/x)^(5/2)*(-a*c*x+c)^(9/2)/a/(1-1/a/x)^(9/2)-856/1155*(1+1/a/x)
^(5/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+16/21*(1+1/a/x)^(5/2)*(-a*c
*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^3*(1+1/a/x)^(5/2)*x*(-a*c*x+
c)^(9/2)/a^3/(1-1/a/x)^(9/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=-\frac{856\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{1155a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{21a^2x\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

[Out] (-8*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(33*a*(1 - 1/(a*x))^(9/2)) - (856*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(1155*a^3*(1 - 1/(a*x))^(9/2)*x^2) + (16*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(21*a^2*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(9/2))/(11*a^3*(1 - 1/(a*x))^(9/2))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(c-acx)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3}\left(1+\frac{x}{a}\right)^{3/2}}{x^{13/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}\\
&= \frac{2\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}\\
&\quad+\frac{\left(12\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2}\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}\\
&= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}\\
&\quad+\frac{\left(8\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{11}{a}+\frac{9x}{2a^{2}}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{33a\left(1-\frac{1}{ax}\right)^{9/2}}\\
&= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^{2}\left(1-\frac{1}{ax}\right)^{9/2}x}\\
&\quad+\frac{2\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}\\
&\quad+\frac{\left(428\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{231a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}
\end{aligned}
$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2c^4\sqrt{1+\frac{1}{ax}}(1+ax)^2\sqrt{c-acx}(-533+755ax-455a^2x^2+105a^3x^3)}{1155a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455*
a^2*x^2 + 105*a^3*x^3))/(1155*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2(ax+1)(105a^3x^3-455a^2x^2+755ax-533)(-acx+c)^{\frac{9}{2}}}{1155a(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64
default	$\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^4(105a^3x^3-455a^2x^2+755ax-533)}{1155\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	66
risch	$-\frac{2c^5(ax-1)(105a^5x^5-245a^4x^4-50a^3x^3+522a^2x^2-311ax-533)}{1155\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}\,a}$	77

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/1155*(a*x+1)*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(a*
x-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(105\,a^6c^4x^6-140\,a^5c^4x^5-295\,a^4c^4x^4+472\,a^3c^4x^3+211\,a^2c^4x^2-844\,ac^4x-533\,c^4\right)\sqrt{-acx}}{1155\,(a^2x-a)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^6*c^4*x^6 - 140*a^5*c^4*x^5 - 295*a^4*c^4*x^4 + 472*a^3*c^4*x
^3 + 211*a^2*c^4*x^2 - 844*a*c^4*x - 533*c^4)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(105\,a^5\sqrt{-c}c^4x^5-455\,a^4\sqrt{-c}c^4x^4+650\,a^3\sqrt{-c}c^4x^3-78\,a^2\sqrt{-c}c^4x^2-755\,a\sqrt{-c}c^4x+533\right.}{1155\,(ax-1)a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*a^5*sqrt(-c)*c^4*x^5 - 455*a^4*sqrt(-c)*c^4*x^4 + 650*a^3*sqrt(
-c)*c^4*x^3 - 78*a^2*sqrt(-c)*c^4*x^2 - 755*a*sqrt(-c)*c^4*x + 533*sqrt(-c)
*c^4)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{2\left(512\sqrt{2}\sqrt{-c}c^3+\frac{105\,(acx+c)^5\sqrt{-acx-c}-770\,(acx+c)^4\sqrt{-acx-c}c+1980\,(acx+c)^3\sqrt{-acx-c}c^2-1848\,(acx+c)^2\sqrt{-acx-c}c^3}{c^2}\right)c^2}{1155\,a|c|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -2/1155*(512*sqrt(2)*sqrt(-c)*c^3 + (105*(a*c*x + c)^5*sqrt(-a*c*x - c) - 7
70*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 1980*(a*c*x + c)^3*sqrt(-a*c*x - c)*c
^2 - 1848*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3)/c^2)*c^2/(a*abs(c)*sgn(a*x +
1))
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(105\,a^5\,x^5-35\,a^4\,x^4-330\,a^3\,x^3+142\,a^2\,x^2+353\,a\,x-491\right)}{1155\,a} - \frac{2048\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{1155\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(353*a*x + 142*a^2*x^2
 - 330*a^3*x^3 - 35*a^4*x^4 + 105*a^5*x^5 - 491))/(1155*a) - (2048*c^4*(c -
 a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(1155*a*(a*x - 1))
```

3.244 $\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -44/63*(1+1/a/x)^(5/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)+214/315*(1+1/a/x)
^(5/2)*(-a*c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(1+1/a/x)^(5/2)*x*(-a*c*x
+c)^(7/2)/(1-1/a/x)^(7/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 91, 79, 37}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{214\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{315a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{44\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-44*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(63*a*(1 - 1/(a*x))^(7/2)) + (2
14*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(315*a^2*(1 - 1/(a*x))^(7/2)*x) +
 (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(7/2))/(9*(1 - 1/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(-\frac{11}{a}+\frac{9x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}dx, x, \frac{1}{x}\right)}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(107\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}}dx, x, \frac{1}{x}\right)}{63a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{1+\frac{1}{ax}}(1+ax)^2\sqrt{c-acx}(107-110ax+35a^2x^2)}{315a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*a
^2*x^2))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(ax+1)(35a^2x^2-110ax+107)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	56
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^3(35a^2x^2-110ax+107)}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
risch	$\frac{2c^4(ax-1)(35a^4x^4-40a^3x^3-78a^2x^2+104ax+107)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	69

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/315*(a*x+1)*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/((a*x-1)/(a*x+1))^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(35\,a^5c^3x^5 - 5\,a^4c^3x^4 - 118\,a^3c^3x^3 + 26\,a^2c^3x^2 + 211\,ac^3x + 107\,c^3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^2x-a)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")

[Out] -2/315*(35*a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 118*a^3*c^3*x^3 + 26*a^2*c^3*x^2 + 211*a*c^3*x + 107*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(7/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(35\,a^4\sqrt{-c}c^3x^4 - 110\,a^3\sqrt{-c}c^3x^3 + 72\,a^2\sqrt{-c}c^3x^2 + 110\,a\sqrt{-c}c^3x - 107\,\sqrt{-c}c^3\right)(ax+1)^{\frac{3}{2}}}{315\,(ax-1)a}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")

[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 110*a^3*sqrt(-c)*c^3*x^3 + 72*a^2*sqrt(-c)*c^3*x^2 + 110*a*sqrt(-c)*c^3*x - 107*sqrt(-c)*c^3)*(a*x + 1)^(3/2)/((a*x - 1)*a)

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^4\,x^4+30\,a^3\,x^3-88\,a^2\,x^2-62\,a\,x+149\right)}{315\,a} - \frac{512\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(30*a^3*x^3 - 88*a^2
*x^2 - 62*a*x + 35*a^4*x^4 + 149))/(315*a) - (512*c^3*(c - a*c*x)^(1/2)*((a
*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.245 $\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -18/35*(1+1/a/x)^(5/2)*(-a*c*x+c)^(5/2)/a/(1-1/a/x)^(5/2)+2/7*(1+1/a/x)^(5/
2)*x*(-a*c*x+c)^(5/2)/(1-1/a/x)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 79, 37}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{18\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-18*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (2
*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-acx)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(9\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-9+5ax)\sqrt{c-acx}(c+acx)^2}{35a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(-9 + 5*a*x)*Sqrt[c - a*c*x]*(c + a*c*x)^2)/(35*a*Sqrt
[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{2(ax+1)(5ax-9)(-acx+c)^{\frac{5}{2}}}{35a(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	48
default	$\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^2(5ax-9)}{35\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
risch	$-\frac{2c^3(ax-1)(5a^3x^3+a^2x^2-13ax-9)}{35\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	60

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/35*(a*x+1)*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^4c^2x^4+6\,a^3c^2x^3-12\,a^2c^2x^2-22\,ac^2x-9\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/35*(5*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 22*a*c^2*x - 9*c^2)*
sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^3\sqrt{-c}c^2x^3-9\,a^2\sqrt{-c}c^2x^2-5\,a\sqrt{-c}c^2x+9\,\sqrt{-c}c^2\right)(ax+1)^{\frac{3}{2}}}{35\,(ax-1)a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] 2/35*(5*a^3*sqrt(-c)*c^2*x^3 - 9*a^2*sqrt(-c)*c^2*x^2 - 5*a*sqrt(-c)*c^2*x
+ 9*sqrt(-c)*c^2)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{2\left(16\,\sqrt{2}\sqrt{-c}c+\frac{5\,(acx+c)^3\sqrt{-acx-c}-14\,(acx+c)^2\sqrt{-acx-c}c}{c^2}\right)c^2}{35\,a|c|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] -2/35*(16*sqrt(2)*sqrt(-c)*c + (5*(a*c*x + c)^3*sqrt(-a*c*x - c) - 14*(a*c*
x + c)^2*sqrt(-a*c*x - c)*c)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-5\,a^3\,x^3-11\,a^2\,x^2+a\,x+23\right)}{35\,a}-\frac{64\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a*x - 11*a^2*x^2 -
5*a^3*x^3 + 23))/(35*a) - (64*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(
1/2))/(35*a*(a*x - 1))
```

3.246 $\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

```
[Out] 2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6309}

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]

[Out] (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	35
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	42
risch	$\frac{2c^{2}(ax-1)\left(a^{2}x^{2}+2ax+1\right)}{5\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	52

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^{3}cx^{3}+3\,a^{2}cx^{2}+3\,acx+c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^{2}x-a\right)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")

[Out] -2/5*(a^3*c*x^3 + 3*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^2*x - a)

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^2\sqrt{-c}cx^2-\sqrt{-c}c\right)(ax+1)^{\frac{3}{2}}}{5\,(ax-1)a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2/5*(a^2*sqrt(-c)*c*x^2 - sqrt(-c)*c)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=$$
$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2+4\,a\,x+7\right)}{5\,a}-\frac{16\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + a^2*x^2 + 7))
/(5*a) - (16*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x - 1
))
```

3.247 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1
/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$	107
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	121

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(1/
2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{}\right.}{3\,(a^2x-a)}\right.$$
$$\left.-\frac{2\left(6\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.248 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}+\frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] 2*a*(1+1/a/x)^(3/2)*x*(1-1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-6*(1-1/a/x)^
(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-3*arctanh(2^(1/2)*(1/x)^(1/2
)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2)/(-a*
c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}+\frac{2ax\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (-6*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - a*c*x]) + (
2*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x)/((a - x^(-1))*Sqrt[c - a*c*x])
 - (3*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqr
t[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int\frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66

$$\int\frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-2+ax) - 3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}(-1+ax)\sqrt{c-acx}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 3*Sqrt[2]*Sq
rt[x^(-1)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(Sqrt[a]*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	s
default	$-\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx+2ax\sqrt{c}\sqrt{-c(ax+1)}+3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)c-4\sqrt{-c(ax+1)}\sqrt{c}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)c^{\frac{3}{2}}\sqrt{-c(ax+1)}\,a}$	1
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}+\frac{\left(-\frac{2\sqrt{-acx-c}}{a(-acx+c)}+\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{a\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$	1

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/
2))*a*c*x+2*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1)
)^(1/2)*2^(1/2)/c^(1/2))*c-4*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^
(3/2)/(a*x+1)/c^(3/2)/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \left[\frac{3\sqrt{2}(a^2cx^2-2\,acx+c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-4\left(a^2x^2-ax-2\right)\sqrt{-ac}}{2\left(a^3cx^2-2\,a^2cx+ac\right)}\right.$$

$$\left.-\frac{2\left(a^2x^2-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}-\frac{3\sqrt{2}(a^2cx^2-2\,acx+c)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{\sqrt{c}}}{a^3cx^2-2\,a^2cx+ac}\right]$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/2*(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c), -(2*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)) - 3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)

[Out] Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1))), x)

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{3\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-2\sqrt{-acx-c}+\frac{2\sqrt{-acx-c}c}{acx-c}}{a|c|}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")

[Out] -(3*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 2*sqrt(-a*c*x - c) + 2*sqrt(-a*c*x - c)*c/(a*c*x - c))/(a*abs(c))

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.249 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 187

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x/(a-1/x)^2/(-a*c*x+c)^(3/2)-3/8*(
1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/2)
/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-3/4*a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^(1/
2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}-\frac{3ax\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))*(c - a*c*x)^
(3/2)) - (a^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x)/(2*(a - x^(-1))^2*
(c - a*c*x)^(3/2)) - (3*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)
^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^{3}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^{2}}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.67

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+5ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{8\sqrt{a}c(-1+ax)^2\sqrt{c-acx}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 + 5*a*x) + 3*Sqrt[2]*
Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 +
1/(a*x)])]))/(8*Sqrt[a]*c*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2-6\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx+10ax\sqrt{c}\sqrt{-c(ax+1)}+3\sqrt{2}\arctan\left(\frac{\sqrt{-c(\ldots}}{}\right.\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)c^{\frac{5}{2}}\sqrt{-c(ax+1)}\,a}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*2^(1/2)*
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+10*a*x*c^(1/2)*(-c*(a*
x+1))^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(-
c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.82

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\ldots}{a^2x^2-2ax+1}\right.}{16\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)\ldots}\right.$$

$$-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+2\left(5a^2x^2+4ax-1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\ldots}{8\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*
c^2*x - a*c^2), -1/8*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) + 2*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5(-acx-c)^{\frac{3}{2}}+6\sqrt{-acx-c}c\right)}{(acx-c)^2}}{8a|c|}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - 2*(5*
(-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/(a*c*x - c)^2)/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\int \frac{1}{{\left(c-a\,c\,x\right)}^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.250 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] 1/24*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)-1/6
*a^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(5/2)+1/32*a^
(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))/
(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/16*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/a/x)
^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}} - \frac{a^4x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(16*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(24*(a - x^(-1
))^2*(c - a*c*x)^(5/2)) - (a^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^2)
/(6*(a - x^(-1))^3*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))
^(5/2)*(c - a*c*x)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^4}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{32\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}(7+22ax+3a^2x^2)-\frac{3\sqrt{2}(-1+ax)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{x}\right)}{96\sqrt{a}c^2\left(\frac{1}{x}\right)^{3/2}(-1+ax)^3\sqrt{c-acx}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -1/96*(Sqrt[1 - 1/(a*x)]*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(7 + 22*
a*x + 3*a^2*x^2) - (3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])])/x))/(Sqrt[a]*c^2*(x^(-1))^(3/2)*(-1 + a*x)^3*S
qrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+9\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2+6a^2x^2\sqrt{-c(ax+1)}\sqrt{c}-9\sqrt{2}\arctan\left(\frac{\sqrt{}}{}\right.\right.}{96\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)^2(ax+1)c^{\frac{7}{2}}\sqrt{-c(ax+}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c
^(1/2))*a^3*c*x^3+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*
a^2*c*x^2+6*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c*(a*
x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+44*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+3*2^(1
/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+14*(-c*(a*x+1))^(1/2)*
c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)/c^(7/2)/(-c*(a*x+1))^(1/
2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\cdots}}{a^2x^2-2ax+1}\right.}{192\,(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-\cdots}\right.$$

$$-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3a^3x^3+25a^2x^2+29ax+7)\cdots}{96\,(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 + 25*a
^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x
^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/96*(3*sqrt(2)
*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3 +
 25*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*
c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{2\left(3(acx+c)^2\sqrt{-acx-c}-16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c}}{96\,a|c|}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")

[Out] 1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - 2*(3*(a*c*x + c)^2*sqrt(-a*c*x - c) - 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*x - c)*c^2)/((a*c*x - c)^3*c))/(a*abs(c))

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

[In] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)

[Out] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

3.251 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 307

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{256\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -1/8*a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^4/(-a*c*x+c)^(7/2)-1/1
28*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)+1/32*
a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^3/(a-1/x)^3/(-a*c*x+c)^(7/2)-3/512*a^
(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))/
(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-3/256*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/a/x
)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{6311, 6316, 96, 95, 212\}$

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{256\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}} + \frac{a^5x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^5x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{3a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -1/8*(a^5*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^2)/((a - x^(-1))^4*(c -
 a*c*x)^(7/2)) - (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(256*(a
- x^(-1))*(c - a*c*x)^(7/2)) - (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)
*x^3)/(128*(a - x^(-1))^2*(c - a*c*x)^(7/2)) + (a^5*(1 - 1/(a*x))^(7/2)*(1
+ 1/(a*x))^(5/2)*x^3)/(32*(a - x^(-1))^3*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1
 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(256*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2} \operatorname{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{8\left(a-\frac{1}{x}\right)^4 (c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{8\left(a-\frac{1}{x}\right)^4 (c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2} x^3}{32\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{64\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{8\left(a-\frac{1}{x}\right)^4 (c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2} x^3}{128\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2} x^3}{32\left(a-\frac{1}{x}\right)^3 (c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{256\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{512\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{256\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{256\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}(39+79ax+13a^2x^2-3a^3x^3)}{\sqrt{\frac{1}{x}}} + 3\sqrt{2}(-1+ax)^4\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{512\sqrt{a}c^3\sqrt{\frac{1}{x}}(-1+ax)^4\sqrt{c-acx}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(39 + 79*a*x + 13*a^2*x^2
- 3*a^3*x^3))/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^4*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(512*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 +
a*x)^4*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.91

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{4}c\,x^{4}+12\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{3}c\,x^{3}+6a^{3}x^{3}\sqrt{-c(ax+1)}\,\sqrt{c}-18\sqrt{2}\,\arctan\left(\frac{\sqrt{}}{}\right.}{512\left(\frac{ax-1}{ax+1}\right.}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*a^4*c*x^4+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2)
)*a^3*c*x^3+6*a^3*x^3*(-c*(a*x+1))^(1/2)*c^(1/2)-18*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-26*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(
1/2)+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-158*a*
x*c^(1/2)*(-c*(a*x+1))^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2
)/c^(1/2))*c-78*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)
^3/(a*x+1)/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx}}{a^2x^2-2}\right.}{1024\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4}\right.$$

$$-\frac{3\sqrt{2}(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3\,a^4x^4-10\,a^3x^3}{512\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x
- 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x +
 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3
*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4), -1/512*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10
*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^4*x^4 - 10*a^3*x^3
```

```
- 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a
^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x
- a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-a*c*x + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}} - \frac{2\left(3(acx+c)^{3}\sqrt{-acx-c}-22(acx+c)^{2}\sqrt{-acx-c}c+44(-acx-c)^{\frac{3}{2}}c^{2}+24\sqrt{-acx-c}c^{3}\right)}{(acx-c)^{4}c^{2}}}{512\,a|c|}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
[Out] 1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*(
3*(a*c*x + c)^3*sqrt(-a*c*x - c) - 22*(a*c*x + c)^2*sqrt(-a*c*x - c)*c + 44
*(-a*c*x - c)^(3/2)*c^2 + 24*sqrt(-a*c*x - c)*c^3)/((a*c*x - c)^4*c^2))/(a*
abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.252 $\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{16384c^5\sqrt{1-\frac{1}{a^2x^2}}x}{693\sqrt{c-acx}} + \frac{4096}{693}c^4\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx} + \frac{512}{231}c^3\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2} + \frac{640}{693}c^2\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{5/2} + \frac{40}{99}c\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{7/2} + \frac{2}{11}\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{9/2}$$

```
[Out] 512/231*c^3*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+640/693*c^2*x*(-a*c*x+c)
^(5/2)*(1-1/a^2/x^2)^(1/2)+40/99*c*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+2
/11*x*(-a*c*x+c)^(9/2)*(1-1/a^2/x^2)^(1/2)+16384/693*c^5*x*(1-1/a^2/x^2)^(1
/2)/(-a*c*x+c)^(1/2)+4096/693*c^4*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.60, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{22016\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{693a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{40\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{640\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{693a^5x\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{1024\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{99a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{512\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{231a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[In] Int[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-40*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^5*(1 - 1/(a*
x))^(9/2)) - (22016*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(693*a^5*(1 - 1/(a
*x))^(9/2)*x^4) + (1024*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1
/(a*x))^(9/2)*x^3) - (512*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(231*a^3*(1
- 1/(a*x))^(9/2)*x^2) + (640*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(
9/2))/(693*a^5*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^5*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^(9/2))/(11*a^5*(1 - 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{13/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(20\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(320\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{99a^2\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(1280\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{512\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(512\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{1024\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{512\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(11008\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{22016\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x^4} + \frac{1024\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3} - \frac{512\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2c^4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-11531+5419ax-3198a^2x^2+1510a^3x^3-455a^4x^4+63a^5x^5)}{693a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-11531 + 5419*a*x - 3198*a^2*x^2
+ 1510*a^3*x^3 - 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.40

method	result	size
risch	$-\frac{2c^5\sqrt{\frac{ax-1}{ax+1}}\,(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531)(ax+1)}{693\sqrt{-c(ax-1)}\,a}$	77
gosper	$\frac{2(ax+1)(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531)(-acx+c)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{693a(ax-1)^5}$	80
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c^4(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531)}{693(ax-1)a}$	84

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/693*c^5*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(63*a^5*x^5-455*a^4*x
^4+1510*a^3*x^3-3198*a^2*x^2+5419*a*x-11531)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,(63\,a^6c^4x^6-392\,a^5c^4x^5+1055\,a^4c^4x^4-1688\,a^3c^4x^3+2221\,a^2c^4x^2-6112\,ac^4x-11531\,c^4)\cdots}{693\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^6*c^4*x^6 - 392*a^5*c^4*x^5 + 1055*a^4*c^4*x^4 - 1688*a^3*c^4*x
^3 + 2221*a^2*c^4*x^2 - 6112*a*c^4*x - 11531*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\left(63\,a^6\sqrt{-c}c^4x^6-392\,a^5\sqrt{-c}c^4x^5+1055\,a^4\sqrt{-c}c^4x^4-1688\,a^3\sqrt{-c}c^4x^3+2221\,a^2\sqrt{-c}c^4x^2\cdots\right.}{693\,(a^2x-a)\sqrt{ax+1}}$$

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*a^6*sqrt(-c)*c^4*x^6 - 392*a^5*sqrt(-c)*c^4*x^5 + 1055*a^4*sqrt(-
c)*c^4*x^4 - 1688*a^3*sqrt(-c)*c^4*x^3 + 2221*a^2*sqrt(-c)*c^4*x^2 - 6112*a
*sqrt(-c)*c^4*x - 11531*sqrt(-c)*c^4)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(63\,a^5\,x^5-329\,a^4\,x^4+726\,a^3\,x^3-962\,a^2\,x^2+1259\,a\,x-4853\right)}{693\,a} - \frac{32768\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{693\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(1259*a*x - 962*a^2*x^
2 + 726*a^3*x^3 - 329*a^4*x^4 + 63*a^5*x^5 - 4853))/(693*a) - (32768*c^4*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(693*a*(a*x - 1))
```

3.253 $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4096c^4\sqrt{1-\frac{1}{a^2x^2}}x}{315\sqrt{c-acx}}+\frac{1024}{315}c^3\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}+\frac{128}{105}c^2\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}+\frac{32}{63}c\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{5/2}+\frac{2}{9}\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{7/2}$$

```
[Out] 128/105*c^2*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+32/63*c*x*(-a*c*x+c)^(5/
2)*(1-1/a^2/x^2)^(1/2)+2/9*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+4096/315*
c^4*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+1024/315*c^3*x*(1-1/a^2/x^2)^(1/
2)*(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{5504\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{315a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{32\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{256\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{45a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{128\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{105a^2x\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-32*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*
x))^(7/2)) + (5504*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(315*a^4*(1 - 1/(a*
x))^(7/2)*x^3) - (256*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(45*a^3*(1 - 1/(
a*x))^(7/2)*x^2) + (128*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(105*a^2*(1 -
1/(a*x))^(7/2)*x) + (2*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(7/2)
)/(9*a^4*(1 - 1/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(64\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{21a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(128\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{256\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{45a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + \frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(2752\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{315a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{5504\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{315a^4\left(1-\frac{1}{ax}\right)^{7/2}x^3} - \frac{256\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{45a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + \frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(2867-1276ax+642a^2x^2-220a^3x^3+35a^4x^4)}{315a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(2867 - 1276*a*x + 642*a^2*x^2 -
220*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{2c^{4}\sqrt{\frac{ax-1}{ax+1}}\,(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)\,(ax+1)}{315\sqrt{-c(ax-1)}\,a}$	69
gosper	$\frac{2(ax+1)\,(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)\,(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{315a(ax-1)^{4}}$	72
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c^{3}\,(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)}{315(ax-1)a}$	76

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/315*c^4*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(35*a^4*x^4-220*a^3*x^
3+642*a^2*x^2-1276*a*x+2867)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx =$$
$$-\frac{2\,(35\,a^{5}c^{3}x^{5}-185\,a^{4}c^{3}x^{4}+422\,a^{3}c^{3}x^{3}-634\,a^{2}c^{3}x^{2}+1591\,ac^{3}x+2867\,c^{3})\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^{2}x-a)}$$

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] -2/315*(35*a^5*c^3*x^5 - 185*a^4*c^3*x^4 + 422*a^3*c^3*x^3 - 634*a^2*c^3*x^
2 + 1591*a*c^3*x + 2867*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(35\,a^5\sqrt{-c}c^3x^5-185\,a^4\sqrt{-c}c^3x^4+422\,a^3\sqrt{-c}c^3x^3-634\,a^2\sqrt{-c}c^3x^2+1591\,a\sqrt{-c}c^3x+2867\,\sqrt{-c}c^3\right)(}{315\,(a^2x-a)\sqrt{ax+1}}$$

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^5*sqrt(-c)*c^3*x^5 - 185*a^4*sqrt(-c)*c^3*x^4 + 422*a^3*sqrt(-
c)*c^3*x^3 - 634*a^2*sqrt(-c)*c^3*x^2 + 1591*a*sqrt(-c)*c^3*x + 2867*sqrt(-
c)*c^3)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^4\,x^4-150\,a^3\,x^3+272\,a^2\,x^2-362\,a\,x+1229\right)}{315\,a} - \frac{8192\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(272*a^2*x^2 - 362*a
*x - 150*a^3*x^3 + 35*a^4*x^4 + 1229))/(315*a) - (8192*c^3*(c - a*c*x)^(1/2
)*((a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.254 $\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{256c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}{35\sqrt{c-acx}}+\frac{64}{35}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{24}{35}c\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}+\frac{2}{7}\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{5/2}$$

```
[Out] 24/35*c*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+2/7*x*(-a*c*x+c)^(5/2)*(1-1/
a^2/x^2)^(1/2)+256/35*c^3*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+64/35*c^2*
x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{344\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}{35a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{16\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}{5a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-24*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) - (344
*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a^3*(1 - 1/(a*x))^(5/2)*x^2) + (1
```

```
6*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(5*a^2*(1 - 1/(a*x))^(5/2)*x) + (2*(
a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(
5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-acx)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{24\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}\\
&\quad+\frac{\left(24\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\operatorname{Subst}\left(\int\frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{35a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{24\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{16\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x}\\
&\quad+\frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}\\
&\quad+\frac{\left(172\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\operatorname{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{35a^3\left(1-\frac{1}{ax}\right)^{5/2}}
\end{aligned}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{344\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a^3\left(1-\frac{1}{ax}\right)^{5/2}x^2} + \frac{16\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2c^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-177+71ax-27a^2x^2+5a^3x^3)}{35a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a^
3*x^3))/(35*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{2c^3\sqrt{\frac{ax-1}{ax+1}}\,(5a^3x^3-27a^2x^2+71ax-177)(ax+1)}{35\sqrt{-c(ax-1)}\,a}$	61
gosper	$\frac{2(ax+1)(5a^3x^3-27a^2x^2+71ax-177)(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{35a(ax-1)^3}$	64
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c^2(5a^3x^3-27a^2x^2+71ax-177)}{35(ax-1)a}$	68

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/35*c^3*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(5*a^3*x^3-27*a^2*x^2+
71*a*x-177)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,(5\,a^4c^2x^4-22\,a^3c^2x^3+44\,a^2c^2x^2-106\,ac^2x-177\,c^2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 22*a^3*c^2*x^3 + 44*a^2*c^2*x^2 - 106*a*c^2*x - 177*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.75

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,\left(5\,a^4\sqrt{-c}c^2x^4-22\,a^3\sqrt{-c}c^2x^3+44\,a^2\sqrt{-c}c^2x^2-106\,a\sqrt{-c}c^2x-177\,\sqrt{-c}c^2\right)(ax-1)}{35\,(a^2x-a)\sqrt{ax+1}}$$

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^4*sqrt(-c)*c^2*x^4 - 22*a^3*sqrt(-c)*c^2*x^3 + 44*a^2*sqrt(-c)*c^
2*x^2 - 106*a*sqrt(-c)*c^2*x - 177*sqrt(-c)*c^2)*(a*x - 1)/((a^2*x - a)*sqr
t(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-17\,a^2\,x^2+27\,a\,x-79\right)}{35\,a} - \frac{512\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(27*a*x - 17*a^2*x^2 +
 5*a^3*x^3 - 79))/(35*a) - (512*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))
^(1/2))/(35*a*(a*x - 1))
```

3.255 $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{64c^2\sqrt{1-\frac{1}{a^2x^2}}x}{15\sqrt{c-acx}}+\frac{16}{15}c\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}+\frac{2}{5}\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}$$

```
[Out] 2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+64/15*c^2*x*(1-1/a^2/x^2)^(1/2)/
(-a*c*x+c)^(1/2)+16/15*c*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 91, 79, 37}

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{86\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{15a^2x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{28\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Int[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-28*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (86*
Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a^2*(1 - 1/(a*x))^(3/2)*x) + (2*Sq
rt[1 + 1/(a*x)]*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{2\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{15a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{86\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a^2\left(1-\frac{1}{ax}\right)^{3/2}x} + \frac{2\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2c\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(43-14ax+3a^2x^2)}{15a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*Sq
rt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{2c^2\sqrt{\frac{ax-1}{ax+1}}\,(3a^2x^2-14ax+43)(ax+1)}{15\sqrt{-c(ax-1)}\,a}$	53
gosper	$\frac{2(ax+1)(3a^2x^2-14ax+43)(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{15a(ax-1)^2}$	56
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c(3a^2x^2-14ax+43)}{15(ax-1)a}$	58

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*c^2*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(3*a^2*x^2-14*a*x+43)/a
*(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,(3\,a^3cx^3-11\,a^2cx^2+29\,acx+43\,c)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - 11*a^2*c*x^2 + 29*a*c*x + 43*c)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,\left(3\,a^3\sqrt{-c}cx^3-11\,a^2\sqrt{-c}cx^2+29\,a\sqrt{-c}cx+43\,\sqrt{-c}c\right)(ax-1)}{15\,(a^2x-a)\sqrt{ax+1}}$$

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^3*sqrt(-c)*c*x^3 - 11*a^2*sqrt(-c)*c*x^2 + 29*a*sqrt(-c)*c*x + 4
3*sqrt(-c)*c)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(3\,a^2\,x^2-8\,a\,x+21)}{15\,a}-\frac{128\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,(a\,x-1)}$$

```
[In] int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 8*a*x + 2
1))/(15*a) - (128*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(15*a*(a
*x - 1))
```

3.256 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{8c\sqrt{1-\frac{1}{a^2x^2}}\,x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}$$

```
[Out] 8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a*
c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 79, 37}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c(ax-1)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-5)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(a^2x^2-4\,ax-5)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x-5\,\sqrt{-c}\right)(ax-1)}{3\,(a^2x-a)\sqrt{ax+1}}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2}+\frac{4\,\sqrt{-acx-c}|c|}{ac}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] 2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.257 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

```
[Out] 2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6309}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
[Out] (2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] && !IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - a*c*x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-acx+c}}$	35
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c(ax-1)}\,a}$	36
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}{(ax-1)ca}$	46

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)}{\sqrt{ax+1}ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))/(sqrt(a*x + 1)*a*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(2\,x+\frac{2}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(1/2),x)
[Out] ((2*x + 2/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

3.258 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx = -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/
2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 95, 212}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx = -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}\\
&= -\frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}\\
&= -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{(ax-1)\sqrt{-c(ax+1)}\,c^{\frac{3}{2}}\,a}$	78

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)/c^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.86

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)}{2\,ac},\right.$$
$$\left.-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{ac^{\frac{3}{2}}}\right]$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/c)*log(-(a^2*x^2 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1))/
(a*c), -sqrt(2)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(
(a*x - 1)*sqrt(c)))/(a*c^(3/2))]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{a\sqrt{c}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] (sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*sqrt(c)) - sqrt(2)
*arctan(sqrt(-c)/sqrt(c))/(a*sqrt(c)))*abs(c)*sgn(a*x + 1)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{3/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2), x)
```

3.259 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] 1/4*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(5/2)*x^2*(1+1
/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/((a - x^(-1))*(c - a*c
*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(S
qrt[a]*Sqrt[1 + 1/(a*x)])])/(2*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{4\sqrt{a}c^2(-1+ax)\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(
-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(4*
Sqrt[a]*c^2*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{4c^{\frac{7}{2}}(ax-1)^2\sqrt{-c(ax+1)}\,a}$	123

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(7/2)/(a*x-1)^2/(-
c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.07

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{-a}\cdots}{8\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right.$$

$$\left.-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{4\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right]$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), -1/4*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a\,c\,x\right)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2), x)
```

3.260 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 193

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -3/32*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)
^(1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/4*a^3*(1-1/a/x)^(7/2)*x^2*(1
+1/a/x)^(1/2)/(a-1/x)^2/(-a*c*x+c)^(7/2)+3/16*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/
a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}} + \frac{3a^3x^3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^3x^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/4*(a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^2)/((a - x^(-1))^2*(c - a
*c*x)^(7/2)) + (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(16*(a - x
^(-1))*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]
*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))^(7/2)*(c
- a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral}=\frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$=-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^{3}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$=-\frac{a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{2}}{4\left(a-\frac{1}{x}\right)^{2}(c-acx)^{7/2}}+\frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^{2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$=-\frac{a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{2}}{4\left(a-\frac{1}{x}\right)^{2}(c-acx)^{7/2}}+\frac{3a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{3}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{\left(3a^{2}\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{32\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$=-\frac{a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{2}}{4\left(a-\frac{1}{x}\right)^{2}(c-acx)^{7/2}}+\frac{3a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{3}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{\left(3a^{2}\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^{2}}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$=-\frac{a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{2}}{4\left(a-\frac{1}{x}\right)^{2}(c-acx)^{7/2}}+\frac{3a^{3}\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^{3}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(7-3ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{32\sqrt{a}c^3(-1+ax)^2\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 - 3*a*x) + 3*Sqrt[2]*S
qrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1
/(a*x)])]))/(32*Sqrt[a]*c^3*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2+6\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+6ax\sqrt{c}\,\sqrt{-c(ax+1)}-3\sqrt{2}\right.}{32c^{\frac{9}{2}}(ax-1)^3\sqrt{-c(ax+1)}\,a}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(
1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2+6*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+6*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)-3*2^
(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^(1/2
)*c^(1/2))/c^(9/2)/(a*x-1)^3/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.77

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\left[-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3}{a^2x^2-2ax+1}\right.}{64\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}\right.$$

$$-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\left(3a^2x^2-4ax-7\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{32\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*
c^4*x - a*c^4), -1/32*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*
arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x -
c)) - 2*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))
/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3(-acx-c)^{\frac{3}{2}}+10\sqrt{-acx-c}c\right)}{(acx-c)^{2}c^{2}}\right)|c|\text{sgn}(ax+1)}{32\,ac^{2}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) + 2*(3
*(-a*c*x - c)^(3/2) + 10*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c^2))*abs(c)*sg
n(a*x + 1)/(a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a\,c\,x\right)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2), x)
```

3.261 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{8c(c-acx)^{5/2}}{5a}-\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -16/3*c^2*(-a*c*x+c)^(3/2)/a-8/5*c*(-a*c*x+c)^(5/2)/a-4/7*(-a*c*x+c)^(7/2)/
a-2/9*(-a*c*x+c)^(9/2)/a/c+32*c^(7/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/
c^(1/2))*2^(1/2)/a-32*c^3*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}-\frac{8c(c-acx)^{5/2}}{5a}$$

```
[In] Int[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-32*c^3*Sqrt[c - a*c*x])/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c -
a*c*x)^(5/2))/(5*a) - (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(
9*a*c) + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}(c-acx)^{7/2}\,dx$$

$$= -\int \frac{(1-ax)(c-acx)^{7/2}}{1+ax}\,dx$$

$$
\begin{aligned}
&= -\frac{\int \frac{(c-acx)^{9/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{9/2}}{9ac} - 2\int \frac{(c-acx)^{7/2}}{1+ax}\,dx\\
&= -\frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (4c)\int \frac{(c-acx)^{5/2}}{1+ax}\,dx\\
&= -\frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (8c^2)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx\\
&= -\frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a}\\
&\quad - \frac{2(c-acx)^{9/2}}{9ac} - (16c^3)\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a}\\
&\quad - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (32c^4)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a}\\
&\quad - \frac{2(c-acx)^{9/2}}{9ac} + \frac{(64c^3)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}\\
&= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a}\\
&\quad - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} + \frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

$$
\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2c^3\left(\sqrt{c-acx}(-6257+1754ax-732a^2x^2+230a^3x^3-35a^4x^4)+5040\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a}
$$

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
[Out] (2*c^3*(Sqrt[c - a*c*x]*(-6257 + 1754*a*x - 732*a^2*x^2 + 230*a^3*x^3 - 35*
a^4*x^4) + 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
)/(315*a)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

method	result
pseudoelliptic	$\frac{32\left(\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-\frac{\left(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257\right)\sqrt{-c(ax-1)}}{5040}\right)c^3}{a}$
risch	$\frac{2(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257)(ax-1)c^4}{315a\sqrt{-c(ax-1)}}+\frac{32c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+16c^4\sqrt{-acx+c}-16c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$
default	$\frac{-\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{4c(-acx+c)^{\frac{7}{2}}}{7}-\frac{8c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{16c^3(-acx+c)^{\frac{3}{2}}}{3}-32c^4\sqrt{-acx+c}+32c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$

```
[In] int((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 32*(c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-1/5040*
(35*a^4*x^4-230*a^3*x^3+732*a^2*x^2-1754*a*x+6257)*(-c*(a*x-1))^(1/2))*c^3/
a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\left[\frac{2\left(2520\,\sqrt{2}c^{\frac{7}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)-(35\,a^4c^3x^4-230\,a^3c^3x^3+732\,a^2c^3x^2-1754\,ac\right.}{315\,a}\right.$$

$$-\frac{2\left(5040\,\sqrt{2}\sqrt{-c}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)+(35\,a^4c^3x^4-230\,a^3c^3x^3+732\,a^2c^3x^2-1754\,ac^3x+6257\,c^3)\right.}{315\,a}$$

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) - (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 -
 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, -2/315*(5040*sqrt(2)*sqrt(-c
)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4 - 2
30*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c
))/a]
```

Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{16\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+16c^4\sqrt{-acx+c}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{ac} & \text{for } ac \neq 0 \\ c^{\frac{7}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*c*x+c)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((-2*(16*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 16*c**4*sqrt(-a*c*x + c) + 8*c**3*(-a*c*x + c)**(3/2)/3 + 4*c**2
*(-a*c*x + c)**(5/2)/5 + 2*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)
/(a*c), Ne(a*c, 0)), (c**(7/2)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x
+ 1) + 1)/a, True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(2520\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}+90(-acx+c)^{\frac{7}{2}}c+252(-acx+c)^{\frac{5}{2}}c^2+840(-acx+c)^{\frac{3}{2}}c^3+5040\sqrt{-acx+c}\,c^4\right)}{315\,ac}$$

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/315*(2520*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqr
t(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) + 90*(-a*c*x + c)
^(7/2)*c + 252*(-a*c*x + c)^(5/2)*c^2 + 840*(-a*c*x + c)^(3/2)*c^3 + 5040*s
qrt(-a*c*x + c)*c^4)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{32\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^8c^8 - 90\,(acx-c)^3\sqrt{-acx+c}a^8c^9 + 252\,(acx-c)^2\sqrt{-acx+c}a^8c^{10} + 840\,(\cdots\right.}{315\,a^9c^9}$$

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
- 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*(-a
*c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{4\,(c-a\,c\,x)^{7/2}}{7\,a} - \frac{8\,c\,(c-a\,c\,x)^{5/2}}{5\,a} - \frac{32\,c^3\,\sqrt{c-a\,c\,x}}{a} - \frac{16\,c^2\,(c-a\,c\,x)^{3/2}}{3\,a} - \frac{2\,(c-a\,c\,x)^{9/2}}{9\,a\,c} - \frac{\sqrt{2}\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,32\mathrm{i}}{a}$$

```
[In] int(((c - a*c*x)^(7/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(7/2))/(7*a) - (8*c*(c - a*c*x)^(5/2))/(5*a) - (32*c^3*(c
- a*c*x)^(1/2))/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(9/2)
)/(9*a*c) - (2^(1/2)*c^(7/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)
))*32i)/a
```

3.262 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{16c^2\sqrt{c-acx}}{a}-\frac{8c(c-acx)^{3/2}}{3a}-\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}+\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -8/3*c*(-a*c*x+c)^(3/2)/a-4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c+1
6*c^(5/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-16*c^2*(-
a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{16c^2\sqrt{c-acx}}{a}-\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}-\frac{8c(c-acx)^{3/2}}{3a}$$

```
[In] Int[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-16*c^2*Sqrt[c - a*c*x])/a - (8*c*(c - a*c*x)^(3/2))/(3*a) - (4*(c - a*c*x
)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c) + (16*Sqrt[2]*c^(5/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}(c-acx)^{5/2}\,dx$$

$$= -\int \frac{(1-ax)(c-acx)^{5/2}}{1+ax}\,dx$$

$$= -\frac{\int \frac{(c-acx)^{7/2}}{1+ax}\,dx}{c}$$

$$= -\frac{2(c-acx)^{7/2}}{7ac} - 2\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$

$$= -\frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (4c)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$

$$= -\frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (8c^2)\int \frac{\sqrt{c-acx}}{1+ax}\,dx$$

$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (16c^3)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$

$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{(32c^2)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}$$

$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{16\sqrt{2}c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2c^2\left(\sqrt{c-acx}(-1037+269ax-87a^2x^2+15a^3x^3)+840\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
[Out] (2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*S
qrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{2\left(56\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\left(15a^{3}x^{3}-87a^{2}x^{2}+269ax-1037\right)\sqrt{-c(ax-1)}}{15}\right)c^{2}}{7a}$	71
risch	$-\frac{2(15a^{3}x^{3}-87a^{2}x^{2}+269ax-1037)(ax-1)c^{3}}{105a\sqrt{-c(ax-1)}}+\frac{16c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$	76
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c^{2}(-acx+c)^{\frac{3}{2}}}{3}+8c^{3}\sqrt{-acx+c}-8c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	87
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}-\frac{8c^{2}(-acx+c)^{\frac{3}{2}}}{3}-16c^{3}\sqrt{-acx+c}+16c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	87

```
[In] int((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(56*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+1/1
5*(15*a^3*x^3-87*a^2*x^2+269*a*x-1037)*(-c*(a*x-1))^(1/2))*c^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.57

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\left[\frac{2\left(420\,\sqrt{2}c^{\frac{5}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+(15\,a^{3}c^{2}x^{3}-87\,a^{2}c^{2}x^{2}+269\,ac^{2}x-1037\,c^{2})\sqrt{-\cdots}\right.}{105\,a}\right.$$
$$\left.-\frac{2\left(840\,\sqrt{2}\sqrt{-c}c^{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-(15\,a^{3}c^{2}x^{3}-87\,a^{2}c^{2}x^{2}+269\,ac^{2}x-1037\,c^{2})\sqrt{-acx+c}\right)}{105\,a}\right]$$

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 1037*
c^2)*sqrt(-a*c*x + c))/a, -2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*
c^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

Sympy [A] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{8\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+8c^3\sqrt{-acx+c}+\frac{4c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{5}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*c*x+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((-2*(8*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 8*c**3*sqrt(-a*c*x + c) + 4*c**2*(-a*c*x + c)**(3/2)/3 + 2*c*(-a*
c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a*c), Ne(a*c, 0)), (c**(5/2)*Pi
ecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2\left(420\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+15(-acx+c)^{\frac{7}{2}}+42(-acx+c)^{\frac{5}{2}}c+140(-acx+c)^{\frac{3}{2}}c^2+840\sqrt{-acx+c}\,c^3\right)}{105\,ac}$$

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/105*(420*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 15*(-a*c*x + c)^(7/2) + 42*(-a*c*x + c)^
(5/2)*c + 140*(-a*c*x + c)^(3/2)*c^2 + 840*sqrt(-a*c*x + c)*c^3)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{16\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} + \frac{2\left(15\,(acx-c)^3\sqrt{-acx+c}a^6c^6 - 42\,(acx-c)^2\sqrt{-acx+c}a^6c^7 - 140\,(-acx+c)^{\frac{3}{2}}a^6c^8 - 840\sqrt{-acx+c}a^6c^9\right)}{105\,a^7c^7}$$

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
+ 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqrt(
-a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x + c)
*a^6*c^9)/(a^7*c^7)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{4\,(c-a\,c\,x)^{5/2}}{5\,a} - \frac{8\,c\,(c-a\,c\,x)^{3/2}}{3\,a} - \frac{16\,c^2\,\sqrt{c-a\,c\,x}}{a} - \frac{2\,(c-a\,c\,x)^{7/2}}{7\,a\,c} - \frac{\sqrt{2}\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,16\mathrm{i}}{a}$$

```
[In] int(((c - a*c*x)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(5/2))/(5*a) - (8*c*(c - a*c*x)^(3/2))/(3*a) - (16*c^2*(c
- a*c*x)^(1/2))/a - (2*(c - a*c*x)^(7/2))/(7*a*c) - (2^(1/2)*c^(5/2)*atan((
2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*16i)/a
```

3.263 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{8c\sqrt{c-acx}}{a}-\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}+\frac{8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c+8*c^(3/2)*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-8*c*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}-\frac{8c\sqrt{c-acx}}{a}$$

```
[In] Int[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-8*c*Sqrt[c - a*c*x])/a - (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/
2))/(5*a*c) + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}(c-acx)^{3/2}\,dx$$

$$= -\int \frac{(1-ax)(c-acx)^{3/2}}{1+ax}\,dx$$

$$= -\frac{\int \frac{(c-acx)^{5/2}}{1+ax}\,dx}{c}$$

$$= -\frac{2(c-acx)^{5/2}}{5ac} - 2\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$

$$= -\frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} - (4c)\int \frac{\sqrt{c-acx}}{1+ax}\,dx$$

$$= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} - (8c^2)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$

$$= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{(16c)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}$$

$$= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{-2c\sqrt{c-acx}(73-16ax+3a^2x^2)+120\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a}$$

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
[Out] (-2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) + 120*Sqrt[2]*c^(3/2)*ArcTa
nh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{2\left(-20\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\left(3a^{2}x^{2}-16ax+73\right)\sqrt{-c(ax-1)}}{3}\right)c}{5a}$	61
risch	$\frac{2(3a^2x^2-16ax+73)(ax-1)c^2}{15a\sqrt{-c(ax-1)}}+\frac{8c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$	68
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	73
default	$\frac{-\frac{2(-acx+c)^{\frac{5}{2}}}{5}-\frac{4c(-acx+c)^{\frac{3}{2}}}{3}-8c^{2}\sqrt{-acx+c}+8c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	73

```
[In] int((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-20*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+1
/3*(3*a^2*x^2-16*a*x+73)*(-c*(a*x-1))^(1/2))*c/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\left[\frac{2\left(30\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-(3a^2cx^2-16acx+73c)\sqrt{-acx+c}\right)}{15a},\ -\frac{2\left(60\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)+(3a^2cx^2-16acx+73c)\sqrt{-acx+c}\right)}{15a}\right]$$

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, -2/
15*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) +
 (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{4\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+4c^2\sqrt{-acx+c}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*c*x+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((-2*(4*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 4*c**2*sqrt(-a*c*x + c) + 2*c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c
)**(5/2)/5)/(a*c), Ne(a*c, 0)), (c**(3/2)*Piecewise((-x, Eq(a, 0)), ((a*x -
 2*log(a*x + 1) + 1)/a, True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(30\sqrt{2}c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{5}{2}}+10(-acx+c)^{\frac{3}{2}}c+60\sqrt{-acx+c}c^2\right)}{15\,ac}$$

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/15*(30*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 10*(-a*c*x + c)^(3/
2)*c + 60*sqrt(-a*c*x + c)*c^2)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}a^4c^4 + 10\,(-acx+c)^{\frac{3}{2}}a^4c^5 + 60\,\sqrt{-acx+c}a^4c^6\right)}{15\,a^5c^5}$$

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a^4
*c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{4\,{(c-a\,c\,x)}^{3/2}}{3\,a} - \frac{8\,c\,\sqrt{c-a\,c\,x}}{a} - \frac{2\,{(c-a\,c\,x)}^{5/2}}{5\,a\,c} - \frac{\sqrt{2}\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,8\mathrm{i}}{a}$$

```
[In] int(((c - a*c*x)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^
(5/2))/(5*a*c) - (2^(1/2)*c^(3/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*8i)/a
```

3.264 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\operatorname{arctanh}(ax)}\sqrt{c-acx}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{3/2}}{3ac} - 2\int \frac{\sqrt{c-acx}}{1+ax}\,dx
\end{aligned}$$

$$= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} - (4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$

$$= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}$$

$$= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2(-7+ax)\sqrt{c-acx} + 12\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}} + \frac{4\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}-4c\sqrt{-acx+c}+4c^{\frac{3}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	59
pseudoelliptic	$\frac{4\sqrt{c}\,\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2ax\sqrt{-c(ax-1)}}{3}-\frac{14\sqrt{-c(ax-1)}}{3}}{a}$	59

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -2/3*(a*x-7)*(a*x-1)/a/(-c*(a*x-1))^(1/2)*c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((-2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c, 0)),
 (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}c\right)}{3\,ac}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(
a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3\,a^3c^3}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c} - \frac{4\sqrt{c-a\,c\,x}}{a}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*(
c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

3.265 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac}+\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)-2*(-a*c*x
+c)^(1/2)/a/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-acx}}\,dx\\ &= -\int \frac{1-ax}{(1+ax)\sqrt{c-acx}}\,dx\\ &= -\frac{\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{c}\\ &= -\frac{2\sqrt{c-acx}}{ac}-2\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\end{aligned}$$

$$= -\frac{2\sqrt{c-acx}}{ac} + \frac{4\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac}$$

$$= -\frac{2\sqrt{c-acx}}{ac} + \frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac} + \frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}-\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}\right)}{ca}$	45
default	$\frac{-2\sqrt{-acx+c}+2\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{ac}$	46
pseudoelliptic	$\frac{-2\sqrt{-c(ax-1)}+2\sqrt{c}\,\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	48
risch	$\frac{2ax-2}{a\sqrt{-c(ax-1)}} + \frac{2\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\sqrt{c}}$	51

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)*c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)-2\sqrt{-acx+c}}{ac}, \frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x +
 1)) - 2*sqrt(-a*c*x + c))/(a*c), 2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*sq
rt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \begin{cases} -\frac{2\left(\frac{\sqrt{2}c\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\sqrt{-acx+c}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((-2*(sqrt(2)*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-
c) + sqrt(-a*c*x + c))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x
 - 2*log(a*x + 1) + 1)/a, True))/sqrt(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+2\sqrt{-acx+c}}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c))) + 2*sqrt(-a*c*x + c))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\sqrt{-acx+c}}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2*s
qrt(-a*c*x + c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{c-a\,c\,x}}{a\,c}$$

```
[In] int((a*x - 1)/((c - a*c*x)^(1/2)*(a*x + 1)),x)
```

```
[Out] (2*2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(1/2)) - (2
*(c - a*c*x)^(1/2))/(a*c)
```

3.266 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6265, 21, 65, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{3/2}}\,dx\\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{3/2}}\,dx\\
&= -\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{c}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^2}\\
&= \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$	29
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$	29
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{c^{\frac{3}{2}}\,a}$	30

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.38

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}}, \frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] [1/2*sqrt(2)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))/
(a*c^(3/2)), sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c)/
(a*x - 1))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \begin{cases} -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(3/2),x)
[Out] Piecewise((-sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(-
c)), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a,
 True))/c**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{2\,ac^{\frac{3}{2}}}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + s
qrt(-a*c*x + c)))/(a*c^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a\,c^{3/2}}$$

```
[In] int((a*x - 1)/((c - a*c*x)^(3/2)*(a*x + 1)),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(3/2))
```

3.267 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{1}{ac^2\sqrt{c-acx}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

```
[Out] 1/2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-1/a/c^2
/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 53, 65, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}-\frac{1}{ac^2\sqrt{c-acx}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(1/(a*c^2*Sqrt[c - a*c*x])) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(
Sqrt[2]*a*c^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{5/2}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{5/2}}\,dx \\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{c} \\
&= -\frac{1}{ac^2\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{2c^2}
\end{aligned}$$

$$= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^3}$$

$$= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x]))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}+\frac{1}{2c\sqrt{-acx+c}}\right)}{ca}$	50
default	$\frac{-\frac{1}{c\sqrt{-acx+c}}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{2c^{\frac{3}{2}}}}{ac}$	50
pseudoelliptic	$\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax-1)}-2\sqrt{c}}{2c^{\frac{5}{2}}\sqrt{-c(ax-1)}\,a}$	58

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*(-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))+
1/2/c/(-a*c*x+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)},\right.$$
$$\left.-\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-2\sqrt{-acx+c}}{2\left(a^2c^3x-ac^3\right)}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), -1/2*(sqr
t(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2
*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{2c\sqrt{-acx+c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4c\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*(1/(2*c*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x
+ c)/(2*sqrt(-c)))/(4*c*sqrt(-c)))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a
, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True))/c**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}+\frac{4}{\sqrt{-acx+c}c}}{4\,ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
sqrt(-a*c*x + c)))/c^(3/2) + 4/(sqrt(-a*c*x + c)*c))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2\,a\sqrt{-c}c^2} - \frac{1}{\sqrt{-acx+c}ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
 - 1/(sqrt(-a*c*x + c)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{2\,a\,c^{5/2}} - \frac{1}{a\,c^2\,\sqrt{c-a\,c\,x}}$$

```
[In] int((a*x - 1)/((c - a*c*x)^(5/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(2*a*c^(5/2)) - 1/
(a*c^2*(c - a*c*x)^(1/2))
```

3.268 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

```
[Out] -1/3/a/c^2/(-a*c*x+c)^(3/2)+1/4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2
))/a/c^(7/2)*2^(1/2)-1/2/a/c^3/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 53, 65, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{1}{3ac^2(c-acx)^{3/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/3*1/(a*c^2*(c - a*c*x)^(3/2)) - 1/(2*a*c^3*Sqrt[c - a*c*x]) + ArcTanh[Sq
rt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}}{(c-acx)^{7/2}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{7/2}}\,dx \\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{c} \\
&= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{2c^2}
\end{aligned}$$

$$= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{4c^3}$$

$$= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{2ac^4}$$

$$= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} + \frac{\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(3/2)
)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}+\frac{1}{4c^2\sqrt{-acx+c}}+\frac{1}{6c(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	64
default	$\frac{-\frac{1}{2c^2\sqrt{-acx+c}}-\frac{1}{3c(-acx+c)^{\frac{3}{2}}}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{ac}$	64
pseudoelliptic	$\frac{\sqrt{2}\,\sqrt{-c(ax-1)}\,(ax-1)\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\left(-2ax+\frac{10}{3}\right)}{4c^{\frac{7}{2}}\sqrt{-c(ax-1)}\,(ax-1)a}$	75

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*(-1/8/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))+
1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.36

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+4\sqrt{-acx+c}(3\,ax-5)}{24\,(a^3c^4x^2-2\,a^2c^4x+ac^4)},\right.$$
$$\left.-\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-2\sqrt{-acx+c}(3\,ax-5)}{12\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(a^
3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)
*(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{6c(-acx+c)^{\frac{3}{2}}}+\frac{1}{4c^2\sqrt{-acx+c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8c^2\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log{(ax+1)}+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Piecewise((-2*(1/(6*c*(-a*c*x + c)**(3/2)) + 1/(4*c**2*sqrt(-a*c*x + c)) +
sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(8*c**2*sqrt(-c)))/(a*c
), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, T
rue))/c**(7/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}} - \frac{4\,(3\,acx-5\,c)}{(-acx+c)^{\frac{3}{2}}c^2}}{24\,ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c)
 + sqrt(-a*c*x + c)))/c^(5/2) - 4*(3*a*c*x - 5*c)/((-a*c*x + c)^(3/2)*c^2))
/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}c^3} - \frac{3\,acx-5\,c}{6\,(acx-c)\sqrt{-acx+c}ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3)
 - 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{4\,a\,c^{7/2}} - \frac{\frac{c-a\,c\,x}{2\,c^2}+\frac{1}{3\,c}}{a\,c\,{\left(c-a\,c\,x\right)}^{3/2}}$$

```
[In] int((a*x - 1)/((c - a*c*x)^(7/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(4*a*c^(7/2)) - ((
c - a*c*x)/(2*c^2) + 1/(3*c))/(a*c*(c - a*c*x)^(3/2))
```

3.269 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

```
[Out] -1/5/a/c^2/(-a*c*x+c)^(5/2)-1/6/a/c^3/(-a*c*x+c)^(3/2)+1/8*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-1/4/a/c^4/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 53, 65, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}} - \frac{1}{4ac^4\sqrt{c-acx}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{5ac^2(c-acx)^{5/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -1/5*1/(a*c^2*(c - a*c*x)^(5/2)) - 1/(6*a*c^3*(c - a*c*x)^(3/2)) - 1/(4*a*c
^4*Sqrt[c - a*c*x]) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]
*a*c^(9/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}}{(c-acx)^{9/2}}\,dx\\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{9/2}}\,dx\\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{7/2}}\,dx}{c}
\end{aligned}$$

$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{2c^2}$$

$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{4c^3}$$

$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{8c^4}$$

$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{4ac^5}$$

$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, \frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(5/2)
)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}+\frac{1}{8c^{3}\sqrt{-acx+c}}+\frac{1}{12c^{2}(-acx+c)^{\frac{3}{2}}}+\frac{1}{10c(-acx+c)^{\frac{5}{2}}}\right)}{ca}$	78
default	$\frac{-\frac{1}{4c^{3}\sqrt{-acx+c}}-\frac{1}{6c^{2}(-acx+c)^{\frac{3}{2}}}-\frac{1}{5c(-acx+c)^{\frac{5}{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{7}{2}}}}{ac}$	78
pseudoelliptic	$\frac{\sqrt{2}\,\sqrt{-c(ax-1)}\,(ax-1)^{2}\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-2\left(a^{2}x^{2}-\frac{8}{3}ax+\frac{37}{15}\right)\sqrt{c}}{8c^{\frac{9}{2}}\sqrt{-c(ax-1)}\,(ax-1)^{2}a}$	85

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c)^(5/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \left[\frac{15\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+4\,(15\,a^2x^2-40\,ax+}{240\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}\right.$$
$$\left.-\frac{15\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-2\,(15\,a^2x^2-40\,ax+37)\sqrt{-acx+c}}{120\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*(15*a^2*x^2 - 40*a*
x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*sqrt(
-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{10c(-acx+c)^{\frac{5}{2}}}+\frac{1}{12c^2(-acx+c)^{\frac{3}{2}}}+\frac{1}{8c^3\sqrt{-acx+c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{16c^3\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(9/2),x)
```

```
[Out] Piecewise((-2*(1/(10*c*(-a*c*x + c)**(5/2)) + 1/(12*c**2*(-a*c*x + c)**(3/2
)) + 1/(8*c**3*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2
*sqrt(-c)))/(16*c**3*sqrt(-c)))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0
)), ((a*x - 2*log(a*x + 1) + 1)/a, True))/c**(9/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}+\frac{4\left(15\,(acx-c)^2-10\,(acx-c)c+12\,c^2\right)}{(-acx+c)^{\frac{5}{2}}c^3}}{240\,ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c)))/c^(7/2) + 4*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c +
12*c^2)/((-a*c*x + c)^(5/2)*c^3))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8\,a\sqrt{-c}c^4}-\frac{15\,(acx-c)^2-10\,(acx-c)c+12\,c^2}{60\,(acx-c)^2\sqrt{-acx+c}ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4)
 - 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqrt(
-a*c*x + c)*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{8\,a\,c^{9/2}}-\frac{\frac{c-a\,c\,x}{6\,c^2}+\frac{1}{5\,c}+\frac{(c-a\,c\,x)^2}{4\,c^3}}{a\,c\,(c-a\,c\,x)^{5/2}}$$

```
[In] int((a*x - 1)/((c - a*c*x)^(9/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(8*a*c^(9/2)) - ((
c - a*c*x)/(6*c^2) + 1/(5*c) + (c - a*c*x)^2/(4*c^3))/(a*c*(c - a*c*x)^(5/2
))
```

3.270 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 368

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{94208(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^5} - \frac{40960(c-acx)^{9/2}}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^4} + \frac{4096(c-acx)^{9/2}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^3} - \frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^2} + \frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^6x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -16/33*(a-1/x)^5*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)-94208
/231*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/x^5/(1+1/a/x)^(1/2)-40960/231*(-a
*c*x+c)^(9/2)/a^5/(1-1/a/x)^(9/2)/x^4/(1+1/a/x)^(1/2)+4096/231*(-a*c*x+c)^(
9/2)/a^4/(1-1/a/x)^(9/2)/x^3/(1+1/a/x)^(1/2)-1024/231*(a-1/x)^3*(-a*c*x+c)^
(9/2)/a^6/(1-1/a/x)^(9/2)/x^2/(1+1/a/x)^(1/2)+320/231*(a-1/x)^4*(-a*c*x+c)^
(9/2)/a^6/(1-1/a/x)^(9/2)/x/(1+1/a/x)^(1/2)+2/11*(a-1/x)^6*x*(-a*c*x+c)^(9/
2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{94208(c-acx)^{9/2}}{231a^6x^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}} - \frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6x^2\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}} + \frac{2x\left(a-\frac{1}{x}\right)^6(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}} - \frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}} + \frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6x\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}} - \frac{40960(c-acx)^{9/2}}{231a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}} + \frac{4096(c-acx)^{9/2}}{231a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-16*(a - x^(-1))^5*(c - a*c*x)^(9/2))/(33*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 +
 1/(a*x)]) - (94208*(c - a*c*x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1
+ 1/(a*x)]*x^5) - (40960*(c - a*c*x)^(9/2))/(231*a^5*(1 - 1/(a*x))^(9/2)*Sq
rt[1 + 1/(a*x)]*x^4) + (4096*(c - a*c*x)^(9/2))/(231*a^4*(1 - 1/(a*x))^(9/2
)*Sqrt[1 + 1/(a*x)]*x^3) - (1024*(a - x^(-1))^3*(c - a*c*x)^(9/2))/(231*a^6
*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x^2) + (320*(a - x^(-1))^4*(c - a*c*
x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1
))^6*x*(c - a*c*x)^(9/2))/(11*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^6}{x^{13/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(24\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(160\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{33a^2\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(2560\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2048\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{77a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{4096(c-acx)^{9/2}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^3}-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(4096\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5 (c-acx)^{9/2}}{33a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}} - \frac{40960(c-acx)^{9/2}}{231a^5 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^4} + \frac{4096(c-acx)^{9/2}}{231a^4 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^3} - \frac{1024\left(a-\frac{1}{x}\right)^3 (c-acx)^{9/2}}{231a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^2} + \frac{320\left(a-\frac{1}{x}\right)^4 (c-acx)^{9/2}}{231a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}} - \frac{\left(47104\left(\frac{1}{x}\right)^{9/2} (c-acx)^{9/2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}} \, dx, x, \frac{1}{x}\right)}{231a^6 \left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5 (c-acx)^{9/2}}{33a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}} - \frac{94208(c-acx)^{9/2}}{231a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^5} - \frac{40960(c-acx)^{9/2}}{231a^5 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^4} + \frac{4096(c-acx)^{9/2}}{231a^4 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^3} - \frac{1024\left(a-\frac{1}{x}\right)^3 (c-acx)^{9/2}}{231a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x^2} + \frac{320\left(a-\frac{1}{x}\right)^4 (c-acx)^{9/2}}{231a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6 \left(1-\frac{1}{ax}\right)^{9/2} \sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2} \, dx = \frac{2c^4\sqrt{c-acx}(-46355-23062ax+5419a^2x^2-2132a^3x^3+755a^4x^4-182a^5x^5+21a^6x^6)}{231a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
[Out] (2*c^4*Sqrt[c - a*c*x]*(-46355 - 23062*a*x + 5419*a^2*x^2 - 2132*a^3*x^3 +
755*a^4*x^4 - 182*a^5*x^5 + 21*a^6*x^6))/(231*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.24

method	result	size
gosper	$\frac{2(ax+1)(21a^6x^6-182a^5x^5+755a^4x^4-2132a^3x^3+5419a^2x^2-23062ax-46355)(-acx+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{231a(ax-1)^6}$	88
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^4(21a^6x^6-182a^5x^5+755a^4x^4-2132a^3x^3+5419a^2x^2-23062ax-46355)}{231(ax-1)^2a}$	92
risch	$-\frac{2(21a^5x^5-203a^4x^4+958a^3x^3-3090a^2x^2+8509ax-31571)(ax+1)c^5\sqrt{\frac{ax-1}{ax+1}}}{231a\sqrt{-c(ax-1)}}+\frac{128c^5\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	111

[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/231*(a*x+1)*(21*a^6*x^6-182*a^5*x^5+755*a^4*x^4-2132*a^3*x^3+5419*a^2*x^2-23062*a*x-46355)*(-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^6

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(21\,a^6c^4x^6-182\,a^5c^4x^5+755\,a^4c^4x^4-2132\,a^3c^4x^3+5419\,a^2c^4x^2-23062\,ac^4x-46355\,c^4\right.}{231\,(a^2x-a)}$$

[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 2/231*(21*a^6*c^4*x^6 - 182*a^5*c^4*x^5 + 755*a^4*c^4*x^4 - 2132*a^3*c^4*x^3 + 5419*a^2*c^4*x^2 - 23062*a*c^4*x - 46355*c^4)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(21\,a^7\sqrt{-c}c^4x^7-161\,a^6\sqrt{-c}c^4x^6+573\,a^5\sqrt{-c}c^4x^5-1377\,a^4\sqrt{-c}c^4x^4+3287\,a^3\sqrt{-c}c^4x^3-\cdots\right.}{231\,(a^3x^2-2\,a^2x+a)(ax+1)\cdots}$$

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*a^7*sqrt(-c)*c^4*x^7 - 161*a^6*sqrt(-c)*c^4*x^6 + 573*a^5*sqrt(-c
)*c^4*x^5 - 1377*a^4*sqrt(-c)*c^4*x^4 + 3287*a^3*sqrt(-c)*c^4*x^3 - 17643*a
^2*sqrt(-c)*c^4*x^2 - 69417*a*sqrt(-c)*c^4*x - 46355*sqrt(-c)*c^4)*(a*x - 1
)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(21\,a^5\,x^5-161\,a^4\,x^4+594\,a^3\,x^3-1538\,a^2\,x^2+3881\,a\,x-19181\right)}{231\,a} - \frac{131072\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{231\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3881*a*x - 1538*a^2*x
^2 + 594*a^3*x^3 - 161*a^4*x^4 + 21*a^5*x^5 - 19181))/(231*a) - (131072*c^4
*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(231*a*(a*x - 1))
```

3.271 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 311

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^4}+\frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}-\frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -40/63*(a-1/x)^4*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+11776
/63*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/x^4/(1+1/a/x)^(1/2)+5120/63*(-a*c*
x+c)^(7/2)/a^4/(1-1/a/x)^(7/2)/x^3/(1+1/a/x)^(1/2)-512/63*(-a*c*x+c)^(7/2)/
a^3/(1-1/a/x)^(7/2)/x^2/(1+1/a/x)^(1/2)+128/63*(a-1/x)^3*(-a*c*x+c)^(7/2)/a
^5/(1-1/a/x)^(7/2)/x/(1+1/a/x)^(1/2)+2/9*(a-1/x)^5*x*(-a*c*x+c)^(7/2)/a^5/(
1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{11776(c-acx)^{7/2}}{63a^5x^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} + \frac{2x\left(a-\frac{1}{x}\right)^5(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} - \frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} + \frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5x\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} + \frac{5120(c-acx)^{7/2}}{63a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} - \frac{512(c-acx)^{7/2}}{63a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-40*(a - x^(-1))^4*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]) + (11776*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]*x^4) + (5120*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*x))^(7/2)*Sqrt[
1 + 1/(a*x)]*x^3) - (512*(c - a*c*x)^(7/2))/(63*a^3*(1 - 1/(a*x))^(7/2)*Sqr
t[1 + 1/(a*x)]*x^2) + (128*(a - x^(-1))^3*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1
/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1))^5*x*(c - a*c*x)^(7/2))
/(9*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(20\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(320\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{63a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(256\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{21a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(512\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}-\frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(5888\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^4}+\frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}-\frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.24

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{c-acx}(5797+2867ax-638a^2x^2+214a^3x^3-55a^4x^4+7a^5x^5)}{63a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
[Out] (-2*c^3*Sqrt[c - a*c*x]*(5797 + 2867*a*x - 638*a^2*x^2 + 214*a^3*x^3 - 55*a
^4*x^4 + 7*a^5*x^5))/(63*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

method	result	size
gosper	$\frac{2(ax+1)(7a^5x^5-55a^4x^4+214a^3x^3-638a^2x^2+2867ax+5797)(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{63a(ax-1)^5}$	80
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^3(7a^5x^5-55a^4x^4+214a^3x^3-638a^2x^2+2867ax+5797)}{63(ax-1)^2a}$	84
risch	$\frac{2(7a^4x^4-62a^3x^3+276a^2x^2-914ax+3781)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{63a\sqrt{-c(ax-1)}}+\frac{64c^4\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	103

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/63*(a*x+1)*(7*a^5*x^5-55*a^4*x^4+214*a^3*x^3-638*a^2*x^2+2867*a*x+5797)*(
-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(7\,a^5c^3x^5-55\,a^4c^3x^4+214\,a^3c^3x^3-638\,a^2c^3x^2+2867\,ac^3x+5797\,c^3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{63\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^5*c^3*x^5 - 55*a^4*c^3*x^4 + 214*a^3*c^3*x^3 - 638*a^2*c^3*x^2 +
 2867*a*c^3*x + 5797*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(7\,a^{6}\sqrt{-c}c^{3}x^{6}-48\,a^{5}\sqrt{-c}c^{3}x^{5}+159\,a^{4}\sqrt{-c}c^{3}x^{4}-424\,a^{3}\sqrt{-c}c^{3}x^{3}+2229\,a^{2}\sqrt{-c}c^{3}x^{2}+8664\,a\sqrt{-c}c^{3}x\right.}{63\,(a^{3}x^{2}-2\,a^{2}x+a)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*a^6*sqrt(-c)*c^3*x^6 - 48*a^5*sqrt(-c)*c^3*x^5 + 159*a^4*sqrt(-c)*
c^3*x^4 - 424*a^3*sqrt(-c)*c^3*x^3 + 2229*a^2*sqrt(-c)*c^3*x^2 + 8664*a*sqr
t(-c)*c^3*x + 5797*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x
+ 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.33

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(7\,a^4\,x^4-48\,a^3\,x^3+166\,a^2\,x^2-472\,a\,x+2395\right)}{63\,a} - \frac{16384\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{63\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(166*a^2*x^2 - 472*a
*x - 48*a^3*x^3 + 7*a^4*x^4 + 2395))/(63*a) - (16384*c^3*(c - a*c*x)^(1/2)*
((a*x - 1)/(a*x + 1))^(1/2))/(63*a*(a*x - 1))
```

3.272 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 254

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{2944(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3}-\frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -32/35*(a-1/x)^3*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-2944/
35*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/x^3/(1+1/a/x)^(1/2)-256/7*(-a*c*x+c
)^(5/2)/a^3/(1-1/a/x)^(5/2)/x^2/(1+1/a/x)^(1/2)+128/35*(-a*c*x+c)^(5/2)/a^2
/(1-1/a/x)^(5/2)/x/(1+1/a/x)^(1/2)+2/7*(a-1/x)^4*x*(-a*c*x+c)^(5/2)/a^4/(1-
1/a/x)^(5/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{6311, 6316, 96, 91, 79, 37\}$

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2944(c-acx)^{5/2}}{35a^4x^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}} + \frac{2x\left(a-\frac{1}{x}\right)^4(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}} - \frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}} - \frac{256(c-acx)^{5/2}}{7a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}} + \frac{128(c-acx)^{5/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-32*(a - x^(-1))^3*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]) - (2944*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
1/(a*x)]*x^3) - (256*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]*x^2) + (128*(c - a*c*x)^(5/2))/(35*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1
 + 1/(a*x)]*x) + (2*(a - x^(-1))^4*x*(c - a*c*x)^(5/2))/(7*a^4*(1 - 1/(a*x)
)^(5/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1]))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-acx)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}\\
&\quad-\frac{\left(192\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}}
\end{aligned}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}}{35a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}} + \frac{128(c-acx)^{5/2}}{35a^2 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^4 x(c-acx)^{5/2}}{7a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}} - \frac{\left(128\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}\right) \text{Subst}\left(\int \frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}} \, dx, x, \frac{1}{x}\right)}{35a^2 \left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}}{35a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}} - \frac{256(c-acx)^{5/2}}{7a^3 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x^2} + \frac{128(c-acx)^{5/2}}{35a^2 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^4 x(c-acx)^{5/2}}{7a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}} - \frac{\left(1472\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}} \, dx, x, \frac{1}{x}\right)}{35a^4 \left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}}{35a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}} - \frac{2944(c-acx)^{5/2}}{35a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x^3} - \frac{256(c-acx)^{5/2}}{7a^3 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x^2} + \frac{128(c-acx)^{5/2}}{35a^2 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^4 x(c-acx)^{5/2}}{7a^4 \left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2} \, dx = \frac{2c^2\sqrt{c-acx}(-1451-708ax+142a^2x^2-36a^3x^3+5a^4x^4)}{35a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*Sqrt[c - a*c*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4*
x^4))/(35*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(ax+1)(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a(ax-1)^4}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^2(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)}{35(ax-1)^2a}$	76
risch	$-\frac{2(5a^3x^3-41a^2x^2+183ax-891)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{35a\sqrt{-c(ax-1)}}+\frac{32c^3\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	95

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*(a*x+1)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-708*a*x-1451)*(-a*c*x+c)^(5/
2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,(5\,a^4c^2x^4-36\,a^3c^2x^3+142\,a^2c^2x^2-708\,ac^2x-1451\,c^2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 1451
*c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.47

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\left(5\,a^5\sqrt{-c}c^2x^5-31\,a^4\sqrt{-c}c^2x^4+106\,a^3\sqrt{-c}c^2x^3-566\,a^2\sqrt{-c}c^2x^2-2159\,a\sqrt{-c}c^2x-145\right.}{35\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^5*sqrt(-c)*c^2*x^5 - 31*a^4*sqrt(-c)*c^2*x^4 + 106*a^3*sqrt(-c)*c
^2*x^3 - 566*a^2*sqrt(-c)*c^2*x^2 - 2159*a*sqrt(-c)*c^2*x - 1451*sqrt(-c)*c
^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.37

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-31\,a^2\,x^2+111\,a\,x-597\right)}{35\,a} - \frac{4096\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(111*a*x - 31*a^2*x^2
+ 5*a^3*x^3 - 597))/(35*a) - (4096*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x +
1))^(1/2))/(35*a*(a*x - 1))
```

3.273 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -8/5*(-a*c*x+c)^(3/2)/a/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+184/5*(-a*c*x+c)^(3
/2)/a^3/(1-1/a/x)^(3/2)/x^2/(1+1/a/x)^(1/2)+16*(-a*c*x+c)^(3/2)/a^2/(1-1/a/
x)^(3/2)/x/(1+1/a/x)^(1/2)+2/5*(a-1/x)^3*x*(-a*c*x+c)^(3/2)/a^3/(1-1/a/x)^(
3/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{184(c-acx)^{3/2}}{5a^3x^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^3(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{16(c-acx)^{3/2}}{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-8*(c - a*c*x)^(3/2))/(5*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + (184*(
c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x^2) + (16*(
c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x
^(-1))^3*x*(c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(12\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(8\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(92\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2} + \frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{c-acx}(91+43ax-7a^2x^2+a^3x^3)}{5a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

[In] Integrate[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]

[Out] (-2*c*Sqrt[c - a*c*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2(ax+1)(a^3x^3-7a^2x^2+43ax+91)(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{5a(ax-1)^3}$	63
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c(a^3x^3-7a^2x^2+43ax+91)}{5(ax-1)^2a}$	65
risch	$\frac{2(a^2x^2-8ax+51)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{5a\sqrt{-c(ax-1)}}+\frac{16c^2\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	86

[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/5*(a*x+1)*(a^3*x^3-7*a^2*x^2+43*a*x+91)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^3

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.32

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^3cx^3-7\,a^2cx^2+43\,acx+91\,c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^2x-a\right)}$$

[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] -2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(a^4\sqrt{-c}cx^4-6\,a^3\sqrt{-c}cx^3+36\,a^2\sqrt{-c}cx^2+134\,a\sqrt{-c}cx+91\,\sqrt{-c}c\right)(ax-1)^2}{5\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^4*sqrt(-c)*c*x^4 - 6*a^3*sqrt(-c)*c*x^3 + 36*a^2*sqrt(-c)*c*x^2 + 1
34*a*sqrt(-c)*c*x + 91*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2-6\,a\,x+37\right)}{5\,a}-\frac{256\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a^2*x^2 - 6*a*x + 37)
)/(5*a) - (256*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x -
 1))
```

3.274 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(1
/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 91, 79, 37}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a(ax-1)^{2}}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,\left(a^{2}x^{2}-10ax-23\right)}{3(ax-1)^{2}a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a/
(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^{2}x^{2}-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{2}x-a\right)}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^3\sqrt{-c}x^3 - 9\,a^2\sqrt{-c}x^2 - 33\,a\sqrt{-c}x - 23\,\sqrt{-c}\right)(ax-1)^2}{3\left(a^3x^2 - 2\,a^2x + a\right)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,(a\,x-9)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{64\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.275 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{2\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

```
[Out] 6*(1-1/a/x)^(1/2)/a/(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)+2*x*(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 79, 37}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}+\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1
/(a*x)]*x)/(Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int\frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}\\
&= -\frac{\sqrt{1-\frac{1}{ax}}\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}\\
&= \frac{2\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}\\
&= \frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{2\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}(3+ax)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
[Out] (2*Sqrt[1 - 1/(a*x)]*(3 + a*x))/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{2(ax+1)(ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(ax-1)\sqrt{-acx+c}}$	47
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(ax+3)}{(ax-1)^{2}ca}$	51
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c(ax-1)}\,a}+\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	67

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(a*x+1)*(a*x+3)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)/(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=-\frac{2\sqrt{-acx+c}(ax+3)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}cx-ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x + 3)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\left(a^2x^2+4\,ax+3\right)(ax-1)}{\left(a^2\sqrt{-c}x-a\sqrt{-c}\right)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2*(a^2*x^2 + 4*a*x + 3)*(a*x - 1)/((a^2*sqrt(-c)*x - a*sqrt(-c))*(a*x + 1)^
(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = 2\left(\frac{\sqrt{-acx-c}}{ac^2}-\frac{2}{\sqrt{-acx-c}ac}\right)|c|$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(-a*c*x - c)/(a*c^2) - 2/(sqrt(-a*c*x - c)*a*c))*abs(c)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{\left(2\,x+\frac{6}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(1/2),x)
[Out] ((2*x + 6/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

3.276 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

```
[Out] -2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6309}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}x}{\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
[Out] (-2*(1 - 1/(a*x))^(3/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(-acx+c)^{\frac{3}{2}}}$	35
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}}{(ax-1)^{2}c^{2}a}$	46

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)(ax-1)}{(a^2c^2x-ac^2)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))*(a*x - 1)/((a^2*c^2*x - a*c^2)*(a*x + 1)^(3/2)
)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\frac{\sqrt{2}}{a\sqrt{-c}}-\frac{2}{\sqrt{-acx-c}a}\right)|c|\text{sgn}(ax+1)}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] (sqrt(2)/(a*sqrt(-c)) - 2/(sqrt(-a*c*x - c)*a))*abs(c)*sgn(a*x + 1)/c^2
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c\,\sqrt{c-a\,c\,x}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(3/2),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c*(c - a*c*x)^(1/2))
```

3.277 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] -1/2*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^
(1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+a*(1-1/a/x)^(5/2)*x^2/(-a*c*x+c
)^(5/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{ax^2\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (a*(1 - 1/(a*x))^(5/2)*x^2)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2)) - (a^(3/2
)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])])/(Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}} - \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}-\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2ac^2\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)] - Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*ArcT
anh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*a*c^2*Sqrt[1 +
 1/(a*x)]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c(ax+1)}+2\sqrt{c}\right)}{2(ax-1)^2c^{\frac{7}{2}}a}$	85

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)/c^(7/2)*(
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)+2
*c^(1/2))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}\sqrt{}}{4\,(a^2c^3x-ac^3)}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] [-1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
- 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x -
a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*sqrt((a*x -
 1)/(a*x + 1)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a\,c\,x)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2), x)
```

3.278 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] 3/8*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(7/2)*x^2/(a-1
/x)/(-a*c*x+c)^(7/2)/(1+1/a/x)^(1/2)-3/4*a^2*(1-1/a/x)^(7/2)*x^3/(-a*c*x+c)
^(7/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}} - \frac{3a^2x^3\left(1-\frac{1}{ax}\right)^{7/2}}{4\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}} - \frac{a^2x^2\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a*x))^(7/2)*x^2)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - a*c
*x)^(7/2)) - (3*a^2*(1 - 1/(a*x))^(7/2)*x^3)/(4*Sqrt[1 + 1/(a*x)]*(c - a*c*
x)^(7/2)) + (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right) \int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\, dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2} \operatorname{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^2 \left(1+\frac{x}{a}\right)^{3/2}}\, dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\, dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2} x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\, dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2} x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\, dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{4\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2} x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(-2+6ax-\frac{3\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+ax)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{\frac{1}{x}}}\right)}{8ac^3\sqrt{1+\frac{1}{ax}}(-1+ax)\sqrt{c-acx}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(-2 + 6*a*x - (3*Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 +
 a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[x^(
-1)]))/(8*a*c^3*Sqrt[1 + 1/(a*x)]*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)ax\sqrt{-c(ax+1)}-3\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c(ax+1)}+6\sqrt{c}\,ax-\right.}{8(ax-1)^3c^{\frac{9}{2}}a}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(-c*(a*x-1))^(1/2)/c^(9/2)*(
3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*x*(-c*(a*x+1))^(
1/2)-3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^
(1/2)+6*c^(1/2)*a*x-2*c^(1/2))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\sqrt{-}}{16\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1
)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^2*x^2
- 2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}}-\frac{2(3acx-c)}{\left((-acx-c)^{\frac{3}{2}}+2\sqrt{-acx-c}c\right)ac^2}\right)|c|\text{sgn}(ax+1)}{8c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) -
2*(3*a*c*x - c)/(((-a*c*x - c)^(3/2) + 2*sqrt(-a*c*x - c)*c)*a*c^2))*abs(c)
*sgn(a*x + 1)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a\,c\,x\right)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2), x)
```

3.279 $\int e^{\coth^{-1}(x)} x(1+x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 99

$$\int e^{\coth^{-1}(x)} x(1+x)\,dx = \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+1/3*(1+1/x)^(3/2)*x^2*((-1+x)/x)^(1
/2)+1/3*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6310, 6315, 98, 96, 94, 212}

$$\int e^{\coth^{-1}(x)} x(1+x)\,dx = \operatorname{arctanh}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)+\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{1}{3}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

```
[In] Int[E^ArcCoth[x]*x*(1 + x),x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x + ((1 + x^(-1))^(3/2)*Sqrt[(-1 + x)/x]*
x^2)/3 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + ArcTanh[Sqrt[1 + x^(
-1)]*Sqrt[(-1 + x)/x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x^2\,dx\\
&= -\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^4}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3-\frac{2}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3-\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2\\
&\quad+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2\\
&\quad+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)\\
&= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2\\
&\quad+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$
\int e^{\coth^{-1}(x)}x(1+x)\,dx=\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\left(5+3x+x^2\right)+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)
$$

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x),x]
[Out] (Sqrt[1 - x^(-2)]*x*(5 + 3*x + x^2))/3 + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

method	result	size
trager	$\frac{(1+x)(x^2+3x+5)\sqrt{-\frac{1-x}{1+x}}}{3}+\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	62
risch	$\frac{(x^2+3x+5)(x-1)}{3\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	62
default	$\frac{(x-1)\left(((x-1)(1+x))^{\frac{3}{2}}+3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)+6\sqrt{x^2-1}\right)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	67

```
[In] int(1/((x-1)/(1+x))^(1/2)*x*(1+x),x,method=_RETURNVERBOSE)
[Out] 1/3*(1+x)*(x^2+3*x+5)*(-(1-x)/(1+x))^(1/2)+ln((-(1-x)/(1+x))^(1/2)*x+(-(1-x
)/(1+x))^(1/2)+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\frac{1}{3}\left(x^3+4x^2+8x+5\right)\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="fricas")
[Out] 1/3*(x^3 + 4*x^2 + 8*x + 5)*sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1
)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\int\frac{x(x+1)}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x),x)
[Out] Integral(x*(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = -\frac{2\left(3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="maxima")
```

```
[Out] -2/3*(3*((x - 1)/(x + 1))^(5/2) - 8*((x - 1)/(x + 1))^(3/2) + 9*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = \frac{1}{3}\sqrt{x^2-1}\left(x\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{3}{\operatorname{sgn}(x+1)}\right)+\frac{5}{\operatorname{sgn}(x+1)}\right) - \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^2 - 1)*(x*(x/sgn(x + 1) + 3/sgn(x + 1)) + 5/sgn(x + 1)) - log(ab
s(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{6\sqrt{\frac{x-1}{x+1}}-\frac{16\left(\frac{x-1}{x+1}\right)^{3/2}}{3}+2\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

```
[In] int((x*(x + 1))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - (6*((x - 1)/(x + 1))^(1/2) - (16*((x - 1
)/(x + 1))^(3/2))/3 + 2*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*
(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.280 $\int e^{\coth^{-1}(x)}(1+x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 79

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{3}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] 3/2*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+1/2*(1+1/x)^(3/2)*x^2*((-1+x)/x
)^(1/2)+3/2*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6310, 6315, 96, 94, 212}

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{3}{2}\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)+\frac{1}{2}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{3}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

```
[In] Int[E^ArcCoth[x]*(1 + x),x]
```

```
[Out] (3*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + ((1 + x^(-1))^(3/2)*Sqrt[(-1 +
x)/x]*x^2)/2 + (3*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x\,dx \\
&= -\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2 - \frac{3}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$=\frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2-\frac{3}{2}\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{3}{2}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

$$=\frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{3}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x(4+x)+\frac{3}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]*(1 + x),x]
[Out] (Sqrt[1 - x^(-2)]*x*(4 + x))/2 + (3*Log[(1 + Sqrt[1 - x^(-2)])*x])/2
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{(x-1)\left(x\sqrt{x^2-1}+4\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{x-1}{1+x}}\sqrt{(x-1)(1+x)}}$	57
risch	$\frac{(x+4)(x-1)}{2\sqrt{\frac{x-1}{1+x}}}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	58
trager	$\frac{(1+x)(x+4)\sqrt{-\frac{1-x}{1+x}}}{2}-\frac{3\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	62

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1+x),x,method=_RETURNVERBOSE)
[Out] 1/2*(x-1)*(x*(x^2-1)^(1/2)+4*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/((x-1)/(1
+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{1}{2}\left(x^2+5\,x+4\right)\sqrt{\frac{x-1}{x+1}} + \frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 5*x + 4)*sqrt((x - 1)/(x + 1)) + 3/2*log(sqrt((x - 1)/(x + 1)) +
 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \int \frac{x+1}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x),x)
[Out] Integral((x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 5\sqrt{\frac{x-1}{x+1}}}{\frac{2\,(x-1)}{x+1} - \frac{(x-1)^2}{(x+1)^2} - 1} + \frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="maxima")
[Out] (3*((x - 1)/(x + 1))^(3/2) - 5*sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) -
(x - 1)^2/(x + 1)^2 - 1) + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(sqr
t((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{1}{2}\sqrt{x^2-1}\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 - 1)*(x/sgn(x + 1) + 4/sgn(x + 1)) - 3/2*log(abs(-x + sqrt(x^2
 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = 3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)+\frac{5\sqrt{\frac{x-1}{x+1}}-3\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}$$

```
[In] int((x + 1)/((x - 1)/(x + 1))^(1/2),x)
[Out] 3*atanh(((x - 1)/(x + 1))^(1/2)) + (5*((x - 1)/(x + 1))^(1/2) - 3*((x - 1)/
(x + 1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

3.281 $\int e^{\coth^{-1}(x)}(1-x)x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[Out] -1/3*(1-1/x^2)^(3/2)*x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6310, 6313, 270}

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[In] Int[E^ArcCoth[x]*(1 - x)*x,x]
```

```
[Out] -1/3*((1 - x^(-2))^(3/2)*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
```

```
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x^2\,dx\\ &= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\left(-1+x^2\right)$$

```
[In] Integrate[E^ArcCoth[x]*(1 - x)*x,x]
[Out] -1/3*(Sqrt[1 - x^(-2)]*x*(-1 + x^2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
gosper	$-\frac{(x-1)^{2}(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	22
default	$-\frac{(x-1)^{2}(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	22
risch	$-\frac{(x^{2}-1)(x-1)}{3\sqrt{\frac{x-1}{1+x}}}$	22
trager	$-\frac{(1+x)(x^{2}-1)\sqrt{-\frac{1-x}{1+x}}}{3}$	25

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x,method=_RETURNVERBOSE)
[Out] -1/3*(x-1)^2*(1+x)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\frac{1}{3}\left(x^{3}+x^{2}-x-1\right)\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="fricas")
[Out] -1/3*(x^3 + x^2 - x - 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\int\left(-\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx-\int\frac{x^{2}}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)*x,x)
[Out] -Integral(-x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(x**2/sqrt(x/(x + 1)
 - 1/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = \frac{8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}{3\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="maxima")
[Out] 8/3*((x - 1)/(x + 1))^(3/2)/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x
 - 1)^3/(x + 1)^3 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{(x^2-1)^{\frac{3}{2}}}{3\,\mathrm{sgn}\,(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="giac")
[Out] -1/3*(x^2 - 1)^(3/2)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{\left(\frac{x-1}{x+1}\right)^{3/2}(x+1)^3}{3}$$

```
[In] int(-(x*(x - 1))/((x - 1)/(x + 1))^(1/2),x)
[Out] -(((x - 1)/(x + 1))^(3/2)*(x + 1)^3)/3
```

3.282 $\int e^{\coth^{-1}(x)}(1-x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] 1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6310, 6313, 272, 43, 65, 212}

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2$$

```
[In] Int[E^ArcCoth[x]*(1 - x),x]
```

```
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + ArcTanh[Sqrt[1 - x^(-2)]]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x\,dx \\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right)
\end{aligned}
$$

$$= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$

$$= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]*(1 - x),x]
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result	size
default	$-\frac{(x-1)\left(x\sqrt{x^2-1}-\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	48
risch	$-\frac{x(x-1)}{2\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	56
trager	$-\frac{(1+x)\sqrt{-\frac{1-x}{1+x}}\,x}{2}+\frac{\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	57

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x),x,method=_RETURNVERBOSE)
[Out] -1/2*(x-1)*(x*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/((x-1)/(1+x))^(1/2)/((x-1)
*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = -\frac{1}{2}\left(x^2+x\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + x)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) -
 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = -\int \frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx-\int \left(-\frac{1}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x),x)
```

```
[Out] -Integral(x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(-1/sqrt(x/(x + 1) -
1/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(27) = 54.

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = \frac{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="maxima")
```

```
[Out] (((x - 1)/(x + 1))^(3/2) + sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x
 - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = -\frac{\sqrt{x^2-1}x}{2\,\mathrm{sgn}\,(x+1)} - \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\,\mathrm{sgn}\,(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="giac")
[Out] -1/2*sqrt(x^2 - 1)*x/sgn(x + 1) - 1/2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x +
1)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = \mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{\sqrt{\frac{x-1}{x+1}} + \left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2} - \frac{2\,(x-1)}{x+1} + 1}$$

```
[In] int(-(x - 1)/((x - 1)/(x + 1))^(1/2),x)
[Out] atanh(((x - 1)/(x + 1))^(1/2)) - ((((x - 1)/(x + 1))^(1/2) + ((x - 1)/(x + 1
))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

3.283 $\int e^{\coth^{-1}(x)}x(1+x)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 133

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4+\frac{15}{8}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] 15/8*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+5/8*(1+1/x)^(3/2)*x^2*((-1+x)/
x)^(1/2)+1/4*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+1/4*(1+1/x)^(7/2)*x^4*((-1+
x)/x)^(1/2)+15/8*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6310, 6315, 98, 96, 94, 212}

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{15}{8}\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)+\frac{1}{4}\left(\frac{1}{x}+1\right)^{7/2}\sqrt{\frac{x-1}{x}}x^4+\frac{1}{4}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{5}{8}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{15}{8}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^2,x]
```

```
[Out] (15*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/8 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
 + x)/x]*x^2)/8 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/4 + ((1 + x^(-1
))^(7/2)*Sqrt[(-1 + x)/x]*x^4)/4 + (15*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]])/8
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
```

```
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^3\,dx \\
&= -\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}x^5}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4-\frac{3}{4}\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4-\frac{5}{4}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 \\
&\quad+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4-\frac{15}{8}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 \\
&\quad+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4-\frac{15}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 \\
&\quad+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4+\frac{15}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right) \\
&= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 \\
&\quad+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4+\frac{15}{8}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x\left(24+15x+8x^2+2x^3\right)+\frac{15}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x*(24 + 15*x + 8*x^2 + 2*x^3))/8 + (15*Log[(1 + Sqrt[1 -
x^(-2)])*x])/8
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

method	result	size
risch	$\frac{(2x^3+8x^2+15x+24)(x-1)}{8\sqrt{\frac{x-1}{1+x}}}+\frac{15\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{8\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	70
trager	$\frac{(1+x)(2x^3+8x^2+15x+24)\sqrt{-\frac{1-x}{1+x}}}{8}-\frac{15\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{8}$	74
default	$\frac{(x-1)\left(2x(x^2-1)^{\frac{3}{2}}+8((x-1)(1+x))^{\frac{3}{2}}+17x\sqrt{x^2-1}+32\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{8\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	79

```
[In] int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*(2*x^3+8*x^2+15*x+24)*(x-1)/((x-1)/(1+x))^(1/2)+15/8*ln(x+(x^2-1)^(1/2)
)/((x-1)/(1+x))^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.50

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{1}{8}\left(2\,x^4+10\,x^3+23\,x^2+39\,x+24\right)\sqrt{\frac{x-1}{x+1}}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="fricas")
[Out] 1/8*(2*x^4 + 10*x^3 + 23*x^2 + 39*x + 24)*sqrt((x - 1)/(x + 1)) + 15/8*log(
sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\int\frac{x(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**2,x)
[Out] Integral(x*(x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-55\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+73\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="maxima")
[Out] 1/4*(15*((x - 1)/(x + 1))^(7/2) - 55*((x - 1)/(x + 1))^(5/2) + 73*((x - 1)/
(x + 1))^(3/2) - 49*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 15/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{1}{8}\left(\left(2x\left(\frac{x}{\mathrm{sgn}(x+1)}+\frac{4}{\mathrm{sgn}(x+1)}\right)+\frac{15}{\mathrm{sgn}(x+1)}\right)x+\frac{24}{\mathrm{sgn}(x+1)}\right)\sqrt{x^2-1}-\frac{15\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\,\mathrm{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="giac")
[Out] 1/8*((2*x*(x/sgn(x + 1) + 4/sgn(x + 1)) + 15/sgn(x + 1))*x + 24/sgn(x + 1))
*sqrt(x^2 - 1) - 15/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{15\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}+\frac{\frac{49\,\sqrt{\frac{x-1}{x+1}}}{4}-\frac{73\,{\left(\frac{x-1}{x+1}\right)}^{3/2}}{4}+\frac{55\,{\left(\frac{x-1}{x+1}\right)}^{5/2}}{4}-\frac{15\,{\left(\frac{x-1}{x+1}\right)}^{7/2}}{4}}{\frac{6\,{\left(x-1\right)}^2}{{\left(x+1\right)}^2}-\frac{4\,\left(x-1\right)}{x+1}-\frac{4\,{\left(x-1\right)}^3}{{\left(x+1\right)}^3}+\frac{{\left(x-1\right)}^4}{{\left(x+1\right)}^4}+1}$$

```
[In] int((x*(x + 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (15*atanh(((x - 1)/(x + 1))^(1/2)))/4 + ((49*((x - 1)/(x + 1))^(1/2))/4 - (
73*((x - 1)/(x + 1))^(3/2))/4 + (55*((x - 1)/(x + 1))^(5/2))/4 - (15*((x -
1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(
x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

3.284 $\int e^{\coth^{-1}(x)}(1+x)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\frac{5}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] 5/2*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+5/6*(1+1/x)^(3/2)*x^2*((-1+x)/x
)^(1/2)+1/3*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+5/2*x*(1+1/x)^(1/2)*((-1+x)/
x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6315, 96, 94, 212}

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \frac{5}{2}\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)+\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{5}{6}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{5}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

```
[In] Int[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (5*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
+ x)/x]*x^2)/6 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + (5*ArcTanh[S
qrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^2\,dx\\
&= -\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \frac{5}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$=\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3-\frac{5}{2}\text{Subst}\left(\int\frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3-\frac{5}{2}\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\frac{5}{2}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

$$=\frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x+\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\frac{5}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x(22+9x+2x^2)+\frac{5}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x*(22 + 9*x + 2*x^2))/6 + (5*Log[(1 + Sqrt[1 - x^(-2)])*x
])/2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{(2x^2+9x+22)(x-1)}{6\sqrt{\frac{x-1}{1+x}}}+\frac{5\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^2+9x+22)\sqrt{-\frac{1-x}{1+x}}}{6}+\frac{5\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	66
default	$\frac{(x-1)\left(2((x-1)(1+x))^{\frac{3}{2}}+9x\sqrt{x^2-1}+24\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	69

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*x^2+9*x+22)*(x-1)/((x-1)/(1+x))^(1/2)+5/2*ln(x+(x^2-1)^(1/2))/((x-1)
/(1+x))^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \frac{1}{6}\left(2\,x^3+11\,x^2+31\,x+22\right)\sqrt{\frac{x-1}{x+1}} + \frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 + 11*x^2 + 31*x + 22)*sqrt((x - 1)/(x + 1)) + 5/2*log(sqrt((x -
1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \int \frac{(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**2,x)
```

```
[Out] Integral((x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = -\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} - 40\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + 33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3\,(x-1)}{x+1} - \frac{3\,(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} + \frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*((x - 1)/(x + 1))^(5/2) - 40*((x - 1)/(x + 1))^(3/2) + 33*sqrt((x
- 1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x +
1)^3 - 1) + 5/2*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x +
1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2x}{\operatorname{sgn}(x+1)}+\frac{9}{\operatorname{sgn}(x+1)}\right)+\frac{22}{\operatorname{sgn}(x+1)}\right)-\frac{5\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) + 9/sgn(x + 1)) + 22/sgn(x + 1)) - 5/2
*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=5\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{11\sqrt{\frac{x-1}{x+1}}-\frac{40\left(\frac{x-1}{x+1}\right)^{3/2}}{3}+5\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

```
[In] int((x + 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 5*atanh(((x - 1)/(x + 1))^(1/2)) - (11*((x - 1)/(x + 1))^(1/2) - (40*((x -
1)/(x + 1))^(3/2))/3 + 5*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3
*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.285 $\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx = \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 - \frac{1}{8}\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -1/3*(1-1/x^2)^(3/2)*x^3+1/4*(1-1/x^2)^(3/2)*x^4-1/8*arctanh((1-1/x^2)^(1/2
))+1/8*x^2*(1-1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6310, 6313, 849, 821, 272, 43, 65, 212}

$$\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx = -\frac{1}{8}\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right) + \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[In] Int[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x^2)/8 - ((1 - x^(-2))^(3/2)*x^3)/3 + ((1 - x^(-2))^(3/2)
*x^4)/4 - ArcTanh[Sqrt[1 - x^(-2)]]/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2 x^3\,dx$$

$$= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{(4-x)\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{8}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 + \frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$

$$= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{8}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{1}{24}\sqrt{1-\frac{1}{x^2}}x\left(8-3x-8x^2+6x^3\right)-\frac{1}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(8 - 3*x - 8*x^2 + 6*x^3))/24 - Log[(1 + Sqrt[1 - x^(-2
)])*x]/8
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{(x-1)\left(6x(x^2-1)^{\frac{3}{2}}-8((x-1)(1+x))^{\frac{3}{2}}+3x\sqrt{x^2-1}-3\ln\left(x+\sqrt{x^2-1}\right)\right)}{24\sqrt{\frac{x-1}{1+x}}\sqrt{(x-1)(1+x)}}$	70
risch	$\frac{(6x^3-8x^2-3x+8)(x-1)}{24\sqrt{\frac{x-1}{1+x}}}-\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{8\sqrt{\frac{x-1}{1+x}}(1+x)}$	70
trager	$\frac{(1+x)(6x^3-8x^2-3x+8)\sqrt{-\frac{1-x}{1+x}}}{24}-\frac{\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{8}$	71

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(x-1)*(6*x*(x^2-1)^(3/2)-8*((x-1)*(1+x))^(3/2)+3*x*(x^2-1)^(1/2)-3*ln(
x+(x^2-1)^(1/2)))/((x-1)/(1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{1}{24}\left(6\,x^4-2\,x^3-11\,x^2+5\,x+8\right)\sqrt{\frac{x-1}{x+1}}-\frac{1}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 - 11*x^2 + 5*x + 8)*sqrt((x - 1)/(x + 1)) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\int\frac{x(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2*x,x)
[Out] Integral(x*(x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(55)=110$.

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}+53\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-11\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}-\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="maxima")
[Out] -1/12*(3*((x - 1)/(x + 1))^(7/2) + 53*((x - 1)/(x + 1))^(5/2) - 11*((x - 1)
/(x + 1))^(3/2) + 3*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{1}{24}\left(\left(2x\left(\frac{3x}{\operatorname{sgn}(x+1)}-\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3}{\operatorname{sgn}(x+1)}\right)x+\frac{8}{\operatorname{sgn}(x+1)}\right)\sqrt{x^2-1}+\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="giac")
[Out] 1/24*((2*x*(3*x/sgn(x + 1) - 4/sgn(x + 1)) - 3/sgn(x + 1))*x + 8/sgn(x + 1)
)*sqrt(x^2 - 1) + 1/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{\frac{\sqrt{\frac{x-1}{x+1}}}{4}-\frac{11\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{53\left(\frac{x-1}{x+1}\right)^{5/2}}{12}+\frac{\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2}-\frac{4\,(x-1)}{x+1}-\frac{4\,(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}-\frac{\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}$$

```
[In] int((x*(x - 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((((x - 1)/(x + 1))^(1/2)/4 - (11*((x - 1)/(x + 1))^(3/2))/12 + (53*((x - 1)
/(x + 1))^(5/2))/12 + ((x - 1)/(x + 1))^(7/2)/4)/((6*(x - 1)^2)/(x + 1)^2 -
 (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1) -
 atanh(((x - 1)/(x + 1))^(1/2))/4
```

3.286 $\int e^{\coth^{-1}(x)}(1-x)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] 1/3*(1-1/x^2)^(3/2)*x^3+1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6310, 6313, 821, 272, 43, 65, 212}

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[In] Int[E^ArcCoth[x]*(1 - x)^2,x]
```

```
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + ((1 - x^(-2))^(3/2)*x^3)/3 + ArcTanh[Sqrt[1 -
 x^(-2)]]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2 x^2\,dx$$

$$= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$

$$= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(-2-3x+2x^2\right) + \frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x*(-2 - 3*x + 2*x^2))/6 + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{(x-1)\left(2((x-1)(1+x))^{\frac{3}{2}}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	60
risch	$\frac{(2x^2-3x-2)(x-1)}{6\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^2-3x-2)\sqrt{-\frac{1-x}{1+x}}}{6}-\frac{\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	69

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x-1)*(2*((x-1)*(1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/(
(x-1)/(1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = \frac{1}{6}\left(2\,x^3-x^2-5\,x-2\right)\sqrt{\frac{x-1}{x+1}} + \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 - x^2 - 5*x - 2)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x
 + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = \int \frac{(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2,x)
```

```
[Out] Integral((x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(41) = 82.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = -\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)} + \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*((x - 1)/(x + 1))^(5/2) + 8*((x - 1)/(x + 1))^(3/2) - 3*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1))
 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{6}\,\sqrt{x^2-1}\left(x\left(\frac{2\,x}{\mathrm{sgn}\,(x+1)}-\frac{3}{\mathrm{sgn}\,(x+1)}\right)-\frac{2}{\mathrm{sgn}\,(x+1)}\right)-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\,\mathrm{sgn}\,(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) - 3/sgn(x + 1)) - 2/sgn(x + 1)) - 1/2*
log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{3/2}}{3}-\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

```
[In] int((x - 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] atanh(((x - 1)/(x + 1))^(1/2)) - ((8*((x - 1)/(x + 1))^(3/2))/3 - ((x - 1)/
(x + 1))^(1/2) + ((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*(x - 1)
^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.287 $\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx=\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x$$

```
[Out] x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6310, 6314, 97}

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx=\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x),x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{e^{\coth^{-1}(x)}}{1+\frac{1}{x}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^2\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = x\sqrt{\frac{-1+x^2}{x^2}}$$

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x),x]
[Out] x*Sqrt[(-1 + x^2)/x^2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
risch	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
trager	$(1+x)\sqrt{-\frac{1-x}{1+x}}$	19
default	$\frac{(x-1)\sqrt{x^2-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	32

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1+x),x,method=_RETURNVERBOSE)
[Out] (x-1)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="fricas")
[Out] (x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x),x)
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="maxima")
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="giac")
[Out] sqrt(x^2 - 1)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)
```

3.288 $\int \frac{e^{\coth^{-1}(x)}}{1+x} dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(x)}}{1+x} dx = \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6310, 6315, 94, 212}

$$\int \frac{e^{\coth^{-1}(x)}}{1+x} dx = \operatorname{arctanh}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[In] Int[E^ArcCoth[x]/(1 + x),x]
```

```
[Out] ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)x}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right) \\
&= \text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(x\left(1+\sqrt{\frac{-1+x^2}{x^2}}\right)\right)$$

```
[In] Integrate[E^ArcCoth[x]/(1 + x),x]
[Out] Log[x*(1 + Sqrt[(-1 + x^2)/x^2])]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{(x-1)\ln\left(x+\sqrt{x^{2}-1}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	35
trager	$-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	39

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1+x),x,method=_RETURNVERBOSE)
[Out] 1/((x-1)/(1+x))^(1/2)*(x-1)/((x-1)*(1+x))^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = -\log\left(\sqrt{1-\frac{2}{x+1}}-1\right)+\log\left(\sqrt{1-\frac{2}{x+1}}+1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x),x)
[Out] -log(sqrt(1 - 2/(x + 1)) - 1) + log(sqrt(1 - 2/(x + 1)) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2))
```

3.289 $\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x - 2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -2*arctanh((1-1/x^2)^(1/2))+2*(1+1/x)/(1-1/x^2)^(1/2)-x*(1-1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6310, 6312, 866, 1819, 821, 272, 65, 212}

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right) + \frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x$$

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x),x]
```

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1 -
 x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{\coth^{-1}(x)}}{1-\frac{1}{x}}\,dx \\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2x^2}\,dx, x, \frac{1}{x}\right) \\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x^2\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{-1-2x}{x^2\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x + 2\text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x - 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x - 2\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}(-3+x)x}{-1+x} - 2\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x),x]
[Out] -((Sqrt[1 - x^(-2)]*(-3 + x)*x)/(-1 + x)) - 2*Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

method	result	size
trager	$-\frac{(1+x)(-3+x)\sqrt{-\frac{1-x}{1+x}}}{x-1}-2\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	64
risch	$-\frac{x^2-2x-3}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}-\frac{2\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
default	$\frac{(x^2-1)^{\frac{3}{2}}-2x^2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2-1}\right)x^2+4x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2-1}\right)}{(x-1)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	106

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1-x),x,method=_RETURNVERBOSE)
[Out] -(1+x)*(-3+x)/(x-1)*(-(1-x)/(1+x))^(1/2)-2*ln((-(1-x)/(1+x))^(1/2)*x+(-(1-x
)/(1+x))^(1/2)+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{2\,(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-2\,(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)+(x^2-2\,x-3)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="fricas")
[Out] -(2*(x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 2*(x - 1)*log(sqrt((x - 1)/(x
+ 1)) - 1) + (x^2 - 2*x - 3)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\int \frac{x}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x),x)
[Out] -Integral(x/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-\sqrt{\frac{x-1}{x+1}}} - 2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + 2\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="maxima")
[Out] 2*(2*(x - 1)/(x + 1) - 1)/(((x - 1)/(x + 1))^(3/2) - sqrt((x - 1)/(x + 1)))
 - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)} - \frac{4}{\left(x-\sqrt{x^2-1}-1\right)\operatorname{sgn}(x+1)} - 2\operatorname{sgn}(x+1)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="giac")
[Out] 2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - sqrt(x^2 - 1)/sgn(x + 1) - 4/((
x - sqrt(x^2 - 1) - 1)*sgn(x + 1)) - 2*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{2\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)\,\sqrt{\frac{x-1}{x+1}}-6}{2\,\sqrt{\frac{x-1}{x+1}}}$$

```
[In] int(-x/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
[Out] -(2*x + 8*atanh(((x - 1)/(x + 1))^(1/2))*((x - 1)/(x + 1))^(1/2) - 6)/(2*((
x - 1)/(x + 1))^(1/2))
```

3.290 $\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx=\frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -arctanh((1-1/x^2)^(1/2))+2*(1+1/x)/(1-1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6310, 6313, 866, 1819, 272, 65, 212}

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx=\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[In] Int[E^ArcCoth[x]/(1 - x),x]
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)x}\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2x}\,dx, x, \frac{1}{x}\right)\\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} + \text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2\sqrt{1-\frac{1}{x^2}}x}{-1+x} - \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[E^ArcCoth[x]/(1 - x),x]
[Out] (2*Sqrt[1 - x^(-2)]*x)/(-1 + x) - Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

method	result	size
risch	$\frac{2}{\sqrt{\frac{x-1}{1+x}}}-\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	52
trager	$\frac{2(1+x)\sqrt{-\frac{1-x}{1+x}}}{x-1}+\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	62
default	$\frac{(x^{2}-1)^{\frac{3}{2}}-x^{2}\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}+2x\sqrt{x^{2}-1}+2\ln\left(x+\sqrt{x^{2}-1}\right)x-\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)}{(x-1)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	106

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1-x),x,method=_RETURNVERBOSE)
[Out] 2/((x-1)/(1+x))^(1/2)-ln(x+(x^2-1)^(1/2))/((x-1)/(1+x))^(1/2)/(1+x)*((x-1)*
(1+x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx=-\frac{(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-2\,(x+1)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="fricas")
[Out] -((x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - (x - 1)*log(sqrt((x - 1)/(x + 1)
) - 1) - 2*(x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx=-\int \frac{1}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x),x)
[Out] -Integral(1/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2}{\sqrt{\frac{x-1}{x+1}}} - \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="maxima")
[Out] 2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(sqrt((x - 1)
/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{4}{\left(x-\sqrt{x^2-1}-1\right)\operatorname{sgn}(x+1)} - 2\operatorname{sgn}(x+1)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="giac")
[Out] log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 4/((x - sqrt(x^2 - 1) - 1)*sgn(x
+ 1)) - 2*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2}{\sqrt{\frac{x-1}{x+1}}} - 2\,\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

```
[In] int(-1/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
[Out] 2/((x - 1)/(x + 1))^(1/2) - 2*atanh(((x - 1)/(x + 1))^(1/2))
```

3.291 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))-((-1+x)/x)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {6310, 6315, 98, 94, 212}

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = \operatorname{arctanh}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right) - \frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

```
[Out] -(Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}$$

$$= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}x}{1+x} + \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^2,x]
[Out] -((Sqrt[1 - x^(-2)]*x)/(1 + x)) + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

method	result	size
trager	$-\sqrt{-\frac{1-x}{1+x}} - \ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x - \sqrt{-\frac{1-x}{1+x}} + x\right)$	56
risch	$-\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)} + \frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	59
default	$\frac{(x-1)\left((x^2-1)^{\frac{3}{2}}-x^2\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)x^2-2x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}\,(1+x)^2}$	110

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] -(-(1-x)/(1+x))^(1/2)-ln(-(-(1-x)/(1+x))^(1/2)*x-(-(1-x)/(1+x))^(1/2)+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\sqrt{\frac{x-1}{x+1}} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="fricas")
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\sqrt{\frac{x-1}{x+1}} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 2/((x - sqrt(x^2 - 1) + 1)*sgn(x
 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \sqrt{\frac{x-1}{x+1}}$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - ((x - 1)/(x + 1))^(1/2)
```

3.292 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx = \frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}$$

```
[Out] ((-1+x)/x)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6310, 6315, 37}

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx = \frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

```
[In] Int[E^ArcCoth[x]/(1 + x)^2,x]
```

```
[Out] Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x^2}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}\,x}{1+x}$$

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x)/(1 + x)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\sqrt{\frac{x-1}{1+x}}$	12
trager	$\sqrt{-\frac{1-x}{1+x}}$	15
gosper	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
risch	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
default	$\frac{\sqrt{x^2-1}\,(x-1)}{(1+x)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	37

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] ((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="fricas")
[Out] sqrt((x - 1)/(x + 1))
```

Sympy [A] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**2,x)
[Out] sqrt((x - 1)/(x + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="maxima")
[Out] sqrt((x - 1)/(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="giac")
[Out] 2/((x - sqrt(x^2 - 1) + 1)*sgn(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{1-\frac{2}{x+1}}$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
[Out] (1 - 2/(x + 1))^(1/2)
```

3.293 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2} dx = -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -4/3*(1+1/x)/(1-1/x^2)^(3/2)+arctanh((1-1/x^2)^(1/2))+1/3*(-3-5/x)/(1-1/x^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6310, 6313, 866, 1819, 837, 12, 272, 65, 212}

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2} dx = \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right) - \frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{\frac{5}{x}+3}{3\sqrt{1-\frac{1}{x^2}}}$$

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) - (3 + 5/x)/(3*Sqrt[1 - x^(-2)]) +
 ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x}\,dx$$

$$= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3 x}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{(1+x)^3}{x\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{1}{3}\text{Subst}\left(\int \frac{-3-5x}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \frac{1}{3}\text{Subst}\left(\int -\frac{3}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}(5-7x)x}{3(-1+x)^2} + \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*(5 - 7*x)*x)/(3*(-1 + x)^2) + Log[(1 + Sqrt[1 - x^(-2)])*
x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

method	result
trager	$-\frac{(1+x)(7x-5)\sqrt{-\frac{1-x}{1+x}}}{3(x-1)^2} + \ln\left(\sqrt{-\frac{1-x}{1+x}}\,x + \sqrt{-\frac{1-x}{1+x}} + x\right)$
risch	$-\frac{7x^2+2x-5}{3(x-1)\sqrt{\frac{x-1}{1+x}}\,(1+x)} + \frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$
default	$-\frac{3x\left(x^2-1\right)^{\frac{3}{2}}-3\sqrt{x^2-1}\,x^3-3\ln\left(x+\sqrt{x^2-1}\right)x^3-2\left(x^2-1\right)^{\frac{3}{2}}+9x^2\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)x^2-9x\sqrt{x^2-1}-9\ln\left(x+\sqrt{x^2-1}\right)x+3\sqrt{x}}{3(x-1)^2\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*(1+x)*(7*x-5)/(x-1)^2*(-(1-x)/(1+x))^(1/2)+ln((-(1-x)/(1+x))^(1/2)*x+(
-(1-x)/(1+x))^(1/2)+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \frac{3\,(x^2-2\,x+1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-3\,(x^2-2\,x+1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-(7\,x^2+2\,x-5)\sqrt{\frac{x-1}{x+1}}}{3\,(x^2-2\,x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x^2 - 2*x + 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 3*(x^2 - 2*x + 1)*l
og(sqrt((x - 1)/(x + 1)) - 1) - (7*x^2 + 2*x - 5)*sqrt((x - 1)/(x + 1)))/(x
^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{\frac{6\,(x-1)}{x+1}+1}{3\,\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*(x - 1)/(x + 1) + 1)/((x - 1)/(x + 1))^(3/2) + log(sqrt((x - 1)/(x
+ 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} + \frac{2\left(9\left(x-\sqrt{x^2-1}\right)^2-12\,x+12\,\sqrt{x^2-1}+7\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)} + \frac{7}{3}\operatorname{sgn}(x+1)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) + 2/3*(9*(x - sqrt(x^2 - 1))^2 - 1
2*x + 12*sqrt(x^2 - 1) + 7)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) + 7/3*sg
n(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{\frac{2\,(x-1)}{x+1}+\frac{1}{3}}{{\left(\frac{x-1}{x+1}\right)}^{3/2}}$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - ((2*(x - 1))/(x + 1) + 1/3)/((x - 1)/(x
+ 1))^(3/2)
```

3.294 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} dx = -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

```
[Out] -1/3*(1-1/x^2)^(3/2)/(1-1/x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6310, 6313, 665}

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} dx = -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

```
[In] Int[E^ArcCoth[x]/(1 - x)^2,x]
```

```
[Out] -1/3*(1 - x^(-2))^(3/2)/(1 - x^(-1))^3
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x^2}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}x(1+x)}{3(-1+x)^2}$$

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^2,x]
[Out] -1/3*(Sqrt[1 - x^(-2)]*x*(1 + x))/(-1 + x)^2
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1+x}{3(x-1)\sqrt{\frac{x-1}{1+x}}}$	22
trager	$-\frac{(1+x)^{2} \sqrt{-\frac{1-x}{1+x}}}{3(x-1)^{2}}$	27
risch	$-\frac{x^{2}+2x+1}{3\sqrt{\frac{x-1}{1+x}}\,(1+x)(x-1)}$	32
default	$-\frac{(x^{2}-1)^{\frac{3}{2}}}{3\sqrt{\frac{x-1}{1+x}}\,(x-1)^{2}\sqrt{(x-1)(1+x)}}$	35

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*(1+x)/(x-1)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{(x^2+2\,x+1)\sqrt{\frac{x-1}{x+1}}}{3\,(x^2-2\,x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="fricas")
[Out] -1/3*(x^2 + 2*x + 1)*sqrt((x - 1)/(x + 1))/(x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**2,x)
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="maxima")
[Out] -1/3/((x - 1)/(x + 1))^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)} + \frac{1}{3}\operatorname{sgn}(x+1)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="giac")
[Out] 2/3*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) +
1/3*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{1}{3\left(\frac{x-1}{x+1}\right)^{3/2}}$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
[Out] -1/(3*((x - 1)/(x + 1))^(3/2))
```

3.295 $\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \frac{2x^{1+m}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, -3/2-m],[-1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/(3+2*m
)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6311, 6316, 66}

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \frac{2x^{m+1}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{3}{2},-m-\frac{1}{2},-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\\ &= -\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int x^{-\frac{5}{2}-m}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\ &= \frac{2x^{1+m}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = -\frac{x^{1+m}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{\left(-\frac{3}{2}-m\right)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
[Out] -((x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((-3/2 - m)*Sqrt[1 - 1/(a*x)]))
```

Maple [F]

$$\int \frac{x^m\sqrt{-acx+c}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
[Out] integral(sqrt(-a*c*x + c)*(a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="giac
")
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int\frac{x^m\sqrt{c-a\,c\,x}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.296 $\int e^{\coth^{-1}(ax)} x^2 \sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int e^{\coth^{-1}(ax)} x^2 \sqrt{c-acx}\,dx = \frac{16\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{8\left(1+\frac{1}{ax}\right)^{3/2} x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 16/105*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-8/35*(1+1/a/x
)^(3/2)*x^2*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*(1+1/a/x)^(3/2)*x^3*(-a*
c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6311, 6316, 47, 37}

$$\int e^{\coth^{-1}(ax)} x^2 \sqrt{c-acx}\,dx = \frac{16x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{2x^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{8x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (16*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (8
*(1 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*(1
+ 1/(a*x))^(3/2)*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}dx, x, \frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}dx, x, \frac{1}{x}\right)}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{16\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(8-4ax+3a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(8 - 4*a*x + 3*a^2*x^2 + 15*a^3*x^3))/
(105*a^3*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{2(ax+1)(15a^2x^2-12ax+8)\sqrt{-acx+c}}{105a^3\sqrt{\frac{ax-1}{ax+1}}}$	49
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(15a^2x^2-12ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,a^3}$	50
risch	$-\frac{2c(ax-1)(15a^3x^3+3a^2x^2-4ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^3}$	59

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(a*x+1)*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/((a*x-1)/(a*x+1))^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\left(15\,a^4x^4+18\,a^3x^3-a^2x^2+4\,ax+8\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/105*(15*a^4*x^4 + 18*a^3*x^3 - a^2*x^2 + 4*a*x + 8)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\left(15\,a^3\sqrt{-c}x^3+3\,a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x+8\,\sqrt{-c}\right)\sqrt{ax+1}}{105\,a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*x^3 + 3*a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x + 8*sqrt(-
c))*sqrt(a*x + 1)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-12\,a\,x+8\right)}{105\,a^3\,\left(a\,x-1\right)}$$

```
[In] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^2 -
12*a*x + 8))/(105*a^3*(a*x - 1))
```

3.297 $\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=-\frac{4\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -4/15*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(3
/2)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6311, 6316, 47, 37}

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]) + (2*(1
 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{4\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(1+ax)(-2+3ax)\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1
 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{2(ax+1)(3ax-2)\sqrt{-acx+c}}{15a^2\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(3ax-2)}{15\sqrt{\frac{ax-1}{ax+1}}\,a^2}$	42
risch	$-\frac{2c(ax-1)\left(3a^2x^2+ax-2\right)}{15\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^2}$	50

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(a*x+1)*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left(3\,a^3x^3+4\,a^2x^2-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^3x-a^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
[Out] 2/15*(3*a^3*x^3 + 4*a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a*c*x+c)**(1/2),x)

[Out] Integral(x*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left(3\,a^2\sqrt{-c}x^2+a\sqrt{-c}x-2\,\sqrt{-c}\right)\sqrt{ax+1}}{15\,a^2}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")

[Out] 2/15*(3*a^2*sqrt(-c)*x^2 + a*sqrt(-c)*x - 2*sqrt(-c))*sqrt(a*x + 1)/a^2

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,c^2\left(\frac{2\,\sqrt{2}\sqrt{-c}}{ac}-\frac{3\,(acx+c)^2\sqrt{-acx-c}+5\,(-acx-c)^{\frac{3}{2}}c}{ac^3}\right)}{15\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")

[Out] 2/15*c^2*(2*sqrt(2)*sqrt(-c)/(a*c) - (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5*
(-a*c*x - c)^(3/2)*c)/(a*c^3))/(a*abs(c)*sgn(a*x + 1))

Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a^2\,\left(a\,x-1\right)}$$

```
[In] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(
15*a^2*(a*x - 1))
```

3.298 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6309}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] && !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	35
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	42

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,c^2\left(\frac{2\,\sqrt{2}\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x +
 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1
))
```

3.299 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6311, 6316, 49, 56, 221}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] - (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,\sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int\frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}-\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)-\sqrt{-c(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}$	70

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.20

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")

[Out] integrate(sqrt(-a*c*x + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{2}\sqrt{-c}\sqrt{c}}{c^{\frac{5}{2}}} - \frac{\sqrt{-acx-c}}{c^2}\right)}{|c|\mathrm{sgn}\left(ax+1\right)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")

[Out] 2*c^3*(arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - (c*arctan(sqrt(2)*sqrt(-c
)/sqrt(c)) - sqrt(2)*sqrt(-c)*sqrt(c))/c^(5/2) - sqrt(-a*c*x - c)/c^2)/(abs
(c)*sgn(a*x + 1))

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

[In] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)

[Out] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)

3.300 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-arcsinh((1/x)^(1/2)/a^(
1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6311, 6316, 52, 56, 221}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\sqrt{a}\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x)) - (Sqrt[a]*Sqr
t[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}dx = -\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\left(\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}\,x\sqrt{c}}$	78
risch	$\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	106

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))*(-c*
(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.36

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2(ax^2-x)}, -\frac{(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -((a^2*x^2 -
a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c
*x - c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x
)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c(ax-1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)+\sqrt{2}a^2\sqrt{-c}\sqrt{c}}{c^{\frac{3}{2}}} + \frac{\sqrt{-acx-c}a}{cx}\right)c^2}{a|c|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] (a^2*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - (a^2*c*arctan(sqrt(2)*sqrt(
-c)/sqrt(c)) + sqrt(2)*a^2*sqrt(-c)*sqrt(c))/c^(3/2) + sqrt(-a*c*x - c)*a/(
c*x))*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.301 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}$$

```
[Out] -14/3*(-a*c*x+c)^(3/2)/a^4/c+18/5*(-a*c*x+c)^(5/2)/a^4/c^2-10/7*(-a*c*x+c)^
(7/2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4+4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6302, 6265, 21, 78}

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^
(9/2))/(9*a^4*c^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)} x^3 \sqrt{c-acx}\, dx \\
&= -\int \frac{x^3(1+ax)\sqrt{c-acx}}{1-ax}\, dx \\
&= -\left(c\int \frac{x^3(1+ax)}{\sqrt{c-acx}}\, dx\right) \\
&= -\left(c\int \left(\frac{2}{a^3\sqrt{c-acx}}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{(c-acx)^{7/2}}{a^3c^4}\right) dx\right) \\
&= \frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(272+136ax+102a^2x^2+85a^3x^3+35a^4x^4)}{315a^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4))
/(315*a^4)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
trager	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	46
risch	$-\frac{2c(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)(ax-1)}{315a^4\sqrt{-c(ax-1)}}$	52
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}}{a^4c^4}$	75
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}}{a^4c^4}$	75

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2\,(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272)\sqrt{-acx+c}}{315\,a^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x +
 c)/a^4
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\begin{cases}\frac{2\cdot\left(2c^4\sqrt{-acx+c}-\frac{7c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{9c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{5c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4} & \text{for } ac\neq 0\\ \sqrt{c}\left(\frac{x^4}{4}+\frac{2x^3}{3a}+\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\left(\begin{cases}-x & \text{for } a=0\\ \frac{\log(ax-1)}{a} & \text{otherwise}\end{cases}\right)}{a^3}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((2*(2*c**4*sqrt(-a*c*x + c) - 7*c**3*(-a*c*x + c)**(3/2)/3 + 9*c*
*2*(-a*c*x + c)**(5/2)/5 - 5*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/
9)/(a**4*c**4), Ne(a*c, 0)), (sqrt(c)*(x**4/4 + 2*x**3/(3*a) + x**2/a**2 +
2*x/a**3 + 2*Piecewise((-x, Eq(a, 0)), (log(a*x - 1)/a, True))/a**3), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(35(-acx+c)^{\frac{9}{2}}-225(-acx+c)^{\frac{7}{2}}c+567(-acx+c)^{\frac{5}{2}}c^2-735(-acx+c)^{\frac{3}{2}}c^3+630\sqrt{-acx+c}c^4\right)}{315\,a^4c^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)^
(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^4
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \frac{2\left(\frac{9\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}c^3\right)}{a^3c^3}+\frac{35\,(acx-c)^4\sqrt{-acx+c}+180\,(acx-c)^3\sqrt{-acx+c}c}{}\right.}{315\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/(a^3*c^3) + (3
5*(a*c*x - c)^4*sqrt(-a*c*x + c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 3
78*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sq
rt(-a*c*x + c)*c^4)/(a^3*c^4))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^4} - \frac{14\,(c-a\,c\,x)^{3/2}}{3\,a^4\,c} + \frac{18\,(c-a\,c\,x)^{5/2}}{5\,a^4\,c^2} - \frac{10\,(c-a\,c\,x)^{7/2}}{7\,a^4\,c^3} + \frac{2\,(c-a\,c\,x)^{9/2}}{9\,a^4\,c^4}$$

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c
*x)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x
)^(9/2))/(9*a^4*c^4)
```

3.302 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}$$

```
[Out] -10/3*(-a*c*x+c)^(3/2)/a^3/c+8/5*(-a*c*x+c)^(5/2)/a^3/c^2-2/7*(-a*c*x+c)^(7
/2)/a^3/c^3+4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6302, 6265, 21, 78}

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*x)
^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}x^2\sqrt{c-acx}\,dx\\
&= -\int \frac{x^2(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= -\left(c\int \frac{x^2(1+ax)}{\sqrt{c-acx}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{a^2\sqrt{c-acx}}-\frac{5\sqrt{c-acx}}{a^2c}+\frac{4(c-acx)^{3/2}}{a^2c^2}-\frac{(c-acx)^{5/2}}{a^2c^3}\right)dx\right)\\
&= \frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(104+52ax+39a^2x^2+15a^3x^3)}{105a^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
trager	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	38
risch	$-\frac{2c(15a^3x^3+39a^2x^2+52ax+104)(ax-1)}{105a^3\sqrt{-c(ax-1)}}$	44
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-2c^3\sqrt{-acx+c}\right)}{c^3a^3}$	61
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{8c(-acx+c)^{\frac{5}{2}}}{5}-\frac{10c^2(-acx+c)^{\frac{3}{2}}}{3}+4c^3\sqrt{-acx+c}}{a^3c^3}$	61

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.45

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\,(15\,a^3x^3+39\,a^2x^2+52\,ax+104)\sqrt{-acx+c}}{105\,a^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\left(-2c^3\sqrt{-acx+c}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^3}{3}+\frac{x^2}{a}+\frac{2x}{a^2}+\frac{2\left(\begin{cases}-x & \text{for } a=0\\ \frac{\log(ax-1)}{a} & \text{otherwise}\end{cases}\right)}{a^2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((-2*(-2*c**3*sqrt(-a*c*x + c) + 5*c**2*(-a*c*x + c)**(3/2)/3 - 4*
c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a**3*c**3), Ne(a*c, 0)),
(sqrt(c)*(x**3/3 + x**2/a + 2*x/a**2 + 2*Piecewise((-x, Eq(a, 0)), (log(a*x
 - 1)/a, True))/a**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\left(15(-acx+c)^{\frac{7}{2}}-84(-acx+c)^{\frac{5}{2}}c+175(-acx+c)^{\frac{3}{2}}c^2-210\sqrt{-acx+c}c^3\right)}{105\,a^3c^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)^
(3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(66) = 132.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(\frac{7\left(3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}c^2\right)}{a^2c^2}+\frac{3\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-c\ldots}\right.}{a^2c^3}\right.}{105\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*s
qrt(-a*c*x + c)*c^2)/(a^2*c^2) + 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(
a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*
x + c)*c^3)/(a^2*c^3))/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{10\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c} + \frac{8\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^3\,c^2}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}$$

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*
x)^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

3.303 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}$$

```
[Out] -2*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2+4*(-a*c*x+c)^(1/2)/a
^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6302, 6265, 21, 78}

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{4\sqrt{c-acx}}{a^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(a^2*c) + (2*(c - a*c*x)^(5
/2))/(5*a^2*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}x\sqrt{c-acx}\,dx\\ &= -\int \frac{x(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{x(1+ax)}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{a\sqrt{c-acx}}-\frac{3\sqrt{c-acx}}{ac}+\frac{(c-acx)^{3/2}}{ac^2}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(6+3ax+a^2x^2)}{5a^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(a^{2}x^{2}+3ax+6)}{5a^{2}}$	28
trager	$\frac{2\sqrt{-acx+c}\,(a^{2}x^{2}+3ax+6)}{5a^{2}}$	28
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(a^{2}x^{2}+3ax+6)}{5a^{2}}$	29
risch	$-\frac{2c(a^{2}x^{2}+3ax+6)(ax-1)}{5a^{2}\sqrt{-c(ax-1)}}$	35
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2c(-acx+c)^{\frac{3}{2}}+4c^{2}\sqrt{-acx+c}}{a^{2}c^{2}}$	47
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2c(-acx+c)^{\frac{3}{2}}+4c^{2}\sqrt{-acx+c}}{a^{2}c^{2}}$	47

```
[In] int(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,(a^{2}x^{2}+3\,ax+6)\sqrt{-acx+c}}{5\,a^{2}}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\begin{cases}\frac{2\cdot\left(2c^{2}\sqrt{-acx+c}-c(-acx+c)^{\frac{3}{2}}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^{2}c^{2}} & \text{for } ac\neq 0\\ \sqrt{c}\left(\frac{x^{2}}{2}+\frac{2x}{a}+\frac{2\left(\begin{cases}-x & \text{for } a=0\\ \frac{\log(ax-1)}{a} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((2*(2*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2) + (-a*c*x + c
)**(5/2)/5)/(a**2*c**2), Ne(a*c, 0)), (sqrt(c)*(x**2/2 + 2*x/a + 2*Piecewis
e((-x, Eq(a, 0)), (log(a*x - 1)/a, True))/a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}c^2\right)}{5\,a^2c^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2)
/(a^2*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$= -\frac{2\left(\frac{5\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c\right)}{ac}-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}c^2}{ac^2}\right)}{15\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/(a*c) - (3*(a*c*x - c)
^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/(a
*c^2))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}-10\,c\,{\left(c-a\,c\,x\right)}^{3/2}+20\,c^2\,\sqrt{c-a\,c\,x}}{5\,a^2\,c^2}$$

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] (2*(c - a*c*x)^(5/2) - 10*c*(c - a*c*x)^(3/2) + 20*c^2*(c - a*c*x)^(1/2))/(
5*a^2*c^2)
```

3.304 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-acx}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\right)\\
&= \frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+5)}{3a}$	21
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4c\sqrt{-acx+c}}{ac}$	33

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{-acx+c}(ax+5)}{3\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\begin{cases}-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0\\ \sqrt{c}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((-2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c
, 0)), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, Tr
ue)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left((-acx+c)^{\frac{3}{2}} - 6\sqrt{-acx+c}c\right)}{3\,ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(3\sqrt{-acx+c} - \frac{(-acx+c)^{\frac{3}{2}} - 3\sqrt{-acx+c}c}{c}\right)}{3\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a} - \frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] (4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

3.305 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6302, 6265, 21, 81, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+2\sqrt{c-acx}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{c-acx}}{x}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x\sqrt{c-acx}}\,dx\right)\\
&= 2\sqrt{c-acx}-c\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= 2\sqrt{c-acx}+\frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= 2\sqrt{c-acx}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-acx+c}$	32
default	$2\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-acx+c}$	32
pseudoelliptic	$2\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-c\,(ax-1)}$	34

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\sqrt{-acx+c},\; -2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\sqrt{-acx+c}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
[Out] [sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x
+ c), -2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x + c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(34) = 68$.

Time = 3.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \begin{cases} -\frac{2c\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c} & \text{for } ac \neq 0 \\ \sqrt{c}\left(-\frac{3a\left(\frac{\log\left(\frac{2}{x}\right)}{a} - \frac{\log\left(2a-\frac{2}{x}\right)}{a}\right)}{2} + \frac{\log\left(\frac{a}{x}-\frac{1}{x^2}\right)}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x,x)
[Out] Piecewise((-2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x +
c), Ne(a*c, 0)), (sqrt(c)*(-3*a*(log(2/x)/a - log(2*a - 2/x)/a)/2 + log(a/x
 - 1/x**2)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -\sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right) + 2\sqrt{-acx+c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
[Out] -sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c))) + 2
*sqrt(-a*c*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -2c\left(\frac{\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{-acx+c}}{c}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
[Out] -2*c*(arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/c)
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\sqrt{c-a\,c\,x}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2)
```

3.306 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6302, 6265, 21, 79, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = 3a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + \frac{\sqrt{c-acx}}{x}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^2}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^2(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x^2\sqrt{c-acx}}\,dx\right)\\
&= \frac{\sqrt{c-acx}}{x}-\frac{1}{2}(3ac)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{x}+3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)\\
&= \frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}} + 3a\ \text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}$	43
pseudoelliptic	$\frac{3\ \text{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax-1)}\sqrt{c}}{x\sqrt{c}}$	43
derivativedivides	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx} - \frac{3\ \text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45
default	$2ca\left(\frac{\sqrt{-acx+c}}{2acx} + \frac{3\ \text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(a*x-1)/x/(-c*(a*x-1))^(1/2)*c+3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.31

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{3\,a\sqrt{c}x\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\sqrt{-acx+c}}{2\,x}, -\frac{3\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c))/x, -(3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) -
sqrt(-a*c*x + c))/x]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^2\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{1}{2}\,ac\left(\frac{3\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\sqrt{-acx+c}}{acx}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima")
[Out] -1/2*a*c*(3*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/
sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\frac{3\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{-acx+c}a}{x}}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
[Out] -(3*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)*a/x
)/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\frac{\sqrt{c-a\,c\,x}}{x}+3\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
[Out] (c - a*c*x)^(1/2)/x + 3*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))
```

3.307 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{7}{4}a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 7/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2+
7/4*a*(-a*c*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 79, 44, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{7}{4}a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] Sqrt[c - a*c*x]/(2*x^2) + (7*a*Sqrt[c - a*c*x])/(4*x) + (7*a^2*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{c-acx}}{x^3}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^3(1-ax)}\,dx
\end{aligned}
$$

$$= -\left(c\int \frac{1+ax}{x^3\sqrt{c-acx}}\,dx\right)$$

$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{1}{4}(7ac)\int \frac{1}{x^2\sqrt{c-acx}}\,dx$$

$$= \frac{\sqrt{c-acx}}{2x^2} + \frac{7a\sqrt{c-acx}}{4x} - \frac{1}{8}\left(7a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx$$

$$= \frac{\sqrt{c-acx}}{2x^2} + \frac{7a\sqrt{c-acx}}{4x} + \frac{1}{4}(7a)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)$$

$$= \frac{\sqrt{c-acx}}{2x^2} + \frac{7a\sqrt{c-acx}}{4x} + \frac{7}{4}a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{(2+7ax)\sqrt{c-acx}}{4x^2} + \frac{7}{4}a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
[Out] ((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c
*x]/Sqrt[c]])/4
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\frac{7\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)a^2c\,x^2}{4}+\frac{7\sqrt{c}\,\sqrt{-c(ax-1)}\,\left(ax+\frac{2}{7}\right)}{4}}{\sqrt{c}\,x^2}$	52
risch	$-\frac{\left(7a^2x^2-5ax-2\right)c}{4x^2\sqrt{-c(ax-1)}}+\frac{7a^2\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}}{4}$	54
derivativedivides	$2a^2c^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{7\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65
default	$2a^2c^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{7\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 7/4/c^(1/2)*(arctanh((-c*(a*x-1))^(1/2)/c^(1/2))*a^2*c*x^2+c^(1/2)*(-c*(a*x
-1))^(1/2)*(a*x+2/7))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \left[\frac{7\,a^2\sqrt{c}x^2\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}(7\,ax+2)}{8\,x^2}, -\frac{7\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}(7\,ax+2)}{4\,x^2}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas")
[Out] [1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) +
 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, -1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt(
-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^3\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**3,x)
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{1}{8}\,a^2c^2\left(\frac{2\left(7\,(-acx+c)^{\frac{3}{2}}-9\,\sqrt{-acx+c}c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{7\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima")
[Out] -1/8*a^2*c^2*(2*(7*(-a*c*x + c)^(3/2) - 9*sqrt(-a*c*x + c)*c)/((a*c*x - c)^
2*c + 2*(a*c*x - c)*c^2 + c^3) + 7*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-
a*c*x + c) + sqrt(c)))/c^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{\frac{7\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{7\,(-acx+c)^{\frac{3}{2}}a^3c-9\,\sqrt{-acx+c}a^3c^2}{a^2c^2x^2}}{4\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(7*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + (7*(-a*c*x + c)^
(3/2)*a^3*c - 9*sqrt(-a*c*x + c)*a^3*c^2)/(a^2*c^2*x^2))/a
```

Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{9\,\sqrt{c-a\,c\,x}}{4\,x^2} + \frac{7\,a^2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)}{4} - \frac{7\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] (9*(c - a*c*x)^(1/2))/(4*x^2) + (7*a^2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1
/2)))/4 - (7*(c - a*c*x)^(3/2))/(4*c*x^2)
```

3.308 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 11/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3
+11/12*a*(-a*c*x+c)^(1/2)/x^2+11/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 79, 44, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) + (11*a*Sqrt[c - a*c*x])/(12*x^2) + (11*a^2*Sqrt[c
- a*c*x])/(8*x) + (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^4}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^4(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x^4\sqrt{c-acx}}\,dx\right) \\
&= \frac{\sqrt{c-acx}}{3x^3} - \frac{1}{6}(11ac)\int \frac{1}{x^3\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} - \frac{1}{8}\left(11a^2c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} + \frac{11a^2\sqrt{c-acx}}{8x} - \frac{1}{16}\left(11a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} + \frac{11a^2\sqrt{c-acx}}{8x} + \frac{1}{8}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} + \frac{11a^2\sqrt{c-acx}}{8x} + \frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(8+22ax+33a^2x^2\right)}{24x^3} + \frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
[Out] (Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) + (11*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\left(33a^{3}x^{3}-11a^{2}x^{2}-14ax-8\right)c}{24x^{3}\sqrt{-c\left(ax-1\right)}}+\frac{11a^{3}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}}{8}$	62
pseudoelliptic	$\frac{\frac{\sqrt{-c\left(ax-1\right)}\left(33a^{2}x^{2}+22ax+8\right)\sqrt{c}}{24}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt{-c\left(ax-1\right)}}{\sqrt{c}}\right)a^{3}c\,x^{3}}{8}}{\sqrt{c}\,x^{3}}$	62
default	$2c^{3}a^{3}\left(\frac{\frac{11\left(-acx+c\right)^{\frac{5}{2}}}{16c^{2}}-\frac{11\left(-acx+c\right)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{a^{3}c^{3}x^{3}}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$	79
derivativedivides	$-2c^{3}a^{3}\left(-\frac{\frac{11\left(-acx+c\right)^{\frac{5}{2}}}{16c^{2}}-\frac{11\left(-acx+c\right)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{a^{3}c^{3}x^{3}}-\frac{11\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$	80

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(33*a^3*x^3-11*a^2*x^2-14*a*x-8)/x^3/(-c*(a*x-1))^(1/2)*c+11/8*a^3*ar
ctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.49

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \left[\frac{33\,a^{3}\sqrt{c}x^{3}\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(33\,a^{2}x^{2}+22\,ax+8)\sqrt{-acx+c}}{48\,x^{3}},\ -\frac{33\,a^{3}\sqrt{-c}x^{3}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(33\,a^{2}x^{2}+22\,ax+8)\sqrt{-acx+c}}{24\,x^{3}}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
 + 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, -1/24*(33*a^3*sqrt(-c
)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sqrt(
-a*c*x + c))/x^3]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^4\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**4,x)
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{1}{48}\,a^3c^3\left(\frac{2\left(33\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+63\,\sqrt{-acx+c}c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{33\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/48*a^3*c^3*(2*(33*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 63*sqrt(
-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*c
^4 + c^5) - 33*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)
))/c^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{\frac{33\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}a^4c-88\,(-acx+c)^{\frac{3}{2}}a^4c^2+63\,\sqrt{-acx+c}a^4c^3}{a^3c^3x^3}}{24\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/24*(33*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (33*(a*c*x - c
)^2*sqrt(-a*c*x + c)*a^4*c - 88*(-a*c*x + c)^(3/2)*a^4*c^2 + 63*sqrt(-a*c*x
 + c)*a^4*c^3)/(a^3*c^3*x^3))/a
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{21\sqrt{c-a\,c\,x}}{8\,x^3} - \frac{11\,(c-a\,c\,x)^{3/2}}{3\,c\,x^3} + \frac{11\,(c-a\,c\,x)^{5/2}}{8\,c^2\,x^3} - \frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,11\mathrm{i}}{8}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] (21*(c - a*c*x)^(1/2))/(8*x^3) - (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c
^(1/2))*11i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (11*(c - a*c*x)^(5/2))/
(8*c^2*x^3)
```

3.309 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^
4+5/8*a*(-a*c*x+c)^(1/2)/x^3+25/32*a^2*(-a*c*x+c)^(1/2)/x^2+75/64*a^3*(-a*c
*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 79, 44, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{75}{64}a^4\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) + (5*a*Sqrt[c - a*c*x])/(8*x^3) + (25*a^2*Sqrt[c -
a*c*x])/(32*x^2) + (75*a^3*Sqrt[c - a*c*x])/(64*x) + (75*a^4*Sqrt[c]*ArcTan
h[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^5}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^5(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x^5\sqrt{c-acx}}\,dx\right)\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{1}{8}(15ac)\int \frac{1}{x^4\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}-\frac{1}{16}\left(25a^2c\right)\int \frac{1}{x^3\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{1}{64}\left(75a^3c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}-\frac{1}{128}\left(75a^4c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}\\
&\quad+\frac{1}{64}\left(75a^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx &= \frac{\sqrt{c-acx}\left(16+40ax+50a^2x^2+75a^3x^3\right)}{64x^4}\\
&\quad+\frac{75}{64}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) + (75*a^
4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\left(75 a^{4} x^{4}-25 a^{3} x^{3}-10 a^{2} x^{2}-24 a x -16\right) c}{64 x^{4} \sqrt{-c \left(a x -1\right)}}+\frac{75 a^{4} \operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right) \sqrt{c}}{64}$	70
pseudoelliptic	$\frac{\frac{\sqrt{-c \left(a x -1\right)}\, \left(75 a^{3} x^{3}+50 a^{2} x^{2}+40 a x +16\right) \sqrt{c}}{64}+\frac{75 \operatorname{arctanh}\left(\frac{\sqrt{-c \left(a x -1\right)}}{\sqrt{c}}\right) c \,a^{4} x^{4}}{64}}{\sqrt{c}\, x^{4}}$	70
derivativedivides	$2 c^{4} a^{4} \left(\frac{-\frac{75 \left(-a c x +c \right)^{\frac{7}{2}}}{128 c^{3}}+\frac{275 \left(-a c x +c \right)^{\frac{5}{2}}}{128 c^{2}}-\frac{365 \left(-a c x +c \right)^{\frac{3}{2}}}{128 c}+\frac{181 \sqrt{-a c x +c}}{128}}{a^{4} c^{4} x^{4}}+\frac{75 \operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right)}{128 c^{\frac{7}{2}}}\right)$	93
default	$2 c^{4} a^{4} \left(\frac{-\frac{75 \left(-a c x +c \right)^{\frac{7}{2}}}{128 c^{3}}+\frac{275 \left(-a c x +c \right)^{\frac{5}{2}}}{128 c^{2}}-\frac{365 \left(-a c x +c \right)^{\frac{3}{2}}}{128 c}+\frac{181 \sqrt{-a c x +c}}{128}}{a^{4} c^{4} x^{4}}+\frac{75 \operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right)}{128 c^{\frac{7}{2}}}\right)$	93

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(75*a^4*x^4-25*a^3*x^3-10*a^2*x^2-24*a*x-16)/x^4/(-c*(a*x-1))^(1/2)*c
+75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \left[\frac{75\,a^{4}\sqrt{c}x^{4}\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(75\,a^{3}x^{3}+50\,a^{2}x^{2}+40\,ax+16)\sqrt{-acx+c}}{128\,x^{4}}, -\frac{75\,a^{4}\sqrt{-c}x^{4}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(75\,a^{3}x^{3}+50\,a^{2}x^{2}+40\,ax+16)\sqrt{-acx+c}}{64\,x^{4}}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) + 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, -1/64*
(75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 + 50
*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^5\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**5,x)
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{1}{128}\,a^4c^4\left(\frac{2\left(75\,(-acx+c)^{\frac{7}{2}}-275\,(-acx+c)^{\frac{5}{2}}c+365\,(-acx+c)^{\frac{3}{2}}c^2-181\,\sqrt{-acx+c}c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{75\log}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/128*a^4*c^4*(2*(75*(-a*c*x + c)^(7/2) - 275*(-a*c*x + c)^(5/2)*c + 365*(
-a*c*x + c)^(3/2)*c^2 - 181*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*(a
*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 75*log((
sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{\frac{75\,a^5c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{75\,(acx-c)^3\sqrt{-acx+c}a^5c+275\,(acx-c)^2\sqrt{-acx+c}a^5c^2-365\,(-acx+c)^{\frac{3}{2}}a^5c^3+181\,\sqrt{-acx+c}a^5c^4}{a^4c^4x^4}}{64\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/64*(75*a^5*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (75*(a*c*x - c
)^3*sqrt(-a*c*x + c)*a^5*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^5*c^2 - 3
65*(-a*c*x + c)^(3/2)*a^5*c^3 + 181*sqrt(-a*c*x + c)*a^5*c^4)/(a^4*c^4*x^4)
)/a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{181\sqrt{c-a\,c\,x}}{64\,x^4} - \frac{365\,{(c-a\,c\,x)}^{3/2}}{64\,c\,x^4} + \frac{275\,{(c-a\,c\,x)}^{5/2}}{64\,c^2\,x^4} - \frac{75\,{(c-a\,c\,x)}^{7/2}}{64\,c^3\,x^4} - \frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,75\mathrm{i}}{64}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] (181*(c - a*c*x)^(1/2))/(64*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*75i)/64 - (365*(c - a*c*x)^(3/2))/(64*c*x^4) + (275*(c - a*c*x)^(
5/2))/(64*c^2*x^4) - (75*(c - a*c*x)^(7/2))/(64*c^3*x^4)
```

3.310 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1576/315*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(1-1/a/x)^(1/2)+472/315*x*(1+
1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+92/105*x^2*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+38/63*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/a/(1-1/a/x)^(1/2)+2/9*x^4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)
^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)
^(1/2)*(-a*c*x+c)^(1/2)/a^(9/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6311, 6316, 100, 157, 12, 95, 212}

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^{9/2}\sqrt{1-\frac{1}{ax}}}+\frac{1576\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}+\frac{38x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (1576*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(315*a^4*Sqrt[1 - 1/(a*x)]) + (472
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(315*a^3*Sqrt[1 - 1/(a*x)]) + (92*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) + (38*Sqrt[
1 + 1/(a*x)]*x^3*Sqrt[c - a*c*x])/(63*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/
(a*x)]*x^4*Sqrt[c - a*c*x])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]
*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{19}{2a}-\frac{17x}{2a^2}}{x^{9/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{9\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\frac{69}{2a^2}+\frac{57x}{2a^3}}{x^{7/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{63\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{177}{2a^3}-\frac{69x}{a^4}}{x^{5/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{315\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(16\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\frac{591}{4a^4}+\frac{177x}{2a^5}}{x^{3/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{945\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \frac{\left(32\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int-\frac{945}{8a^5\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{945\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.42

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(788+236ax+138a^2x^2+95a^3x^3+35a^4x^4)-630\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{315a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(788 + 236*a*x + 138*a^2*x^2
+ 95*a^3*x^3 + 35*a^4*x^4) - 630*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt
[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(315*a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.47

method	result
risch	$-\frac{2(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788)c(ax-1)}{315a^4\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(35a^4x^4\sqrt{-c(ax+1)}+95a^3x^3\sqrt{-c(ax+1)}+138a^2x^2\sqrt{-c(ax+1)}+236ax\sqrt{-c(ax+1)}-630\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{}\right.\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^4}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(35*a^4*x^4+95*a^3*x^3+138*a^2*x^2+236*a*x+788)/a^4*c/((a*x-1)/(a*x+
1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)-4/a^4*2^(1/2)*c^(1/2)*arctan(1/2*(-a*c
*x-c)^(1/2)*2^(1/2)/c^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x+1))^(
1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.98

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$

$$=\left[\frac{2\left(315\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(35\,a^5x^5+130\,a^4x^4+233\,a^3\right.}{315\,(a^5x-a^4)}\right.$$

$$-\frac{2\left(630\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(35\,a^5x^5+130\,a^4x^4+233\,a^3x^3+374\,a^2x^2+1024\right.}{315\,(a^5x-a^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/315*(315*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2
)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (35*a^5*x^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x^2
+ 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x - a^4)
, -2/315*(630*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (35*a^5*x^5 + 130*a^4*x^4 + 2
33*a^3*x^3 + 374*a^2*x^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/(a^5*x - a^4)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int \frac{x^3\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a*c*x + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac
")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int \frac{x^3\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.311 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 261

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 104/21*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+32/21*x*(1+1/a/
x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+6/7*x^2*(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1
/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*
(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(7/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{6311, 6316, 100, 157, 12, 95, 212\}$

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^{7/2}\sqrt{1-\frac{1}{ax}}} + \frac{104\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{6x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (104*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(21*a^3*Sqrt[1 - 1/(a*x)]) + (32*Sq
rt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(21*a^2*Sqrt[1 - 1/(a*x)]) + (6*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(7*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*x
)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(
a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{15}{2a}-\frac{13x}{2a^2}}{x^{7/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{20}{a^2}+\frac{15x}{a^3}}{x^{5/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{35\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{65}{2a^3}-\frac{20x}{a^4}}{x^{3/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{105\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(16\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{105}{4a^4\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{105\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(52+16ax+9a^2x^2+3a^3x^3)-42\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{21a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(52 + 16*a*x + 9*a^2*x^2 + 3*
a^3*x^3) - 42*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*
Sqrt[1 + 1/(a*x)])]))/(21*a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2(3a^3x^3+9a^2x^2+16ax+52)c(ax-1)}{21a^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3a^3x^3\sqrt{-c(ax+1)}-9a^2x^2\sqrt{-c(ax+1)}+42\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-16ax\sqrt{-c(ax+1)}-52\sqrt{-c(a}\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}a^3}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(3*a^3*x^3+9*a^2*x^2+16*a*x+52)/a^3*c/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*
x-1))^(1/2)*(a*x-1)-4/a^3*2^(1/2)*c^(1/2)*arctan(1/2*(-a*c*x-c)^(1/2)*2^(1/
2)/c^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x+1))^(1/2)/(-c*(a*x-1))
^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= \left[\frac{2\left(21\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2 -\right.}{21\,(a^4x-a^3)}\right.$$
$$-\frac{2\left(42\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2+68\,ax+52)\sqrt{-acx\right.}}{21\,(a^4x-a^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/21*(21*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*sqrt
(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3), -2/21*(42*sqrt(2)*(a
*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
 + 1))/(a*c*x - c)) - (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*s
qrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{\sqrt{-acx+c}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a*c*x + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\,c^2\left(\frac{2\,\sqrt{2}\left(21\,\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-40\,\sqrt{-c}\right)}{a^2c}-\frac{42\,\sqrt{2}c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-3\,(acx+c)^3\sqrt{-acx-c}+7\,(-acx-c)^{\frac{3}{2}}c^2-42\,\sqrt{-acx-c}c^3}{a^2c^4}\right)}{21\,a|c|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
[Out] -2/21*c^2*(2*sqrt(2)*(21*sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - 40*sqrt(-c))/(a
^2*c) - (42*sqrt(2)*c^(7/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) -
3*(a*c*x + c)^3*sqrt(-a*c*x - c) + 7*(-a*c*x - c)^(3/2)*c^2 - 42*sqrt(-a*c*
x - c)*c^3)/(a^2*c^4))/(a*abs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2\sqrt{c-a\,c\,x}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.312 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}x\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(5/2
)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a
^2/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2
^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(5/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{6311, 6316, 98, 96, 95, 212\}$

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^{5/2}\sqrt{1-\frac{1}{ax}}} + \frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}} + \frac{2x^2\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(
a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*x))^(
5/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])
/(a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} \\
&\quad-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(38+11ax+3a^2x^2)-30\sqrt{2}\sqrt{\frac{1}{x}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{15a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(38 + 11*a*x + 3*a^2*x^2) - 3
0*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(
a*x)])]))/(15*a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3a^2x^2\sqrt{-c(ax+1)}+30\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-11ax\sqrt{-c(ax+1)}-38\sqrt{-c(ax+1)}\right)}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^2}$	125
risch	$-\frac{2(3a^2x^2+11ax+38)c(ax-1)}{15a^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$	130

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*a^2*x^
2*(-c*(a*x+1))^(1/2)+30*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/
2)/c^(1/2))-11*a*x*(-c*(a*x+1))^(1/2)-38*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(
1/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.29

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$= \left[\frac{2\left(15\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(3\,a^3x^3+14\,a^2x^2+49\,ax+3\right.}{15\,(a^3x-a^2)}\right.$$
$$-\frac{2\left(30\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3\,a^3x^3+14\,a^2x^2+49\,ax+38)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right.}{15\,(a^3x-a^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] [2/15*(15*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*
sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2), -2/15*(30*sqrt(2)*(a*x - 1)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a*c*x+c)**(1/2),x)
[Out] Integral(x*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+cx}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a*c*x + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\sqrt{c-a\,c\,x}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.313 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1
/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$	107
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	121

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(1/2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)},\right.$$
$$\left.-\frac{2\left(6\,\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")

[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.314 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(1/
2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2
)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6311, 6316, 100, 163, 56, 221, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)]) -
 (4*Sqrt[2]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{3}{2a}-\frac{x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,\sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-2\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]] - 2*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-2\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}}$	107

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*x-1)*(-c*(a*x-1))^(1/2)*(-2*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1
/2)*2^(1/2)/c^(1/2))+c^(1/2)*arctan((-c*(a*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1)
)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.07

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{2\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-}}{}\right.}{ax-1}\right.$$

$$-\frac{2\left(2\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\right.}{ax-1}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2
*a*x + 1)) + (a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt
(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*(2*sqrt(2)*
(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a
*x + 1))/(a*c*x - c)) - (a*x - 1)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*s
qrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*
x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-c(ax-1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(a*x - 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
[Out] integrate(sqrt(-a*c*x + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.315 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 172

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+5*arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(1/
2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*a^(1/2)*(1/x)^(1/2)*(-a*c*x
+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6311, 6316, 104, 163, 56, 221, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{5\sqrt{a}\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (5*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)] -
 (4*Sqrt[2]*Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}+\frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{3}{2a}-\frac{5x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}+\frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}+\frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}dx,x,\sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}+\frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}dx=\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+5\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-4\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + 5*Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]] - 4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/Sqrt[1 - 1/(a*x)]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-4\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx+5\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\sqrt{c}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\sqrt{c}\,x}$	117
risch	$-\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{5a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{-c(ax+1)}(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$	140

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-4*2^(1/2)*ar
ctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+5*arctan((-c*(a*x+1))^(1
/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/c^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.27

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= \left[\frac{4\sqrt{2}(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+5\,(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+\cdots}{\cdots}\right.}{2\,(ax^2-x)}\right.$$
$$-\frac{4\sqrt{2}(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-5\,(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-ac\cdots}}{ax^2-x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt
(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + 5*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x -
 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*
x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
- x), -(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*s
```

```
qrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 5*(a^2*x^2 - a*x)*sqrt(c)*a
rctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a*c*x + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.316 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1/2*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7/4*a*(1+1/a/x)^(1
/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+23/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(
1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(3/2)*arctanh(2^(1/2
)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)
/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6311, 6316, 103, 159, 163, 56, 221, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{23a^{3/2}\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{3/2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{2x\sqrt{1-\frac{1}{ax}}} + \frac{7a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (7*a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (a*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x) + (23*a^(3/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(3/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
```

```
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
```

```
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}\left(\frac{1}{2}+\frac{7x}{2a}\right)}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{\left(a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{9}{4a}-\frac{23x}{4a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8\sqrt{1-\frac{1}{ax}}} - \frac{\left(4a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,\sqrt{\frac{1}{x}}\right)}{4\sqrt{1-\frac{1}{ax}}} - \frac{\left(8a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(2+9ax) + \frac{23a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}} - \frac{16\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{4\sqrt{1-\frac{1}{ax}}x^2}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(2 + 9*a*x) + (23*a^(3/2)*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]])/(x^(-1))^(3/2) - (16*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqr
t[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(3/2)))/(4*Sqrt[1 - 1/(a*
x)]*x^2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.64

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-16\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+23c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^{2}x^{2}+9ax\sqrt{c}\sqrt{-c(ax+1)}+2\sqrt{-c(ax+1)}\sqrt{c}\right.}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c(ax+1)}\,x^{2}}$
risch	$-\frac{(9a^{2}x^{2}+11ax+2)c(ax-1)}{4x^{2}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^{2}\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{23a^{2}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a*x-1)*(-c*(a*x-1))^(1/2)*(-16*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2
^(1/2)/c^(1/2))*a^2*c*x^2+23*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2+9
*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+2*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x
+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*x+1))^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.91

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^{3}}\,dx$$

$$=\left[\frac{16\sqrt{2}(a^{3}x^{3}-a^{2}x^{2})\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+23\,(a^{3}x^{3}-a^{2}x^{2})\sqrt{-c}\log\left(-\right.}{8\,(ax^{3}-x^{2})}\right.$$

$$-\frac{16\sqrt{2}(a^{3}x^{3}-a^{2}x^{2})\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-23\,(a^{3}x^{3}-a^{2}x^{2})\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-}{4\,(ax^{3}-x^{2})}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2
 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
- 2*c)/(a*x^2 - x)) + 2*(9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt((a*x
```

```
 - 1)/(a*x + 1)))/(a*x^3 - x^2), -1/4*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqr
t((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\int \frac{\sqrt{-acx+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.317 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 274

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2}+\frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x}\\
&+\frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x}\\
&+\frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}\\
&-\frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

```
[Out] 1/3*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+3/4*a^2*(1+1/a/x
)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+13/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)/x/(1-1/a/x)^(1/2)+45/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)
^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(5/2)*arctanh(2^(1/2)*(1/x)^(1/
2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6311, 6316, 103, 159, 163, 56, 221, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{45a^{5/2}\sqrt{\frac{1}{x}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{3a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}} + \frac{13a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}} + \frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3x^2\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^2) + (13*a^2
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (3*a^2*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (45*a^(5/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(5/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}\left(\frac{3}{2}+\frac{9x}{2a}\right)}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(-\frac{9}{4a}-\frac{39x}{4a^2}\right)\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{\left(a^4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{57}{8a^2}+\frac{135x}{8a^3}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{\left(45a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16\sqrt{1-\frac{1}{ax}}} - \frac{\left(4a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{\left(45a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{8\sqrt{1-\frac{1}{ax}}} - \frac{\left(8a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.51

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\left(8+26ax+57a^2x^2\right) + \frac{135a^{5/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}} - \frac{96\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{24\sqrt{1-\frac{1}{ax}}x^3}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(8 + 26*a*x + 57*a^2*x^2) + (135*a^(5/2
)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2) - (96*Sqrt[2]*a^(5/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(5/2)))/(2
4*Sqrt[1 - 1/(a*x)]*x^3)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-96\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+135c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^3x^3+57a^2x^2\sqrt{-c(ax+1)}\sqrt{c}+26ax\sqrt{c}\sqrt{-}\right.}{24\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c(ax+1)}x^3}$
risch	$-\frac{(57a^3x^3+83a^2x^2+34ax+8)c(ax-1)}{24x^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^3\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{45a^3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{-c(ax+1)}(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-96*2^(1/2
)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3+135*c*arctan((-c
*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3+57*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)+26*a
*x*c^(1/2)*(-c*(a*x+1))^(1/2)+8*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(-c*(a*
x+1))^(1/2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.62

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=\left[\frac{96\sqrt{2}(a^4x^4-a^3x^3)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+135\,(a^4x^4-a^3x^3)\sqrt{-c}\log\left(\right.}{48\,(ax^4-x^3}\right.$$
$$-\frac{96\sqrt{2}(a^4x^4-a^3x^3)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-135\,(a^4x^4-a^3x^3)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-}{24\,(ax^4-x^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3
*c)/(a^2*x^2 - 2*a*x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x
^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)
) - 2*c)/(a*x^2 - x)) + 2*(57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3), -1/24*(96*sqrt(2)*(a^4*x^4
 - a^3*x^3)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(sqrt(-a*c*
x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (57*a^3*x^3 + 83*a^
2*x^2 + 34*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^
3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-acx+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.318 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3}+\frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2}+\frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x}+\frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x}+\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1/4*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+11/24*a^2*(1+1/a
/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+21/32*a^3*(1+1/a/x)^(3/2)*(-
a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+107/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2
)/x/(1-1/a/x)^(1/2)+363/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)
*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6311, 6316, 103, 159, 163, 56, 221, 95, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{363a^{7/2}\sqrt{\frac{1}{x}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{7/2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{21a^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{32x\sqrt{1-\frac{1}{ax}}} + \frac{107a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}} + \frac{11a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{24x^2\sqrt{1-\frac{1}{ax}}} + \frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x^3\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^3) + (11*a^2
*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^2) + (107*a^3
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) + (21*a^3*(1 +
 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(32*Sqrt[1 - 1/(a*x)]*x) + (363*a^(7/2)*Sq
rt[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a
*x)]) - (4*Sqrt[2]*a^(7/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}\left(\frac{5}{2}+\frac{11x}{2a}\right)}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(-\frac{33}{4a}-\frac{63x}{4a^2}\right)\sqrt{1+\frac{x}{a}}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{12\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} - \frac{\left(a^5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(\frac{63}{8a^2}+\frac{321x}{8a^3}\right)\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{24\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} + \frac{\left(a^6\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{447}{16a^3}-\frac{1089x}{16a^4}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{24\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} + \frac{\left(363a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{128\sqrt{1-\frac{1}{ax}}} - \frac{\left(4a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} + \frac{\left(363a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{64\sqrt{1-\frac{1}{ax}}} - \frac{\left(8a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} + \frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(48+136ax+214a^2x^2+447a^3x^3)+\frac{1089a^{7/2}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}-\frac{768\sqrt{2}a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{192\sqrt{1-\frac{1}{ax}}x^4}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*x
^3) + (1089*a^(7/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2) - (768*Sq
rt[2]*a^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(x^(-1))^(7/2)))/(192*Sqrt[1 - 1/(a*x)]*x^4)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{(447a^4x^4+661a^3x^3+350a^2x^2+184ax+48)c(ax-1)}{192x^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^4\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{363a^4\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{-c(ax+1)}(a}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-768\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^4cx^4+1089c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4+447a^3x^3\sqrt{-c(ax+1)}\sqrt{c}+214a^2x^2\right.}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c(ax+1)}x^4}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(447*a^4*x^4+661*a^3*x^3+350*a^2*x^2+184*a*x+48)/x^4*c/(a*x+1)/((a*x
-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)-(4*a^4*2^(1/2)/c^(1/2)*arctan
(1/2*(-a*c*x-c)^(1/2)*2^(1/2)/c^(1/2))-363/64*a^4/c^(1/2)*arctan((-a*c*x-c)
^(1/2)/c^(1/2)))*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x+1))^(1/2)/(-c*(
a*x-1))^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.43

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \left[\frac{768\sqrt{2}(a^5x^5-a^4x^4)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+1089\,(a^5x^5-a^4x^4)\sqrt{-c}\,\mathrm{lo}}{38}\right.$$
$$-\frac{768\sqrt{2}(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-1089\,(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right.}{192\,(ax^5-x^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*
c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x +
 1)) - 2*c)/(a*x^2 - x)) + 2*(447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184
*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/19
2*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 1089*(a^5*x^5 - a^4*x^4)*s
qrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184*a*x + 48)*sqrt(-a*c*x +
 c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-acx+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.319 $\int e^{\coth^{-1}(x)} x(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int e^{\coth^{-1}(x)} x(1+x)^{3/2}\,dx = \frac{46\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{92\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

```
[Out] 46/21*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+92/21*(1+x)^(3/2)*((-1+x)/
x)^(1/2)/(1+1/x)^(3/2)/x+8/7*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+2
/7*x^2*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6311, 6316, 91, 79, 47, 37}

$$\int e^{\coth^{-1}(x)} x(1+x)^{3/2}\,dx = \frac{2\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}x^2}{7\left(\frac{1}{x}+1\right)^{3/2}} + \frac{8\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}x}{7\left(\frac{1}{x}+1\right)^{3/2}} + \frac{46\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}} + \frac{92\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}x}$$

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (46*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)) + (92*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)*x) + (8*Sqrt[-((1 - x)/x)
```

```
]*x*(1 + x)^(3/2))/(7*(1 + x^(-1))^(3/2)) + (2*Sqrt[-((1 - x)/x)]*x^2*(1 +
x)^(3/2))/(7*(1 + x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{5/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{(1+x)^2}{\sqrt{1-x}x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{10+\frac{7x}{2}}{\sqrt{1-x}x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(23\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x^{5/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{46\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(46\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x^{3/2}}\,dx,x,\frac{1}{x}\right)}{21\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{46\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{92\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(46+23x+12x^2+3x^3)}{21\sqrt{1+\frac{1}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(46 + 23*x + 12*x^2 + 3*x^3))/(21*Sqrt[1 + 
x^(-1)])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

method	result	size
gosper	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
default	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
risch	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37

```
[In] int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/21*(x-1)*(3*x^3+12*x^2+23*x+46)/(1+x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.23

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \frac{2}{21}\left(3\,x^3+12\,x^2+23\,x+46\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\int \frac{x(x+1)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(3/2),x)
[Out] Integral(x*(x + 1)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.19

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{2\,(3\,x^4+9\,x^3+11\,x^2+23\,x-46)}{21\,\sqrt{x-1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="maxima")
[Out] 2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(x - 1)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\sqrt{\frac{x-1}{x+1}}\left(\frac{46\,x\,\sqrt{x+1}}{21}+\frac{92\,\sqrt{x+1}}{21}+\frac{8\,x^2\,\sqrt{x+1}}{7}+\frac{2\,x^3\,\sqrt{x+1}}{7}\right)$$

```
[In] int((x*(x + 1)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
[Out] ((x - 1)/(x + 1))^(1/2)*((46*x*(x + 1)^(1/2))/21 + (92*(x + 1)^(1/2))/21 +
(8*x^2*(x + 1)^(1/2))/7 + (2*x^3*(x + 1)^(1/2))/7)
```

3.320 $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

```
[Out] 28/15*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+86/15*(1+x)^(3/2)*((-1+x)/
x)^(1/2)/(1+1/x)^(3/2)/x+2/5*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6311, 6316, 91, 79, 37}

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{2\sqrt{-\frac{1-x}{x}}x(x+1)^{3/2}}{5\left(\frac{1}{x}+1\right)^{3/2}}+\frac{28\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}x}$$

```
[In] Int[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

```
[Out] (28*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)) + (86*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)*x) + (2*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(5*(1 + x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{(1+x)^2}{\sqrt{1-x}x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{7+\frac{5x}{2}}{\sqrt{1-x}x^{5/2}}\,dx, x, \frac{1}{x}\right)}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx, x, \frac{1}{x}\right)}{15\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{86\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(43+14x+3x^2)}{15\sqrt{1+\frac{1}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^(3/2),x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(43 + 14*x + 3*x^2))/(15*Sqrt[1 + x^(-1)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

method	result	size
gosper	$\frac{2(x-1)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
default	$\frac{2(x-1)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
risch	$\frac{2(x-1)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(x-1)*(3*x^2+14*x+43)/(1+x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.26

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{2}{15}\left(3\,x^2+14\,x+43\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + 14*x + 43)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A] (verification not implemented)

Time = 42.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = 2\left(\begin{cases}4\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{5}{2}}}{40}+\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{6}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for } \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(3/2),x)
[Out] 2*Piecewise((4*sqrt(2)*(sqrt(2)*(x - 1)**(5/2)/40 + sqrt(2)*(x - 1)**(3/2)/
6 + sqrt(2)*sqrt(x - 1)/2), (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(
2))))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{2\left(3\,x^3+11\,x^2+29\,x-43\right)}{15\,\sqrt{x-1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
[Out] 2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(x - 1)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \sqrt{\frac{x-1}{x+1}}\left(\frac{28\,x\,\sqrt{x+1}}{15}+\frac{86\,\sqrt{x+1}}{15}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

```
[In] int((x + 1)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((28*x*(x + 1)^(1/2))/15 + (86*(x + 1)^(1/2))/15 +
(2*x^2*(x + 1)^(1/2))/5)
```

3.321 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx=\frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] 44/105*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)-22/35*(1+1/x)^(3/2)*(1-x)^(3
/2)*x/(1-1/x)^(3/2)+2/7*(1+1/x)^(3/2)*(1-x)^(3/2)*x^2/(1-1/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6311, 6316, 79, 47, 37}

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx=\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{44\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (44*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(105*(1 - x^(-1))^(3/2)) - (22*(1 + x
^(-1))^(3/2)*(1 - x)^(3/2)*x)/(35*(1 - x^(-1))^(3/2)) + (2*(1 + x^(-1))^(3/
2)*(1 - x)^(3/2)*x^2)/(7*(1 - x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(1-x)^{3/2} \int e^{\coth^{-1}(x)} \left(1-\frac{1}{x}\right)^{3/2} x^{5/2} \, dx}{\left(1-\frac{1}{x}\right)^{3/2} x^{3/2}}$$

$$= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{(1-x)\sqrt{1+x}}{x^{9/2}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}} + \frac{\left(11(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^{7/2}}\,dx, x, \frac{1}{x}\right)}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}} - \frac{\left(22(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^{5/2}}\,dx, x, \frac{1}{x}\right)}{35\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= \frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}} - \frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = -\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(22-11x-18x^2+15x^3)}{105\sqrt{\frac{-1+x}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(22 - 11*x - 18*x^2 + 15*x^3))/(105*Sqrt[(
-1 + x)/x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

method	result	size
gosper	$-\frac{2(1+x)\sqrt{1-x}\,(15x^2-33x+22)}{105\sqrt{\frac{x-1}{1+x}}}$	34
default	$-\frac{2(1+x)\sqrt{1-x}\,(15x^2-33x+22)}{105\sqrt{\frac{x-1}{1+x}}}$	34
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)(15x^3-18x^2-11x+22)}{105\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	62

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(1+x)*(1-x)^(1/2)*(15*x^2-33*x+22)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.43

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = -\frac{2\left(15\,x^4-3\,x^3-29\,x^2+11\,x+22\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{105\,(x-1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
[Out] -2/105*(15*x^4 - 3*x^3 - 29*x^2 + 11*x + 22)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \text{Timed out}$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2)*x,x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2}{105}\left(-15i\,x^3+18i\,x^2+11i\,x-22i\right)\sqrt{x+1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
[Out] 2/105*(-15*I*x^3 + 18*I*x^2 + 11*I*x - 22*I)*sqrt(x + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{16}{105}i\sqrt{2}\mathrm{sgn}(x+1) - \frac{2\left(15\,(x+1)^3\sqrt{-x-1} - 63\,(x+1)^2\sqrt{-x-1} - 70\,(-x-1)^{\frac{3}{2}} - 8i\sqrt{2}\right)\mathrm{sgn}(x)}{105\,\mathrm{sgn}\,(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
[Out] 16/105*I*sqrt(2)*sgn(x + 1) - 2/105*(15*(x + 1)^3*sqrt(-x - 1) - 63*(x + 1)
^2*sqrt(-x - 1) - 70*(-x - 1)^(3/2) - 8*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2\,(15\,x^2-33\,x+22)}{105\,\sqrt{1-x}}$$

```
[In] int((x*(1 - x)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2*(15*x^2 - 33*x + 22))/(105*(1 - x)^(1/
2))
```

3.322 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] -14/15*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)+2/5*(1+1/x)^(3/2)*(1-x)^(3/2
)*x/(1-1/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6311, 6316, 79, 37}

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}-\frac{14\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

```
[Out] (-14*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(15*(1 - x^(-1))^(3/2)) + (2*(1 + x^
(-1))^(3/2)*(1 - x)^(3/2)*x)/(5*(1 - x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(1-x)^{3/2} \int e^{\coth^{-1}(x)} \left(1-\frac{1}{x}\right)^{3/2} x^{3/2}\, dx}{\left(1-\frac{1}{x}\right)^{3/2} x^{3/2}} \\
&= -\frac{\left((1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{(1-x)\sqrt{1+x}}{x^{7/2}}\, dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}} \\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2} (1-x)^{3/2} x}{5\left(1-\frac{1}{x}\right)^{3/2}} + \frac{\left(7(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^{5/2}}\, dx, x, \frac{1}{x}\right)}{5\left(1-\frac{1}{x}\right)^{3/2}} \\
&= -\frac{14\left(1+\frac{1}{x}\right)^{3/2} (1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2} (1-x)^{3/2} x}{5\left(1-\frac{1}{x}\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = -\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(-7-4x+3x^2)}{15\sqrt{\frac{-1+x}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2),x]
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(-7 - 4*x + 3*x^2))/(15*Sqrt[(-1 + x)/x])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
default	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)(3x^2-4x-7)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	57

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/15*(1+x)*(3*x-7)*(1-x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = -\frac{2\,(3\,x^3-x^2-11\,x-7)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
[Out] -2/15*(3*x^3 - x^2 - 11*x - 7)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=\int \frac{(1-x)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2),x)
[Out] Integral((1 - x)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=\frac{2}{15}\left(-3i\,x^2+4i\,x+7i\right)\sqrt{x+1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
[Out] 2/15*(-3*I*x^2 + 4*I*x + 7*I)*sqrt(x + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{16}{15}i\sqrt{2}\mathrm{sgn}(x+1)-\frac{2\left(3\,(x+1)^2\sqrt{-x-1}+10\,(-x-1)^{\frac{3}{2}}+8i\sqrt{2}\right)\mathrm{sgn}(x)}{15\,\mathrm{sgn}\,(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="giac")
[Out] -16/15*I*sqrt(2)*sgn(x + 1) - 2/15*(3*(x + 1)^2*sqrt(-x - 1) + 10*(-x - 1)^
(3/2) + 8*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=\frac{2\,(3\,x-7)\,\sqrt{\frac{x-1}{x+1}}\,{(x+1)}^2}{15\,\sqrt{1-x}}$$

```
[In] int((1 - x)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
[Out] (2*(3*x - 7)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

3.323 $\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{6\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

```
[Out] 12/5*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+6/5*x*((-1+x)/x)^(1/2)*(1+x
)^(1/2)/(1+1/x)^(1/2)+2/5*x^2*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6311, 6316, 79, 47, 37}

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x^2}{5\sqrt{\frac{1}{x}+1}}+\frac{6\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x}{5\sqrt{\frac{1}{x}+1}}+\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{5\sqrt{\frac{1}{x}+1}}$$

```
[In] Int[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

```
[Out] (12*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (6*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x)/x)]*x^2*Sqrt[
1 + x])/(5*Sqrt[1 + x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}x^{3/2}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1+x}{\sqrt{1-x}x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} - \frac{\left(9\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx, x, \frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$

$$= \frac{6\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} - \frac{\left(6\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx, x, \frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$

$$= \frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{6\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(6+3x+x^2)}{5\sqrt{1+\frac{1}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*x*Sqrt[1 + x],x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(6 + 3*x + x^2))/(5*Sqrt[1 + x^(-1)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
default	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
risch	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30

```
[In] int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(x-1)*(x^2+3*x+6)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \frac{2}{5}\left(x^2+3\,x+6\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(x^2 + 3*x + 6)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \int \frac{x\sqrt{x+1}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(1/2),x)
[Out] Integral(x*sqrt(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.19

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \frac{2\left(x^3+2\,x^2+3\,x-6\right)}{5\sqrt{x-1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(x - 1)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\sqrt{\frac{x-1}{x+1}}\left(\frac{6\,x\,\sqrt{x+1}}{5}+\frac{12\,\sqrt{x+1}}{5}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

```
[In] int((x*(x + 1)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
[Out] ((x - 1)/(x + 1))^(1/2)*((6*x*(x + 1)^(1/2))/5 + (12*(x + 1)^(1/2))/5 + (2*
x^2*(x + 1)^(1/2))/5)
```

3.324 $\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

```
[Out] 10/3*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+2/3*x*((-1+x)/x)^(1/2)*(1+x
)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6311, 6316, 79, 37}

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x}{3\sqrt{\frac{1}{x}+1}}+\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{3\sqrt{\frac{1}{x}+1}}$$

```
[In] Int[E^ArcCoth[x]*Sqrt[1 + x],x]
```

```
[Out] (10*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}\sqrt{x}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1+x}{\sqrt{1-x}x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}} - \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1+\frac{1}{x}}}$$

$$= \frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(5+x)}{3\sqrt{1+\frac{1}{x}}}$$

[In] Integrate[E^ArcCoth[x]*Sqrt[1 + x],x]

[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(5 + x))/(3*Sqrt[1 + x^(-1)])

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

method	result	size
gosper	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25

[In] int(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*(x-1)*(x+5)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{2}{3}\,(x+5)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="fricas")

[Out] 2/3*(x + 5)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))

Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$$
$$= 2\left(\begin{cases} 2\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{12}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for } \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2} \end{cases}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(1/2),x)
[Out] 2*Piecewise((2*sqrt(2)*(sqrt(2)*(x - 1)**(3/2)/12 + sqrt(2)*sqrt(x - 1)/2),
 (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2))))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2\,(x^2+4\,x-5)}{3\,\sqrt{x-1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^2 + 4*x - 5)/sqrt(x - 1)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+5)}{3}$$

```
[In] int((x + 1)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 5))/3
```

3.325 $\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=-\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}$$

```
[Out] -4/15*(1+1/x)^(3/2)*x*(1-x)^(1/2)/(1-1/x)^(1/2)+2/5*(1+1/x)^(3/2)*x^2*(1-x)
^(1/2)/(1-1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6311, 6316, 47, 37}

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}-\frac{4\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}$$

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (-4*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(15*Sqrt[1 - x^(-1)]) + (2*(1 + x^(-1
))^(3/2)*Sqrt[1 - x]*x^2)/(5*Sqrt[1 - x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-x}\int e^{\coth^{-1}(x)}\sqrt{1-\frac{1}{x}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{x}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{1-x}\sqrt{\frac{1}{x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x}}} \\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}+\frac{\left(2\sqrt{1-x}\sqrt{\frac{1}{x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1-\frac{1}{x}}} \\
&= -\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(-2+x+3x^2)}{15\sqrt{\frac{-1+x}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(-2 + x + 3*x^2))/(15*Sqrt[(-1 + x)/x])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
default	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
risch	$-\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)\,(3x^2+x-2)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	55

```
[In] int(1/((x-1)/(1+x))^(1/2)*x*(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+x)*(3*x-2)*(1-x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=\frac{2\,(3\,x^3+4\,x^2-x-2)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^3 + 4*x^2 - x - 2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=\int\frac{x\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1-x)**(1/2),x)
[Out] Integral(x*sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=-\frac{2}{15}\left(-3i\,x^2-i\,x+2i\right)\sqrt{x+1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="maxima")
[Out] -2/15*(-3*I*x^2 - I*x + 2*I)*sqrt(x + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=-\frac{4}{15}i\sqrt{2}\mathrm{sgn}(x+1)+\frac{2\left(3\,(x+1)^2\sqrt{-x-1}+5\,(-x-1)^{\frac{3}{2}}-2i\,\sqrt{2}\right)\mathrm{sgn}(x)}{15\,\mathrm{sgn}\,(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="giac")
[Out] -4/15*I*sqrt(2)*sgn(x + 1) + 2/15*(3*(x + 1)^2*sqrt(-x - 1) + 5*(-x - 1)^(3
/2) - 2*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=-\frac{2\,(3\,x-2)\,\sqrt{\frac{x-1}{x+1}}\,{(x+1)}^2}{15\,\sqrt{1-x}}$$

```
[In] int((x*(1 - x)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
[Out] -(2*(3*x - 2)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

3.326 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}(1+x)$$

```
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6309}

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2}{3}\sqrt{1-x}(x+1)e^{\coth^{-1}(x)}$$

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x],x]
```

```
[Out] (2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}(1+x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{3\sqrt{1-\frac{1}{x}}}$$

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x],x]
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(3*Sqrt[1 - x^(-1)])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result	size
gosper	$\frac{2(1+x)\sqrt{1-x}}{3\sqrt{\frac{x-1}{1+x}}}$	24
default	$\frac{2(1+x)\sqrt{1-x}}{3\sqrt{\frac{x-1}{1+x}}}$	24
risch	$-\frac{2(1+x)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	50

```
[In] int(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((x-1)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2\,(x^2+2\,x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{3\,(x-1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^2 + 2*x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\int \frac{\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(1/2),x)
[Out] Integral(sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=-\frac{2}{3}\sqrt{x+1}(-i\,x-i)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(x + 1)*(-I*x - I)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=-\frac{4}{3}i\sqrt{2}\mathrm{sgn}(x+1)-\frac{2\left((-x-1)^{\frac{3}{2}}+2i\sqrt{2}\right)\mathrm{sgn}(x)}{3\,\mathrm{sgn}\left(x+1\right)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="giac")
[Out] -4/3*I*sqrt(2)*sgn(x + 1) - 2/3*((-x - 1)^(3/2) + 2*I*sqrt(2))*sgn(x)/sgn(x
 + 1)
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{3\sqrt{1-x}}$$

```
[In] int((1 - x)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
[Out] -(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(3*(1 - x)^(1/2))
```

3.327 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\frac{4\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{1+x}}+\frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{1+x}}$$

```
[Out] 4/3*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)+2/3*x^2*(1+1/x)^(1/2)*((-1
+x)/x)^(1/2)/(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6311, 6316, 47, 37}

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{x+1}}+\frac{4\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{x+1}}$$

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (4*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/(3*Sqrt[1 + x]) + (2*Sqrt[1 + x^(
-1)]*Sqrt[-((1 - x)/x)]*x^2)/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1+\frac{1}{x}}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}\\
&= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{1+x}}-\frac{\left(2\sqrt{1+\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{\frac{1}{x}}\sqrt{1+x}}\\
&= \frac{4\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{3\sqrt{1+x}}+\frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\sqrt{1-\frac{1}{x^2}}x(2+x)}{3\sqrt{1+x}}$$

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
[Out] (2*Sqrt[1 - x^(-2)]*x*(2 + x))/(3*Sqrt[1 + x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

method	result	size
gosper	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x-1)*(x+2)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+2)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x + 2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \begin{cases} \frac{2x\sqrt{x-1}}{3} + \frac{4\sqrt{x-1}}{3} & \text{for } |x| > 1 \\ \frac{2ix\sqrt{1-x}}{3} + \frac{4i\sqrt{1-x}}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(x) > 1), (2*I*x*sqrt(1
- x)/3 + 4*I*sqrt(1 - x)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\,(x^2+x-2)}{3\,\sqrt{x-1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + x - 2)/sqrt(x - 1)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+2)}{3}$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 2))/3
```

3.328 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} \, dx = \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{\sqrt{1+x}}$$

```
[Out] 2*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 37}

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} \, dx = \frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x}{\sqrt{x+1}}$$

```
[In] Int[E^ArcCoth[x]/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/Sqrt[1 + x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1+x}}$$

$$= -\frac{\sqrt{1+\frac{1}{x}}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}$$

$$= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{\sqrt{1+x}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = \frac{2\sqrt{1-\frac{1}{x^2}}x}{\sqrt{1+x}}$$

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 + x],x]
[Out] (2*Sqrt[1 - x^(-2)]*x)/Sqrt[1 + x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
default	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
risch	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x-1)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = \begin{cases} 2\sqrt{x-1} & \text{for } |x|>1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.21

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = -\frac{2\left(i\sqrt{2}-\sqrt{x-1}\right)\operatorname{sgn}(x)}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*(I*sqrt(2) - sqrt(x - 1))*sgn(x)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{\frac{x-1}{x+1}}\sqrt{x+1}$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
[Out] 2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)
```

3.329 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] 2/3*(1+1/x)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(1/2)+2*x*(1-1/x)^(1/2)*(1+1/x)^(
1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))*2^(1/2)*(1-1/
x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6311, 6316, 98, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = -\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}} + \frac{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{3\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{1-x}}$$

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] + (2*Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2)/(3*Sqrt[1 - x]) - (2*Sqrt[2]*Sqrt[1 - x^(-1)]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[x^(-1)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1-\frac{1}{x}}}\,dx}{\sqrt{1-x}}\\
&= -\frac{\sqrt{1-\frac{1}{x}}\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{\sqrt{1-\frac{1}{x}}\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}x\left(\sqrt{1+\frac{1}{x}}(4+x)-3\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{3\sqrt{1-x}}$$

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)]*(4 + x) - 3*Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(3*Sqrt[1 - x])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{2\sqrt{1-x}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-\sqrt{-1-x}\,x-4\sqrt{-1-x}\right)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{-1-x}}$	66
risch	$\frac{2(x+4)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{3\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	111

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*(3*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1
/2))-(-1-x)^(1/2)*x-4*(-1-x)^(1/2))/(-1-x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx=\frac{2\left(3\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x^2+5x+4)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{3(x-1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x
 - 1)) - (x^2 + 5*x + 4)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx=\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\sqrt{1-x}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: (-12*
atan(i)+20*i)*1/3/sqrt(2)*sign(sageVARx+1)-(-2/3*sqrt(-sageVARx-1)*(-sageVA
Rx-1)+2*sqrt(-sageVARx-1)+1/3*(12*atan(i)-20*i)/sqrt(2)-4*atan(sqrt(-sageVA
Rx-1)/sqrt(2))/s
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

3.330 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] 2*x*(1-1/x)^(1/2)*(1+1/x)^(1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(
1+1/x)^(1/2))*2^(1/2)*(1-1/x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[In] Int[E^ArcCoth[x]/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] - (2*Sqrt[2]*Sqrt[1 - x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[
x^(-1)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1-x}}\\
&= -\frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}
\end{aligned}
$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}x\left(\sqrt{1+\frac{1}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{\sqrt{1-x}}$$

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 - x],x]
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)] - Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[Sqrt
[2]*Sqrt[(1 + x)^(-1)]]))/Sqrt[1 - x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2\sqrt{1-x}\left(\sqrt{2}\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-\sqrt{-1-x}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{-1-x}}$	55
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	108

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))
-(-1-x)^(1/2))/(-1-x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\left(\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{x-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] 2*(sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1
)) - (x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = -2\left(\begin{cases}\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2}-\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) & \text{for } \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(1/2),x)
[Out] -2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x))),
 (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: (-4*a
tan(i)+4*i)/sqrt(2)*sign(sageVARx+1)-(2*sqrt(-sageVARx-1)+(4*atan(i)-4*i)/s
qrt(2)-4*atan(sqrt(-sageVARx-1)/sqrt(2))/sqrt(2))*sign(sageVARx)/sign(sageV
ARx+1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

3.331 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx=\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}}+\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

```
[Out] (1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(3
/2)/(1+x)^(3/2)+2*(1+1/x)^(3/2)*x^2*((-1+x)/x)^(1/2)/(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6311, 6316, 98, 95, 209}

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx=\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}+\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(x+1)^{3/2}}$$

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[-((1 - x)/x)]*x^2)/(1 + x)^(3/2) + (Sqrt[2]*(1 +
 x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/x)]])/((x^(-1))
^(3/2)*(1 + x)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int\frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1+x)^{3/2}}\\
&= -\frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x^{3/2}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(1+x)^{3/2}}+\frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{x}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}
\end{aligned}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(1+x)^{3/2}} + \frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(1+x)^{3/2}} + \frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{\sqrt{1+\frac{1}{x}}x\left(2\sqrt{\frac{-1+x}{x}} - \sqrt{2}\sqrt{\frac{1}{x}}\arctan\left(\frac{\sqrt{\frac{-1+x}{x^2}}x}{\sqrt{2}}\right)\right)}{\sqrt{1+x}}$$

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
[Out] (Sqrt[1 + x^(-1)]*x*(2*Sqrt[(-1 + x)/x] - Sqrt[2]*Sqrt[x^(-1)]*ArcTan[(Sqrt
[(-1 + x)/x^2]*x)/Sqrt[2]]))/Sqrt[1 + x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\sqrt{x-1}\left(\sqrt{2}\arctan\left(\frac{\sqrt{x-1}\sqrt{2}}{2}\right)-2\sqrt{x-1}\right)}{\sqrt{\frac{x-1}{1+x}}\sqrt{1+x}}$	47
risch	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\sqrt{1+x}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{x-1}\sqrt{2}}{2}\right)\sqrt{x-1}}{\sqrt{\frac{x-1}{1+x}}\sqrt{1+x}}$	60

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(x-1)^(1/2)*(2^(1/2)*arctan(1/2*(x-1)^(1/2)*2^(1/2))-2*(x-1)^(1/2))/((x-1)
/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)+2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))) + 2*sqrt(x +
 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(3/2),x)
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: 2*(sq
rt(sageVARx-1)+(atan(i)-2*i)/sqrt(2)-atan(sqrt(sageVARx-1)/sqrt(2))/sqrt(2)
)*sign(sageVARx)/sign(sageVARx+1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

3.332 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = -\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

```
[Out] -(1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(
3/2)/(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6311, 6316, 95, 209}

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

```
[In] Int[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] -((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/
x)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1+\frac{1}{x}\right)^{3/2} x^{3/2}\right) \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2} x^{3/2}} \, dx}{(1+x)^{3/2}} \\
&= -\frac{\left(1+\frac{1}{x}\right)^{3/2} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{x}(1+x)} \, dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2} (1+x)^{3/2}} \\
&= -\frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{1}{1+2x^2} \, dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2} (1+x)^{3/2}} \\
&= -\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2} \arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2} (1+x)^{3/2}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \sqrt{2}\sqrt{\frac{1}{1+x}}\sqrt{1+x}\arctan\left(\frac{\sqrt{\frac{-1+x}{x^2}}x}{\sqrt{2}}\right)$$

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] Sqrt[2]*Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcTan[(Sqrt[(-1 + x)/x^2]*x)/Sqrt[2
]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)\sqrt{x-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	37

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*arctan(1/2*(x-1)^(1/2)*2^(1/2))/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)*(x-
1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1)))
```

Sympy [A] (verification not implemented)

Time = 40.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = 2\left(\begin{cases} \frac{\sqrt{2}\,\mathrm{acos}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{for } \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(3/2),x)
[Out] 2*Piecewise((sqrt(2)*acos(sqrt(2)/sqrt(x + 1))/2, (sqrt(x + 1) < sqrt(2)) &
 (sqrt(x + 1) > -sqrt(2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: 2*(-1
/2*sqrt(2)*atan(i)+1/2*sqrt(2)*atan(sqrt(sageVARx-1)/sqrt(2)))*sign(sageVAR
x)/sign(sageVARx+1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

3.333 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{5\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -5/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(3/2)/(
1/x)^(3/2)*2^(1/2)-1/2*(1+1/x)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(3/2)+5/2*(1-1
/x)^(3/2)*x^2*(1+1/x)^(1/2)/(1-x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6311, 6316, 98, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = -\frac{5\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{2(1-x)^{3/2}} + \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{x}+1}x^2}{2(1-x)^{3/2}}$$

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (5*(1 - x^(-1))^(3/2)*Sqrt[1 + x^(-1)]*x^2)/(2*(1 - x)^(3/2)) - (Sqrt[1 - x
^(-1)]*(1 + x^(-1))^(3/2)*x^2)/(2*(1 - x)^(3/2)) - (5*(1 - x^(-1))^(3/2)*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*(x^(
-1))^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{x}\right)^{3/2} x^{3/2}\right) \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2} \sqrt{x}}\, dx}{(1-x)^{3/2}} \\
&= -\frac{\left(1-\frac{1}{x}\right)^{3/2} \operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2 x^{3/2}}\, dx, x, \frac{1}{x}\right)}{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}} \\
&= -\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2} x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x) x^{3/2}}\, dx, x, \frac{1}{x}\right)}{4(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}} \\
&= \frac{5\left(1-\frac{1}{x}\right)^{3/2} \sqrt{1+\frac{1}{x}} x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2} x^2}{2(1-x)^{3/2}} \\
&\quad - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\, dx, x, \frac{1}{x}\right)}{2(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}} \\
&= \frac{5\left(1-\frac{1}{x}\right)^{3/2} \sqrt{1+\frac{1}{x}} x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2} x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{1}{1-2x^2}\, dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}} \\
&= \frac{5\left(1-\frac{1}{x}\right)^{3/2} \sqrt{1+\frac{1}{x}} x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2} x^2}{2(1-x)^{3/2}} - \frac{5\left(1-\frac{1}{x}\right)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)} x}{(1-x)^{3/2}}\, dx = -\frac{\sqrt{\frac{-1+x}{x}}\, x\left(2\sqrt{1+\frac{1}{x}}(3-2x)+5\sqrt{2}(-1+x)\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
[Out] -1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)]*(3 - 2*x) + 5*Sqrt[2]*(-1 + x)
*Sqrt[x^(-1)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{\sqrt{1-x}\,\left(5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)x-5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-4\sqrt{-1-x}\,x+6\sqrt{-1-x}\right)}{2\sqrt{\frac{x-1}{1+x}}\,(x-1)\sqrt{-1-x}}$	90
risch	$-\frac{(2x^{2}-x-3)\sqrt{\frac{(1+x)(1-x)}{x-1}}}{\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	120

```
[In] int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((x-1)/(1+x))^(1/2)/(x-1)*(1-x)^(1/2)*(5*2^(1/2)*arctan(1/2*(-1-x)^(1/
2)*2^(1/2))*x-5*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))-4*(-1-x)^(1/2)*x+6
*(-1-x)^(1/2))/(-1-x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = -\frac{5\,\sqrt{2}(x^{2}-2\,x+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-2\,(2\,x^{2}-x-3)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x^{2}-2\,x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(5*sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x
 + 1))/(x - 1)) - 2*(2*x^2 - x - 3)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^
2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 101.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = 2\left(\begin{cases}\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2}-\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right) - 2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(3/2),x)
```

```
[Out] 2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x))),
(sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2)))) - 2*Piecewise((sqrt(2)
*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1 - x))/(2*sqrt(1 - x))
)/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \int \frac{x}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right) - 2\sqrt{-x-1} + \frac{\sqrt{-x-1}}{x-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - 2*sqrt(-x - 1) + sqrt(-x - 1
)/(x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

3.334 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx=-\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -1/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(3/2)/(
1/x)^(3/2)*2^(1/2)-x*(1-1/x)^(1/2)*(1+1/x)^(1/2)/(1-x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6311, 6316, 96, 95, 212}

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx=-\frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}-\frac{\sqrt{\frac{1}{x}+1}x\sqrt{1-\frac{1}{x}}}{(1-x)^{3/2}}$$

```
[In] Int[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] -((Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/(1 - x)^(3/2)) - ((1 - x^(-1))^(3/2
)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*
(x^(-1))^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{x}\right)^{3/2} x^{3/2}\right) \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2} x^{3/2}} \, dx}{(1-x)^{3/2}} \\
&= -\frac{\left(1-\frac{1}{x}\right)^{3/2} \operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2 \sqrt{x}} \, dx, x, \frac{1}{x}\right)}{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}} \\
&= -\frac{\sqrt{1-\frac{1}{x}} \sqrt{1+\frac{1}{x}} x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2} \operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}} \, dx, x, \frac{1}{x}\right)}{2(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}}
\end{aligned}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}x\left(2\sqrt{1+\frac{1}{x}}+\sqrt{2}(-1+x)\sqrt{\frac{1}{x}}\text{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)] + Sqrt[2]*(-1 + x)*Sqrt[x^(-1)
]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{1-x}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)x-\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)+2\sqrt{-1-x}\right)}{2\sqrt{\frac{x-1}{1+x}}\,(x-1)\sqrt{-1-x}}$	79
risch	$\frac{\sqrt{\frac{(1+x)(1-x)}{x-1}}}{\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	104

```
[In] int(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((x-1)/(1+x))^(1/2)/(x-1)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/2)
*2^(1/2))*x-2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))+2*(-1-x)^(1/2))/(-1-x)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{2}(x^2-2\,x+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)+2\,(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2\,x+1)}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)) + 2*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^2 - 2*x +
1)
```

Sympy [A] (verification not implemented)

Time = 97.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(3/2),x)
```

```
[Out] -2*Piecewise((sqrt(2)*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1
- x))/(2*sqrt(1 - x)))/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2)
)))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \int \frac{1}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right) + \frac{\sqrt{-x-1}}{x-1}$$

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) + sqrt(-x - 1)/(x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

3.335 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} - \frac{2(5+4m)x^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -2*(5+4*m)*x^m*hypergeom([1/2, -1/2-m],[1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/a/(
4*m^2+8*m+3)/(1-1/a/x)^(1/2)+2*x^(1+m)*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(3+
2*m)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6311, 6316, 80, 66}

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx = \frac{2\sqrt{\frac{1}{ax}+1}x^{m+1}\sqrt{c-acx}}{(2m+3)\sqrt{1-\frac{1}{ax}}} - \frac{2(4m+5)x^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{1}{2}-m,-\frac{1}{ax}\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*x^(1 + m)*Sqrt[c - a*c*x])/((3 + 2*m)*Sqrt[1 - 1/(a*x)
]) - (2*(5 + 4*m)*x^m*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -1/2 - m, 1/2
- m, -(1/(a*x))])/(a*(1 + 2*m)*(3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{x^{-\frac{5}{2}-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} + \frac{\left((5+4m)\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{x^{-\frac{3}{2}-m}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a(3+2m)\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} - \frac{2(5+4m)x^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.78

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \frac{2x^m\sqrt{c-acx}\left(a(1+2m)\sqrt{1+\frac{1}{ax}}x-(5+4m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
[Out] (2*x^m*Sqrt[c - a*c*x]*(a*(1 + 2*m)*Sqrt[1 + 1/(a*x)]*x - (5 + 4*m)*Hyperge
ometric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))]))/(a*(1 + 2*m)*(3 + 2*m)*Sqr
t[1 - 1/(a*x)])
```

Maple [F]

$$\int x^m\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}dx$$

```
[In] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c - acx} \, dx = \int \sqrt{-acx + c} x^m \sqrt{\frac{ax - 1}{ax + 1}} \, dx$$

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] integral(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c - acx} \, dx = \text{Timed out}$$

```
[In] integrate(x**m*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c - acx} \, dx = \int \sqrt{-acx + c} x^m \sqrt{\frac{ax - 1}{ax + 1}} \, dx$$

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c - acx} \, dx = \int \sqrt{-acx + c} x^m \sqrt{\frac{ax - 1}{ax + 1}} \, dx$$

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx = \int x^m\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.336 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{152c\sqrt{1-\frac{1}{a^2x^2}}x}{105a^2\sqrt{c-acx}}+\frac{38\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}}{105a^2}+\frac{6\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}}{35a^2c}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x^2(c-acx)^{3/2}}{7ac}$$

```
[Out] 6/35*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a^2/c-2/7*x^2*(-a*c*x+c)^(3/2)*
(1-1/a^2/x^2)^(1/2)/a/c+152/105*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(-a*c*x+c)^(1/2
)+38/105*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6311, 6316, 79, 47, 37}

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=-\frac{208\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{104x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{26x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (-208*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) + (104
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (26*Sqr
```

```
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 +
1/(a*x)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(13\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{26\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(52\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{26\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(104\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{208\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} + \frac{104\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{26\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-104+52ax-39a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]

[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3*Sqrt[1 - 1/(a*x)])

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(15a^3x^3-39a^2x^2+52ax-104)(ax+1)}{105\sqrt{-c(ax-1)}\,a^3}$	59
gosper	$\frac{2(ax+1)(15a^3x^3-39a^2x^2+52ax-104)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105a^3(ax-1)}$	64
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(15a^3x^3-39a^2x^2+52ax-104)}{105(ax-1)a^3}$	65

[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/105*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(15*a^3*x^3-39*a^2*x^2+52*a*x-104)/a^3*(a*x+1)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\frac{2\left(15\,a^4x^4-24\,a^3x^3+13\,a^2x^2-52\,ax-104\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

[Out] 2/105*(15*a^4*x^4 - 24*a^3*x^3 + 13*a^2*x^2 - 52*a*x - 104)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int x^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^4\sqrt{-c}x^4-24\,a^3\sqrt{-c}x^3+13\,a^2\sqrt{-c}x^2-52\,a\sqrt{-c}x-104\,\sqrt{-c}\right)(ax-1)}{105\,(a^4x-a^3)\sqrt{ax+1}}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^4*sqrt(-c)*x^4 - 24*a^3*sqrt(-c)*x^3 + 13*a^2*sqrt(-c)*x^2 - 52
*a*sqrt(-c)*x - 104*sqrt(-c))*(a*x - 1)/((a^4*x - a^3)*sqrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-9\,a^2\,x^2+4\,a\,x-48\right)}{105\,a^3} - \frac{304\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

```
[In] int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x - 9*a^2*x^2 + 15*a^
3*x^3 - 48))/(105*a^3) - (304*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)
)/(105*a^3*(a*x - 1))
```

3.337 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = -\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{5a\sqrt{c-acx}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}}{5a}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}}{5ac}$$

```
[Out] -2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a/c-8/5*c*x*(1-1/a^2/x^2)^(1/2)
/a/(-a*c*x+c)^(1/2)-2/5*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6311, 6316, 79, 47, 37}

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{12\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{6x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (12*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(5*a^2*Sqrt[1 - 1/(a*x)]) - (6*Sqrt[
1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*
x)]*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{\left(9\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(6\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{12\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}} - \frac{6\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(6-3ax+a^2x^2)}{5a^2\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[1 -
 1/(a*x)])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-3ax+6)(ax+1)}{5\sqrt{-c(ax-1)}\,a^2}$	50
gosper	$\frac{2(ax+1)(a^2x^2-3ax+6)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5a^2(ax-1)}$	55
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(a^2x^2-3ax+6)}{5(ax-1)a^2}$	56

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a^2*x^2-3*a*x+6)/a^2*(a*
x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left(a^3x^3-2\,a^2x^2+3\,ax+6\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^3x-a^2\right)}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^3*x^3 - 2*a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left(a^3\sqrt{-c}x^3 - 2\,a^2\sqrt{-c}x^2 + 3\,a\sqrt{-c}x + 6\,\sqrt{-c}\right)(ax-1)}{5\,(a^3x-a^2)\sqrt{ax+1}}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a^3*sqrt(-c)*x^3 - 2*a^2*sqrt(-c)*x^2 + 3*a*sqrt(-c)*x + 6*sqrt(-c))*(
a*x - 1)/((a^3*x - a^2)*sqrt(a*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = -\frac{4\sqrt{-acx-c}|c|}{a^2c} - \frac{2\left((acx+c)^2\sqrt{-acx-c}|c| + 5\,(-acx-c)^{\frac{3}{2}}c|c|\right)}{5\,a^2c^3}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-a*c*x - c)*abs(c)/(a^2*c) - 2/5*((a*c*x + c)^2*sqrt(-a*c*x - c)*ab
s(c) + 5*(-a*c*x - c)^(3/2)*c*abs(c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^3\,x^3-2\,a^2\,x^2+3\,a\,x+6\right)}{5\,a^2\,\left(a\,x-1\right)}$$

```
[In] int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a*x - 2*a^2*x^2 + a^3*x
^3 + 6))/(5*a^2*(a*x - 1))
```

3.338 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}$$

```
[Out] 8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a*
c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 79, 37}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c(ax-1)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-5)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x-5\,\sqrt{-c}\right)(ax-1)}{3\,(a^2x-a)\sqrt{ax+1}}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2}+\frac{4\,\sqrt{-acx-c}|c|}{ac}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] 2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.339 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6311, 6316, 79, 56, 221}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}} + \sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{(ax-1)\sqrt{-c(ax+1)}}$	80

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a
*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.19

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, \frac{2\left((ax-1\right.}{}\right.$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), 2*((a*x - 1)*sqrt(c)*arct
an(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) + sqrt(-
a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}{x}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{-acx-c}}{c}\right)|c|$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] -2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + sqrt(-a*c*x - c)/c)*abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.340 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-3*arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6311, 6316, 81, 56, 221}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) - (3*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}-\frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,\sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}-3\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - 3*Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-3\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{(ax-1)\sqrt{-c(ax+1)}\,x\sqrt{c}}$	90
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c(ax-1)}}-\frac{3a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	95

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
/x/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.42

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{3\,(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^2-x)},\right.$$
$$\left.-\frac{3\,(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(3*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -(3*(a^2*x^
2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/
(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
 - x)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}{x^2}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima
")

[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = a\left(\frac{3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{\sqrt{-acx-c}}{acx}\right)|c|$$

[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")

[Out] a*(3*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - sqrt(-a*c*x - c)/(a*c*x))*a
bs(c)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)

[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)

3.341 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^4}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^4/c+2/5*(-a*c*x+c)^(5/2)/a^4/c^2-2/7*(-a*c*x+c)^(7/2
)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4-4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/
2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4+4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 90, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}+\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x)^
(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(9/
2))/(9*a^4*c^4) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[
c])])/a^4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)} x^3 \sqrt{c-acx}\, dx \\
&= -\int \frac{x^3(1-ax)\sqrt{c-acx}}{1+ax}\, dx \\
&= -\frac{\int \frac{x^3(c-acx)^{3/2}}{1+ax}\, dx}{c} \\
&= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^3} - \frac{(c-acx)^{3/2}}{a^3(1+ax)} - \frac{(c-acx)^{5/2}}{a^3c} + \frac{(c-acx)^{7/2}}{a^3c^2}\right) dx}{c} \\
&= \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\, dx}{a^3c} \\
&= \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\, dx}{a^3} \\
&= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} \\
&\quad - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\, dx}{a^3} \\
&= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} \\
&\quad + \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\, dx, x, \sqrt{c-acx}\right)}{a^4} \\
&= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} \\
&\quad + \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$
\int e^{-2\coth^{-1}(ax)} x^3 \sqrt{c-acx}\, dx = \frac{2\left(\sqrt{c-acx}(788-236ax+138a^2x^2-95a^3x^3+35a^4x^4) - 630\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a^4}
$$

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4)
 - 630*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(315*a^
4)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$\frac{\frac{2\left(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788\right)\sqrt{-c(ax-1)}}{315}-4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^4}$	75
risch	$-\frac{2(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788)(ax-1)c}{315a^4\sqrt{-c(ax-1)}}-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^4}$	82
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^4c^4}$	101
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^4c^4}$	101

```
[In] int(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*((35*a^4*x^4-95*a^3*x^3+138*a^2*x^2-236*a*x+788)*(-c*(a*x-1))^(1/2)-6
30*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2)))/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.21

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \left[\frac{2\left(315\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788)\sqrt{-acx+c}\right)}{315a^4},\right.$$

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 788
)*sqrt(-a*c*x + c))/a^4, 2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*
x + 788)*sqrt(-a*c*x + c))/a^4]
```

Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \begin{cases} \frac{2\cdot\left(\frac{2\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^4\sqrt{-acx+c}+\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^4}{4}-\frac{2x^3}{3a}+\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise} \end{cases}\right)}{a^3}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise(((2*(2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sq
rt(-c) + 2*c**4*sqrt(-a*c*x + c) + c**3*(-a*c*x + c)**(3/2)/3 + c**2*(-a*c*
x + c)**(5/2)/5 - c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c*
*4), Ne(a*c, 0)), (sqrt(c)*(x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 +
2*Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \frac{2\left(315\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}-45(-acx+c)^{\frac{7}{2}}c+63(-acx+c)^{\frac{5}{2}}c^2+105(-acx+\right.}{315\,a^4c^4}$$

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) - 45*(-a*c*x + c)^(
7/2)*c + 63*(-a*c*x + c)^(5/2)*c^2 + 105*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(
-a*c*x + c)*c^4)/(a^4*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}} + \frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^{32}c^{32}+45\,(acx-c)^3\sqrt{-acx+c}a^{32}c^{33}+63\,(acx-c)^2\sqrt{-acx+c}a^{32}c^{34}+\right.}{315\,a^{36}c^{36}}$$

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) +
2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 105*(
-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^36)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^4} + \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^4\,c} + \frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^4\,c^2} - \frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^4\,c^3} + \frac{2\,(c-a\,c\,x)^{9/2}}{9\,a^4\,c^4} + \frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^4}$$

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(
9/2))/(9*a^4*c^4) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2
*c^(1/2)))*4i)/a^4
```

3.342 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=-\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a^3/c-2/7*(-a*c*x+c)^(7/2)/a^3/c^3+4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3-4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 90, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}$$

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c*x)
^(7/2))/(7*a^3*c^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}x^2\sqrt{c-acx}\,dx\\
&= -\int \frac{x^2(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{x^2(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(1+ax)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}\\
&= -\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^2c}\\
&= -\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^2}\\
&= -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a^2}\\
&= -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a^3}\\
&= -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$
\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-52+16ax-9a^2x^2+3a^3x^3)+84\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}
$$

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(-52 + 16*a*x - 9*a^2*x^2 + 3*a^3*x^3) + 84*Sqrt[2]*Sqrt
[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{84\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+2\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)\sqrt{-c(ax-1)}}{21a^{3}}$	68
risch	$-\frac{2\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)(ax-1)c}{21a^{3}\sqrt{-c(ax-1)}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^{3}}$	74
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{c^{2}(-acx+c)^{\frac{3}{2}}}{3}+2c^{3}\sqrt{-acx+c}-2c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^{3}a^{3}}$	75
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c^{2}(-acx+c)^{\frac{3}{2}}}{3}-4c^{3}\sqrt{-acx+c}+4c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{3}c^{3}}$	75

```
[In] int(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*(84*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+2*
(3*a^3*x^3-9*a^2*x^2+16*a*x-52)*(-c*(a*x-1))^(1/2))/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{-acx+c}\right)}{21a^3},\right.$$
$$\left.-\frac{2\left(42\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{-acx+c}\right)}{21a^3}\right]$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/
a^3, -2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c
)/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

Sympy [A] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^3\sqrt{-acx+c}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^3}{3}-\frac{x^2}{a}+\frac{2x}{a^2}-\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise}\end{cases}\right)}{a^2}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((-2*(2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c**3*sqrt(-a*c*x + c) + c**2*(-a*c*x + c)**(3/2)/3 + (-a*c*x +
c)**(7/2)/7)/(a**3*c**3), Ne(a*c, 0)), (sqrt(c)*(x**3/3 - x**2/a + 2*x/a**2
 - 2*Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\left(21\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{7}{2}}+7(-acx+c)^{\frac{3}{2}}c^2+42\sqrt{-acx+c}c^3\right)}{21a^3c^3}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/21*(21*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(7/2) + 7*(-a*c*x + c)^(3/2
)*c^2 + 42*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}$$
$$+ \frac{2\left(3\,(acx-c)^3\sqrt{-acx+c}a^{18}c^{18} - 7\,(-acx+c)^{\frac{3}{2}}a^{18}c^{20} - 42\sqrt{-acx+c}a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c)) +
 2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*a^
18*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{4\sqrt{c-a\,c\,x}}{a^3} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^3\,c}$$
$$- \frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^3\,c^3} - \frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^3}$$

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(1/2))/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c
*x)^(7/2))/(7*a^3*c^3) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1
i)/(2*c^(1/2)))*4i)/a^3
```

3.343 $\int e^{-2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int e^{-2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2-4*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2+4*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6265, 21, 81, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2} + \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{4\sqrt{c-acx}}{a^2}$$

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x)^
(5/2))/(5*a^2*c^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)} x\sqrt{c-acx}\,dx \\
&= -\int \frac{x(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{\int \frac{x(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{ac} \\
&= \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a} \\
&= \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a} \\
&= \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a^2} \\
&= \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$
\int e^{-2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(38-11ax+3a^2x^2) - 60\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a^2}
$$

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) - 60*Sqrt[2]*Sqrt[c]*ArcTanh[S
qrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{-60\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\left(6a^{2}x^{2}-22ax+76\right)\sqrt{-c(ax-1)}}{15a^{2}}$	59
risch	$-\frac{2\left(3a^{2}x^{2}-11ax+38\right)(ax-1)c}{15a^{2}\sqrt{-c(ax-1)}}-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^{2}}$	66
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{2}c^{2}}$	73
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{2}c^{2}}$	73

```
[In] int(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-60*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+(
6*a^2*x^2-22*a*x+76)*(-c*(a*x-1))^(1/2))/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^{2}x^{2}-11ax+38\right)\sqrt{-acx+c}\right)}{15a^{2}},\ \frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\right.\right.}{}\right.$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, 2/15*(3
0*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*a^2
*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\begin{cases}\frac{2\cdot\left(\frac{2\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^2\sqrt{-acx+c}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2} & \text{for } ac\neq 0\\ \sqrt{c}\left(\frac{x^2}{2}-\frac{2x}{a}+\frac{2\left(\begin{cases}x & \text{for } a=0\\ \frac{\log(ax+1)}{a} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((2*(2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sq
rt(-c) + 2*c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**
(5/2)/5)/(a**2*c**2), Ne(a*c, 0)), (sqrt(c)*(x**2/2 - 2*x/a + 2*Piecewise((
x, Eq(a, 0)), (log(a*x + 1)/a, True))/a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\frac{2\left(15\sqrt{2}c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{5}{2}}+5(-acx+c)^{\frac{3}{2}}c+30\sqrt{-acx+c}c^2\right)}{15a^2c^2}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 5*(-a*c*x + c)^(3/2)
*c + 30*sqrt(-a*c*x + c)*c^2)/(a^2*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}} + \frac{2\left(3(acx-c)^2\sqrt{-acx+c}a^8c^8+5(-acx+c)^{\frac{3}{2}}a^8c^9+30\sqrt{-acx+c}a^8c^{10}\right)}{15\,a^{10}c^{10}}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) +
2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8*c
^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^2} + \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^2\,c} + \frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^2\,c^2} + \frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^2}$$

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x
)^(5/2))/(5*a^2*c^2) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)
/(2*c^(1/2)))*4i)/a^2
```

3.344 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\sqrt{c-acx}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{3/2}}{3ac} - 2\int \frac{\sqrt{c-acx}}{1+ax}\,dx
\end{aligned}
$$

$$= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} - (4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$

$$= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}$$

$$= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2(-7+ax)\sqrt{c-acx} + 12\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}}+\frac{4\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}-4c\sqrt{-acx+c}+4c^{\frac{3}{2}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	59
pseudoelliptic	$\frac{4\sqrt{c}\,\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2ax\sqrt{-c(ax-1)}}{3}-\frac{14\sqrt{-c(ax-1)}}{3}}{a}$	59

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(a*x-7)*(a*x-1)/a/(-c*(a*x-1))^(1/2)*c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise((-2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c, 0)),
 (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}c\right)}{3\,ac}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(
a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3\,a^3c^3}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c} - \frac{4\sqrt{c-a\,c\,x}}{a}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*(
c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

3.345 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6302, 6265, 21, 86, 162, 65, 214, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+2\sqrt{c-acx}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}\sqrt{c-acx}}{x}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x(1+ax)}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x(1+ax)}\,dx}{c}\\
&= 2\sqrt{c-acx}-\frac{\int \frac{ac^2-3a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{ac}\\
&= 2\sqrt{c-acx}-c\int \frac{1}{x\sqrt{c-acx}}\,dx+(4ac)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= 2\sqrt{c-acx}-8\operatorname{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)+\frac{2\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= 2\sqrt{c-acx}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) -4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right) \sqrt{c}-4 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2}\,\sqrt{c}+2 \sqrt{-acx+c}$	58
default	$2 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right) \sqrt{c}-4 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2}\,\sqrt{c}+2 \sqrt{-acx+c}$	58
pseudoelliptic	$2 \operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right) \sqrt{c}-4 \sqrt{c}\,\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2 \sqrt{c}}\right)+2 \sqrt{-c\,(ax-1)}$	61

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[2\,\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+\sqrt{c}\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c},4\,\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-2\,\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\,\sqrt{-acx+c}\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(
a*x + 1)) + sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(-c)/c) - 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x +
 c)]
```

Sympy [A] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.65

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \begin{cases} -\frac{2c\,\text{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{4\sqrt{2}c\,\text{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c} & \text{for}\ ac \neq 0 \\ \sqrt{c}\left(-\frac{3a\left(\frac{\log\left(-\frac{2}{x}\right)}{a} - \frac{\log\left(2a+\frac{2}{x}\right)}{a}\right)}{2} + \frac{\log\left(\frac{a}{x}+\frac{1}{x^2}\right)}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Piecewise((-2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan
(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(-a*c*x + c), Ne(a
*c, 0)), (sqrt(c)*(-3*a*(log(-2/x)/a - log(2*a + 2/x)/a)/2 + log(a/x + x**(
-2))/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right) - \sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right) + 2\sqrt{-acx+c}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c))) - sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*
c*x + c) + sqrt(c))) + 2*sqrt(-a*c*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\,c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*c*ar
ctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right) + 2\sqrt{c-a\,c\,x} - 4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

```
[Out] 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2) - 4*2^(1/2
)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

3.346 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -5*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*arctanh(1/2*(-a*c*x+c)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6302, 6265, 21, 100, 162, 65, 214, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -5a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{\sqrt{c-acx}}{x}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^2}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^2(1+ax)}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^2(1+ax)}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{x}+\frac{\int \frac{\frac{5ac^2}{2}-\frac{3}{2}a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{x}+\frac{1}{2}(5ac)\int \frac{1}{x\sqrt{c-acx}}\,dx-\left(4a^2c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{x}-5\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)+(8a)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-5\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax-1)}\,\sqrt{c}}{x\sqrt{c}}$	70
derivativedivides	$-2ca\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx+c}}{2acx}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	71
default	$2ca\left(\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{-acx+c}}{2acx}-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	71
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}}+\frac{a\left(-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{8\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{2}$	73

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-5*arctanh(
(-c*(a*x-1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x-1))^(1/2)*c^(1/2))/x/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$=\left[\frac{4\sqrt{2}a\sqrt{c}x\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+5a\sqrt{c}x\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}}{2x},\right.$$
$$\left.-\frac{4\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-5a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c)
- 2*c)/x) + 2*sqrt(-a*c*x + c))/x, -(4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt
(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*s
qrt(-c)/c) - sqrt(-a*c*x + c))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^2\,(ax+1)}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{1}{2}\,ac\left(\frac{4\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{\sqrt{c}}-\frac{5\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\,\sqrt{-acx+c}}{acx}\right)$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*(4*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt
(c) + sqrt(-a*c*x + c)))/sqrt(c) - 5*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt
(-a*c*x + c) + sqrt(c)))/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{4\,\sqrt{2}ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 5*a
*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-a\,c\,x}}{x} - 5\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right) + 4\,\sqrt{2}\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
[Out] (c - a*c*x)^(1/2)/x - 5*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 4*2^(1
/2)*a*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

3.347 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} + \frac{23}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 23/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^2*arctanh(1/2*(-a*c*
x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2-9/4*a*
(-a*c*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6302, 6265, 21, 100, 156, 162, 65, 214, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{23}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] Sqrt[c - a*c*x]/(2*x^2) - (9*a*Sqrt[c - a*c*x])/(4*x) + (23*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^3}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^3(1+ax)}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^3(1+ax)}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{2x^2}+\frac{\int \frac{\frac{9ac^2}{2}-\frac{7}{2}a^2c^2x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{2c} \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}-\frac{\int \frac{\frac{23a^2c^3}{4}-\frac{9}{4}a^3c^3x}{x(1+ax)\sqrt{c-acx}}\,dx}{2c^2} \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}-\frac{1}{8}\left(23a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx+\left(4a^3c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{1}{4}(23a)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right) \\
&\quad -\left(8a^2\right)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{23}{4}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{(2-9ax)\sqrt{c-acx}}{4x^2} + \frac{23}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]

[Out] ((2 - 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\sqrt{-c(ax-1)}\,(9ax-2)\sqrt{c}+a^2cx^2\left(16\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-23\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{4\sqrt{c}\,x^2}$	80
risch	$\frac{(9a^2x^2-11ax+2)c}{4x^2\sqrt{-c(ax-1)}}-\frac{a^2\left(-\frac{46\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{8}$	84
derivativedivides	$2c^2a^2\left(\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7c\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$	94
default	$2c^2a^2\left(\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7c\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$	94

[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/4/c^(1/2)*((-c*(a*x-1))^(1/2)*(9*a*x-2)*c^(1/2)+a^2*c*x^2*(16*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-23*arctanh((-c*(a*x-1))^(1/2)/c^(1/2))))/x^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \left[\frac{16\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+23a^2\sqrt{c}x^2\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}(9ax-2)}{8x^2}\right.$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c
)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, 1/4*(16*sqrt(2)*
a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a^2*s
qrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*a*x -
 2))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-c(ax-1)}(ax-1)}{x^3(ax+1)}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \frac{1}{8}a^2c^2\left(\frac{2\left(9(-acx+c)^{\frac{3}{2}}-7\sqrt{-acx+c}c\right)}{(acx-c)^2c+2(acx-c)c^2+c^3}+\frac{16\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}-\frac{23\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*c^2*(2*(9*(-a*c*x + c)^(3/2) - 7*sqrt(-a*c*x + c)*c)/((a*c*x - c)^2
*c + 2*(a*c*x - c)*c^2 + c^3) + 16*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*
c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(3/2) - 23*log((sqrt(-a*c
*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{4\sqrt{2}a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{9\,(-acx+c)^{\frac{3}{2}}a^2c - 7\sqrt{-acx+c}a^2c^2}{4\,a^2c^2x^2}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 23
/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(9*(-a*c*x + c)^(
3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{9\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2} - \frac{a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,23\mathrm{i}}{4} - \frac{7\sqrt{c-a\,c\,x}}{4\,x^2} + \sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] (9*(c - a*c*x)^(3/2))/(4*c*x^2) - (a^2*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*23i)/4 - (7*(c - a*c*x)^(1/2))/(4*x^2) + 2^(1/2)*a^2*c^(1/2)*atan(
(2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

3.348 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} - \frac{45}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -45/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a^3*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3-13/12
*a*(-a*c*x+c)^(1/2)/x^2+19/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6302, 6265, 21, 100, 156, 162, 65, 214, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{45}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{19a^2\sqrt{c-acx}}{8x} + \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) - (13*a*Sqrt[c - a*c*x])/(12*x^2) + (19*a^2*Sqrt[c
- a*c*x])/(8*x) - (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}\sqrt{c-acx}}{x^4}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^4(1+ax)}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^4(1+ax)}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{3x^3} + \frac{\int \frac{\frac{13ac^2}{2}-\frac{11}{2}a^2c^2x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{3c} \\
&= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} - \frac{\int \frac{\frac{57a^2c^3}{4}-\frac{39}{4}a^3c^3x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{6c^2} \\
&= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} + \frac{\int \frac{\frac{135a^3c^4}{8}-\frac{57}{8}a^4c^4x}{x(1+ax)\sqrt{c-acx}}\,dx}{6c^3} \\
&= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} \\
&\quad + \frac{1}{16}\left(45a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx - \left(4a^4c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} \\
&\quad - \frac{1}{8}\left(45a^2\right) \mathrm{Subst}\left(\int \frac{1}{\frac{1}{a} - \frac{x^2}{ac}}\, dx, x, \sqrt{c-acx}\right) \\
&\quad + \left(8a^3\right) \mathrm{Subst}\left(\int \frac{1}{2 - \frac{x^2}{c}}\, dx, x, \sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} \\
&\quad - \frac{45}{8}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\, dx = & \frac{\sqrt{c-acx}\left(8 - 26ax + 57a^2x^2\right)}{24x^3} \\
& - \frac{45}{8}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) \\
& + 4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
[Out] (Sqrt[c - a*c*x]*(8 - 26*a*x + 57*a^2*x^2))/(24*x^3) - (45*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{\sqrt{-c(ax-1)}\,(57a^2x^2-26ax+8)\sqrt{c}}{3}+a^3c\,x^3\left(32\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-45\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{8\sqrt{c}\,x^3}$
risch	$-\frac{(57a^3x^3-83a^2x^2+34ax-8)c}{24x^3\sqrt{-c(ax-1)}}+\frac{a^3\left(-\frac{90\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{64\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{16}$
derivativedivides	$-2c^3a^3\left(-\frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13c^2\sqrt{-acx+c}}{16}}{a^3c^3x^3}+\frac{45\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^2}\right.$
default	$2c^3a^3\left(\frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13c^2\sqrt{-acx+c}}{16}}{a^3c^3x^3}+\frac{45\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^2}\right)$

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c^(1/2)*(1/3*(-c*(a*x-1))^(1/2)*(57*a^2*x^2-26*a*x+8)*c^(1/2)+a^3*c*x^3
*(32*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-45*arctanh((-c
*(a*x-1))^(1/2)/c^(1/2))))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$

$$= \left[\frac{96\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+135\,a^3\sqrt{c}x^3\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\,(57\,a^2x^2-26\,ax+\right.}{48\,x^3}$$

$$-\frac{96\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-135\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(57\,a^2x^2-26\,ax+8)\sqrt{}}{24\,x^3}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) + 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3,
-1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt
(-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (57*a^
2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^4\,(ax+1)}\,dx$$

[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)

[Out] Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**4*(a*x + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{1}{48}\,a^3c^3\left(\frac{2\left(57\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+39\,\sqrt{-acx+c}c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{96\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}+\frac{135}{}\right.$$

[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")

[Out] 1/48*a^3*c^3*(2*(57*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 39*sqrt(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*c^4 + c^5) - 96*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(5/2) + 135*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(5/2))

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{4\,\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{45\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}}+\frac{57\,(acx-c)^2\sqrt{-acx+c}a^3c-88\,(-acx+c)^{\frac{3}{2}}a^3c^2+39\,\sqrt{-acx+c}a^3c^3}{24\,a^3c^3x^3}$$

[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")

```
[Out] -4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4
5/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(57*(a*c*x - c)
^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c*x
+ c)*a^3*c^3)/(a^3*c^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{13\sqrt{c-a\,c\,x}}{8\,x^3}+\frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,45\mathrm{i}}{8}-\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}+\frac{19\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}-\sqrt{2}\,a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

```
[Out] (13*(c - a*c*x)^(1/2))/(8*x^3) + (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c
^(1/2))*45i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (19*(c - a*c*x)^(5/2))/
(8*c^2*x^3) - 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*4i
```

3.349 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{363}{64}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 363/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^4-17/2
4*a*(-a*c*x+c)^(1/2)/x^3+107/96*a^2*(-a*c*x+c)^(1/2)/x^2-149/64*a^3*(-a*c*x
+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6302, 6265, 21, 100, 156, 162, 65, 214, 212}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{363}{64}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) - (17*a*Sqrt[c - a*c*x])/(24*x^3) + (107*a^2*Sqrt[c
 - a*c*x])/(96*x^2) - (149*a^3*Sqrt[c - a*c*x])/(64*x) + (363*a^4*Sqrt[c]*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
- a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}\sqrt{c-acx}}{x^5}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^5(1+ax)}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^5(1+ax)}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{4x^4} + \frac{\int \frac{\frac{17ac^2}{2}-\frac{15}{2}a^2c^2x}{x^4(1+ax)\sqrt{c-acx}}\,dx}{4c} \\
&= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} - \frac{\int \frac{\frac{107a^2c^3}{4}-\frac{85}{4}a^3c^3x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{12c^2} \\
&= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{\int \frac{\frac{447a^3c^4}{8}-\frac{321}{8}a^4c^4x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{24c^3} \\
&= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} - \frac{\int \frac{\frac{1089a^4c^5}{16}-\frac{447}{16}a^5c^5x}{x(1+ax)\sqrt{c-acx}}\,dx}{24c^4}
\end{aligned}
$$

$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} - \frac{1}{128}(363a^4c)\int \frac{1}{x\sqrt{c-acx}}\,dx + (4a^5c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$

$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{1}{64}(363a^3)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) - (8a^4)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)$$

$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{363}{64}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}(48-136ax+214a^2x^2-447a^3x^3)}{192x^4} + \frac{363}{64}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a*c*x]*(48 - 136*a*x + 214*a^2*x^2 - 447*a^3*x^3))/(192*x^4) + (3
63*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{-c(ax-1)}\left(447a^3x^3-214a^2x^2+136ax-48\right)\sqrt{c}}{3}+a^4cx^4\left(256\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-363\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{64\sqrt{c}\,x^4}$
risch	$\frac{\left(447a^4x^4-661a^3x^3+350a^2x^2-184ax+48\right)c}{192x^4\sqrt{-c(ax-1)}}-\frac{a^4\left(-\frac{726\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{512\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{128}$
derivativedivides	$2c^4a^4\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107c^3\sqrt{-acx+c}}{128}}{a^4c^4x^4}+\frac{363\ a}{}}{c^3}\right.$
default	$2c^4a^4\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107c^3\sqrt{-acx+c}}{128}}{a^4c^4x^4}+\frac{363\ a}{}}{c^3}\right.$

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/c^(1/2)*(1/3*(-c*(a*x-1))^(1/2)*(447*a^3*x^3-214*a^2*x^2+136*a*x-48)*
c^(1/2)+a^4*c*x^4*(256*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/
2))-363*arctanh((-c*(a*x-1))^(1/2)/c^(1/2))))/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$=\left[\frac{768\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+1089\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\left(447\,a^3x^3-214\,a^2\right.}{384\,x^4}\right.$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)
*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*
x + c)*sqrt(c) - 2*c)/x) - 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqr
t(-a*c*x + c))/x^4, 1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*
sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c
)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*x + c)
)/x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^5\,(ax+1)}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \frac{1}{384}\,a^4c^4\left(\frac{2\left(447\,(-acx+c)^{\frac{7}{2}}-1127\,(-acx+c)^{\frac{5}{2}}c+1049\,(-acx+c)^{\frac{3}{2}}c^2-321\,\sqrt{-acx+c}c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{768\,\sqrt{}}{}\right.$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
[Out] 1/384*a^4*c^4*(2*(447*(-a*c*x + c)^(7/2) - 1127*(-a*c*x + c)^(5/2)*c + 1049
*(-a*c*x + c)^(3/2)*c^2 - 321*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*
(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 768*sq
rt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*
c*x + c)))/c^(7/2) - 1089*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c
) + sqrt(c)))/c^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{363\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}}$$
$$-\frac{447\,(acx-c)^3\sqrt{-acx+c}a^4c+1127\,(acx-c)^2\sqrt{-acx+c}a^4c^2-1049\,(-acx+c)^{\frac{3}{2}}a^4c^3+321\,\sqrt{-acx}}{192\,a^4c^4x^4}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
[Out] 4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 36
3/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/192*(447*(a*c*x -
 c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2
- 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4*
x^4)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{1049\,{\left(c-a\,c\,x\right)}^{3/2}}{192\,c\,x^4}-\frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,363{}\mathrm{i}}{64}-\frac{107\,\sqrt{c-a\,c\,x}}{64\,x^4}-\frac{1127\,{\left(c-a\,c\,x\right)}^{5/2}}{192\,c^2\,x^4}+\frac{149\,{\left(c-a\,c\,x\right)}^{7/2}}{64\,c^3\,x^4}+\sqrt{2}\,a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}$$

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] (1049*(c - a*c*x)^(3/2))/(192*c*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)
*1i)/c^(1/2))*363i)/64 - (107*(c - a*c*x)^(1/2))/(64*x^4) - (1127*(c - a*c*
x)^(5/2))/(192*c^2*x^4) + (149*(c - a*c*x)^(7/2))/(64*c^3*x^4) + 2^(1/2)*a^
4*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

3.350 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 281

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -82/9*x^2*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-8/9*x^3*(-a*
c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/9*x^4*(-a*c*x+c)^(1/2)/(1-
1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1312/45*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(
1-1/a/x)^(1/2)-656/45*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2
)+164/15*x^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{6311, 6316, 91, 79, 47, 37\}$

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{1312\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}} - \frac{656x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}} + \frac{164x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (1312*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(45*a^4*Sqrt[1 - 1/(a*x)]) - (656*
Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(45*a^3*Sqrt[1 - 1/(a*x)]) - (82*x^2*S
qrt[c - a*c*x])/(9*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (164*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (8*x^3*Sqrt[c -
 a*c*x])/(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^4*Sqrt[c - a*c*x]
)/(9*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{14}{a}+\frac{9x}{2a^2}}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{9\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

$$= -\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(41\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{9a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(82\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(328\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{15a^3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}} - \frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(656\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{45a^4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}} - \frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}} - \frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-acx}(656+328ax-82a^2x^2+41a^3x^3-20a^4x^4+5a^5x^5)}{45a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
5*a^5*x^5))/(45*a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(ax+1)(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45a^4(ax-1)^2}$	80
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656)}{45(ax-1)^2a^4}$	81
risch	$-\frac{2(5a^4x^4-25a^3x^3+66a^2x^2-148ax+476)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{45a^4\sqrt{-c(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^4\sqrt{-c(ax-1)}}$	99

```
[In] int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45*(a*x+1)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-82*a^2*x^2+328*a*x+656)*(-a*c
*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{45\left(a^5x-a^4\right)}$$

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^5*x - a^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(5\,a^6\sqrt{-c}x^6-15\,a^5\sqrt{-c}x^5+21\,a^4\sqrt{-c}x^4-41\,a^3\sqrt{-c}x^3+246\,a^2\sqrt{-c}x^2+984\,a\sqrt{-c}x+656\,\sqrt{-c}\right)(ax}{45\,(a^6x^2-2\,a^5x+a^4)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
[Out] 2/45*(5*a^6*sqrt(-c)*x^6 - 15*a^5*sqrt(-c)*x^5 + 21*a^4*sqrt(-c)*x^4 - 41*a
^3*sqrt(-c)*x^3 + 246*a^2*sqrt(-c)*x^2 + 984*a*sqrt(-c)*x + 656*sqrt(-c))*(
a*x - 1)^2/((a^6*x^2 - 2*a^5*x + a^4)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^5\,x^5-20\,a^4\,x^4+41\,a^3\,x^3-82\,a^2\,x^2+328\,a\,x+656\right)}{45\,a^4\,\left(a\,x-1\right)}$$

```
[In] int(x^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(328*a*x - 82*a^2*x^2 + 41
*a^3*x^3 - 20*a^4*x^4 + 5*a^5*x^5 + 656))/(45*a^4*(a*x - 1))
```

3.351 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 231

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} - \frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -334/35*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-44/35*x^2*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/7*x^3*(-a*c*x+c)^(1/2)/(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2672/105*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^
3/(1-1/a/x)^(1/2)+1336/105*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{6311, 6316, 91, 79, 47, 37\}$

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2672\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} + \frac{1336x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2672*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) - (33
4*x*Sqrt[c - a*c*x])/(35*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1336*S
qrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (44*x^2*S
qrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^3*Sqrt[c
- a*c*x])/(7*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{11}{a}+\frac{7x}{2a^2}}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{7\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(167\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(668\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(1336\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} - \frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-1336-668ax+167a^2x^2-66a^3x^3+15a^4x^4)}{105a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^4
))/(105*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2(ax+1)(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105a^3(ax-1)^2}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)}{105(ax-1)^2a^3}$	73
risch	$-\frac{2(15a^3x^3-81a^2x^2+248ax-916)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{105a^3\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^3\sqrt{-c(ax-1)}}$	91

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(a*x+1)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^2-668*a*x-1336)*(-a*c*x+c)^(
1/2)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\,(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.45

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\frac{2\left(15\,a^5\sqrt{-c}x^5-51\,a^4\sqrt{-c}x^4+101\,a^3\sqrt{-c}x^3-501\,a^2\sqrt{-c}x^2-2004\,a\sqrt{-c}x-1336\,\sqrt{-c}\right)(ax-1)^2}{105\,(a^5x^2-2\,a^4x+a^3)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^5*sqrt(-c)*x^5 - 51*a^4*sqrt(-c)*x^4 + 101*a^3*sqrt(-c)*x^3 - 5
01*a^2*sqrt(-c)*x^2 - 2004*a*sqrt(-c)*x - 1336*sqrt(-c))*(a*x - 1)^2/((a^5*
x^2 - 2*a^4*x + a^3)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-51\,a^2\,x^2+116\,a\,x-552\right)}{105\,a^3}-\frac{3776\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

```
[In] int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(116*a*x - 51*a^2*x^2 + 15
*a^3*x^3 - 552))/(105*a^3) - (3776*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^
(1/2))/(105*a^3*(a*x - 1))
```

3.352 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 182

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -158/15*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32/15*x*(-a*c*
x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/5*x^2*(-a*c*x+c)^(1/2)/(1-1/
a/x)^(1/2)/(1+1/a/x)^(1/2)+316/15*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1
/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6311, 6316, 91, 79, 47, 37}

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{316\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-158*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (316*
Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (32*x*Sqrt[
c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^2*Sqrt[c - a*
c*x])/(5*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{8}{a}+\frac{5x}{2a^2}}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{5\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(79\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(158\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{15a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.31

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(158+79ax-16a^2x^2+3a^3x^3)}{15a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^3*Sqrt[1
 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{2(ax+1)(3a^3x^3-16a^2x^2+79ax+158)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15a^2(ax-1)^2}$	64
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(3a^3x^3-16a^2x^2+79ax+158)}{15(ax-1)^2a^2}$	65
risch	$-\frac{2(3a^2x^2-19ax+98)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{15a^2\sqrt{-c(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^2\sqrt{-c(ax-1)}}$	83

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(a*x+1)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)*(-a*c*x+c)^(1/2)*((a*x-1)/(a
*x+1))^(3/2)/a^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,(3\,a^3x^3 - 16\,a^2x^2 + 79\,ax + 158)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x - a^2)}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] 2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^3*x - a^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \text{Timed out}$$

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,\left(3\,a^4\sqrt{-c}x^4 - 13\,a^3\sqrt{-c}x^3 + 63\,a^2\sqrt{-c}x^2 + 237\,a\sqrt{-c}x + 158\,\sqrt{-c}\right)(ax-1)^2}{15\,(a^4x^2 - 2\,a^3x + a^2)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] 2/15*(3*a^4*sqrt(-c)*x^4 - 13*a^3*sqrt(-c)*x^3 + 63*a^2*sqrt(-c)*x^2 + 237*
a*sqrt(-c)*x + 158*sqrt(-c))*(a*x - 1)^2/((a^4*x^2 - 2*a^3*x + a^2)*(a*x +
1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.32

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^3\,x^3-16\,a^2\,x^2+79\,a\,x+158\right)}{15\,a^2\,\left(a\,x-1\right)}$$

```
[In] int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a*x - 16*a^2*x^2 + 3*a
^3*x^3 + 158))/(15*a^2*(a*x - 1))
```

3.353 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(1
/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 91, 79, 37}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)(a^2x^2-10ax-23)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a(ax-1)^2}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(a^2x^2-10ax-23)}{3(ax-1)^2a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a/
(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(a^2x^2-10\,ax-23)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^3\sqrt{-c}x^3 - 9\,a^2\sqrt{-c}x^2 - 33\,a\sqrt{-c}x - 23\,\sqrt{-c}\right)(ax-1)^2}{3\left(a^3x^2 - 2\,a^2x + a\right)(ax+1)^{\frac{3}{2}}}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-9)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{64\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.354 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+10*(-a*c*x+c)^(1/2)/a/x/
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*
(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6311, 6316, 91, 79, 56, 221}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -\frac{2\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{10\sqrt{c-acx}}{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (10*Sqrt[c - a*
c*x])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (2*Sqrt[x^(-1)]*Sqrt[c -
a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{2}{a}+\frac{x}{2a^2}}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(a+\frac{5}{x}-\sqrt{a}\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
[Out] (2*Sqrt[c - a*c*x]*(a + 5/x - Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*ArcSin
h[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)\sqrt{-c(ax+1)}+acx+5c\right)}{(ax-1)^2c}$	80

[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] 2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*(-c*(a*x+1))^(1/2)+a*c*x+5*c)/c

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")

[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.355 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7*arcsinh((1/x)^(1/2)/a^(1/2))*a^(1/2)*(1/x)^
(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6311, 6316, 91, 81, 56, 221}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (Sqrt[1 + 1/
(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (7*Sqrt[a]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
```

```
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{3}{2a^2}-\frac{x}{2a^3}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\left(-1-9ax+\frac{7a^{3/2}\sqrt{1+\frac{1}{ax}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
[Out] (Sqrt[c - a*c*x]*(-1 - 9*a*x + (7*a^(3/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^
(-1)]/Sqrt[a]])/(x^(-1))^(3/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(7\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)ax\sqrt{-c(ax+1)}+9\sqrt{c}\,ax+\sqrt{c}\right)\sqrt{-c(ax-1)}}{(ax-1)^{2}\sqrt{c}\,x}$	86
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c(ax-1)}}-\frac{\left(-\frac{8a}{\sqrt{-acx-c}}-\frac{7a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	112

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(7*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*x*
(-c*(a*x+1))^(1/2)+9*c^(1/2)*a*x+c^(1/2))*(-c*(a*x-1))^(1/2)/(a*x-1)^2/c^(1
/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.66

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{7\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\sqrt{-acx+c}(9\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, \frac{7\,(a^2\right.$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(7*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqr
t(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), (7*(a^2*x
^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))
/(a*c*x - c)) - sqrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^2 - x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^2}\,dx$$

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.356 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2}+\frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x}-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/2*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+47/4*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/
2)/x/(1-1/a/x)^(1/2)-47/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*
(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6311, 6316, 91, 81, 52, 56, 221}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{47a^{3/2}\sqrt{\frac{1}{x}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{2x^2\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^2) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x^2) + (47*a*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) - (47*a^(3/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(\frac{11}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{\left(47\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{\left(47a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{\left(47a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{\sqrt{c-acx}\left(2-13ax-47a^2x^2+\frac{47a^{5/2}\sqrt{1+\frac{1}{ax}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{4a\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/4*(Sqrt[c - a*c*x]*(2 - 13*a*x - 47*a^2*x^2 + (47*a^(5/2)*Sqrt[1 + 1/(a*
x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2)))/(a*Sqrt[1 - 1/(a^2*x^2)
]*x^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(47\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^2x^2\sqrt{-c(ax+1)}+47\sqrt{c}\,a^2x^2+13\sqrt{c}\,ax-2\sqrt{c}\right)}{4(ax-1)^2\sqrt{c}\,x^2}$	103
risch	$-\frac{(15a^2x^2+13ax-2)c\sqrt{\frac{ax-1}{ax+1}}}{4x^2\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^2}{\sqrt{-acx-c}}+\frac{47a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	126

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(47*arctan
((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2*(-c*(a*x+1))^(1/2)+47*c^(1/2)*a^2*x^2+
13*c^(1/2)*a*x-2*c^(1/2))/c^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.38

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \left[\frac{47\,(a^3x^3-a^2x^2)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,(47\,a^2x^2+13\,ax-2)\sqrt{-acx+}}{8\,(ax^3-x^2)} \right.$$
$$\left. -\frac{47\,(a^3x^3-a^2x^2)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(47\,a^2x^2+13\,ax-2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{4\,(ax^3-x^2)}\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas
")
```

```
[Out] [1/8*(47*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c
*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) +
2*(47*a^2*x^2 + 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^3 - x^2), -1/4*(47*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (47*a^2*x^2 + 13*a*x - 2)*sq
rt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima
")
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.357 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2} - \frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/3*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+119/12*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x^2/(1-1/a/x)^(1/2)-119/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/
a/x)^(1/2)+119/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c
)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6311, 6316, 91, 81, 52, 56, 221}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{119a^{5/2}\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}} - \frac{119a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3x^3\sqrt{1-\frac{1}{ax}}} - \frac{8\sqrt{c-acx}}{x^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{119a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{12x^2\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^3) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^3) + (119*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(12*Sqrt[1 - 1/(a*x)]*x^2) - (119*a^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (119*a^(5/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(\frac{19}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3}+\frac{\left(119\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{x^{3/2}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3}+\frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2}-\frac{\left(119a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3}+\frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2}-\frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x}+\frac{\left(119a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{16\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3}+\frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2}-\frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x}+\frac{\left(119a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,\sqrt{\frac{1}{x}}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3}+\frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2}-\frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x}+\frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(-8+38ax-119a^2x^2-357a^3x^3+\frac{357a^{7/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{24a\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]

[Out] (Sqrt[c - a*c*x]*(-8 + 38*a*x - 119*a^2*x^2 - 357*a^3*x^3 + (357*a^(7/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2)))/(24*a*Sqrt[1 - 1/(a^2*x^2)]*x^4)

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.48

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(357\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^3x^3\sqrt{-c(ax+1)}+357a^3x^3\sqrt{c}+119\sqrt{c}\,a^2x^2-38\sqrt{c}\,ax+8\sqrt{c}\right)}{24(ax-1)^2\sqrt{c}\,x^3}$	114
risch	$\frac{(165a^3x^3+119a^2x^2-38ax+8)c\sqrt{\frac{ax-1}{ax+1}}}{24x^3\sqrt{-c(ax-1)}}-\frac{\left(-\frac{8a^3}{\sqrt{-acx-c}}-\frac{119a^3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	134

[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/24*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(357*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3*(-c*(a*x+1))^(1/2)+357*a^3*x^3*c^(1/2)+119*c^(1/2)*a^2*x^2-38*c^(1/2)*a*x+8*c^(1/2))/c^(1/2)/x^3

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \left[\frac{357\,(a^4x^4-a^3x^3)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,(357\,a^3x^3+119\,a^2x^2-38\,ax+\right.}{48\,(ax^4-x^3)}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas
")
```

```
[Out] [1/48*(357*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a
*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x))
- 2*(357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/(a*x^4 - x^3), 1/24*(357*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(s
qrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (357*a^3*x
^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a*x^4 - x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.358 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 286

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4}+\frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3}-\frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2}+\frac{1115a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x}-\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/4*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)+223/24*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x^3/(1-1/a/x)^(1/2)-1115/96*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x^2/(
1-1/a/x)^(1/2)+1115/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/
2)-1115/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2
)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6311, 6316, 91, 81, 52, 56, 221}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}} + \frac{1115a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}} - \frac{1115a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{96x^2\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x^4\sqrt{1-\frac{1}{ax}}} - \frac{8\sqrt{c-acx}}{x^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{223a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{24x^3\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^4) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^4) + (223*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^3) - (1115*a^2*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(96*Sqrt[1 - 1/(a*x)]*x^2) + (1115*a^3*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) - (1115*a^(7/2)*Sqrt[x^(-1)]*Sq
rt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a*x)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(\frac{27}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{\left(223\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{\left(1115a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{48\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2} + \frac{\left(1115a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{64\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2} + \frac{1115a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} - \frac{\left(1115a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{128\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2} + \frac{1115a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} - \frac{\left(1115a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{64\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2} + \frac{1115a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} - \frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{\sqrt{c-acx}\left(48-200ax+446a^2x^2-1115a^3x^3-3345a^4x^4+\frac{3345a^{9/2}\sqrt{1+\frac{1}{ax}}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{9/2}}\right)}{192a\sqrt{1-\frac{1}{a^2x^2}}x^5}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/192*(Sqrt[c - a*c*x]*(48 - 200*a*x + 446*a^2*x^2 - 1115*a^3*x^3 - 3345*a
^4*x^4 + (3345*a^(9/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^
(-1))^(9/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.44

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(3345\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4\sqrt{-c(ax+1)}+3345a^4x^4\sqrt{c}+1115a^3x^3\sqrt{c}-446\sqrt{c}\,a^2x^2+200\sqrt{c}\,ax-\right.}{192(ax-1)^2\sqrt{c}\,x^4}$
risch	$-\frac{(1809a^4x^4+1115a^3x^3-446a^2x^2+200ax-48)c\sqrt{\frac{ax-1}{ax+1}}}{192x^4\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^4}{\sqrt{-acx-c}}+\frac{1115a^4\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(3345*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^4*x^4*(-c*(a*x+1))^(1/2)+3345*a^4*x^4*c^
(1/2)+1115*a^3*x^3*c^(1/2)-446*c^(1/2)*a^2*x^2+200*c^(1/2)*a*x-48*c^(1/2))/
c^(1/2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.03

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$

$$=\left[\frac{3345\,(a^5x^5-a^4x^4)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,(3345\,a^4x^4+1115\,a^3x^3-446}{384\,(ax^5-x^4)}\right.$$

$$-\frac{3345\,(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3345\,a^4x^4+1115\,a^3x^3-446\,a^2x^2+200\,ax-48)}{192\,(ax^5-x^4)}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas
")
```

```
[Out] [1/384*(3345*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)
) + 2*(3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/192*(3345*(a^5*x^5 - a^
4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a
*c*x - c)) - (3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima
")
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^5}\,dx$$

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.359 $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{2(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a^2(6+n)(8+6n+n^2)x} + \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a}$$

```
[Out] -(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(2+1/2*n
)/a/(4+n)/(6+n)+2*(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-
a*c*x+c)^(2+1/2*n)/a^2/(6+n)/(n^2+6*n+8)/x+(8+n)*(1-1/a/x)^(-2-1/2*n)*(1+1/
a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/(6+n)-(a-1/x)*(1-1/a/x)^(-2-1/2*n)*(1
+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/a
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{6311, 6316, 92, 80, 47, 37\}$

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \frac{2(n^2+14n+56)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a^2(n+6)\left(n^2+6n+8\right)x} - \frac{(n^2+14n+56)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a(n+4)(n+6)} + \frac{(n+8)x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{n+6} - \frac{x\left(a-\frac{1}{x}\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] -(((56 + 14*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c
- a*c*x)^((4 + n)/2))/(a*(4 + n)*(6 + n))) + (2*(56 + 14*n + n^2)*(1 - 1/(a
*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^((4 + n)/2))/(a^2*(6
+ n)*(8 + 6*n + n^2)*x) + ((8 + n)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/(6 + n) - ((a - x^(-1))*(1 - 1/(a*x))
^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/a
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}} x^{-2-\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{2+\frac{n}{2}} x^{2+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-4-\frac{n}{2}}\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a}+\left(a\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\left(-\frac{8+n}{2a}+\frac{(4+n)x}{2a^2}\right)dx,x,\frac{1}{x}\right)$$

$$= \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a} + \frac{\left((56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{2a(6+n)}$$

$$= -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a} - \frac{\left((56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\mathrm{Subst}\left(\int x^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{a^2(4+n)(6+n)}$$

$$= -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{2(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a^2(2+n)(4+n)(6+n)x} + \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.42

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}\left(n^2(-1+ax)^2+8(7-4ax+a^2x^2)+2n(7-10ax+3a^2\right.}{a(2+n)(4+n)(6+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(n^2*(-1 + a*x)^2 +
8*(7 - 4*a*x + a^2*x^2) + 2*n*(7 - 10*a*x + 3*a^2*x^2)))/(a*(2 + n)*(4 + n)
*(6 + n)*(1 - 1/(a*x))^(n/2))
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

method	result	size
gosper	$\frac{2(ax+1)\left(a^{2} n^{2} x^{2}+6 n\, x^{2} a^{2}+8 a^{2} x^{2}-2 n^{2} x a -20 a n x -32 a x +n^{2}+14 n +56\right) {\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} \left(-a c x +c \right)^{2+\frac{n}{2}}}{\left(a x -1\right)^{2} a \left(n^{3}+12 n^{2}+44 n +48\right)}$	104

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*x+1)*(a^2*n^2*x^2+6*a^2*n*x^2+8*a^2*x^2-2*a*n^2*x-20*a*n*x-32*a*x+n^2+
14*n+56)*exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n)/(a*x-1)^2/a/(n^3+12*n^2+4
4*n+48)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.67

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$
$$=\frac{2\left((a^3n^2+6\,a^3n+8\,a^3)x^3-(a^2n^2+14\,a^2n+24\,a^2)x^2+n^2-(an^2+6\,an-24\,a)x+14\,n+56\right)(-acx+}{an^3+12\,an^2+(a^3n^3+12\,a^3n^2+44\,a^3n+48\,a^3)x^2+44\,an-2\,(a^2n^3+12\,a^2n^2+44\,a^2n+48\,a^2}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
```

```
[Out] 2*((a^3*n^2 + 6*a^3*n + 8*a^3)*x^3 - (a^2*n^2 + 14*a^2*n + 24*a^2)*x^2 + n^
2 - (a*n^2 + 6*a*n - 24*a)*x + 14*n + 56)*(-a*c*x + c)^(1/2*n + 2)*((a*x +
1)/(a*x - 1))^(1/2*n)/(a*n^3 + 12*a*n^2 + (a^3*n^3 + 12*a^3*n^2 + 44*a^3*n
+ 48*a^3)*x^2 + 44*a*n - 2*(a^2*n^3 + 12*a^2*n^2 + 44*a^2*n + 48*a^2)*x + 4
8*a)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx=\int\left(-c(ax-1)\right)^{\frac{n}{2}+2}e^{n\,\operatorname{acoth}(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(2+1/2*n),x)
```

```
[Out] Integral((-c*(a*x - 1))**(n/2 + 2)*exp(n*acoth(a*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.44

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \frac{2\left((n^2+6n+8)a^3(-c)^{\frac{1}{2}n}c^2x^3-(n^2+14n+24)a^2(-c)^{\frac{1}{2}n}c^2x^2-(n^2+6n-24)a(-c)^{\frac{1}{2}n}c^2x+(n^2+14n+56)(-c)^{\frac{1}{2}n}c^2\right)(ax+1)^{\frac{1}{2}n}}{(n^3+12n^2+44n+48)a}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="maxima")
[Out] 2*((n^2 + 6*n + 8)*a^3*(-c)^(1/2*n)*c^2*x^3 - (n^2 + 14*n + 24)*a^2*(-c)^(1
/2*n)*c^2*x^2 - (n^2 + 6*n - 24)*a*(-c)^(1/2*n)*c^2*x + (n^2 + 14*n + 56)*(
-c)^(1/2*n)*c^2)*(a*x + 1)^(1/2*n)/((n^3 + 12*n^2 + 44*n + 48)*a)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \int (-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n + 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{x^3\,(c-a\,c\,x)^{\frac{n}{2}+2}\,(2\,n^2+12\,n+16)}{n^3+12\,n^2+44\,n+48}+\frac{(c-a\,c\,x)^{\frac{n}{2}+2}\,(2\,n^2+28\,n+112)}{a^3\,(n^3+12\,n^2+44\,n+48)}-\frac{2\,x\,(c-a\,c\,x)^{\frac{n}{2}+2}\,(n^2+6\,n-24)}{a^2\,(n^3+12\,n^2+44\,n+48)}-\frac{x^2\,(c-a\,c\,x)^{\frac{n}{2}+2}\,(2\,n^2+28\,n+48)}{a\,(n^3+12\,n^2+44\,n+48)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 2),x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((x^3*(c - a*c*x)^(n/2 + 2)*(12*n + 2*n^2 + 16))/(
44*n + 12*n^2 + n^3 + 48) + ((c - a*c*x)^(n/2 + 2)*(28*n + 2*n^2 + 112))/(a
^3*(44*n + 12*n^2 + n^3 + 48)) - (2*x*(c - a*c*x)^(n/2 + 2)*(6*n + n^2 - 24
))/(a^2*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(c - a*c*x)^(n/2 + 2)*(28*n + 2*
n^2 + 48))/(a*(44*n + 12*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^2
 - (2*x)/a + x^2))
```

3.360 $\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx=-\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)}+\frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}$$

```
[Out] -2*(6+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(1+1/2*n)/a/(n
^2+6*n+8)+2*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(1+1/2*n)
/(4+n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6311, 6316, 80, 37}

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx=\frac{2x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{n+4}-\frac{2(n+6)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{a(n+2)(n+4)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*(6 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^
((2 + n)/2))/(a*(2 + n)*(4 + n)) + (2*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x)
)^((2 + n)/2)*x*(c - a*c*x)^((2 + n)/2))/(4 + n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}} x^{-1-\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{1+\frac{n}{2}} x^{1+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-3-\frac{n}{2}}\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n} + \frac{\left((6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{a(4+n)}$$

$$= -\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)} + \frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}(-6+2ax+n(-1+ax))}{a(2+n)(4+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(-6 + 2*a*x + n*(-1 + a*x)))/(a*(2 + n)*(4 + n)*(1 - 1/(a*x))^(n/2))
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result
gosper	$\frac{2(-acx+c)^{1+\frac{n}{2}} e^{n \operatorname{arccoth}(ax)}(anx+2ax-n-6)(ax+1)}{(ax-1)a(n^2+6n+8)}$
parallelrisch	$-\frac{-2x^2(-acx+c)^{1+\frac{n}{2}} e^{n \operatorname{arccoth}(ax)} a^2 n-4x^2(-acx+c)^{1+\frac{n}{2}} e^{n \operatorname{arccoth}(ax)} a^2+8 e^{n \operatorname{arccoth}(ax)} x(-acx+c)^{1+\frac{n}{2}} a+2(-acx+c)^{1}}{(ax-1)a(n^2+6n+8)}$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-a*c*x+c)^(1+1/2*n)*exp(n*arccoth(a*x))*(a*n*x+2*a*x-n-6)*(a*x+1)/(a*x-1
)/a/(n^2+6*n+8)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx=-\frac{2\left((a^2n+2\,a^2)x^2-4\,ax-n-6\right)(-acx+c)^{\frac{1}{2}\,n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an^2+6\,an-(a^2n^2+6\,a^2n+8\,a^2)x+8\,a}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

```
[Out] -2*((a^2*n + 2*a^2)*x^2 - 4*a*x - n - 6)*(-a*c*x + c)^(1/2*n + 1)*((a*x + 1
)/(a*x - 1))^(1/2*n)/(a*n^2 + 6*a*n - (a^2*n^2 + 6*a^2*n + 8*a^2)*x + 8*a)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$$
$$=\begin{cases} c^{\frac{n}{2}+1}xe^{\frac{i\pi n}{2}} \\ 0^{\frac{n}{2}+1}xe^{\infty n} \\ -\frac{\int \frac{1}{axe^{4\operatorname{acoth}(ax)}-e^{4\operatorname{acoth}(ax)}}\,dx}{c} \\ \int e^{-2\operatorname{acoth}(ax)}\,dx \\ \frac{2a^2nx^2(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a}+\frac{4a^2x^2(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a}-\frac{8ax(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a}-\frac{2n(-acx+c)^{\frac{n}{2}+\cdots}}{a^2n^2x+6a^2nx+8a^2\cdots} \end{cases}$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1+1/2*n),x)
```

```
[Out] Piecewise((c**(n/2 + 1)*x*exp(I*pi*n/2), Eq(a, 0)), (0**(n/2 + 1)*x*exp(oo*
n), Eq(a, 1/x)), (-Integral(1/(a*x*exp(4*acoth(a*x)) - exp(4*acoth(a*x))),
x)/c, Eq(n, -4)), (Integral(exp(-2*acoth(a*x)), x), Eq(n, -2)), (2*a**2*n*x
**2*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8
*a**2*x - a*n**2 - 6*a*n - 8*a) + 4*a**2*x**2*(-a*c*x + c)**(n/2 + 1)*exp(n
*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2*x - a*n**2 - 6*a*n - 8*a) -
 8*a*x*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x
+ 8*a**2*x - a*n**2 - 6*a*n - 8*a) - 2*n*(-a*c*x + c)**(n/2 + 1)*exp(n*acot
h(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2*x - a*n**2 - 6*a*n - 8*a) - 12*(
-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2
*x - a*n**2 - 6*a*n - 8*a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2\left(a^2(-c)^{\frac{1}{2}n}c(n+2)x^2-4\,a(-c)^{\frac{1}{2}n}cx-(-c)^{\frac{1}{2}n}c(n+6)\right)(ax+1)^{\frac{1}{2}n}}{(n^2+6\,n+8)a}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="maxima")
```

```
[Out] -2*(a^2*(-c)^(1/2*n)*c*(n + 2)*x^2 - 4*a*(-c)^(1/2*n)*c*x - (-c)^(1/2*n)*c*
(n + 6))*(a*x + 1)^(1/2*n)/((n^2 + 6*n + 8)*a)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = \int (-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{\left(\frac{(2\,n+12)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a^2\,(n^2+6\,n+8)}-\frac{x^2\,(2\,n+4)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{n^2+6\,n+8}+\frac{8\,x\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a\,(n^2+6\,n+8)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\left(x-\frac{1}{a}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 1),x)
```

```
[Out] -((((2*n + 12)*(c - a*c*x)^(n/2 + 1))/(a^2*(6*n + n^2 + 8)) - (x^2*(2*n + 4
)*(c - a*c*x)^(n/2 + 1))/(6*n + n^2 + 8) + (8*x*(c - a*c*x)^(n/2 + 1))/(a*(
6*n + n^2 + 8)))*((a*x + 1)/(a*x))^(n/2))/((x - 1/a)*((a*x - 1)/(a*x))^(n/2
))
```

3.361 $\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2e^{n \coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

```
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6309}

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2(ax+1)(c-acx)^{n/2}e^{n \coth^{-1}(ax)}}{a(n+2)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] (2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{2e^{n \coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=-\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}}x(c-acx)^{n/2}}{-1-\frac{n}{2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
[Out] -(((1 + 1/(a*x))^(1 + n/2)*x*(c - a*c*x)^(n/2))/((-1 - n/2)*(1 - 1/(a*x))^(
n/2)))
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{2\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}(ax+1)(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
parallelrisch	$-\frac{-2\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}x(-acx+c)^{\frac{n}{2}}a-2\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
risch	$\frac{2(ax+1)(ax+1)^{\frac{n}{2}}(ax-1)^{-\frac{n}{2}}(ax-1)^{\frac{n}{2}}c^{\frac{n}{2}}\,\mathrm{e}^{-\frac{i\pi n\left(-\operatorname{csgn}(i(ax-1))\operatorname{csgn}(ic(ax-1))^{2}+\operatorname{csgn}(i(ax-1))\operatorname{csgn}(ic(ax-1))\operatorname{csgn}(ic)-\operatorname{csgn}(ic(ax}{4}}}{a(2+n)}$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x,method=_RETURNVERBOSE)
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\,(ax+1)(-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an+2\,a}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
[Out] 2*(a*x + 1)*(-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*n + 2*a)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c - acx)^{n/2}\,dx = \begin{cases} -\frac{x}{c} & \text{for } a = 0 \wedge n = -2 \\ c^{\frac{n}{2}} x e^{\frac{i\pi n}{2}} & \text{for } a = 0 \\ -\frac{\int \frac{1}{axe^{2\operatorname{acoth}(ax)} - e^{2\operatorname{acoth}(ax)}}\,dx}{c} & \text{for } n = -2 \\ \frac{2ax(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an+2a} + \frac{2(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an+2a} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2*n),x)
[Out] Piecewise((-x/c, Eq(a, 0) & Eq(n, -2)), (c**(n/2)*x*exp(I*pi*n/2), Eq(a, 0)
), (-Integral(1/(a*x*exp(2*acoth(a*x)) - exp(2*acoth(a*x))), x)/c, Eq(n, -2
)), (2*a*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a) + 2*(-a*c*x +
c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int e^{n\coth^{-1}(ax)}(c - acx)^{n/2}\,dx = \frac{2\left(a(-c)^{\frac{1}{2}n}x + (-c)^{\frac{1}{2}n}\right)(ax+1)^{\frac{1}{2}n}}{a(n+2)}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="maxima")
[Out] 2*(a*(-c)^(1/2*n)*x + (-c)^(1/2*n))*(a*x + 1)^(1/2*n)/(a*(n + 2))
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c - acx)^{n/2}\,dx = \int (-acx+c)^{\frac{1}{2}n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\left(\frac{1}{a\,x}+1\right)^{n/2}\,\left(c-a\,c\,x\right)^{n/2}\,\left(a\,x+1\right)}{a\,\left(1-\frac{1}{a\,x}\right)^{n/2}\,\left(n+2\right)}$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2),x)
```

```
[Out] (2*(1/(a*x) + 1)^(n/2)*(c - a*c*x)^(n/2)*(a*x + 1))/(a*(1 - 1/(a*x))^(n/2)*
(n + 2))
```

3.362 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

```
[Out] 2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x*(-a*c*x+c)^(-1+1/2*n)*hypergeom([
1, -1/2*n],[1-1/2*n],2/(a+1/x)/x)/n
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6311, 6316, 133}

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \frac{2x\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x*(c - a*c*x)^((-2 + n)/2)*H
ypergeometric2F1[1, -1/2*n, 1 - n/2, 2/((a + x^(-1))*x)])/n
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} x^{1-\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-1+\frac{n}{2}} x^{-1+\frac{n}{2}}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{x}\right)^{-1+\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\text{Subst}\left(\int\frac{x^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{1-\frac{x}{a}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$$

$$=-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{1+ax}\right)}{acn}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 -
n/2, 2/(1 + a*x)])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{-1+\frac{n}{2}}\,dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-c(ax-1))^{\frac{n}{2}-1}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-1+1/2*n),x)
[Out] Integral((-c*(a*x - 1))**(n/2 - 1)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-1}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1), x)
```

3.363 $\int e^{n \coth^{-1}(ax)} (c - acx)^{-2+\frac{n}{2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{-2+\frac{n}{2}} \, dx = -\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} x (c-acx)^{\frac{1}{2}(-4+n)} \text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

```
[Out] -2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-1+1/2*n)*x*(-a*c*x+c)^(-2+1/2*n)*hyperge
om([2, 1-1/2*n],[2-1/2*n],2/(a+1/x)/x)/(2-n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6311, 6316, 133}

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{-2+\frac{n}{2}} \, dx = -\frac{2x\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}} (c-acx)^{\frac{n-4}{2}} \text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x*(c - a*c*x)^((-4 +
 n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, 2/((a + x^(-1))*x)])/(2 - n)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}} x^{2-\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-2+\frac{n}{2}} x^{-2+\frac{n}{2}}\,dx$$
$$=-\left(\left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{x}\right)^{-2+\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\text{Subst}\left(\int\frac{x^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)\right)$$
$$=-\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{1+ax}\right)}{ac^2(-2+n)(1+ax)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
[Out] (2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, 2/(1 + a*x)])/(a*c^2*(-2 + n)*(1 - 1/(a*x))^(n/2)*(1 + a*x))
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} (-acx + c)^{-2+\frac{n}{2}} \, dx$$

[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)

[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)

Fricas [F]

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{-2+\frac{n}{2}} \, dx = \int (-acx + c)^{\frac{1}{2}n-2} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="fricas")

[Out] integral((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Sympy [F]

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{-2+\frac{n}{2}} \, dx = \int (-c(ax - 1))^{\frac{n}{2}-2} e^{n \operatorname{acoth}(ax)} \, dx$$

[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-2+1/2*n),x)

[Out] Integral((-c*(a*x - 1))**(n/2 - 2)*exp(n*acoth(a*x)), x)

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{-2+\frac{n}{2}} \, dx = \int (-acx + c)^{\frac{1}{2}n-2} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="maxima")

[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-2}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2), x)
```

3.364 $\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2*n-p)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^p*hypergeom([-
1-p, 1/2*n-p],[-p],2/(a+1/x)/x)/(p+1)/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6311, 6316, 134}

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-p-1,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c -
 a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])/(
(1 + p)*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-p} x^{-p}(c-acx)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-1-p,-p,\right.}{1+p}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(n-2p)}(1+ax)(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,\frac{n}{2}-p,-p,\frac{2}{1+ax}\right.}{a(1+p)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((n - 2*p)/2)*(1 + a*x)*(c - a*
c*x)^p*Hypergeometric2F1[-1 - p, n/2 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*(1 -
 1/(a*x))^(n/2))
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)}(-acx+c)^p\, dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\, dx = \int (-acx+c)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] integral((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\, dx = \int (-c(ax-1))^p e^{n \operatorname{acoth}(ax)}\, dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\, dx = \int (-acx+c)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^p\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^p,x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^p, x)
```

3.365 $\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] -32*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([5, 4-1/2*n],[5-
1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6315, 133}

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric2
F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(a^3c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)\\ &= \left(a^3c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^5}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(81) = 162.

Time = 1.92 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{c^3e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\left(-48+44n-12n^2+n^3\right)\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -1/24*(c^3*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(-48 + 44*n - 12*n^2 +
n^3)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(
a*n^3*x + n^2*(-1 - 12*a*x + a^2*x^2) + 2*n*(6 + 21*a*x - 6*a^2*x^2 + a^3*x
^3) + 6*(-7 - 4*a*x + 6*a^2*x^2 - 4*a^3*x^3 + a^4*x^4) + (-48 + 44*n - 12*n
^2 + n^3)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*(2 +
 n))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^3\,dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int -(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=-c^3\left(\int 3axe^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx+\int a^3x^3e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**3,x)
[Out] -c**3*(Integral(3*a*x*exp(n*acoth(a*x)), x) + Integral(-3*a**2*x**2*exp(n*a
coth(a*x)), x) + Integral(a**3*x**3*exp(n*acoth(a*x)), x) + Integral(-exp(n
*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int -(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -integrate((a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int -(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
[Out] integrate(-(a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^3\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^3,x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^3, x)
```

3.366 $\int e^{n \coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int e^{n \coth^{-1}(ax)}(c-acx)^2\,dx = \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] 16*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([4, 3-1/2*n],[4-1
/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6315, 133}

$$\int e^{n \coth^{-1}(ax)}(c-acx)^2\,dx = \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx$$
$$= \frac{c^2e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(8-6n+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(6-\right.\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(8 - 6*n + n^2)*Hypergeometri
c2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(6 + 6*a*x + a*n^2*
x - 6*a^2*x^2 + 2*a^3*x^3 + n*(-1 - 6*a*x + a^2*x^2) + (8 - 6*n + n^2)*Hype
rgeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(6*a*(2 + n))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^2\,dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = \int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int \left(-2axe^{n\,\mathrm{acoth}\,(ax)}\right)\,dx + \int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**2,x)
[Out] c**2*(Integral(-2*a*x*exp(n*acoth(a*x)), x) + Integral(a**2*x**2*exp(n*acot
h(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = \int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^2\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^2,x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^2, x)
```

3.367 $\int e^{n \coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int e^{n \coth^{-1}(ax)}(c-acx)\,dx = -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] -8*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([3, 2-1/2*n],[3-1/2
*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6310, 6315, 133}

$$\int e^{n \coth^{-1}(ax)}(c-acx)\,dx = -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1[
3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\text{integral} = -\left((ac)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(3, 2-\frac{n}{2}, 3-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx =$$
$$-\frac{ce^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}(-2+n)n\,\text{Hypergeometric2F1}\left(1, 1+\frac{n}{2}, 2+\frac{n}{2}, e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\right.\right.}{2a(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -1/2*(c*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1
[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*(-2 + n)*x + a^
2*x^2 + (-2 + n)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/
(a*(2 + n))
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}(-acx+c)\,dx$$

[In] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)

[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int-(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="fricas")

[Out] integral(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=-c\left(\int axe^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c),x)

[Out] -c*(Integral(a*x*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)), x))

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int-(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="maxima")

[Out] -integrate((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int -(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="giac")

[Out] integrate(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)\,dx$$

[In] int(exp(n*acoth(a*x))*(c - a*c*x),x)

[Out] int(exp(n*acoth(a*x))*(c - a*c*x), x)

3.368 $\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] 2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/
((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6315, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(
a + x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
```

```
|| !SumSimplerQ[p, 1]) && !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\text{Hypergeo}\right.\right.}{acn(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + Hypergeometric2F1[1, n/2, 1 + n
/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n)))
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{-acx+c}dx$$

[In] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)

[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="fricas")

[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c),x)

[Out] -Integral(exp(n*acoth(a*x))/(a*x - 1), x)/c

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="maxima")

[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - acx}\, dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{acx - c}\, dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - acx}\, dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c - a\,c\,x}\, dx$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x), x)
```

3.369 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}$$

```
[Out] -(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6315, 37}

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2x^2}\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^2c^2}\\ &= -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{e^{n\coth^{-1}(ax)}(1+ax)}{ac^2(2+n)(-1+ax)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -((E^(n*ArcCoth[a*x])*(1 + a*x))/(a*c^2*(2 + n)*(-1 + a*x)))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{e^{n\,\mathrm{arccoth}(ax)}(ax+1)}{(ax-1)c^2(2+n)a}$	33
parallelrisch	$\frac{-x\,e^{n\,\mathrm{arccoth}(ax)}a-e^{n\,\mathrm{arccoth}(ax)}}{c^2(ax-1)(2+n)a}$	41

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(n*arccoth(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(ax+1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n+2\,ac^2-(a^2c^2n+2\,a^2c^2)x}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n + 2*a*c^2 - (a^2*c^2*n + 2
*a^2*c^2)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.90

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \begin{cases} -\frac{x}{c^2} & \text{for}\: a=0 \wedge n=-2 \\ \frac{xe^{\frac{i\pi n}{2}}}{c^2} & \text{for}\: a=0 \\ -\frac{ax\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)}-ac^2e^{2\operatorname{acoth}(ax)}}-\frac{\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)}-ac^2e^{2\operatorname{acoth}(ax)}} & \text{for}\: n=-2 \\ -\frac{axe^{n\operatorname{acoth}(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2}-\frac{e^{n\operatorname{acoth}(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**2,x)
```

```
[Out] Piecewise((-x/c**2, Eq(a, 0) & Eq(n, -2)), (x*exp(I*pi*n/2)/c**2, Eq(a, 0))
, (-a*x*acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x)
)) - acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x))),
 Eq(n, -2)), (-a*x*exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**
2*n - 2*a*c**2) - exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2
*n - 2*a*c**2), True))
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{e^{n\,\mathrm{acoth}(a\,x)}\,(a\,x+1)}{a\,c^2\,(a\,x-1)\,(n+2)}$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^2,x)
[Out] -(exp(n*acoth(a*x))*(a*x + 1))/(a*c^2*(a*x - 1)*(n + 2))
```

3.370 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(n^2+6*n+8)+(1-1/a/x)
^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(4+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6310, 6315, 80, 37}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+4)} - \frac{(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+2)(n+4)}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(4 + n)) - ((3
+ n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(2 + n)*(4
+ n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3} \\
&= \frac{\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^3c^3} \\
&= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^2c^3(4+n)} \\
&= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{e^{n \coth^{-1}(ax)}(3+n-ax)\left(\cosh\left(3\coth^{-1}(ax)\right)+\sinh\left(3\coth^{-1}(ax)\right)\right)}{a^2c^3(2+n)(4+n)\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 + n - a*x)*(Cosh[3*ArcCoth[a*x]] + Sinh[3*ArcCoth[a*
x]]))/(a^2*c^3*(2 + n)*(4 + n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{e^{n\ \mathrm{arccoth}(ax)}(ax-n-3)(ax+1)}{(ax-1)^2c^3(n^2+6n+8)a}$	46
parallelrisch	$\frac{2x\,e^{n\ \mathrm{arccoth}(ax)}a+x\,e^{n\ \mathrm{arccoth}(ax)}an-x^2e^{n\ \mathrm{arccoth}(ax)}a^2+e^{n\ \mathrm{arccoth}(ax)}n+3\,e^{n\ \mathrm{arccoth}(ax)}}{c^3(ax-1)^2(n^2+6n+8)a}$	81

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(n*arccoth(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^2x^2-(an+2\,a)x-n-3)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^2+6\,ac^3n+8\,ac^3+(a^3c^3n^2+6\,a^3c^3n+8\,a^3c^3)x^2-2\,(a^2c^3n^2+6\,a^2c^3n+8\,a^2c^3)x}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - (a*n + 2*a)*x - n - 3)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^2
 + 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 + 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*(a
^2*c^3*n^2 + 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 59.89 (sec) , antiderivative size = 1112, normalized size of antiderivative = 10.69

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(I*pi*n/2)/c**3, Eq(a, 0)), (a**2*x**2*acoth(a*x)/(2*a**3*c
**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp
(4*acoth(a*x))) + 2*a*x*acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*
a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - a*x/(2*a**3*c
**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp
(4*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c
**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - 1/(2*a**3*c**3*x**2
*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth
(a*x))), Eq(n, -4)), (-a**2*x**2*acoth(a*x)/(2*a**3*c**3*x**2*exp(2*acoth(a
*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + a*x/
(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a
*c**3*exp(2*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) -
 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + 1/(2*a**3*
c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*ex
p(2*acoth(a*x))), Eq(n, -2)), (-a**2*x**2*exp(n*acoth(a*x))/(a**3*c**3*n**2
*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**
2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + a*n*x*
exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x
**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2
+ 6*a*c**3*n + 8*a*c**3) + 2*a*x*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6
*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*
x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + n*exp(n*acoth(a
*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*
c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n
+ 8*a*c**3) + 3*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2
 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*
x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3), True))
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(acx-c)^3}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(acx-c)^3}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n+3}{a^3\,c^3\,(n^2+6\,n+8)}-\frac{x^2}{a\,c^3\,(n^2+6\,n+8)}+\frac{x\,(n+2)}{a^2\,c^3\,(n^2+6\,n+8)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^3,x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((n + 3)/(a^3*c^3*(6*n + n^2 + 8)) - x^2/(a*c^3*(6
*n + n^2 + 8)) + (x*(n + 2))/(a^2*c^3*(6*n + n^2 + 8))))/(((a*x - 1)/(a*x))
^(n/2)*(1/a^2 - (2*x)/a + x^2))
```

3.371 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 224

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} dx = \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)\left(8+6n+n^2\right)} - \frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}$$

```
[Out] -(n^2+8*n+14)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^2+10*n+24)-
(n^2+8*n+14)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^3+12*n^2+44*
n+48)+(5+n)*(1-1/a/x)^(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(6+n)-(1-1/a/x)^
(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a^2/c^4/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6310, 6315, 92, 80, 47, 37\}$

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{a^2c^4x} - \frac{(n^2+8n+14)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)} - \frac{(n^2+8n+14)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)} + \frac{(n+5)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{ac^4(n+6)}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^4*(6 + n)
) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(
a*c^4*(4 + n)*(6 + n)) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2)) - ((1 - 1/(a*x))^(-3 -
n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a^2*c^4*x)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)}{a^4c^4}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}-\frac{\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{(4+n)x}{a}\right)dx,x,\frac{1}{x}\right)}{a^2c^4}\\
&= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)}-\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}\\
&\quad-\frac{(14+8n+n^2)\,\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{a^2c^4(6+n)}\\
&= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)}-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}\\
&\quad-\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}-\frac{(14+8n+n^2)\,\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{a^2c^4(4+n)(6+n)}
\end{aligned}
$$

$$=\frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)}-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(2+n)(4+n)(6+n)}-\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}\left(-12-8n-n^2+(4+n)^2\cosh\left(2\coth^{-1}(ax)\right)-2(4+n)\sinh\left(2\coth^{-1}(ax)\right)\right)\left(\cosh\left(4\coth\right.\right.}{2ac^4(2+n)(4+n)(6+n)}$$

[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]

[Out] -1/2*(E^(n*ArcCoth[a*x])*(-12 - 8*n - n^2 + (4 + n)^2*Cosh[2*ArcCoth[a*x]] - 2*(4 + n)*Sinh[2*ArcCoth[a*x]])*(Cosh[4*ArcCoth[a*x]] + Sinh[4*ArcCoth[a*x]]))/(a*c^4*(2 + n)*(4 + n)*(6 + n))

Maple [A] (verified)

Time = 12.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.30

method	result
gosper	$-\frac{(ax+1)\left(2a^2x^2-2anx-8ax+n^2+8n+14\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(ax-1)^3c^4a(n^2+8n+12)(4+n)}$
parallelrisch	$\frac{2x^2\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a^2n-x\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a\,n^2-6x\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a-6x\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}an+6x^2\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a^2-8\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}n-14}{c^4(ax-1)^3a(n^2+8n+12)(4+n)}$

[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)

[Out] -(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arccoth(a*x))/(a*x-1)^3/c^4/a/(n^2+8*n+12)/(4+n)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{(2\,a^3x^3 - 2\,(a^2n + 3\,a^2)x^2 + n^2 + (an^2 + 6\,an + 6\,a)x}{ac^4n^3 + 12\,ac^4n^2 + 44\,ac^4n + 48\,ac^4 - (a^4c^4n^3 + 12\,a^4c^4n^2 + 44\,a^4c^4n + 48\,a^4c^4)x^3 + 3\,(a^3c^4n^3 + 12\,a^3c^4}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
[Out] (2*a^3*x^3 - 2*(a^2*n + 3*a^2)*x^2 + n^2 + (a*n^2 + 6*a*n + 6*a)*x + 8*n +
14)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^3 + 12*a*c^4*n^2 + 44*a*c^4*n +
48*a*c^4 - (a^4*c^4*n^3 + 12*a^4*c^4*n^2 + 44*a^4*c^4*n + 48*a^4*c^4)*x^3 +
 3*(a^3*c^4*n^3 + 12*a^3*c^4*n^2 + 44*a^3*c^4*n + 48*a^3*c^4)*x^2 - 3*(a^2*
c^4*n^3 + 12*a^2*c^4*n^2 + 44*a^2*c^4*n + 48*a^2*c^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(acx-c)^4}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="giac")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)

Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,x^3}{a\,c^4\,(n^3+12\,n^2+44\,n+48)}+\frac{n^2+8\,n+14}{a^4\,c^4\,(n^3+12\,n^2+44\,n+48)}-\frac{x^2\,(2\,n+6)}{a^2\,c^4\,(n^3+12\,n^2+44\,n+48)}+\frac{x\,(n^2+6\,n+6)}{a^3\,c^4\,(n^3+12\,n^2+44\,n+48)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{3\,x}{a^2}-\frac{1}{a^3}+x^3-\frac{3\,x^2}{a}\right)}$$

[In] int(exp(n*acoth(a*x))/(c - a*c*x)^4,x)

[Out] -(((a*x + 1)/(a*x))^(n/2)*((2*x^3)/(a*c^4*(44*n + 12*n^2 + n^3 + 48)) + (8*n + n^2 + 14)/(a^4*c^4*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(2*n + 6))/(a^2*c^4*(44*n + 12*n^2 + n^3 + 48)) + (x*(6*n + n^2 + 6))/(a^3*c^4*(44*n + 12*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*((3*x)/a^2 - 1/a^3 + x^3 - (3*x^2)/a))

3.372 $\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}(-5+n),-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/7*((a-1/x)/(a+1/x))^(-5/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(5/2)*h
ypergeom([-7/2, -5/2+1/2*n],[-5/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6311, 6316, 134}

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{n-5}{2},-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/((a + x^(-1))*x)
])/(7*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{5/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}(-5+n),-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^3\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}(-\right.}{7a}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^3*
Sqrt[c - a*c*x]*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/(1 + a*x)])/(7*
a*(1 - 1/(a*x))^(n/2))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{5}{2}}\,dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int(-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(5/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int(-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,6,1,0,0]%%%}+%%%{3,[0,4,1,1,0]%%%}+%%%{-3,[0,2,1,2,0]%%
%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^{5/2}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2), x)
```

3.373 $\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/5*((a-1/x)/(a+1/x))^(-3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(3/2)*h
ypergeom([-5/2, -3/2+1/2*n],[-3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6311, 6316, 134}

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{n-3}{2},-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/((a + x^(-1))*x)
])/(5*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{(c-acx)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx =$$
$$-\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^2\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{1}{\cdot}\right.}{5a}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^2*S
qrt[c - a*c*x]*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/(1 + a*x)])/(5*a
*(1 - 1/(a*x))^(n/2))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{3}{2}}\,dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-c(ax-1))^{\frac{3}{2}}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,4,1,0,0]%%%}+%%%{-2,[0,2,1,1,0]%%%}+%%%{1,[0,0,1,2,0]%%%
} / %%
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^{3/2}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2), x)
```

3.374 $\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/3*((a-1/x)/(a+1/x))^(-1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-3/2, -
1/2+1/2*n],[-1/2],2/(a+1/x)/x)*(-a*c*x+c)^(1/2)/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6311, 6316, 134}

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n-1}{2},-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqr
t[c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/((a + x^(-1))*x)])
/(3*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-acx}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{1+ax}\right)}{3a}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)*Sqrt[c
 - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/(1 + a*x)])/(3*a*(1 -
 1/(a*x))^(n/2))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{-acx+c}dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int \sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int \sqrt{-c\,(ax-1)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a\,c\,x}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2), x)
```

3.375 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-acx}} dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-acx}} dx = \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x \, \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1+n}{2}, \frac{1}{2}, \frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

```
[Out] 2*((a-1/x)/(a+1/x))^(1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-1/2, 1/2+
1/2*n],[1/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6311, 6316, 134}

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-acx}} dx = \frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{n+1}{2}, \frac{1}{2}, \frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hype
rgeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*Sqrt[c - a*c*x])
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}} \\
&= -\frac{\sqrt{1-\frac{1}{ax}}\,\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}} \\
&= \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1+n}{2}, \frac{1}{2}, \frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}(1+ax)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{1+ax}\right)}{a\sqrt{c-acx}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*(1 + a*x)*Hyperge
ometric2F1[-1/2, (1 + n)/2, 1/2, 2/(1 + a*x)])/(a*(1 - 1/(a*x))^(n/2)*Sqrt[
c - a*c*x])
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{-acx+c}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\,(ax-1)}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a\,c\,x}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2), x)
```

3.376 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

```
[Out] -2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([1/2, 3/2+
1/2*n],[3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6311, 6316, 134}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hyp
ergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*(c - a*c*x)^(3/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{1+ax}\right)}{ac\sqrt{c-acx}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1
[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)])/(a*c*(1 - 1/(a*x))^(n/2)*Sqrt[c - a*c*x
])
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{3}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*
c^2*x + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{3/2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2), x)
```

3.377 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{5/2}} \, dx = -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(3+n)(c-acx)^{5/2}}$$

```
[Out] -a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(3+n)/(-a*c*x+c)^(5/2)+a*((a
-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2*hype
rgeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(3+n)/(-a*c*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 96, 134}

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{5/2}} \, dx = \frac{ax^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(n+3)(c-acx)^{5/2}} - \frac{ax^2\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+3)(c-acx)^{5/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((3 + n)*(c -
 a*c*x)^(5/2))) + (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^
```

```
((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2*Hypergeometric2F1[1/2, (3 + n)/2,
 3/2, 2/((a + x^(-1))*x)])/((3 + n)*(c - a*c*x)^(5/2))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right) \int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \operatorname{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}dx, x, \frac{1}{x}\right)}{2(3+n)\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(3+n)(c-acx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(-1-ax+(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3\cdots}{}\right.\right.}{ac^2(3+n)(-1+ax)\sqrt{c-acx}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(-1 - a*x + (-1 + a*x)*((-1 + a*x)/(1 + a*x))^((1 + n)
/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)]))/(a*c^2*(3 + n)*(1
 - 1/(a*x))^(n/2)*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [F]

$$\int\frac{e^{n\,\operatorname{arccoth}(ax)}}{(-acx+c)^{\frac{5}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

Fricas [F]

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx = \int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*x^3 - 3*a
^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(5/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{5/2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2), x)
```

3.378 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 245

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=-\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}}+\frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}}-\frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(15+8n+n^2\right)(c-acx)^{7/2}}$$

```
[Out] -a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(5+n)/(-a*c*x+c)^(7/2)+3/2*a
^2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^3/(n^2+8*n+15)/(-a*c*x+c)^(7/2
)-3/2*a^2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/
2*n)*x^3*hypergeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(n^2+8*n+15)/(-a*c*x
+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 96, 134}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=-\frac{3a^2x^3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}+\frac{3a^2x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}-\frac{ax^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}}{(n+5)(c-acx)^{7/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((5 + n)*(c -
 a*c*x)^(7/2))) + (3*a^2*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2
)*x^3)/(2*(15 + 8*n + n^2)*(c - a*c*x)^(7/2)) - (3*a^2*((a - x^(-1))/(a + x
^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3
*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(2*(15 + 8*n +
 n^2)*(c - a*c*x)^(7/2))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1))))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int x^{3/2}\left(1-\frac{x}{a}\right)^{-\frac{7}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int\sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{2(5+n)\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}dx,x,\frac{1}{x}\right)}{4(3+n)(5+n)\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(15+8n+n^2\right)(c-acx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.56

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left((9+2n-3ax)(1+ax)+3(-1+ax)^2\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hyperge}\right.}{2ac^3(3+n)(5+n)(-1+ax)^2\sqrt{c-acx}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((9 + 2*n - 3*a*x)*(1 + a*x) + 3*(-1 + a*x)^2*((-1 + a
*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*
x)]))/(2*a*c^3*(3 + n)*(5 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x)^2*Sqrt[c - a*
c*x])
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{(-acx+c)^{\frac{7}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a^
3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(7/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{7/2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2), x)
```

3.379 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{c^4\csc^{-1}(ax)}{2a}-\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+c^4*(1-1/a^2/x^2)^(3/2)*x-1/2*c^4*arccsc(a*x
)/a-3*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(6*a-1/x)*(1-1/a^2/x^2)^(1
/2)/a^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6312, 1821, 1823, 829, 858, 222, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{c^4\csc^{-1}(ax)}{2a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] -1/3*(c^4*(1 - 1/(a^2*x^2))^(3/2))/a + (c^4*Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^
(-1)))/(2*a^2) + c^4*(1 - 1/(a^2*x^2))^(3/2)*x - (c^4*ArcCsc[a*x])/(2*a) -
(3*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(\frac{3c^3}{a}-\frac{c^3x}{a^2}+\frac{c^3x^2}{a^3}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{1}{3}\left(a^2c\right)\text{Subst}\left(\int \frac{\left(-\frac{9c^3}{a^3}+\frac{3c^3x}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{1}{6}\left(a^4c\right)\text{Subst}\left(\int \frac{\frac{18c^3}{a^5}-\frac{3c^3x}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} - (3ac^4)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} - \frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.54

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(-2+9ax-14a^2x^2-15a^3x^3+16a^4x^4+6a^5x^5+24a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+9a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{1}{ax}\right)-18a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{6a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(-2 + 9*a*x - 14*a^2*x^2 - 15*a^3*x^3 + 16*a^4*x^4 + 6*a^5*x^5 + 24*a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 9*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 18*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(ax-1)(16a^2x^2-9ax+2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{(ax-1)(ax+1)}}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^4\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+18(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+18\ln\left(\frac{a^2x+}{}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x-1)*(16*a^2*x^2-9*a*x+2)/x^3*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)+(-3*a^
4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/2*a^3*arctan(1/(a^2
*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.37

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{6\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^4x^4+22\,a^3c^4}{6\,a^4x^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^4*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 7*a^2*c^4*x^2 - 7*a*c^4*x + 2*c^
4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a}{x\sqrt{\frac{ax}{ax+1}}}\right.\right.}{a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a/(x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(6*a**2/(x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))), x) + Integral(-4*a**3/(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)
)/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^4\left(\frac{ax-1}{ax+1}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-}{(ax+}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^2 + 9*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21*c
^4*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 37*c^
4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x +
1)^4 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(100) = 200.

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.18

$$
\begin{aligned}
&\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx\\
&=\frac{c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\left(ax+1\right)}\\
&\quad+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4+36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-9\left(x|a|-\sqrt{a^2x^2\cdots}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}
\end{aligned}
$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 3*c^4*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
^4/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) +
12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 + 36*(x*abs(a) - sqrt(a^2*x^2 - 1
))^2*a*c^4 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) + 16*a*c^4)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.61

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx&=\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}\\
&\quad+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}
\end{aligned}
$$

```
[In] int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (17*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^4*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^4
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.380 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}-\frac{2c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x+1/2*c^3*arccsc(a*x)/a-2*c^3*arctanh((1-1/a^2/x^2)
^(1/2))/a+1/2*c^3*(4*a+1/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6312, 1821, 829, 858, 222, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{2c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{c^3\csc^{-1}(ax)}{2a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1)))/(2*a^2) + c^3*(1 - 1/(a^2*x^2))^
(3/2)*x + (c^3*ArcCsc[a*x])/(2*a) - (2*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/
a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + c\text{Subst}\left(\int \frac{\left(\frac{2c^2}{a}+\frac{c^2x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int \frac{-\frac{4c^2}{a^3}-\frac{c^2x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x\\
&\quad + \frac{c^3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} + \frac{c^3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a}\\
&= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a}\\
&\quad - \left(2ac^3\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} - \frac{2c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$=\frac{c^3\left(1-4ax-3a^2x^2+4a^3x^3+2a^4x^4+2a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+2a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{1}{ax}\right)-\right.}{2a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(1 - 4*a*x - 3*a^2*x^2 + 4*a^3*x^3 + 2*a^4*x^4 + 2*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 2*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 4*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

method	result
risch	$\frac{(ax-1)(2a^2x^2+4ax-1)c^3}{2x^2a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{2a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)c^3\sqrt{(ax-1)(ax+1)}}{a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^3\left(-4\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+4\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^3x^2\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x-1)*(2*a^2*x^2+4*a*x-1)/x^2*c^3/a^3/((a*x-1)/(a*x+1))^(1/2)+(-2*a^3
*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+1/2*a^2*arctan(1/(a^2*
x^2-1)^(1/2)))*c^3/a^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x
+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\frac{2\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^3x^3+6\,a^2c^3}{2\,a^3x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 4*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (2*a^3*c^3*x^3 + 6*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = \frac{c^3\left(\int\frac{a^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(a**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a/(x**2*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**2/(x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(78) = 156.

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.28

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-}{(ax+}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 2*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (3*c^3*((a
*x - 1)/(a*x + 1))^(5/2) - 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqrt((
a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2
 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.51

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$= -\frac{c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)}$$
$$+\frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|+4\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3-\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|+4\,ac^3}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2a|a|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 2*c^3*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^3/(a*sgn(a*x + 1)) + ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a) + 4*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - (x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*ab
s(a) + 4*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a)*sgn(a*x
+ 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 3*
c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x - 1
)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^3*atan(((a*x - 1)/(a*x
 + 1))^(1/2)))/a - (4*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.381 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a+1/x)*x*(1-1/a^2
/x^2)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 827, 858, 222, 272, 65, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{1}{2}c\text{Subst}\left(\int \frac{\frac{2c}{a}+\frac{2cx}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - (ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(62) = 124$.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.55

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$= \frac{c^2\left(-2-2ax+2a^2x^2+2a^3x^3-2a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{1}{ax}\right)-2a^2\sqrt{}\right.}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*(-2 - 2*a*x + 2*a^2*x^2 + 2*a^3*x^3 - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*
ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/
(a*x)] - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(
2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{(ax-1)(ax+1)}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)/x*c^2/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(-a*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/
2)))*c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+2\,ac^2x+c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 + 2*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{c^2\left(\int \frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{2a}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**2,x)
[Out] c**2*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a/(x*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.02

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.21

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\,(ax+1)}+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\,(ax+1)}+\frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^2/(a*sgn(a*x + 1)) + 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

3.382 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] c*arccsc(a*x)/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6312, 283, 222}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x + (c*ArcCsc[a*x])/a
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x + \frac{c\csc^{-1}(ax)}{a}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x+\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x)),x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcSin[1/(a*x)]))/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{(ax-1)c\left(\sqrt{a^2x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a}$	63

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/((a*x-1)*(a*x+1))^(1/2)*c/a*((a^2*x^2-1)^
(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*c*x + c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\int\frac{a}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)\,dx\right)}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{c\arctan\left(\sqrt{a^2x^2-1}\right)-\sqrt{a^2x^2-1}c}{a\text{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="giac")
[Out] -(c*arctan(sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)*c)/(a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*atan
(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.383 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] 2*arctanh((1-1/a^2/x^2)^(1/2))/a/c-2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1
-1/a^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (-2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)
/c + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^2-\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3+ax)+2(-1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3 + a*x) + 2*(-1 + a*x)*Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c*(-1 + a*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(64) = 128.

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{2\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a\sqrt{a^{2}}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{a^{3}\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+2\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}-4\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{2}x-((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}-4\sqrt{}}{a\sqrt{a^{2}}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(2/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)
^(1/2))/(a^2)^(1/2)-2/a^3/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a/c/((
a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^{2}x^{2}-2\,ax-3)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}cx-ac}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 2*a*x - 3)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c*x - a*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\int \frac{x}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
[Out] a*Integral(x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.66

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -2a\left(\frac{\frac{2(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
[Out] -2*a*((2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{2\,a\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*a*x + 8*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 6)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.384 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -4/3*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c
^2+1/3*(-9*a-11/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2} - \frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (-4*(a + x^(-1)))/(3*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (9*a + 11/x)/(3*a^2
*c^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (3*ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^3}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^3-\frac{9c^3x}{a}-\frac{8c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^3+\frac{9c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{14-5ax-16a^2x^2+3a^3x^3+9a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^5\left(x-\frac{1}{a}\right)^2}-\frac{13\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{(ax-1)(ax+1)}}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+9\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-27\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-6\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)
-13/3/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c^2/((a*x-1)/(a*x+
1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax\right.}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^
2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**
2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)}-\frac{6\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((11*(a*x - 1))/(3*(a*x +
1)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.385 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -8/5*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+8/x)/a^2/c^3/(1-1/a^2/x^
2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c^3+1/15*(-60*a-79/x)/a^2/c^3/(1-
1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = \frac{4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3} - \frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-8*(a + x^(-1)))/(5*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 8/x))/(15
*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (60*a + 79/x)/(15*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]])/(a*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^4-\frac{20c^4x}{a}-\frac{27c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^4+\frac{60c^4x}{a}+\frac{64c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^4-\frac{60c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac^3}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(4a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^3}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-94+128ax+73a^2x^2-134a^3x^3+15a^4x^4+60a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
[Out] (-94 + 128*a*x + 73*a^2*x^2 - 134*a^3*x^3 + 15*a^4*x^4 + 60*a*Sqrt[1 - 1/(a
^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^3*Sqrt[1
- 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^7\left(x-\frac{1}{a}\right)^3}-\frac{31\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{15a^6\left(x-\frac{1}{a}\right)^2}-\frac{104\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{15a^5\left(x-\frac{1}{a}\right)}\right)a^3\sqrt{(ax}}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-60\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^4x^4-60\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4+45\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+240\sqrt{a^2}\,\sqrt{(ax-1)(ax+}}{}$

[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)

[Out] 1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(4/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-2/5/a^7/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-31/15/a^6/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-104/15/a^5/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^3/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{60\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-60\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")

[Out] 1/15*(60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (15*a^4*x^4 - 134*a^3*x^3 + 73*a^2*x^2 + 128*a*x - 94)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\int \frac{x^3}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{1}{30}a\left(\frac{\frac{22\,(ax-1)}{ax+1}+\frac{155\,(ax-1)^2}{(ax+1)^2}-\frac{240\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{120\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*a*((22*(a*x - 1)/(a*x + 1) + 155*(a*x - 1)^2/(a*x + 1)^2 - 240*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
- 1)/(a*x + 1))^(5/2)) + 120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) -
 120*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^3|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac^3\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^3*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^3*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3} - \frac{\frac{31\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{16\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{22\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((31*(a*x - 1)^2)/(3*(a*x
+ 1)^2) - (16*(a*x - 1)^3)/(a*x + 1)^3 + (22*(a*x - 1))/(15*(a*x + 1)) + 1/
5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(7/
2))
```

3.386 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{5\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] -16/7*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-4/35*(7*a+17/x)/a^2/c^4/(1-1/a^2/
x^2)^(5/2)+1/105*(-175*a-307/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+5*arctanh((1-1/
a^2/x^2)^(1/2))/a/c^4+1/105*(-525*a-719/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1
-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = \frac{5\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4} - \frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (-16*(a + x^(-1)))/(7*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (4*(7*a + 17/x))/(
35*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (175*a + 307/x)/(105*a^2*c^4*(1 - 1/(
a^2*x^2))^(3/2)) - (525*a + 719/x)/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (S
qrt[1 - 1/(a^2*x^2)]*x)/c^4 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^5-\frac{35c^5x}{a}-\frac{61c^5x^2}{a^2}+\frac{7c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{35c^5+\frac{175c^5x}{a}+\frac{272c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-105c^5-\frac{525c^5x}{a}-\frac{614c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{105c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{105c^5+\frac{525c^5x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{105c^9}$$

$$=-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^4}-\frac{5\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{ac^4}$$

$$=-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^4}-\frac{5\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx,x,\frac{1}{x^2}\right)}{2ac^4}$$

$$=-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^4}+\frac{(5a)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c^4}$$

$$=-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^4}+\frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int\frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx=\frac{824-1947ax+485a^2x^2+1812a^3x^3-1339a^4x^4+105a^5x^5+525a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3\text{arctanh}\left(\sqrt{1-\right.}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (824 - 1947*a*x + 485*a^2*x^2 + 1812*a^3*x^3 - 1339*a^4*x^4 + 105*a^5*x^5 +
 525*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.55

method	result
risch	$\frac{ax-1}{a\,c^{4}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{4}\sqrt{a^{2}}}-\frac{57\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{35a^{8}\left(x-\frac{1}{a}\right)^{3}}-\frac{446\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{105a^{7}\left(x-\frac{1}{a}\right)^{2}}-\frac{1024\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{105a^{6}\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{\left(x-\right.}}{}\right.}{c^{4}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{5}x^{5}-525\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{6}x^{5}+420((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}+2625\sqrt{a^{2}}\,\sqrt{(ax-1)(ax}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(5/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-57/35/a^8/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-446/105/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1024/105/a^6/(x-
1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-2/7/a^9/(x-1/a)^4*((x-1/a)^2*a^2+2*(
x-1/a)*a)^(1/2))*a^4/c^4/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a
*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{4}}\,dx$$

$$=\frac{525\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-525\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\sqrt{\right.}\right.}{105\left(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - 525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sq
rt((a*x - 1)/(a*x + 1)) - 1) + (105*a^5*x^5 - 1339*a^4*x^4 + 1812*a^3*x^3 +
 485*a^2*x^2 - 1947*a*x + 824)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{420}a\left(\frac{\frac{111(ax-1)}{ax+1}+\frac{469(ax-1)^2}{(ax+1)^2}+\frac{2765(ax-1)^3}{(ax+1)^3}-\frac{4200(ax-1)^4}{(ax+1)^4}+15}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{2100\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/420*a*((111*(a*x - 1)/(a*x + 1) + 469*(a*x - 1)^2/(a*x + 1)^2 + 2765*(a*x - 1)^3/(a*x + 1)^3 - 4200*(a*x - 1)^4/(a*x + 1)^4 + 15)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 2100*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{67\,(a\,x-1)^2}{15\,(a\,x+1)^2}+\frac{79\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{40\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{37\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

```
[In] int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((67*(a*x - 1)^2)/(15*(a*
x + 1)^2) + (79*(a*x - 1)^3)/(3*(a*x + 1)^3) - (40*(a*x - 1)^4)/(a*x + 1)^4
 + (37*(a*x - 1))/(35*(a*x + 1)) + 1/7)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2
) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2))
```

3.387 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}$$

```
[Out] -1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/a^3/x^2-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 76}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}-\frac{3c^5\log(x)}{a}+c^5x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -1/4*c^5/(a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*
x - (3*c^5*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^5 dx \\
&= \frac{c^5\int\frac{e^{2\text{arctanh}(ax)}(1-ax)^5}{x^5}dx}{a^5} \\
&= \frac{c^5\int\frac{(1-ax)^4(1+ax)}{x^5}dx}{a^5} \\
&= \frac{c^5\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5} \\
&= -\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\left(\frac{5a^4}{4}+\frac{1}{4x^4}-\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{2a^3}{x}-a^5x+3a^4\log(x)\right)}{a^5}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
[Out] -((c^5*((5*a^4)/4 + 1/(4*x^4) - a/x^3 + a^2/x^2 + (2*a^3)/x - a^5*x + 3*a^4
*Log[x]))/a^5)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{c^{5}\left(a^{5}x-3a^{4}\ln(x)-\frac{1}{4x^{4}}+\frac{a}{x^{3}}-\frac{a^{2}}{x^{2}}-\frac{2a^{3}}{x}\right)}{a^{5}}$
risch	$c^{5}x+\frac{-2a^{3}c^{5}x^{3}-a^{2}c^{5}x^{2}+a\,c^{5}x-\frac{1}{4}c^{5}}{a^{5}x^{4}}-\frac{3c^{5}\ln(x)}{a}$
norman	$\frac{c^{5}x+a^{4}c^{5}x^{5}-\frac{c^{5}}{4a}-a\,c^{5}x^{2}-2c^{5}a^{2}x^{3}}{a^{4}x^{4}}-\frac{3c^{5}\ln(x)}{a}$
parallelrisch	$-\frac{-4a^{5}c^{5}x^{5}+12c^{5}\ln(x)a^{4}x^{4}+8a^{3}c^{5}x^{3}+4a^{2}c^{5}x^{2}-4a\,c^{5}x+c^{5}}{4a^{5}x^{4}}$
meijerg	$-\frac{c^{5}(-ax-\ln(-ax+1))}{a}-\frac{4c^{5}\ln(-ax+1)}{a}-\frac{5c^{5}(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{5c^{5}\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{}{2a}\right.}{a}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x,method=_RETURNVERBOSE)

[Out] c^5/a^5*(a^5*x-3*a^4*ln(x)-1/4/x^4+a/x^3-a^2/x^2-2*a^3/x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log(x)-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="fricas")

[Out] 1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2 + 4*a*c^5*x - c^5)/(a^5*x^4)

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{a^5c^5x-3a^4c^5\log(x)+\frac{-8a^3c^5x^3-4a^2c^5x^2+4ac^5x-c^5}{4x^4}}{a^5}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**5,x)

[Out] (a**5*c**5*x - 3*a**4*c**5*log(x) + (-8*a**3*c**5*x**3 - 4*a**2*c**5*x**2 + 4*a*c**5*x - c**5)/(4*x**4))/a**5

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = c^5x - \frac{3\,c^5\log\left(x\right)}{a} - \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - 3*c^5*log(x)/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x + c
^5)/(a^5*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = c^5x - \frac{3\,c^5\log\left(|x|\right)}{a} - \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*x - 3*c^5*log(abs(x))/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*
x + c^5)/(a^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\,\left(4\,a^2\,x^2 - 4\,a\,x + 8\,a^3\,x^3 - 4\,a^5\,x^5 + 12\,a^4\,x^4\,\ln\left(x\right) + 1\right)}{4\,a^5\,x^4}$$

```
[In] int(((c - c/(a*x))^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] -(c^5*(4*a^2*x^2 - 4*a*x + 8*a^3*x^3 - 4*a^5*x^5 + 12*a^4*x^4*log(x) + 1))/
(4*a^5*x^4)
```

3.388 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}$$

```
[Out] 1/3*c^4/a^4/x^3-c^4/a^3/x^2+c^4*x-2*c^4*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 76}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}-\frac{2c^4\log(x)}{a}+c^4x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx\\
&= -\frac{c^4\int\frac{e^{2\text{arctanh}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\
&= -\frac{c^4\int\frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4}\\
&= -\frac{c^4\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}\\
&= \frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(\frac{4a^3}{3}-\frac{1}{3x^3}+\frac{a}{x^2}-a^4x+2a^3\log(x)\right)}{a^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
[Out] -((c^4*((4*a^3)/3 - 1/(3*x^3) + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^4\left(a^4x-2a^3\ln(x)+\frac{1}{3x^3}-\frac{a}{x^2}\right)}{a^4}$
risch	$c^4x+\frac{-a\,c^4x+\frac{1}{3}c^4}{a^4x^3}-\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^3c^4x^4+\frac{c^4}{3a}-c^4x}{a^3x^3}-\frac{2c^4\ln(x)}{a}$
parallelrisch	$-\frac{-3a^4c^4x^4+6c^4\ln(x)a^3x^3+3a\,c^4x-c^4}{3a^4x^3}$
meijerg	$-\frac{c^4(-ax-\ln(-ax+1))}{a}-\frac{3c^4\ln(-ax+1)}{a}-\frac{2c^4(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{2c^4\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
[Out] c^4/a^4*(a^4*x-2*a^3*ln(x)+1/3/x^3-a/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{3\,a^4c^4x^4-6\,a^3c^4x^3\log(x)-3\,ac^4x+c^4}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{a^4c^4x-2a^3c^4\log(x)+\frac{-3ac^4x+c^4}{3x^3}}{a^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**4,x)
[Out] (a**4*c**4*x - 2*a**3*c**4*log(x) + (-3*a*c**4*x + c**4)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x-\frac{2\,c^4\log\left(x\right)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="maxima")
[Out] c^4*x - 2*c^4*log(x)/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x-\frac{2\,c^4\log\left(|x|\right)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="giac")
[Out] c^4*x - 2*c^4*log(abs(x))/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln\left(x\right)-1\right)}{3\,a^4\,x^3}$$

```
[In] int(((c - c/(a*x))^4*(a*x + 1))/(a*x - 1),x)
[Out] -(c^4*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

3.389 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}$$

```
[Out] -1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 76}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(x)}{a}+c^3x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^3 dx \\
&= \frac{c^3\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^3}{x^3}\,dx}{a^3} \\
&= \frac{c^3\int \frac{(1-ax)^2(1+ax)}{x^3}\,dx}{a^3} \\
&= \frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3} \\
&= -\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\left(3a^2+\frac{1}{x^2}-\frac{2a}{x}-2a^3x+2a^2\log(x)\right)}{2a^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
[Out] -1/2*(c^3*(3*a^2 + x^(-2) - (2*a)/x - 2*a^3*x + 2*a^2*Log[x]))/a^3
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^{3}\left(a^{3}x-a^{2}\ln\left(x\right)-\frac{1}{2x^{2}}+\frac{a}{x}\right)}{a^{3}}$
risch	$c^{3}x+\frac{a\,c^{3}x-\frac{1}{2}c^{3}}{a^{3}x^{2}}-\frac{c^{3}\ln\left(x\right)}{a}$
norman	$\frac{c^{3}x+a^{2}c^{3}x^{3}-\frac{c^{3}}{2a}}{a^{2}x^{2}}-\frac{c^{3}\ln\left(x\right)}{a}$
parallelrisch	$-\frac{-2a^{3}c^{3}x^{3}+2c^{3}\ln\left(x\right)a^{2}x^{2}-2a\,c^{3}x+c^{3}}{2a^{3}x^{2}}$
meijerg	$-\frac{c^{3}\left(-ax-\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{3}\ln\left(-ax+1\right)}{a}+\frac{2c^{3}\left(\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)+\frac{1}{ax}\right)}{a}+\frac{c^{3}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)-\right.}{a}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)

[Out] c^3/a^3*(a^3*x-a^2*ln(x)-1/2/x^2+a/x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}\,dx=\frac{2\,a^{3}c^{3}x^{3}-2\,a^{2}c^{3}x^{2}\log\left(x\right)+2\,ac^{3}x-c^{3}}{2\,a^{3}x^{2}}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="fricas")

[Out] 1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}\,dx=\frac{a^{3}c^{3}x-a^{2}c^{3}\log\left(x\right)+\frac{2ac^{3}x-c^{3}}{2x^{2}}}{a^{3}}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**3,x)

[Out] (a**3*c**3*x - a**2*c**3*log(x) + (2*a*c**3*x - c**3)/(2*x**2))/a**3

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = c^3x - \frac{c^3\log(x)}{a} + \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="maxima")
[Out] c^3*x - c^3*log(x)/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = c^3x - \frac{c^3\log(|x|)}{a} + \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="giac")
[Out] c^3*x - c^3*log(abs(x))/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = \frac{c^3\,\left(2\,a\,x+2\,a^3\,x^3-2\,a^2\,x^2\,\ln(x)-1\right)}{2\,a^3\,x^2}$$

```
[In] int(((c - c/(a*x))^3*(a*x + 1))/(a*x - 1),x)
[Out] (c^3*(2*a*x + 2*a^3*x^3 - 2*a^2*x^2*log(x) - 1))/(2*a^3*x^2)
```

3.390 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

```
[Out] c^2/a^2/x+c^2*x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6302, 6266, 6264, 74, 14}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] c^2/(a^2*x) + c^2*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

```
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx\\
&= -\frac{c^2\int\frac{e^{2\text{arctanh}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\frac{(1-ax)(1+ax)}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\frac{1-a^2x^2}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\left(-a^2+\frac{1}{x^2}\right)\,dx}{a^2}\\
&= \frac{c^2}{a^2x}+c^2x
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
[Out] c^2/(a^2*x) + c^2*x
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	s
default	$\frac{c^2\left(a^2x+\frac{1}{x}\right)}{a^2}$	1
risch	$\frac{c^2}{a^2x}+c^2x$	1
gosper	$\frac{c^2\left(a^2x^2+1\right)}{x\,a^2}$	2
parallelrisch	$\frac{a^2c^2x^2+c^2}{a^2x}$	2
norman	$\frac{\frac{c^2}{a}+a\,c^2x^2}{ax}$	2
meijerg	$-\frac{c^2(-ax-\ln(-ax+1))}{a}-\frac{c^2\ln(-ax+1)}{a}+\frac{c^2(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{c^2\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}$	9

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
[Out] c^2/a^2*(a^2*x+1/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2+c^2}{a^2x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="fricas")
[Out] (a^2*c^2*x^2 + c^2)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x+\frac{c^2}{x}}{a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**2,x)
[Out] (a**2*c**2*x + c**2/x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{c^2}{a^2x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="maxima")
[Out] c^2*x + c^2/(a^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{c^2}{a^2x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="giac")
[Out] c^2*x + c^2/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(a^2\,x^2+1\right)}{a^2\,x}$$

```
[In] int(((c - c/(a*x))^2*(a*x + 1))/(a*x - 1),x)
[Out] (c^2*(a^2*x^2 + 1))/(a^2*x)
```

3.391 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 11

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}$$

```
[Out] c*x+c*ln(x)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6266, 6264, 45}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\log(x)}{a}+cx$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x + (c*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)\,dx\\
&= \frac{c\int \frac{e^{2\text{arctanh}(ax)}(1-ax)}{x}\,dx}{a}\\
&= \frac{c\int \frac{1+ax}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\,dx}{a}\\
&= cx+\frac{c\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = cx+\frac{c\log(x)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] c*x + (c*Log[x])/a
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{c(ax+\ln(x))}{a}$	12
norman	$cx+\frac{c\ln(x)}{a}$	12
risch	$cx+\frac{c\ln(x)}{a}$	12
parallelrisch	$\frac{acx+c\ln(x)}{a}$	14
meijerg	$-\frac{c(-ax-\ln(-ax+1))}{a}+\frac{c(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}$	43

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] c/a*(a*x+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+c\log(x)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="fricas")
[Out] (a*c*x + c*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+c\log(x)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
[Out] (a*c*x + c*log(x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = cx + \frac{c\log(x)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="maxima")
[Out] c*x + c*log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = cx + \frac{c\log(|x|)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="giac")
[Out] c*x + c*log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c(\ln(x)+a\,x)}{a}$$

```
[In] int(((c - c/(a*x))*(a*x + 1))/(a*x - 1),x)
[Out] (c*(log(x) + a*x))/a
```

3.392 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}$$

```
[Out] x/c+2/a/c/(-a*x+1)+3*ln(-a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}+\frac{x}{c}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c + 2/(a*c*(1 - a*x)) + (3*Log[1 - a*x])/(a*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{c-\frac{c}{ax}}\,dx\\
&= \frac{a\int \frac{e^{2\text{arctanh}(ax)}x}{1-ax}\,dx}{c}\\
&= \frac{a\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{c}\\
&= \frac{a\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{c}\\
&= \frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] (a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\left(\frac{x}{a}-\frac{2}{a^{2}(ax-1)}+\frac{3\ln(ax-1)}{a^{2}}\right)}{c}$	35
risch	$\frac{x}{c}-\frac{2}{ac(ax-1)}+\frac{3\ln(ax-1)}{ac}$	36
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{3x}{c}}{ax-1}+\frac{3\ln(ax-1)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}+3a\ln(ax-1)x-3ax-3\ln(ax-1)}{c(ax-1)a}$	45

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] a/c*(x/a-2/a^2/(a*x-1)+3/a^2*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a^{2}x^{2}-ax+3\,(ax-1)\log{(ax-1)}-2}{a^{2}cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{2}{a^{2}cx-ac}+\frac{x}{c}+\frac{3\log{(ax-1)}}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
[Out] -2/(a**2*c*x - a*c) + x/c + 3*log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c} - \frac{2}{a^2cx - ac} + \frac{3\log(ax-1)}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="maxima")
[Out] x/c - 2/(a^2*c*x - a*c) + 3*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c} + \frac{3\log(|ax-1|)}{ac} - \frac{2}{(ax-1)ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="giac")
[Out] x/c + 3*log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c} + \frac{2}{a\,(c-a\,c\,x)} + \frac{3\,\ln(a\,x-1)}{a\,c}$$

```
[In] int((a*x + 1)/((c - c/(a*x))*(a*x - 1)),x)
[Out] x/c + 2/(a*(c - a*c*x)) + (3*log(a*x - 1))/(a*c)
```

3.393 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{x}{c^2} - \frac{1}{ac^2(1-ax)^2} + \frac{5}{ac^2(1-ax)} + \frac{4\log(1-ax)}{ac^2}$$

```
[Out] x/c^2-1/a/c^2/(-a*x+1)^2+5/a/c^2/(-a*x+1)+4*ln(-a*x+1)/a/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{5}{ac^2(1-ax)} - \frac{1}{ac^2(1-ax)^2} + \frac{4\log(1-ax)}{ac^2} + \frac{x}{c^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 - 1/(a*c^2*(1 - a*x)^2) + 5/(a*c^2*(1 - a*x)) + (4*Log[1 - a*x])/(a*c
^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx \\
&= -\frac{a^2\int \frac{e^{2\operatorname{arctanh}(ax)}x^2}{(1-ax)^2}\,dx}{c^2} \\
&= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c^2} \\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c^2} \\
&= \frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{a^2\left(-\frac{x}{a^2}+\frac{1}{a^3(1-ax)^2}-\frac{5}{a^3(1-ax)}-\frac{4\log(1-ax)}{a^3}\right)}{c^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
[Out] -((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*x
])/a^3))/c^2)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x}{c^2}+\frac{-5c^2x+\frac{4c^2}{a}}{c^4(ax-1)^2}+\frac{4\ln(ax-1)}{a\,c^2}$	47
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{1}{a^3(ax-1)^2}-\frac{5}{a^3(ax-1)}+\frac{4\ln(ax-1)}{a^3}\right)}{c^2}$	49
norman	$\frac{\frac{a^2x^3}{c}-\frac{6a\,x^2}{c}+\frac{4x}{c}}{c(ax-1)^2}+\frac{4\ln(ax-1)}{a\,c^2}$	53
parallelrisch	$\frac{a^3x^3+4a^2\ln(ax-1)x^2-6a^2x^2-8a\ln(ax-1)x+4ax+4\ln(ax-1)}{c^2(ax-1)^2a}$	67

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
[Out] x/c^2+(-5*c^2*x+4*c^2/a)/c^4/(a*x-1)^2+4/a/c^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log(ax-1)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-5ax+4}{a^3c^2x^2-2a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\log(ax-1)}{ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**2,x)
```

```
[Out] (-5*a*x + 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) + x/c**2 + 4*log(a*x
 - 1)/(a*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\,\log(ax-1)}{ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 + 4*log(a*x - 1)/(
a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}+\frac{4\,\log(|ax-1|)}{ac^2}-\frac{5\,ax-4}{(ax-1)^2ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{5\,x-\frac{4}{a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

```
[In] int((a*x + 1)/((c - c/(a*x))^2*(a*x - 1)),x)
[Out] x/c^2 - (5*x - 4/a)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (4*log(a*x - 1))/(a*c
^2)
```

3.394 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}$$

```
[Out] x/c^3+2/3/a/c^3/(-a*x+1)^3-7/2/a/c^3/(-a*x+1)^2+9/a/c^3/(-a*x+1)+5*ln(-a*x+
1)/a/c^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{9}{ac^3(1-ax)}-\frac{7}{2ac^3(1-ax)^2}+\frac{2}{3ac^3(1-ax)^3}+\frac{5\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 + 2/(3*a*c^3*(1 - a*x)^3) - 7/(2*a*c^3*(1 - a*x)^2) + 9/(a*c^3*(1 - a
*x)) + (5*Log[1 - a*x])/(a*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx \\
&= \frac{a^3 \int \frac{e^{2\text{arctanh}(ax)}x^3}{(1-ax)^3}\,dx}{c^3} \\
&= \frac{a^3 \int \frac{x^3(1+ax)}{(1-ax)^4}\,dx}{c^3} \\
&= \frac{a^3 \int \left(\frac{1}{a^3}+\frac{2}{a^3(-1+ax)^4}+\frac{7}{a^3(-1+ax)^3}+\frac{9}{a^3(-1+ax)^2}+\frac{5}{a^3(-1+ax)}\right)dx}{c^3} \\
&= \frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{-37+81ax-36a^2x^2-18a^3x^3+6a^4x^4+30(-1+ax)^3\log(1-ax)}{6ac^3(-1+ax)^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (-37 + 81*a*x - 36*a^2*x^2 - 18*a^3*x^3 + 6*a^4*x^4 + 30*(-1 + a*x)^3*Log[1
 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x}{c^3}+\frac{-9a\,c^3x^2+\frac{29c^3x}{2}-\frac{37c^3}{6a}}{c^6(ax-1)^3}+\frac{5\ln(ax-1)}{a\,c^3}$	56
default	$\frac{a^3\left(\frac{x}{a^3}-\frac{7}{2a^4(ax-1)^2}-\frac{9}{a^4(ax-1)}-\frac{2}{3a^4(ax-1)^3}+\frac{5\ln(ax-1)}{a^4}\right)}{c^3}$	61
norman	$\frac{\frac{a^3x^4}{c}-\frac{5x}{c}+\frac{25a\,x^2}{2c}-\frac{55a^2x^3}{6c}}{(ax-1)^3c^2}+\frac{5\ln(ax-1)}{a\,c^3}$	64
parallelrisch	$\frac{6a^4x^4+30a^3\ln(ax-1)x^3-55a^3x^3-90a^2\ln(ax-1)x^2+75a^2x^2+90a\ln(ax-1)x-30ax-30\ln(ax-1)}{6c^3(ax-1)^3a}$	91

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^3+(-9*a*c^3*x^2+29/2*c^3*x-37/6*c^3/a)/c^6/(a*x-1)^3+5/a/c^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{6\,a^4x^4-18\,a^3x^3-36\,a^2x^2+81\,ax+30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log(ax-1)-37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
 - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-54a^2x^2+87ax-37}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}+\frac{x}{c^3}+\frac{5\log(ax-1)}{ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] (-54*a**2*x**2 + 87*a*x - 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a*
*2*c**3*x - 6*a*c**3) + x/c**3 + 5*log(a*x - 1)/(a*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{54\,a^2x^2-87\,ax+37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}+\frac{x}{c^3}+\frac{5\,\log(ax-1)}{ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
- a*c^3) + x/c^3 + 5*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{5\,\log(|ax-1|)}{ac^3}-\frac{54\,a^2x^2-87\,ax+37}{6\,(ax-1)^3ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 + 5*log(abs(a*x - 1))/(a*c^3) - 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
- 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{9\,a\,x^2-\frac{29\,x}{2}+\frac{37}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}+\frac{x}{c^3}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c^3}$$

```
[In] int((a*x + 1)/((c - c/(a*x))^3*(a*x - 1)),x)
[Out] (9*a*x^2 - (29*x)/2 + 37/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^
3*x) + x/c^3 + (5*log(a*x - 1))/(a*c^3)
```

3.395 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4} - \frac{1}{2ac^4(1-ax)^4} + \frac{3}{ac^4(1-ax)^3} - \frac{8}{ac^4(1-ax)^2} + \frac{14}{ac^4(1-ax)} + \frac{6\log(1-ax)}{ac^4}$$

```
[Out] x/c^4-1/2/a/c^4/(-a*x+1)^4+3/a/c^4/(-a*x+1)^3-8/a/c^4/(-a*x+1)^2+14/a/c^4/(
-a*x+1)+6*ln(-a*x+1)/a/c^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{14}{ac^4(1-ax)} - \frac{8}{ac^4(1-ax)^2} + \frac{3}{ac^4(1-ax)^3} - \frac{1}{2ac^4(1-ax)^4} + \frac{6\log(1-ax)}{ac^4} + \frac{x}{c^4}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 - 1/(2*a*c^4*(1 - a*x)^4) + 3/(a*c^4*(1 - a*x)^3) - 8/(a*c^4*(1 - a*x
)^2) + 14/(a*c^4*(1 - a*x)) + (6*Log[1 - a*x])/(a*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx \\
&= -\frac{a^4\int \frac{e^{2\operatorname{arctanh}(ax)}x^4}{(1-ax)^4}\,dx}{c^4} \\
&= -\frac{a^4\int \frac{x^4(1+ax)}{(1-ax)^5}\,dx}{c^4} \\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{2}{a^4(-1+ax)^5}-\frac{9}{a^4(-1+ax)^4}-\frac{16}{a^4(-1+ax)^3}-\frac{14}{a^4(-1+ax)^2}-\frac{6}{a^4(-1+ax)}\right)dx}{c^4} \\
&= \frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3}-\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{17-56ax+60a^2x^2-16a^3x^3-8a^4x^4+2a^5x^5+12(-1+ax)^4\log(1-ax)}{2ac^4(-1+ax)^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (17 - 56*a*x + 60*a^2*x^2 - 16*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 12*(-1 + a
*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x}{c^4}+\frac{-14a^2c^4x^3+34a\,c^4x^2-29c^4x+\frac{17c^4}{2a}}{c^8(ax-1)^4}+\frac{6\ln(ax-1)}{a\,c^4}$
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{8}{a^5(ax-1)^2}-\frac{1}{2a^5(ax-1)^4}-\frac{14}{a^5(ax-1)}-\frac{3}{a^5(ax-1)^3}+\frac{6\ln(ax-1)}{a^5}\right)}{c^4}$
norman	$\frac{\frac{a^4x^5}{c}+\frac{6x}{c}-\frac{21a\,x^2}{c}+\frac{26a^2x^3}{c}-\frac{25a^3x^4}{2c}}{(ax-1)^4c^3}+\frac{6\ln(ax-1)}{a\,c^4}$
parallelrisch	$\frac{2a^5x^5+12\ln(ax-1)x^4a^4-25a^4x^4-48a^3\ln(ax-1)x^3+52a^3x^3+72a^2\ln(ax-1)x^2-42a^2x^2-48a\ln(ax-1)x+12ax+12\ln(ax-}{2c^4(ax-1)^4a}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^4+(-14*a^2*c^4*x^3+34*a*c^4*x^2-29*c^4*x+17/2*c^4/a)/c^8/(a*x-1)^4+6/a/
c^4*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{2\,a^5x^5-8\,a^4x^4-16\,a^3x^3+60\,a^2x^2-56\,ax+12\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log(ax-1)+17}{2\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^4
 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4*a
^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-28a^3x^3+68a^2x^2-58ax+17}{2a^5c^4x^4-8a^4c^4x^3+12a^3c^4x^2-8a^2c^4x+2ac^4}+\frac{x}{c^4}+\frac{6\log\left(ax-1\right)}{ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] (-28*a**3*x**3 + 68*a**2*x**2 - 58*a*x + 17)/(2*a**5*c**4*x**4 - 8*a**4*c**
4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) + x/c**4 + 6*log(a*x
 - 1)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}+\frac{x}{c^4}+\frac{6\,\log\left(ax-1\right)}{ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) + x/c^4 + 6*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}+\frac{6\,\log\left(|ax-1|\right)}{ac^4}-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,(ax-1)^4ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 + 6*log(abs(a*x - 1))/(a*c^4) - 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x
 - 17)/((a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}-\frac{29\,x-34\,a\,x^2-\frac{17}{2\,a}+14\,a^2\,x^3}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^4}$$

```
[In] int((a*x + 1)/((c - c/(a*x))^4*(a*x - 1)),x)
```

```
[Out] x/c^4 - (29*x - 34*a*x^2 - 17/(2*a) + 14*a^2*x^3)/(c^4 + 6*a^2*c^4*x^2 - 4*
a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c^4*x) + (6*log(a*x - 1))/(a*c^4)
```

3.396 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\frac{c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 1/3*c^4*(1-1/a^2/x^2)^(3/2)*(3*a+1/x)*x/a+3/2*c^4*arccsc(a*x)/a-c^4*arctanh
((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(2*a+3/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 827, 829, 858, 222, 272, 65, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/(2*a^2) + (c^4*(1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/(3*a) + (3*c^4*ArcCsc[a*x])/(2*a) - (c^4*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{1}{2}c^3\text{Subst}\left(\int \frac{\left(\frac{2c}{a}+\frac{6cx}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} - \frac{1}{4}\left(a^2c^3\right)\text{Subst}\left(\int \frac{-\frac{4c}{a^3}-\frac{6cx}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{\left(3c^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a^2} + \frac{c^4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} + \frac{c^4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} - \left(ac^4\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=$$
$$-\frac{c^4\left(-8+12ax+40a^2x^2+12a^3x^3-32a^4x^4-24a^5x^5+42a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-15a^4\sqrt{1}\right.}{24a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -1/24*(c^4*(-8 + 12*a*x + 40*a^2*x^2 + 12*a^3*x^3 - 32*a^4*x^4 - 24*a^5*x^5
 + 42*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 15*
a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] + 24*a^4*Sqrt[1 - 1/(a^2*x^2)
]*x^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

method	result
risch	$\frac{(ax-1)(8a^2x^2+3ax-2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{(ax-1)(ax+1)}}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^4\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-9a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\frac{a^2x+\sqrt{}}{}\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x-1)*(8*a^2*x^2+3*a*x-2)/x^3*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)+(-a^4*l
n(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+3/2*a^3*arctan(1/(a^2*x^
2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x+1)/((a*x-1)/(a*x+1))^
(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{18\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^4x^4+14\,a^3c^4x^3+\cdots}{6\,a^4x^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^4*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^4*x^4 + 14*a^3*c^4*x^3 + 11*a^2*c^4*x^2 + a*c^4*x - 2*c^4
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(\int\left(-\frac{4a}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\cdots}}{\cdots}}\right)\cdots\right)}{a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(-4*a/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2/(a*x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**3/(a*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(a**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1)), x))/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.17

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = -\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-}{(ax}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3*c
^4*((a*x - 1)/(a*x + 1))^(7/2) + c^4*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^4*(
(a*x - 1)/(a*x + 1))^(3/2) + 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)
*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^
4 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(91) = 182$.

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = -\frac{3\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)}+\frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^4}{a\text{sgn}\left(ax+1\right)} - \frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4-12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-3\left(x|a|-\sqrt{a^2\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\text{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^4/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) -
 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 - 12*(x*abs(a) - sqrt(a^2*x^2 -
1))^2*a*c^4 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) - 8*a*c^4)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^4*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^4*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.397 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{2a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x+3/2*c^3*arccsc(a*x)/a+3/2*c^3*(1-1/a^2/x^2)^(1/2)
/a^2/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6312, 283, 201, 222}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{3c^3\csc^{-1}(ax)}{2a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (3*c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*(1 - 1/(a^2*x^2))^(3/2)*x + (
3*c^3*ArcCsc[a*x])/(2*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{(3c^3)\,\text{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{(3c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(1+2a^2x^2)+3ax\arcsin\left(\frac{1}{ax}\right)\right)}{2a^2x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a^2*x^2) + 3*a*x*ArcSin[1/(a*x)]))/(2*a^
2*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{(ax-1)^2c^3\left(-3\sqrt{a^2x^2-1}\,a^2x^2-3a^2x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+(a^2x^2-1)^{\frac{3}{2}}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^3x^2}$	105
risch	$\frac{(ax-1)c^3}{2x^2a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^2\sqrt{(ax-1)(ax+1)}+\frac{3a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)c^3\sqrt{(ax-1)(ax+1)}}{a^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	110

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(a*x-1)^2*c^3*(-3*(a^2*x^2-1)^(1/2)*a^2*x^2-3*a^2*x^2*arctan(1/(a^2*x^
2-1)^(1/2))+(a^2*x^2-1)^(3/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*
x+1))^(1/2)/a^3/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=-\frac{6\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(2\,a^3c^3x^3+2\,a^2c^3x^2+ac^3x+c^3)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
[Out] -1/2*(6*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^3*c^3*x^3 + 2*
a^2*c^3*x^2 + a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{c^3\left(\int\frac{3a}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}}{a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(3*a/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2/(a*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(a**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**3*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.48

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=-\left(\frac{3\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(3*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - (3*c^3*((a*x - 1)/(a*x + 1)
)^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((a*x - 1)/(a*x + 1
)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^
2/(a*x + 1)^3 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{3\,a^4c^3\arctan\left(\sqrt{a^2x^2-1}\right)-2\,\sqrt{a^2x^2-1}a^4c^3-\frac{\sqrt{a^2x^2-1}a^2c^3}{x^2}}{2\,a^5\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*a^4*c^3*arctan(sqrt(a^2*x^2 - 1)) - 2*sqrt(a^2*x^2 - 1)*a^4*c^3 - s
qrt(a^2*x^2 - 1)*a^2*c^3/x^2)/(a^5*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+c^3\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,x}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^3\,x^2}$$

```
[In] int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/a + c^3*x*((a*x - 1)/(a*x + 1))^(1/2) + (c^3*((a*x - 1)/(a*x + 1))^(1/
2))/(2*a^2*x) + (c^3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a^3*x^2)
```

3.398 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a-1/x)*x*(1-1/a^2
/x^2)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 864, 827, 858, 222, 272, 65, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
```

```
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(c^3\text{Subst}\left(\int \frac{\left(\frac{1}{c}+\frac{x}{ac}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{1}{2}c^3\text{Subst}\left(\int \frac{-\frac{2}{ac}+\frac{2x}{a^2c}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(63) = 126.

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.44

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx =$$
$$-\frac{c^2\left(-1+ax+a^2x^2-a^3x^3+4a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{1}{ax}\right)-a^2\sqrt{1-\right.}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(-1 + a*x + a^2*x^2 - a^3*x^3 + 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcS
in[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/(a*x
)] - a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.08

method	result
risch	$-\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{(ax-1)(ax+1)}}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x-1)/x*c^2/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(a*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/
2)))*c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.81

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) + a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\left(\int\left(-\frac{2a}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(-2*a/(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2/(a*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1)), x) + Integral(1/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(59) = 118.

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(59) = 118.

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.19

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = &-\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}\\&-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\left(ax+1\right)}\\&-\frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}\end{aligned}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^2/(a*sgn(a*x + 1)) - 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{4\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

3.399 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{c\csc^{-1}(ax)}{a}+\frac{2c\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -c*arccsc(a*x)/a+2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6312, 866, 1821, 858, 222, 272, 65, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2c\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{c\csc^{-1}(ax)}{a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a + (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{-\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{(2c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{c\csc^{-1}(ax)}{a} - \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{c\csc^{-1}(ax)}{a} + (2ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{c\csc^{-1}(ax)}{a} + \frac{2c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x - 2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right) - 2\arcsin\left(\frac{1}{ax}\right) + 2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(45) = 90.

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.96

method	result	size
default	$-\frac{(ax-1)^2 c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-2a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)-2\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	145

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^
2)^(1/2)-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-2*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-
1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{c\left(\int\frac{a}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**2*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1)), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$$
$$= -2a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{2c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\operatorname{sgn}(ax+1)} - \frac{2c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)} + \frac{\sqrt{a^2x^2-1}c}{a\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 2*c*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/(
a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

```
[In] int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (4*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1
))
```

3.400 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c-4
/3*(3*a+4/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac} - \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (-8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) - (4*(3*a + 4/x))/(3*a^
2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c + (4*ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^4-\frac{12c^4x}{a}-\frac{13c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^4+\frac{12c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(4a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(19-26ax+3a^2x^2)}{(-1+ax)^2}+12\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{3ac}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 26*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + 12*Lo
g[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(3*a*c)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.74

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^4\left(x-\frac{1}{a}\right)^2}-\frac{20\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^3\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax-1)(ax+1)}}{c(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{12\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+12\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,a^3x^3-36\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-9\sqrt{a^2}\,((ax-1)(ax+1}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(4/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)
^(1/2))/(a^2)^(1/2)-4/3/a^4/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-20/
3/a^3/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a/c/(a*x+1)/((a*x-1)/(a*x+
1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$=\frac{12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-23\,a^2x^2-7\,a}{3\,(a^3cx^2-2\,a^2cx+ac)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] 1/3*(12*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*(a^2*
x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 23*a^2*x
^2 - 7*a*x + 19)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\int \frac{x}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
[Out] a*Integral(x/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2}{3}\,a\left(\frac{\frac{8\,(ax-1)}{ax+1}-\frac{12\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
[Out] 2/3*a*((8*(a*x - 1)/(a*x + 1) - 12*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a*
x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 6*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^
2*c))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{4\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
[Out] -4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1)) + sqrt(a
^2*x^2 - 1)/(a*c*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c} - \frac{\frac{16\,(a\,x-1)}{3\,(a\,x+1)} - \frac{8\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((16*(a*x - 1))/(3*(a*x + 1)
) - (8*(a*x - 1)^2)/(a*x + 1)^2 + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) - a
*c*((a*x - 1)/(a*x + 1))^(5/2))
```

3.401 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}+\frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -16/5*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+11/x)/a^2/c^2/(1-1/a^2/
x^2)^(3/2)+5*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+1/15*(-75*a-103/x)/a^2/c^2/
(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-16*(a + x^(-1)))/(5*a^2*c^2*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 11/x))/(
15*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (75*a + 103/x)/(15*a^2*c^2*Sqrt[1 - 1
/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]])/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^5-\frac{25c^5x}{a}-\frac{39c^5x^2}{a^2}+\frac{5c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^5+\frac{75c^5x}{a}+\frac{88c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^5-\frac{75c^5x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(5a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-118+161ax+91a^2x^2-173a^3x^3+15a^4x^4+75a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-118 + 161*a*x + 91*a^2*x^2 - 173*a^3*x^3 + 15*a^4*x^4 + 75*a*Sqrt[1 - 1/(
a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^2*Sqrt[1
 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{5a^{6}\left(x-\frac{1}{a}\right)^{3}}-\frac{52\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{15a^{5}\left(x-\frac{1}{a}\right)^{2}}-\frac{143\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{15a^{4}\left(x-\frac{1}{a}\right)}\right)a^{2}\sqrt{(ax-}{c^{2}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-75\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{4}x^{4}-75\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{5}x^{4}+60\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}a^{2}x^{2}+300\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)

[Out] 1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(5/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/5/a^6/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-52/15/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-143/15/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$=\frac{75\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-75\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^{4}x}{15\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")

[Out] 1/15*(75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (15*a^4*x^4 - 173*a^3*x^3 + 91*a^2*x^2 + 161*a*x - 118)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2 \int \frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1} - \frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1} + \frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1} - \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1)), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{1}{15}a\left(\frac{\frac{17(ax-1)}{ax+1}+\frac{100(ax-1)^2}{(ax+1)^2}-\frac{150(ax-1)^3}{(ax+1)^3}+3}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a*((17*(a*x - 1)/(a*x + 1) + 100*(a*x - 1)^2/(a*x + 1)^2 - 150*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x
- 1)/(a*x + 1))^(5/2)) + 75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) -
75*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{5\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^2|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac^2\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -5*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.87

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{20\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{10\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{17\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((20*(a*x - 1)^2)/(3*(a*x
 + 1)^2) - (10*(a*x - 1)^3)/(a*x + 1)^3 + (17*(a*x - 1))/(15*(a*x + 1)) + 1
/5)/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

3.402 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 165

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{6\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -32/7*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(7/2)-2/7*(7*a+13/x)/a^2/c^3/(1-1/a^2/x
^2)^(3/2)-16/7/a^2/c^3/(1-1/a^2/x^2)^(5/2)/x+6*arctanh((1-1/a^2/x^2)^(1/2))
/a/c^3+1/7*(-42*a-59/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{6\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}-\frac{16}{7a^2c^3x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (-32*(a + x^(-1)))/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(7/2)) - (2*(7*a + 13/x))/(
7*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (42*a + 59/x)/(7*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) - 16/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)*x) + (Sqrt[1 - 1/(a^2*x^2
)]*x)/c^3 + (6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^6-\frac{42c^6x}{a}-\frac{80c^6x^2}{a^2}+\frac{42c^6x^3}{a^3}+\frac{7c^6x^4}{a^4}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Subst}\left(\int \frac{35c^6+\frac{210c^6x}{a}+\frac{355c^6x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\text{Subst}\left(\int \frac{-105c^6-\frac{630c^6x}{a}-\frac{780c^6x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{105c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Subst}\left(\int \frac{105c^6+\frac{630c^6x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{105c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3} - \frac{6\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{ac^3}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3} + \frac{(6a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^3}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3} + \frac{6\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx = \frac{66-156ax+39a^2x^2+145a^3x^3-109a^4x^4+7a^5x^5+42a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
[Out] (66 - 156*a*x + 39*a^2*x^2 + 145*a^3*x^3 - 109*a^4*x^4 + 7*a^5*x^5 + 42*a*S
qrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(7*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.61

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{7a^8\left(x-\frac{1}{a}\right)^4}-\frac{20\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{7a^7\left(x-\frac{1}{a}\right)^3}-\frac{45\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{7a^6\left(x-\frac{1}{a}\right)^2}-\frac{88\sqrt{\left(x-\right.}}{7}\right.}{c^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-42\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-42\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+35((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+210\sqrt{a^2}\,\sqrt{(ax-1)(ax+}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(6/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-4/7/a^8/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)
-20/7/a^7/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-45/7/a^6/(x-1/a)^2*((
x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-88/7/a^5/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a
)^(1/2))*a^3/c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\right.}\right.}{7\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*(42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((
a*x - 1)/(a*x + 1)) - 1) + (7*a^5*x^5 - 109*a^4*x^4 + 145*a^3*x^3 + 39*a^2*
x^2 - 156*a*x + 66)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3
 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3 \int \frac{x^3}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
[Out] a**3*Integral(x**3/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{1}{14}a\left(\frac{\frac{6(ax-1)}{ax+1}+\frac{21(ax-1)^2}{(ax+1)^2}+\frac{112(ax-1)^3}{(ax+1)^3}-\frac{168(ax-1)^4}{(ax+1)^4}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{84\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
[Out] 1/14*a*((6*(a*x - 1)/(a*x + 1) + 21*(a*x - 1)^2/(a*x + 1)^2 + 112*(a*x - 1)
^3/(a*x + 1)^3 - 168*(a*x - 1)^4/(a*x + 1)^4 + 1)/(a^2*c^3*((a*x - 1)/(a*x
+ 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 84*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/(a^2*c^3) - 84*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3)
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{3\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{16\,{(a\,x-1)}^3}{{(a\,x+1)}^3}-\frac{24\,{(a\,x-1)}^4}{{(a\,x+1)}^4}+\frac{6\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

```
[In] int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (12*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((3*(a*x - 1)^2)/(a*x + 1
)^2 + (16*(a*x - 1)^3)/(a*x + 1)^3 - (24*(a*x - 1)^4)/(a*x + 1)^4 + (6*(a*x
 - 1))/(7*(a*x + 1)) + 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 2*a*c^3*
((a*x - 1)/(a*x + 1))^(9/2))
```

3.403 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{7\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] 16/63*(9*a-5/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-64/9*(a+1/x)/a^2/c^4/(1-1/a^2/x
^2)^(9/2)-8/105*(21*a+41/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/315*(-735*a-1417/
x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+7*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/315*(
-2205*a-3149/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{6312, 866, 1819, 821, 272, 65, 214\}$

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{7\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4} + \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (16*(9*a - 5/x))/(63*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (64*(a + x^(-1)))/(
9*a^2*c^4*(1 - 1/(a^2*x^2))^(9/2)) - (8*(21*a + 41/x))/(105*a^2*c^4*(1 - 1/
(a^2*x^2))^(5/2)) - (735*a + 1417/x)/(315*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2))
- (2205*a + 3149/x)/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*
x^2)]*x)/c^4 + (7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 272</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

<u>Rule 821</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^3 \text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^7}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^{11}}\\
&= -\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{-9c^7-\frac{63c^7x}{a}-\frac{134c^7x^2}{a^2}+\frac{198c^7x^3}{a^3}+\frac{63c^7x^4}{a^4}+\frac{9c^7x^5}{a^5}}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9c^{11}}\\
&= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{\text{Subst}\left(\int \frac{63c^7+\frac{441c^7x}{a}+\frac{921c^7x^2}{a^2}+\frac{63c^7x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63c^{11}}
\end{aligned}
$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-315c^7-\frac{2205c^7x}{a}-\frac{3936c^7x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{315c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{945c^7+\frac{6615c^7x}{a}+\frac{8502c^7x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{945c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-945c^7-\frac{6615c^7x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{945c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{7\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^4}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{7\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^4}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{(7a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^4}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{7\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-3464+11651ax-10232a^2x^2-5567a^3x^3+13241a^4x^4-6224a^5x^5+315a^6x^6+2205a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^4}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-3464 + 11651*a*x - 10232*a^2*x^2 - 5567*a^3*x^3 + 13241*a^4*x^4 - 6224*a^
5*x^5 + 315*a^6*x^6 + 2205*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{9a^{10}\left(x-\frac{1}{a}\right)^5}-\frac{164\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{63a^9\left(x-\frac{1}{a}\right)^4}-\frac{697\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{105a^8\left(x-\frac{1}{a}\right)^3}-\frac{3226\sqrt{(x-}}{31}\right.}{c^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-2205\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^6x^6-2205\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^7x^6+1890((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+13230\sqrt{(ax-1)(ax}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(7/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-4/9/a^10/(x-1/a)^5*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2
)-164/63/a^9/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-697/105/a^8/(x-1/a
)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-3226/315/a^7/(x-1/a)^2*((x-1/a)^2*a^2
```

```
+2*(x-1/a)*a)^(1/2)-4964/315/a^6/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))
*a^4/c^4/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.18

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{2205\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2205\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-\right.}{315\left(a^6c^4x^5-5\,a^5c^4\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 -
10*a^2*x^2 + 5*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^6*x^6 -
 6224*a^5*x^5 + 13241*a^4*x^4 - 5567*a^3*x^3 - 10232*a^2*x^2 + 11651*a*x -
3464)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*
x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 5*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{1260}\,a\left(\frac{\frac{235\,(ax-1)}{ax+1}+\frac{801\,(ax-1)^2}{(ax+1)^2}+\frac{2289\,(ax-1)^3}{(ax+1)^3}+\frac{11760\,(ax-1)^4}{(ax+1)^4}-\frac{17640\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{8820\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*a*((235*(a*x - 1)/(a*x + 1) + 801*(a*x - 1)^2/(a*x + 1)^2 + 2289*(a*
x - 1)^3/(a*x + 1)^3 + 11760*(a*x - 1)^4/(a*x + 1)^4 - 17640*(a*x - 1)^5/(a
*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)) + 8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 882
0*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{7\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^4|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac^4\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -7*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^4*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^4*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{14\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{89\,(a\,x-1)^2}{35\,(a\,x+1)^2}+\frac{109\,(a\,x-1)^3}{15\,(a\,x+1)^3}+\frac{112\,(a\,x-1)^4}{3\,(a\,x+1)^4}-\frac{56\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{47\,(a\,x-1)}{63\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

```
[In] int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (14*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((89*(a*x - 1)^2)/(35*(a*
x + 1)^2) + (109*(a*x - 1)^3)/(15*(a*x + 1)^3) + (112*(a*x - 1)^4)/(3*(a*x
+ 1)^4) - (56*(a*x - 1)^5)/(a*x + 1)^5 + (47*(a*x - 1))/(63*(a*x + 1)) + 1/
9)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(11
/2))
```

3.404 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/a^3/x^2+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx\\
&= -\frac{c^5\int\frac{e^{4\text{arctanh}(ax)}(1-ax)^5}{x^5}\,dx}{a^5}\\
&= -\frac{c^5\int\frac{(1-ax)^3(1+ax)^2}{x^5}\,dx}{a^5}\\
&= -\frac{c^5\int\left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5}\\
&= \frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5dx=\frac{c^5\left(\frac{1}{4x^4}-\frac{a}{3x^3}-\frac{a^2}{x^2}+\frac{2a^3}{x}+a^5x-a^4\log(x)\right)}{a^5}
$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
[Out] (c^5*(1/(4*x^4) - a/(3*x^3) - a^2/x^2 + (2*a^3)/x + a^5*x - a^4*Log[x]))/a^
5
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^5\left(a^5x-a^4\ln(x)+\frac{1}{4x^4}-\frac{a}{3x^3}-\frac{a^2}{x^2}+\frac{2a^3}{x}\right)}{a^5}$
risch	$c^5x+\frac{2a^3c^5x^3-a^2c^5x^2-\frac{1}{3}a\,c^5x+\frac{1}{4}c^5}{a^5x^4}-\frac{c^5\ln(x)}{a}$
parallelrisch	$-\frac{-12a^5c^5x^5+12c^5\ln(x)a^4x^4-24a^3c^5x^3+12a^2c^5x^2+4a\,c^5x-3c^5}{12a^5x^4}$
norman	$\frac{a^4c^5x^5+a^5c^5x^6-\frac{c^5}{4a}+\frac{7c^5x}{12}+\frac{2a\,c^5x^2}{3}-3c^5a^2x^3}{(ax-1)a^4x^4}-\frac{c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{3c^5\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}+\frac{c^5x}{-ax+1}+\frac{5c^5\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a\right.}{a}$

[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x,method=_RETURNVERBOSE)

[Out] c^5/a^5*(a^5*x-a^4*ln(x)+1/4/x^4-1/3*a/x^3-a^2/x^2+2*a^3/x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx = \frac{12\,a^5c^5x^5-12\,a^4c^5x^4\log(x)+24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="fricas")

[Out] 1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx = \frac{a^5c^5x-a^4c^5\log(x)+\frac{24a^3c^5x^3-12a^2c^5x^2-4ac^5x+3c^5}{12x^4}}{a^5}$$

[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**5,x)

[Out] (a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 - 4*a*c**5*x + 3*c**5)/(12*x**4))/a**5

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = c^5x - \frac{c^5\log(x)}{a} + \frac{24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
3*c^5)/(a^5*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(60) = 120.

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = \frac{c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a} + \frac{\left(12\,c^5 + \frac{37\,c^5}{ax-1} + \frac{52\,c^5}{(ax-1)^2} + \frac{42\,c^5}{(ax-1)^3} + \frac{12\,c^5}{(ax-1)^4}\right)(ax-1)}{12\,a\left(\frac{1}{ax-1}+1\right)^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c^5*log(abs(-1/(a*x - 1) - 1
))/a + 1/12*(12*c^5 + 37*c^5/(a*x - 1) + 52*c^5/(a*x - 1)^2 + 42*c^5/(a*x -
 1)^3 + 12*c^5/(a*x - 1)^4)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\,\left(4\,a\,x + 12\,a^2\,x^2 - 24\,a^3\,x^3 - 12\,a^5\,x^5 + 12\,a^4\,x^4\,\ln(x) - 3\right)}{12\,a^5\,x^4}$$

```
[In] int(((c - c/(a*x))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] -(c^5*(4*a*x + 12*a^2*x^2 - 24*a^3*x^3 - 12*a^5*x^5 + 12*a^4*x^4*log(x) - 3
))/(12*a^5*x^4)
```

3.405 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

```
[Out] -1/3*c^4/a^4/x^3+2*c^4/a^2/x+c^4*x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6302, 6266, 6264, 74, 276}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -1/3*c^4/(a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{4\operatorname{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^4 dx \\
&= \frac{c^4\int \frac{e^{4\operatorname{arctanh}(ax)}(1-ax)^4}{x^4}\,dx}{a^4} \\
&= \frac{c^4\int \frac{(1-ax)^2(1+ax)^2}{x^4}\,dx}{a^4} \\
&= \frac{c^4\int \frac{\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4} \\
&= \frac{c^4\int \left(a^4+\frac{1}{x^4}-\frac{2a^2}{x^2}\right)dx}{a^4} \\
&= -\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\left(-\frac{1}{3x^3}+\frac{2a^2}{x}+a^4x\right)}{a^4}$$

[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]

[Out] (c^4*(-1/3*1/x^3 + (2*a^2)/x + a^4*x))/a^4

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^4\left(a^4x-\frac{1}{3x^3}+\frac{2a^2}{x}\right)}{a^4}$
gosper	$\frac{c^4\left(3a^4x^4+6a^2x^2-1\right)}{3x^3a^4}$
risch	$c^4x+\frac{2a^2c^4x^2-\frac{1}{3}c^4}{a^4x^3}$
parallelrisch	$\frac{3a^4c^4x^4+6a^2c^4x^2-c^4}{3a^4x^3}$
norman	$\frac{a^3c^4x^4+a^4c^4x^5+\frac{c^4}{3a}-\frac{c^4x}{3}-2a\,c^4x^2}{(ax-1)a^3x^3}$
meijerg	$-\frac{c^4\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{2c^4\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{c^4x}{-ax+1}+\frac{4c^4\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a\right.}{a}$

[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x,method=_RETURNVERBOSE)

[Out] c^4/a^4*(a^4*x-1/3/x^3+2*a^2/x)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{3\,a^4c^4x^4+6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="fricas")

[Out] 1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{a^4c^4x+\frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**4,x)
[Out] (a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{(ax-1)c^4}{a}-\frac{5\,c^4+\frac{9\,c^4}{ax-1}+\frac{3\,c^4}{(ax-1)^2}}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="giac")
[Out] (a*x - 1)*c^4/a - 1/3*(5*c^4 + 9*c^4/(a*x - 1) + 3*c^4/(a*x - 1)^2)/(a*(1/(
a*x - 1) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(a^4x^4+2a^2x^2-\frac{1}{3}\right)}{a^4x^3}$$

```
[In] int(((c - c/(a*x))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^4*(2*a^2*x^2 + a^4*x^4 - 1/3))/(a^4*x^3)
```

3.406 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

```
[Out] 1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 76}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx\\
&= -\frac{c^3\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int \frac{(1-ax)(1+ax)^2}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int \left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)\,dx}{a^3}\\
&= \frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(\frac{3a^2}{2}+\frac{1}{2x^2}+\frac{a}{x}+a^3x+a^2\log(x)\right)}{a^3}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
[Out] (c^3*((3*a^2)/2 + 1/(2*x^2) + a/x + a^3*x + a^2*Log[x]))/a^3
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^3\left(a^3x+a^2\ln(x)+\frac{1}{2x^2}+\frac{a}{x}\right)}{a^3}$
risch	$c^3x+\frac{a\,c^3x+\frac{1}{2}c^3}{a^3x^2}+\frac{c^3\ln(x)}{a}$
parallelrisch	$\frac{2a^3c^3x^3+2c^3\ln(x)a^2x^2+2a\,c^3x+c^3}{2a^3x^2}$
norman	$\frac{a^3c^3x^4-\frac{c^3}{2a}-\frac{c^3x}{2}}{(ax-1)a^2x^2}+\frac{c^3\ln(x)}{a}$
meijerg	$-\frac{c^3\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{c^3\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{2c^3x}{-ax+1}+\frac{2c^3\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-\right.}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
[Out] c^3/a^3*(a^3*x+a^2*ln(x)+1/2/x^2+a/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{2\,a^3c^3x^3+2\,a^2c^3x^2\log{(x)}+2\,ac^3x+c^3}{2\,a^3x^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
[Out] 1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{a^3c^3x+a^2c^3\log{(x)}+\frac{2ac^3x+c^3}{2x^2}}{a^3}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**3,x)
[Out] (a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3x+\frac{c^3\log(x)}{a}+\frac{2\,ac^3x+c^3}{2\,a^3x^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
[Out] c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(2\,c^3+\frac{c^3}{ax-1}-\frac{2\,c^3}{(ax-1)^2}\right)(ax-1)}{2\,a\left(\frac{1}{ax-1}+1\right)^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="giac")
[Out] -c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + c^3*log(abs(-1/(a*x - 1) -
1))/a + 1/2*(2*c^3 + c^3/(a*x - 1) - 2*c^3/(a*x - 1)^2)*(a*x - 1)/(a*(1/(a*
x - 1) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(a\,x+a^3\,x^3+a^2\,x^2\,\ln(x)+\frac{1}{2}\right)}{a^3\,x^2}$$

```
[In] int(((c - c/(a*x))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^3*(a*x + a^3*x^3 + a^2*x^2*log(x) + 1/2))/(a^3*x^2)
```

3.407 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

```
[Out] -c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 45}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx\\
&= \frac{c^2\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \frac{(1+ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c^2\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)\,dx}{a^2}\\
&= -\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(-\frac{1}{x}+a^2x+2a\log(x)\right)}{a^2}
$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
[Out] (c^2*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{c^{2}\left(a^{2}x+2a\ln\left(x\right)-\frac{1}{x}\right)}{a^{2}}$	24
risch	$-\frac{c^{2}}{a^{2}x}+c^{2}x+\frac{2c^{2}\ln\left(x\right)}{a}$	28
parallelrisch	$\frac{a^{2}c^{2}x^{2}+2c^{2}\ln\left(x\right)ax-c^{2}}{a^{2}x}$	33
norman	$\frac{\frac{c^{2}}{a}-2a\,c^{2}x^{2}+a^{2}c^{2}x^{3}}{\left(ax-1\right)ax}+\frac{2c^{2}\ln\left(x\right)}{a}$	53
meijerg	$-\frac{c^{2}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{2}x}{-ax+1}-\frac{c^{2}\left(-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)-1-2\ln\left(x\right)-2\ln\left(-a\right)+\frac{1}{ax}\right)}{a}$	100

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
[Out] c^2/a^2*(a^2*x+2*a*ln(x)-1/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2+2\,ac^2x\log\left(x\right)-c^2}{a^2x}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
[Out] (a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x+2ac^2\log\left(x\right)-\frac{c^2}{x}}{a^2}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**2,x)
[Out] (a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{2\,c^2\log\left(x\right)}{a}-\frac{c^2}{a^2x}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{2\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{c^2+\frac{2\,c^2}{ax-1}}{a^2\left(\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x - 1
) - 1))/a + (c^2 + 2*c^2/(a*x - 1))/(a^2*(1/((a*x - 1)*a) + 1/((a*x - 1)^2*
a)))
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(a^2\,x^2+2\,a\,x\,\ln\left(x\right)-1\right)}{a^2\,x}$$

```
[In] int(((c - c/(a*x))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^2*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

3.408 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

```
[Out] c*x-c*ln(x)/a+4*c*ln(-a*x+1)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6266, 6264, 84}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)\,dx\\
&= -\frac{c\int \frac{e^{4\text{arctanh}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int \frac{(1+ax)^2}{x(1-ax)}\,dx}{a}\\
&= -\frac{c\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)\,dx}{a}\\
&= cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c(ax-\log(x)+4\log(1-ax))}{a}
$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] (c*(a*x - Log[x] + 4*Log[1 - a*x]))/a
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result
default	$\frac{c(ax-\ln(x)+4\ln(ax-1))}{a}$
parallelrisch	$-\frac{-acx+c\ln(x)-4c\ln(ax-1)}{a}$
risch	$cx-\frac{c\ln(x)}{a}+\frac{4c\ln(-ax+1)}{a}$
norman	$\frac{ac\,x^2-cx}{ax-1}-\frac{c\ln(x)}{a}+\frac{4c\ln(ax-1)}{a}$
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{cx}{-ax+1}-\frac{c\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] c/a*(a*x-ln(x)+4*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+4\,c\log\left(ax-1\right)-c\log\left(x\right)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="fricas")
[Out] (a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\left(-\log\left(x\right)+4\log\left(x-\frac{1}{a}\right)\right)}{a}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x),x)
[Out] c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{4\,c\log\left(ax-1\right)}{a}-\frac{c\log\left(x\right)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="maxima")
[Out] c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{(ax-1)c}{a}-\frac{3\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="giac")
[Out] (a*x - 1)*c/a - 3*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c*log(abs(-1
/(a*x - 1) - 1))/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\,x-\frac{c\,\ln\left(x\right)}{a}+\frac{4\,c\,\ln\left(a\,x-1\right)}{a}$$

```
[In] int(((c - c/(a*x))*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c*x - (c*log(x))/a + (4*c*log(a*x - 1))/a
```

3.409 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}$$

```
[Out] x/c-2/a/c/(-a*x+1)^2+8/a/c/(-a*x+1)+5*ln(-a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 78}

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

<u>Rule 78</u>

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{c-\frac{c}{ax}}\,dx \\
&= -\frac{a\int \frac{e^{4\text{arctanh}(ax)}x}{1-ax}\,dx}{c} \\
&= -\frac{a\int \frac{x(1+ax)^2}{(1-ax)^3}\,dx}{c} \\
&= -\frac{a\int \left(-\frac{1}{a}-\frac{4}{a(-1+ax)^3}-\frac{8}{a(-1+ax)^2}-\frac{5}{a(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\left(-\frac{x}{a}+\frac{2}{a^2(1-ax)^2}-\frac{8}{a^2(1-ax)}-\frac{5\log(1-ax)}{a^2}\right)}{c}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] -((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/a
^2))/c)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{c}+\frac{-8cx+\frac{6c}{a}}{c^{2}(ax-1)^{2}}+\frac{5\ln(ax-1)}{ac}$	43
default	$\frac{a\left(\frac{x}{a}-\frac{2}{a^{2}(ax-1)^{2}}-\frac{8}{a^{2}(ax-1)}+\frac{5\ln(ax-1)}{a^{2}}\right)}{c}$	47
norman	$\frac{\frac{a^{2}x^{3}}{c}-\frac{8a\,x^{2}}{c}+\frac{5x}{c}}{(ax-1)^{2}}+\frac{5\ln(ax-1)}{ac}$	50
parallelrisch	$\frac{a^{3}x^{3}+5a^{2}\ln(ax-1)x^{2}-8a^{2}x^{2}-10a\ln(ax-1)x+5ax+5\ln(ax-1)}{(ax-1)^{2}ca}$	67

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] x/c+(-8*c*x+6*c/a)/c^2/(a*x-1)^2+5/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a^{3}x^{3}-2\,a^{2}x^{2}-7\,ax+5\,(a^{2}x^{2}-2\,ax+1)\log(ax-1)+6}{a^{3}cx^{2}-2\,a^{2}cx+ac}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="fricas")
[Out] (a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{-8ax+6}{a^{3}cx^{2}-2a^{2}cx+ac}+\frac{x}{c}+\frac{5\log(ax-1)}{ac}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x),x)
[Out] (-8*a*x + 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log\left(ax-1\right)}{ac}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{ax-1}{ac}-\frac{5\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{2\left(\frac{4\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}\right)}{a^4c^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - 2*(4*a^3
*c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}-\frac{8\,x-\frac{6}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c}$$

```
[In] int((a*x + 1)^2/((c - c/(a*x))*(a*x - 1)^2),x)
```

```
[Out] x/c - (8*x - 6/a)/(c + a^2*c*x^2 - 2*a*c*x) + (5*log(a*x - 1))/(a*c)
```

3.410 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

```
[Out] x/c^2+4/3/a/c^2/(-a*x+1)^3-6/a/c^2/(-a*x+1)^2+13/a/c^2/(-a*x+1)+6*ln(-a*x+1
)/a/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int \frac{e^{4\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{a^2 \int \frac{e^{4\text{arctanh}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2 \int \frac{x^2(1+ax)^2}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{a^2 \int \left(\frac{1}{a^2}+\frac{4}{a^2(-1+ax)^4}+\frac{12}{a^2(-1+ax)^3}+\frac{13}{a^2(-1+ax)^2}+\frac{6}{a^2(-1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-25+57ax-30a^2x^2-9a^3x^3+3a^4x^4+18(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
[Out] (-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-13a\,c^{2}x^{2}+20c^{2}x-\frac{25c^{2}}{3a}}{c^{4}\left(ax-1\right)^{3}}+\frac{6\ln\left(ax-1\right)}{a\,c^{2}}$	56
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{6}{a^{3}\left(ax-1\right)^{2}}-\frac{13}{a^{3}\left(ax-1\right)}-\frac{4}{3a^{3}\left(ax-1\right)^{3}}+\frac{6\ln\left(ax-1\right)}{a^{3}}\right)}{c^{2}}$	61
norman	$\frac{\frac{a^{3}x^{4}}{c}-\frac{6x}{c}+\frac{15a\,x^{2}}{c}-\frac{34a^{2}x^{3}}{3c}}{\left(ax-1\right)^{3}c}+\frac{6\ln\left(ax-1\right)}{a\,c^{2}}$	64
parallelrisch	$\frac{3a^{4}x^{4}+18a^{3}\ln\left(ax-1\right)x^{3}-34a^{3}x^{3}-54a^{2}\ln\left(ax-1\right)x^{2}+45a^{2}x^{2}+54a\ln\left(ax-1\right)x-18ax-18\ln\left(ax-1\right)}{3\left(ax-1\right)^{3}c^{2}a}$	91

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
[Out] x/c^2+(-13*a*c^2*x^2+20*c^2*x-25/3*c^2/a)/c^4/(a*x-1)^3+6/a/c^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{3\,a^{4}x^{4}-9\,a^{3}x^{3}-30\,a^{2}x^{2}+57\,ax+18\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(ax-1\right)-25}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-39a^{2}x^{2}+60ax-25}{3a^{4}c^{2}x^{3}-9a^{3}c^{2}x^{2}+9a^{2}c^{2}x-3ac^{2}}+\frac{x}{c^{2}}+\frac{6\log\left(ax-1\right)}{ac^{2}}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**2,x)
[Out] (-39*a**2*x**2 + 60*a*x - 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2
*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{39\,a^2x^2-60\,ax+25}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{6\,\log\left(ax-1\right)}{ac^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{ax-1}{ac^2}-\frac{6\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{39\,a^5c^4}{ax-1}+\frac{18\,a^5c^4}{(ax-1)^2}+\frac{4\,a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 6*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(39*a^5*c^4/(a*x - 1) + 18*a^5*c^4/(a*x - 1)^2 + 4*a^5*c^4/(a*x - 1)^3)/(a^
6*c^6)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{13\,a\,x^2-20\,x+\frac{25}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^2}$$

```
[In] int((a*x + 1)^2/((c - c/(a*x))^2*(a*x - 1)^2),x)
```

```
[Out] (13*a*x^2 - 20*x + 25/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x
) + x/c^2 + (6*log(a*x - 1))/(a*c^2)
```

3.411 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

```
[Out] x/c^3-1/a/c^3/(-a*x+1)^4+16/3/a/c^3/(-a*x+1)^3-25/2/a/c^3/(-a*x+1)^2+19/a/c
^3/(-a*x+1)+7*ln(-a*x+1)/a/c^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int \frac{e^{4\operatorname{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$

$$= -\frac{a^3\int \frac{e^{4\operatorname{arctanh}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3(1+ax)^2}{(1-ax)^5}\,dx}{c^3}$$

$$= -\frac{a^3\int \left(-\frac{1}{a^3}-\frac{4}{a^3(-1+ax)^5}-\frac{16}{a^3(-1+ax)^4}-\frac{25}{a^3(-1+ax)^3}-\frac{19}{a^3(-1+ax)^2}-\frac{7}{a^3(-1+ax)}\right)dx}{c^3}$$

$$= \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{65-218ax+243a^2x^2-78a^3x^3-24a^4x^4+6a^5x^5+42(-1+ax)^4\log(1-ax)}{6ac^3(-1+ax)^4}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
+ a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x}{c^3}+\frac{-19a^2c^3x^3+\frac{89a\,c^3x^2}{2}-\frac{112c^3x}{3}+\frac{65c^3}{6a}}{c^6(ax-1)^4}+\frac{7\ln(ax-1)}{a\,c^3}$
default	$\frac{a^3\left(\frac{x}{a^3}-\frac{25}{2a^4(ax-1)^2}-\frac{1}{a^4(ax-1)^4}-\frac{19}{a^4(ax-1)}-\frac{16}{3a^4(ax-1)^3}+\frac{7\ln(ax-1)}{a^4}\right)}{c^3}$
norman	$\frac{\frac{a^4x^5}{c}+\frac{7x}{c}-\frac{49a\,x^2}{2c}+\frac{91a^2x^3}{3c}-\frac{89a^3x^4}{6c}}{(ax-1)^4c^2}+\frac{7\ln(ax-1)}{a\,c^3}$
parallelrisch	$\frac{6a^5x^5+42\ln(ax-1)x^4a^4-89a^4x^4-168a^3\ln(ax-1)x^3+182a^3x^3+252a^2\ln(ax-1)x^2-147a^2x^2-168a\ln(ax-1)x+42ax+42\ln}{6(ax-1)^4c^3a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^3+(-19*a^2*c^3*x^3+89/2*a*c^3*x^2-112/3*c^3*x+65/6*c^3/a)/c^6/(a*x-1)^4
+7/a/c^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.42

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{6\,a^5x^5-24\,a^4x^4-78\,a^3x^3+243\,a^2x^2-218\,ax+42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax-1\right)+6}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

[Out] 1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-114a^3x^3+267a^2x^2-224ax+65}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}+\frac{x}{c^3}+\frac{7\log(ax-1)}{ac^3}$$

[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**3,x)

[Out] (-114*a**3*x**3 + 267*a**2*x**2 - 224*a*x + 65)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*log(a*x - 1)/(a*c**3)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}+\frac{7\,\log(ax-1)}{ac^3}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="maxima")

[Out] -1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{ax-1}{ac^3}-\frac{7\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\frac{114\,a^7c^9}{ax-1}+\frac{75\,a^7c^9}{(ax-1)^2}+\frac{32\,a^7c^9}{(ax-1)^3}+\frac{6\,a^7c^9}{(ax-1)^4}}{6\,a^8c^{12}}$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="giac")

[Out] (a*x - 1)/(a*c^3) - 7*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/6*(114*a^7*c^9/(a*x - 1) + 75*a^7*c^9/(a*x - 1)^2 + 32*a^7*c^9/(a*x - 1)^3 + 6*a^7*c^9/(a*x - 1)^4)/(a^8*c^12)

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{x}{c^3}-\frac{\frac{112\,x}{3}-\frac{89\,a\,x^2}{2}-\frac{65}{6\,a}+19\,a^2\,x^3}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{7\,\ln\left(a\,x-1\right)}{a\,c^3}$$

```
[In] int((a*x + 1)^2/((c - c/(a*x))^3*(a*x - 1)^2),x)
```

```
[Out] x/c^3 - ((112*x)/3 - (89*a*x^2)/2 - 65/(6*a) + 19*a^2*x^3)/(c^3 + 6*a^2*c^3
*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (7*log(a*x - 1))/(a*c^3)
```

3.412 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}$$

```
[Out] x/c^4+4/5/a/c^4/(-a*x+1)^5-5/a/c^4/(-a*x+1)^4+41/3/a/c^4/(-a*x+1)^3-22/a/c^4/(-a*x+1)^2+26/a/c^4/(-a*x+1)+8*ln(-a*x+1)/a/c^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 - a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/(a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx \\
&= \frac{a^4\int \frac{e^{4\text{arctanh}(ax)}x^4}{(1-ax)^4}\,dx}{c^4} \\
&= \frac{a^4\int \frac{x^4(1+ax)^2}{(1-ax)^6}\,dx}{c^4} \\
&= \frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(-1+ax)^6}+\frac{20}{a^4(-1+ax)^5}+\frac{41}{a^4(-1+ax)^4}+\frac{44}{a^4(-1+ax)^3}+\frac{26}{a^4(-1+ax)^2}+\frac{8}{a^4(-1+ax)}\right)dx}{c^4} \\
&= \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3} \\
&\quad -\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-202+890ax-1480a^2x^2+1080a^3x^3-240a^4x^4-75a^5x^5+15a^6x^6+120(-1+ax)^5\log(1-ax)}{15ac^4(-1+ax)^5}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{c^4}+\frac{-26a^3c^4x^4+82a^2c^4x^3-\frac{311a\,c^4x^2}{3}+\frac{181c^4x}{3}-\frac{202c^4}{15a}}{c^8(ax-1)^5}+\frac{8\ln(ax-1)}{a\,c^4}$
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{4}{5a^5(ax-1)^5}-\frac{22}{a^5(ax-1)^2}-\frac{5}{a^5(ax-1)^4}-\frac{26}{a^5(ax-1)}-\frac{41}{3a^5(ax-1)^3}+\frac{8\ln(ax-1)}{a^5}\right)}{c^4}$
norman	$\frac{\frac{a^5x^6}{c}-\frac{8x}{c}+\frac{36a\,x^2}{c}-\frac{188a^2x^3}{3c}+\frac{154a^3x^4}{3c}-\frac{277a^4x^5}{15c}}{(ax-1)^5c^3}+\frac{8\ln(ax-1)}{a\,c^4}$
parallelrisch	$\frac{15a^6x^6+120\ln(ax-1)x^5a^5-277a^5x^5-600\ln(ax-1)x^4a^4+770a^4x^4+1200a^3\ln(ax-1)x^3-940a^3x^3-1200a^2\ln(ax-1)x^2+540a^2}{15(ax-1)^5c^4a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^4+(-26*a^3*c^4*x^4+82*a^2*c^4*x^3-311/3*a*c^4*x^2+181/3*c^4*x-202/15*c^
4/a)/c^8/(a*x-1)^5+8/a/c^4*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+\right.}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2 +
 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*
log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-390a^4x^4+1230a^3x^3-1555a^2x^2+905ax-202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4} + \frac{x}{c^4}+\frac{8\log(ax-1)}{ac^4}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**4,x)
```

```
[Out] (-390*a**4*x**4 + 1230*a**3*x**3 - 1555*a**2*x**2 + 905*a*x - 202)/(15*a**6
*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2 +
75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)} + \frac{x}{c^4}+\frac{8\,\log(ax-1)}{ac^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4*
x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4
) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{ax-1}{ac^4} - \frac{8\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{\frac{390\,a^9c^{16}}{ax-1} + \frac{330\,a^9c^{16}}{(ax-1)^2} + \frac{205\,a^9c^{16}}{(ax-1)^3} + \frac{75\,a^9c^{16}}{(ax-1)^4} + \frac{12\,a^9c^{16}}{(ax-1)^5}}{15\,a^{10}c^{20}}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^4) - 8*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 1/15
*(390*a^9*c^16/(a*x - 1) + 330*a^9*c^16/(a*x - 1)^2 + 205*a^9*c^16/(a*x - 1
)^3 + 75*a^9*c^16/(a*x - 1)^4 + 12*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4} + \frac{\frac{311\,a\,x^2}{3} - \frac{181\,x}{3} + \frac{202}{15\,a} - 82\,a^2\,x^3 + 26\,a^3\,x^4}{-a^5\,c^4\,x^5 + 5\,a^4\,c^4\,x^4 - 10\,a^3\,c^4\,x^3 + 10\,a^2\,c^4\,x^2 - 5\,a\,c^4\,x + c^4} + \frac{8\,\ln(a\,x-1)}{a\,c^4}$$

```
[In] int((a*x + 1)^2/((c - c/(a*x))^4*(a*x - 1)^2),x)
```

```
[Out] x/c^4 + ((311*a*x^2)/3 - (181*x)/3 + 202/(15*a) - 82*a^2*x^3 + 26*a^3*x^4)/
(c^4 + 10*a^2*c^4*x^2 - 10*a^3*c^4*x^3 + 5*a^4*c^4*x^4 - a^5*c^4*x^5 - 5*a*
c^4*x) + (8*log(a*x - 1))/(a*c^4)
```

3.413 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{25c^4\csc^{-1}(ax)}{2a}-\frac{5c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -25/2*c^4*arccsc(a*x)/a-5*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-32/3*c^4*(1-1/
a^2/x^2)^(1/2)/a-1/3*c^4*(1-1/a^2/x^2)^(1/2)/a^3/x^2+5/2*c^4*(1-1/a^2/x^2)^
(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 1821, 1823, 858, 222, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{5c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+c^4x\sqrt{1-\frac{1}{a^2x^2}}-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{25c^4\csc^{-1}(ax)}{2a}$$

```
[In] Int[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-32*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) - (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*
x^2) + (5*c^4*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*
x - (25*c^4*ArcCsc[a*x])/(2*a) - (5*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{\frac{5c^5}{a}-\frac{10c^5x}{a^2}+\frac{10c^5x^2}{a^3}-\frac{5c^5x^3}{a^4}+\frac{c^5x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^2\text{Subst}\left(\int \frac{-\frac{15c^5}{a^3}+\frac{30c^5x}{a^4}-\frac{32c^5x^2}{a^5}+\frac{15c^5x^3}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^4\text{Subst}\left(\int \frac{\frac{30c^5}{a^5}-\frac{75c^5x}{a^6}+\frac{64c^5x^2}{a^7}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6c}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^6\text{Subst}\left(\int \frac{-\frac{30c^5}{a^7}+\frac{75c^5x}{a^8}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6c}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{(25c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(5c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{25c^4\csc^{-1}(ax)}{2a} + \frac{(5c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{25c^4\csc^{-1}(ax)}{2a} - (5ac^4)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{25c^4\csc^{-1}(ax)}{2a} - \frac{5c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(2-15ax+62a^2x^2+9a^3x^3-64a^4x^4+6a^5x^5+90a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-30a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\right.}{6a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

```
[In] Integrate[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(2 - 15*a*x + 62*a^2*x^2 + 9*a^3*x^3 - 64*a^4*x^4 + 6*a^5*x^5 + 90*a^4
*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 30*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 30*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{(ax+1)(64a^2x^2-15ax+2)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4} + \frac{\left(-\frac{5a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}} - \frac{25a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2} + a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{}}{a^4(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)c^4\left(-66\sqrt{a^2x^2-1}\sqrt{a^2}a^4x^4+66(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^2x^2-75\sqrt{a^2x^2-1}\sqrt{a^2}a^3x^3-75a^3x^3\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+66\ln\right.}{6\sqrt{(ax-}}$

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x+1)*(64*a^2*x^2-15*a*x+2)/x^3*c^4/a^4*((a*x-1)/(a*x+1))^(1/2)+(-5*
a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-25/2*a^3*arctan(1/(
```

```
a^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x-1)*((a*x-1)/(a*
x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{150\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^4x^4-58}{6\,a^4x^3}$$

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(150*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 30*a^3*c^4*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) + 30*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^4*x^4 - 58*a^3*c^4*x^3 - 49*a^2*c^4*x^2 + 13*a*c^4*x -
2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{4a^3}{}\right.\right.}{a^4}$$

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-4*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**3, x) + Integral(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x*
*2, x) + Integral(-4*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.65

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{1}{3}\left(\frac{75\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{87\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+61\,c^4\left(\frac{ax-}{ax+}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(a}{(c}}\right.$$

[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

[Out] 1/3*(75*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (87*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 61*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 55*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 45*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.96

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{25\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}$$
$$+\frac{5\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^4\operatorname{sgn}(ax+1)}{a}$$
$$-\frac{15\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|\operatorname{sgn}(ax+1)+60\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4\operatorname{sgn}(ax+1)+132\left(x|a|-\sqrt{a^2x^2-\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right.}$$

[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")

[Out] 25*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 5*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^4*sgn(a*x + 1)/a - 1/3*(15*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) + 60*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 132*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 15*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a)*sgn(a*x + 1) + 64*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.37

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{25\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{15\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{61\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}$$
$$-\frac{10\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (25*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (15*c^4*((a*x - 1)/(a*x + 1)
)^(1/2) + (55*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3 - (61*c^4*((a*x - 1)/(a*x
+ 1))^(5/2))/3 - 29*c^4*((a*x - 1)/(a*x + 1))^(7/2))/(a + (2*a*(a*x - 1))/(
a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1)^4)/(a*x + 1)^4) - (
10*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.414 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{13c^3\csc^{-1}(ax)}{2a}-\frac{4c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -13/2*c^3*arccsc(a*x)/a-4*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*c^3*(1-1/a^2
/x^2)^(1/2)/a+1/2*c^3*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 1821, 1823, 858, 222, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{4c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{13c^3\csc^{-1}(ax)}{2a}$$

```
[In] Int[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-4*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) +
c^3*Sqrt[1 - 1/(a^2*x^2)]*x - (13*c^3*ArcCsc[a*x])/(2*a) - (4*c^3*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{4c^4}{a}-\frac{6c^4x}{a^2}+\frac{4c^4x^2}{a^3}-\frac{c^4x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\text{Subst}\left(\int \frac{-\frac{8c^4}{a^3}+\frac{13c^4x}{a^4}-\frac{8c^4x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^4\text{Subst}\left(\int \frac{\frac{8c^4}{a^5}-\frac{13c^4x}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{(13c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(4c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} + \frac{(2c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} - (4ac^3)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{13c^3\csc^{-1}(ax)}{2a} - \frac{4c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= \frac{c^3\left(-1+8ax-a^2x^2-8a^3x^3+2a^4x^4+10a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-8a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{1}{ax}\right)\right.}{2a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

```
[In] Integrate[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(-1 + 8*a*x - a^2*x^2 - 8*a^3*x^3 + 2*a^4*x^4 + 10*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{(ax+1)(8ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(-\frac{4a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{13a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^2\sqrt{(ax-1)(ax+1)}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(}}{a^3(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)c^3\left(-8\sqrt{a^2x^2-1}\sqrt{a^2}a^3x^3+8(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}ax-13\sqrt{a^2x^2-1}\sqrt{a^2}a^2x^2-13a^2x^2\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+8\ln\left(\right.\right.}{2\sqrt{(ax-1)(ax+1)}a^3x}$

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+1)*(8*a*x-1)/x^2*c^3/a^3*((a*x-1)/(a*x+1))^(1/2)+(-4*a^3*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-13/2*a^2*arctan(1/(a^2*x^2-1)^(
1/2))+a^2*((a*x-1)*(a*x+1))^(1/2))*c^3/a^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*
((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{26\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^3c^3x^3-6\,a^2c^3x^2}{2\,a^3x^2}$$

[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

[Out] 1/2*(26*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (2*a^3*c^3*x^3 - 6*a^2*c^3*x^2 - 7*a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(\int a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^3}$$

[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(1/2),x)

[Out] c**3*(Integral(a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**3, x) + Integral(3*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**3

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(94) = 188.

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.90

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \left(\frac{13\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1}{(ax+}}\right.$$

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] (13*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 4*c^3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^2 + 4*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (11*c^3*
((a*x - 1)/(a*x + 1))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqr
t((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1
)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.19

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{13\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{4\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a} - \frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|\operatorname{sgn}(ax+1)+8\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3\operatorname{sgn}(ax+1)-\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2a|a|}$$

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] 13*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 4*c^3*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
3*sgn(a*x + 1)/a - ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn(a*x + 1
) + 8*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - (x*abs(a) - sqr
t(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 8*a*c^3*sgn(a*x + 1))/(((x*abs(a)
 - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.54

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{2\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-5\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}+11\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}} + \frac{13\,c^3\operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{8\,c^3\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 11
*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (13*c^3*atan(((a*x - 1)/
(a*x + 1))^(1/2)))/a - (8*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.415 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c^2\csc^{-1}(ax)}{a}-\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -3*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-c^2*(1-1/a^2/x^2)
^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 1821, 1823, 858, 222, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{3c^2\csc^{-1}(ax)}{a}$$

```
[In] Int[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] -((c^2*Sqrt[1 - 1/(a^2*x^2)])/a) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x - (3*c^2*Arc
Csc[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{3c^3}{a}-\frac{3c^3x}{a^2}+\frac{c^3x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\text{Subst}\left(\int \frac{-\frac{3c^3}{a^3}+\frac{3c^3x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} - (3ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} - \frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)-3\arcsin\left(\frac{1}{ax}\right)-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

```
[In] Integrate[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]]))/a
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\Big(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-3ax\sqrt{a^2}\,\arctan\Big(}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(a*x+1)/x*c^2/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(-3*a*ln(a^2*x/(a^2)^(1/2)+(
a^2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)-3*arctan(1/(a^2*x^2-1
)^(1/2)))*c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$=\frac{6\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) + 3*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c^2*
x^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^2}$$

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-2*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x, x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{6\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a}-\frac{2\,c^2\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 6*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2
*sgn(a*x + 1)/a - 2*c^2*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1
)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{4\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}+\frac{6\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) + (6*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

3.416 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{c\csc^{-1}(ax)}{a}-\frac{2c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -c*arccsc(a*x)/a-2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 1821, 858, 222, 272, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{2c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{c\csc^{-1}(ax)}{a}$$

```
[In] Int[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a - (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}
\end{aligned}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{(2c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} - (2ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} - \frac{2c\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x - 2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right) - 2\arcsin\left(\frac{1}{ax}\right) - 2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

```
[In] Integrate[(c - c/(a*x))/E^ArcCoth[a*x],x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(45) = 90.

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.78

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}+2a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)-2\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	136

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/
(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2))/(a^2)^(1/2))-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1)
)^(1/2)/a/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(\int a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)\,dx\right)}{a}$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/x, x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(45) = 90.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 2*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

```
[In] int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (4*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1
))
```

3.417 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6312, 270}

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
```

```
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\ &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

method	result	size
gosper	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	30

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\int\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
[Out] a*Integral(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{2\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
[Out] -2*a*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
[Out] sqrt(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x)),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1))
```

3.418 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+2*x*(1-1/a^2/x^2)^(1/2)/c^2-a*x*(1-1/a^2
/x^2)^(1/2)/c^2/(a-1/x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6312, 871, 821, 272, 65, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2} + \frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2} - \frac{ax\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - (a*Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(a - x^(
-1))) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\left(a-\frac{1}{x}\right)} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\left(a-\frac{1}{x}\right)} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\left(a-\frac{1}{x}\right)} + \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\left(a-\frac{1}{x}\right)} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx = \frac{-2-ax+a^2x^2+a\sqrt{1-\frac{1}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
[Out] (-2 - a*x + a^2*x^2 + a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(67) = 134.

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.97

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^2(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+6\sqrt{a^2}\,\sqrt{(ax-1)(}}{2a\sqrt{(ax-1)(ax+1)}\,c^2(ax-}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^
2/c^2*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^2
*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1
), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -a\left(\frac{\frac{3(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c^2*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^
2) + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^2,x)
```

```
[Out] (2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 4)/(2*a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

3.419 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -2/3*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c
^3+1/3*(-6*a-7/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = \frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3} - \frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (-2*(a + x^(-1)))/(3*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (6*a + 7/x)/(3*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (2*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^2-\frac{6c^2x}{a}-\frac{4c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^2+\frac{6c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{10-4ax-11a^2x^2+3a^3x^3+6a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^6\left(x-\frac{1}{a}\right)^2}-\frac{8\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^5\left(x-\frac{1}{a}\right)}\right)a^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^3(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-27\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+15\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+81\sqrt{a^2}\,\cdots}{\cdots}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(2/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/3/a^6/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)
-8/3/a^5/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^3/c^3*((a*x-1)/(a*x+1
))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$=\frac{6\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-11\,a^2x^2-4\,ax\cdots\right.}{3\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 11*a^2*x^2
 - 4*a*x + 10)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^
3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{a^3\int\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x*
*2 + 3*a*x - 1), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$=\frac{1}{6}a\left(\frac{\frac{14\,(ax-1)}{ax+1}-\frac{27\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*a*((14*(a*x - 1)/(a*x + 1) - 27*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 12*log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3} - \frac{\frac{14\,(a\,x-1)}{3\,(a\,x+1)} - \frac{9\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - 2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^3,x)
```

```
[Out] (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((14*(a*x - 1))/(3*(a*x +
1)) - (9*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/
2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.420 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] -4/5*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/5*(-5*a-7/x)/a^2/c^4/(1-1/a^2/x^
2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/5*(-15*a-19/x)/a^2/c^4/(1-1
/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4} - \frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (5*a + 7/x)/(5*a^2*
c^4*(1 - 1/(a^2*x^2))^(3/2)) - (15*a + 19/x)/(5*a^2*c^4*Sqrt[1 - 1/(a^2*x^2
)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^4 + (3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a
*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}\\
&= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{\text{Subst}\left(\int \frac{-5c^3-\frac{15c^3x}{a}-\frac{16c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}\\
&= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{15c^3+\frac{45c^3x}{a}+\frac{42c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}\\
&= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\\
&\quad-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{-15c^3-\frac{45c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}\\
&= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}\\
&\quad+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}-\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^4}
\end{aligned}
$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^4}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{(3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^4}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-24+33ax+18a^2x^2-34a^3x^3+5a^4x^4+15a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-24 + 33*a*x + 18*a^2*x^2 - 34*a^3*x^3 + 5*a^4*x^4 + 15*a*Sqrt[1 - 1/(a^2*
x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(5*a^2*c^4*Sqrt[1 - 1/
(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{4}}+\frac{\left(\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{4}\sqrt{a^{2}}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{5a^{8}\left(x-\frac{1}{a}\right)^{3}}-\frac{6\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{5a^{7}\left(x-\frac{1}{a}\right)^{2}}-\frac{24\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{5a^{6}\left(x-\frac{1}{a}\right)}\right)a^{4}\sqrt{\frac{ax-}{ax+}} \cdots}{c^{4}(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-125\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{4}x^{4}-120\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{5}x^{4}+85\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}a^{2}x^{2}+500\sqrt{\cdots}\right.}{\cdots}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/5/a^8/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)
-6/5/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-24/5/a^6/(x-1/a)*((x-1
/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^4*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x
-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{4}}\,dx$$
$$=\frac{15\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(5\,a^{4}x^{4}\cdots\right.}{5\left(a^{4}c^{4}x^{3}-3\,a^{3}c^{4}x^{2}+3\,a^{2}c^{4}x-ac^{4}\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x*
*3 + 6*a**2*x**2 - 4*a*x + 1), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$= \frac{1}{20}a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.43

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^4|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^4*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{25\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{9\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^4,x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1
)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(4*
a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.421 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}$$

```
[Out] 1/3*c^4/a^4/x^3-3*c^4/a^3/x^2+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(a
*x+1)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

```
[In] Int[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[x])/a - (32*c^4*Log[1 + a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^4 dx\\
&= -\frac{c^4\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\
&= -\frac{c^4\int \frac{(1-ax)^5}{x^4(1+ax)}\,dx}{a^4}\\
&= -\frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{6a}{x^3}+\frac{16a^2}{x^2}-\frac{26a^3}{x}+\frac{32a^4}{1+ax}\right)dx}{a^4}\\
&= \frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{c^4\left(-\frac{1}{3x^3}+\frac{3a}{x^2}-\frac{16a^2}{x}-a^4x-26a^3\log(x)+32a^3\log(1+ax)\right)}{a^4}$$

```
[In] Integrate[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
[Out] -((c^4*(-1/3*1/x^3 + (3*a)/x^2 - (16*a^2)/x - a^4*x - 26*a^3*Log[x] + 32*a^
3*Log[1 + a*x]))/a^4)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^4\left(-32a^3\ln(ax+1)+a^4x+\frac{1}{3x^3}-\frac{3a}{x^2}+\frac{16a^2}{x}+26a^3\ln(x)\right)}{a^4}$
risch	$c^4x+\frac{16a^2c^4x^2-3a\,c^4x+\frac{1}{3}c^4}{a^4x^3}+\frac{26c^4\ln(-x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
norman	$\frac{a^3c^4x^4+\frac{c^4}{3a}-3c^4x+16a\,c^4x^2}{a^3x^3}+\frac{26c^4\ln(x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
parallelrisch	$\frac{3a^4c^4x^4+78c^4\ln(x)a^3x^3-96c^4\ln(ax+1)a^3x^3+48a^2c^4x^2-9a\,c^4x+c^4}{3a^4x^3}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{5c^4\ln(ax+1)}{a}+\frac{10c^4(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{10c^4\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{5c^4\left(-\ln(\right.}{}$

```
[In] int((c-c/a/x)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^4/a^4*(-32*a^3*ln(a*x+1)+a^4*x+1/3/x^3-3*a/x^2+16*a^2/x+26*a^3*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = \frac{3\,a^4c^4x^4-96\,a^3c^4x^3\log(ax+1)+78\,a^3c^4x^3\log(x)+48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = c^4x+\frac{2c^4\cdot\left(13\log(x)-16\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{48a^2c^4x^2-9ac^4x+c^4}{3a^4x^3}$$

```
[In] integrate((c-c/a/x)**4*(a*x-1)/(a*x+1),x)
[Out] c**4*x + 2*c**4*(13*log(x) - 16*log(x + 1/a))/a + (48*a**2*c**4*x**2 - 9*a*
c**4*x + c**4)/(3*a**4*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x - \frac{32\,c^4\log\left(ax+1\right)}{a} + \frac{26\,c^4\log\left(x\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 32*c^4*log(a*x + 1)/a + 26*c^4*log(x)/a + 1/3*(48*a^2*c^4*x^2 - 9*a
*c^4*x + c^4)/(a^4*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x - \frac{32\,c^4\log\left(|ax+1|\right)}{a} + \frac{26\,c^4\log\left(|x|\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 32*c^4*log(abs(a*x + 1))/a + 26*c^4*log(abs(x))/a + 1/3*(48*a^2*c^4
*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4\,x + \frac{16\,a^2\,c^4\,x^2 - 3\,a\,c^4\,x + \frac{c^4}{3}}{a^4\,x^3} + \frac{26\,c^4\,\ln\left(x\right)}{a} - \frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - c/(a*x))^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] c^4*x + (c^4/3 + 16*a^2*c^4*x^2 - 3*a*c^4*x)/(a^4*x^3) + (26*c^4*log(x))/a
- (32*c^4*log(a*x + 1))/a
```

3.422 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}$$

```
[Out] -1/2*c^3/a^3/x^2+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

```
[In] Int[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[x])/a - (16*c^3*
Log[1 + a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx\\
&= \frac{c^3\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= \frac{c^3\int \frac{(1-ax)^4}{x^3(1+ax)}\,dx}{a^3}\\
&= \frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{5a}{x^2}+\frac{11a^2}{x}-\frac{16a^3}{1+ax}\right)dx}{a^3}\\
&= -\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\left(\frac{1}{2x^2}-\frac{5a}{x}-a^3x-11a^2\log(x)+16a^2\log(1+ax)\right)}{a^3}$$

```
[In] Integrate[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
[Out] -((c^3*(1/(2*x^2) - (5*a)/x - a^3*x - 11*a^2*Log[x] + 16*a^2*Log[1 + a*x]))
/a^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^{3}\left(-16a^{2}\ln\left(ax+1\right)+a^{3}x-\frac{1}{2x^{2}}+\frac{5a}{x}+11a^{2}\ln\left(x\right)\right)}{a^{3}}$
risch	$c^{3}x+\frac{5a\,c^{3}x-\frac{1}{2}c^{3}}{a^{3}x^{2}}+\frac{11c^{3}\ln\left(-x\right)}{a}-\frac{16c^{3}\ln\left(ax+1\right)}{a}$
norman	$\frac{a^{2}c^{3}x^{3}-\frac{c^{3}}{2a}+5c^{3}x}{a^{2}x^{2}}+\frac{11c^{3}\ln\left(x\right)}{a}-\frac{16c^{3}\ln\left(ax+1\right)}{a}$
parallelrisch	$\frac{2a^{3}c^{3}x^{3}+22c^{3}\ln\left(x\right)a^{2}x^{2}-32c^{3}\ln\left(ax+1\right)a^{2}x^{2}+10a\,c^{3}x-c^{3}}{2a^{3}x^{2}}$
meijerg	$\frac{c^{3}\left(ax-\ln\left(ax+1\right)\right)}{a}-\frac{4c^{3}\ln\left(ax+1\right)}{a}+\frac{6c^{3}\left(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right)\right)}{a}-\frac{4c^{3}\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{ax}\right)}{a}+\frac{c^{3}\left(-\ln(ax\right.}{}$

```
[In] int((c-c/a/x)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^3*(-16*a^2*ln(a*x+1)+a^3*x-1/2/x^2+5*a/x+11*a^2*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}\,dx=\frac{2\,a^{3}c^{3}x^{3}-32\,a^{2}c^{3}x^{2}\log\left(ax+1\right)+22\,a^{2}c^{3}x^{2}\log\left(x\right)+10\,ac^{3}x-c^{3}}{2\,a^{3}x^{2}}$$

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
10*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}\,dx=c^{3}x+\frac{c^{3}\cdot\left(11\log\left(x\right)-16\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{10ac^{3}x-c^{3}}{2a^{3}x^{2}}$$

```
[In] integrate((c-c/a/x)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*x + c**3*(11*log(x) - 16*log(x + 1/a))/a + (10*a*c**3*x - c**3)/(2*a**
3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3x - \frac{16\,c^3\log\left(ax+1\right)}{a} + \frac{11\,c^3\log\left(x\right)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 16*c^3*log(a*x + 1)/a + 11*c^3*log(x)/a + 1/2*(10*a*c^3*x - c^3)/(a
^3*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3x - \frac{16\,c^3\log\left(|ax+1|\right)}{a} + \frac{11\,c^3\log\left(|x|\right)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 16*c^3*log(abs(a*x + 1))/a + 11*c^3*log(abs(x))/a + 1/2*(10*a*c^3*x
 - c^3)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3\,x - \frac{\frac{c^3}{2} - 5\,a\,c^3\,x}{a^3\,x^2} + \frac{11\,c^3\,\ln\left(x\right)}{a} - \frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - c/(a*x))^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] c^3*x - (c^3/2 - 5*a*c^3*x)/(a^3*x^2) + (11*c^3*log(x))/a - (16*c^3*log(a*x
 + 1))/a
```

3.423 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}$$

```
[Out] c^2/a^2/x+c^2*x+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

```
[In] Int[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[x])/a - (8*c^2*Log[1 + a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx\\
&= -\frac{c^2\int\frac{e^{-2\text{arctanh}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\frac{(1-ax)^3}{x^2(1+ax)}\,dx}{a^2}\\
&= -\frac{c^2\int\left(-a^2+\frac{1}{x^2}-\frac{4a}{x}+\frac{8a^2}{1+ax}\right)dx}{a^2}\\
&= \frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{c^2\left(-\frac{1}{x}-a^2x-4a\log(x)+8a\log(1+ax)\right)}{a^2}$$

```
[In] Integrate[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
[Out] -((c^2*(-x^(-1) - a^2*x - 4*a*Log[x] + 8*a*Log[1 + a*x]))/a^2)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{c^{2}\left(-8a\ln\left(ax+1\right)+a^{2}x+\frac{1}{x}+4a\ln\left(x\right)\right)}{a^{2}}$	31
risch	$\frac{c^{2}}{a^{2}x}+c^{2}x+\frac{4c^{2}\ln\left(-x\right)}{a}-\frac{8c^{2}\ln\left(ax+1\right)}{a}$	43
parallelrisch	$\frac{a^{2}c^{2}x^{2}+4c^{2}\ln\left(x\right)ax-8c^{2}\ln\left(ax+1\right)ax+c^{2}}{a^{2}x}$	44
norman	$\frac{\frac{c^{2}}{a}+a\,c^{2}x^{2}}{ax}+\frac{4c^{2}\ln\left(x\right)}{a}-\frac{8c^{2}\ln\left(ax+1\right)}{a}$	49
meijerg	$\frac{c^{2}\left(ax-\ln\left(ax+1\right)\right)}{a}-\frac{3c^{2}\ln\left(ax+1\right)}{a}+\frac{3c^{2}\left(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right)\right)}{a}-\frac{c^{2}\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{ax}\right)}{a}$	87

```
[In] int((c-c/a/x)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^2/a^2*(-8*a*ln(a*x+1)+a^2*x+1/x+4*a*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2-8\,ac^2x\log\left(ax+1\right)+4\,ac^2x\log\left(x\right)+c^2}{a^2x}$$

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{4c^2\left(\log\left(x\right)-2\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{c^2}{a^2x}$$

```
[In] integrate((c-c/a/x)**2*(a*x-1)/(a*x+1),x)
[Out] c**2*x + 4*c**2*(log(x) - 2*log(x + 1/a))/a + c**2/(a**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = c^2x - \frac{8\,c^2\log\left(ax+1\right)}{a} + \frac{4\,c^2\log\left(x\right)}{a} + \frac{c^2}{a^2x}$$

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c^2*x - 8*c^2*log(a*x + 1)/a + 4*c^2*log(x)/a + c^2/(a^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = c^2x - \frac{8\,c^2\log\left(|ax+1|\right)}{a} + \frac{4\,c^2\log\left(|x|\right)}{a} + \frac{c^2}{a^2x}$$

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c^2*x - 8*c^2*log(abs(a*x + 1))/a + 4*c^2*log(abs(x))/a + c^2/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = c^2\,x + \frac{c^2}{a^2\,x} + \frac{4\,c^2\,\ln\left(x\right)}{a} - \frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - c/(a*x))^2*(a*x - 1))/(a*x + 1),x)
[Out] c^2*x + c^2/(a^2*x) + (4*c^2*log(x))/a - (8*c^2*log(a*x + 1))/a
```

3.424 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

```
[Out] c*x+c*ln(x)/a-4*c*ln(a*x+1)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6266, 6264, 84}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

```
[In] Int[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)\,dx\\
&= \frac{c\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)}{x}\,dx}{a}\\
&= \frac{c\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)\,dx}{a}\\
&= cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

```
[In] Integrate[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{c(ax-4\ln(ax+1)+\ln(x))}{a}$	20
parallelrisch	$\frac{acx+c\ln(x)-4c\ln(ax+1)}{a}$	23
norman	$cx+\frac{c\ln(x)}{a}-\frac{4c\ln(ax+1)}{a}$	24
risch	$cx+\frac{c\ln(-x)}{a}-\frac{4c\ln(ax+1)}{a}$	26
meijerg	$\frac{c(ax-\ln(ax+1))}{a}-\frac{2c\ln(ax+1)}{a}+\frac{c(-\ln(ax+1)+\ln(x)+\ln(a))}{a}$	49

```
[In] int((c-c/a/x)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c/a*(a*x-4*ln(a*x+1)+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx-4\,c\log\left(ax+1\right)+c\log\left(x\right)}{a}$$

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\left(\log\left(x\right)-4\log\left(x+\frac{1}{a}\right)\right)}{a}$$

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x)
[Out] c*x + c*(log(x) - 4*log(x + 1/a))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{4\,c\log\left(ax+1\right)}{a}+\frac{c\log\left(x\right)}{a}$$

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c*x - 4*c*log(a*x + 1)/a + c*log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{4\,c\log\left(|ax+1|\right)}{a}+\frac{c\log\left(|x|\right)}{a}$$

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c*x - 4*c*log(abs(a*x + 1))/a + c*log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\,x+\frac{c\,\ln\left(x\right)}{a}-\frac{4\,c\,\ln\left(a\,x+1\right)}{a}$$

```
[In] int(((c - c/(a*x))*(a*x - 1))/(a*x + 1),x)
[Out] c*x + (c*log(x))/a - (4*c*log(a*x + 1))/a
```

3.425 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}-\frac{\log(1+ax)}{ac}$$

```
[Out] x/c-ln(a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 45}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}-\frac{\log(ax+1)}{ac}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] x/c - Log[1 + a*x]/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{c-\frac{c}{ax}}\,dx \\
&= \frac{a\int \frac{e^{-2\text{arctanh}(ax)}x}{1-ax}\,dx}{c} \\
&= \frac{a\int \frac{x}{1+ax}\,dx}{c} \\
&= \frac{a\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)\,dx}{c} \\
&= \frac{x}{c}-\frac{\log(1+ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\left(\frac{x}{a}-\frac{\log(1+ax)}{a^2}\right)}{c}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
[Out] (a*(x/a - Log[1 + a*x]/a^2))/c
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{ax-\ln(ax+1)}{ac}$	20
norman	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
risch	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
default	$\frac{a\left(\frac{x}{a}-\frac{\ln(ax+1)}{a^2}\right)}{c}$	23

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] (a*x-ln(a*x+1))/a/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{ax-\log(ax+1)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="fricas")
[Out] (a*x - log(a*x + 1))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=a\left(\frac{x}{ac}-\frac{\log(ax+1)}{a^2c}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x)
[Out] a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{\log(ax+1)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="maxima")
[Out] x/c - log(a*x + 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{\log(|ax+1|)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="giac")
[Out] x/c - log(abs(a*x + 1))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{\ln(a\,x+1)-a\,x}{a\,c}$$

```
[In] int((a*x - 1)/((c - c/(a*x))*(a*x + 1)),x)
[Out] -(log(a*x + 1) - a*x)/(a*c)
```

3.426 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{\operatorname{arctanh}(ax)}{ac^2}$$

```
[Out] x/c^2-arctanh(a*x)/a/c^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6302, 6266, 6264, 84, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{\operatorname{arctanh}(ax)}{ac^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx\\
&= -\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= -\frac{a^2\int \frac{x^2}{(1-ax)(1+ax)}\,dx}{c^2}\\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{1}{a^2(-1+a^2x^2)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2}\\
&= \frac{x}{c^2}-\frac{\text{arctanh}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{\operatorname{arctanh}(ax)}{ac^2}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
[Out] x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

method	result	size
parallelrisch	$\frac{2ax+\ln(ax-1)-\ln(ax+1)}{2a\,c^2}$	28
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{\ln(ax+1)}{2a^3}+\frac{\ln(ax-1)}{2a^3}\right)}{c^2}$	36
risch	$\frac{x}{c^2}-\frac{\ln(ax+1)}{2a\,c^2}+\frac{\ln(-ax+1)}{2a\,c^2}$	36
norman	$\frac{\frac{a\,x^2}{c}-\frac{x}{c}}{c(ax-1)}+\frac{\ln(ax-1)}{2a\,c^2}-\frac{\ln(ax+1)}{2a\,c^2}$	56

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*a*x+ln(a*x-1)-ln(a*x+1))/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{2\,ax-\log\left(ax+1\right)+\log\left(ax-1\right)}{2\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
[Out] 1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = a^2\left(\frac{x}{a^2c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^3c^2}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**2,x)
[Out] a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log(ax+1)}{2\,ac^2}+\frac{\log(ax-1)}{2\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
[Out] x/c^2 - 1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log(|ax+1|)}{2\,ac^2}+\frac{\log(|ax-1|)}{2\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
[Out] x/c^2 - 1/2*log(abs(a*x + 1))/(a*c^2) + 1/2*log(abs(a*x - 1))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{\mathrm{atanh}(a\,x)-a\,x}{a\,c^2}$$

```
[In] int((a*x - 1)/((c - c/(a*x))^2*(a*x + 1)),x)
[Out] -(atanh(a*x) - a*x)/(a*c^2)
```

3.427 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

```
[Out] x/c^3+1/2/a/c^3/(-a*x+1)+5/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/(
4*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{a^3\int \frac{e^{-2\text{arctanh}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= \frac{a^3\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c^3}$$
$$= \frac{a^3\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
[Out] x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/(
4*a*c^3)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{a^{3}\left(-\frac{\ln(ax+1)}{4a^{4}}+\frac{x}{a^{3}}-\frac{1}{2a^{4}(ax-1)}+\frac{5\ln(ax-1)}{4a^{4}}\right)}{c^{3}}$	48
risch	$\frac{x}{c^{3}}-\frac{1}{2a(ax-1)c^{3}}+\frac{5\ln(-ax+1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$	51
parallelrisch	$\frac{4a^{2}x^{2}+5a\ln(ax-1)x-a\ln(ax+1)x-6ax-5\ln(ax-1)+\ln(ax+1)}{4c^{3}(ax-1)a}$	63
norman	$\frac{\frac{a^{2}x^{3}}{c}+\frac{3x}{2c}-\frac{5a\,x^{2}}{2c}}{c^{2}(ax-1)^{2}}+\frac{5\ln(ax-1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$	67

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
[Out] a^3/c^3*(-1/4*ln(a*x+1)/a^4+x/a^3-1/2/a^4/(a*x-1)+5/4/a^4*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3}}\,dx=\frac{4\,a^{2}x^{2}-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2}{4\,(a^{2}c^{3}x-ac^{3})}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)/(a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3}}\,dx=a^{3}\left(-\frac{1}{2a^{5}c^{3}x-2a^{4}c^{3}}+\frac{x}{a^{3}c^{3}}+\frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{4}c^{3}}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**3,x)
[Out] a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
- log(x + 1/a)/4)/(a**4*c**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{1}{2\,(a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{\log\left(ax+1\right)}{4\,ac^3}+\frac{5\,\log\left(ax-1\right)}{4\,ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 5/4*log(a*x -
 1)/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{\log\left(|ax+1|\right)}{4\,ac^3}+\frac{5\,\log\left(|ax-1|\right)}{4\,ac^3}-\frac{1}{2\,(ax-1)ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 5/4*log(abs(a*x - 1))/(a*c^3) - 1/2
/((a*x - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}+\frac{5\,\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

```
[In] int((a*x - 1)/((c - c/(a*x))^3*(a*x + 1)),x)
```

```
[Out] x/c^3 + 1/(2*a*(c^3 - a*c^3*x)) + (5*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)
/(4*a*c^3)
```

3.428 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}$$

```
[Out] x/c^4-1/4/a/c^4/(-a*x+1)^2+7/4/a/c^4/(-a*x+1)+17/8*ln(-a*x+1)/a/c^4-1/8*ln(
a*x+1)/a/c^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{7}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] x/c^4 - 1/(4*a*c^4*(1 - a*x)^2) + 7/(4*a*c^4*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx \\
&= -\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}x^4}{(1-ax)^4}\,dx}{c^4} \\
&= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^4} \\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^4} \\
&= \frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{a^4\left(-\frac{x}{a^4}+\frac{1}{4a^5(1-ax)^2}-\frac{7}{4a^5(1-ax)}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(1+ax)}{8a^5}\right)}{c^4}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
[Out] -((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
- a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^4)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{a^{4}\left(-\frac{\ln(ax+1)}{8a^{5}}+\frac{x}{a^{4}}-\frac{1}{4a^{5}(ax-1)^{2}}-\frac{7}{4a^{5}(ax-1)}+\frac{17\ln(ax-1)}{8a^{5}}\right)}{c^{4}}$	60
risch	$\frac{x}{c^{4}}+\frac{-\frac{7c^{4}x}{4}+\frac{3c^{4}}{2a}}{c^{8}(ax-1)^{2}}-\frac{\ln(ax+1)}{8a\,c^{4}}+\frac{17\ln(-ax+1)}{8a\,c^{4}}$	62
norman	$\frac{\frac{a^{3}x^{4}}{c}-\frac{9x}{4c}+\frac{23a\,x^{2}}{4c}-\frac{9a^{2}x^{3}}{2c}}{c^{3}(ax-1)^{3}}+\frac{17\ln(ax-1)}{8a\,c^{4}}-\frac{\ln(ax+1)}{8a\,c^{4}}$	78
parallelrisch	$\frac{8a^{3}x^{3}+17a^{2}\ln(ax-1)x^{2}-a^{2}\ln(ax+1)x^{2}-28a^{2}x^{2}-34a\ln(ax-1)x+2a\ln(ax+1)x+18ax+17\ln(ax-1)-\ln(ax+1)}{8c^{4}(ax-1)^{2}a}$	101

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^4*(-1/8*ln(a*x+1)/a^5+x/a^4-1/4/a^5/(a*x-1)^2-7/4/a^5/(a*x-1)+17/8/a^
5*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{4}}\,dx = \frac{8\,a^{3}x^{3}-16\,a^{2}x^{2}-6\,ax-(a^{2}x^{2}-2\,ax+1)\log(ax+1)+17\,(a^{2}x^{2}-2\,ax+1)\log(ax-1)+12}{8\,(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4})}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*
c^4)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{4}}\,dx = a^{4}\left(\frac{-7ax+6}{4a^{7}c^{4}x^{2}-8a^{6}c^{4}x+4a^{5}c^{4}}+\frac{x}{a^{4}c^{4}}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^{5}c^{4}}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] a**4*((-7*a*x + 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a*
*4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{7\,ax-6}{4\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{\log{(ax+1)}}{8\,ac^4}+\frac{17\,\log{(ax-1)}}{8\,ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 1/8*log(a*x
+ 1)/(a*c^4) + 17/8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}-\frac{\log{(|ax+1|)}}{8\,ac^4}+\frac{17\,\log{(|ax-1|)}}{8\,ac^4}-\frac{7\,ax-6}{4\,(ax-1)^2ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 1/8*log(abs(a*x + 1))/(a*c^4) + 17/8*log(abs(a*x - 1))/(a*c^4) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}+\frac{17\,\ln{(a\,x-1)}}{8\,a\,c^4}-\frac{\ln{(a\,x+1)}}{8\,a\,c^4}$$

```
[In] int((a*x - 1)/((c - c/(a*x))^4*(a*x + 1)),x)
```

```
[Out] x/c^4 - ((7*x)/4 - 3/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) + (17*log(a*x -
 1))/(8*a*c^4) - log(a*x + 1)/(8*a*c^4)
```

3.429 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 164

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{91c^4\csc^{-1}(ax)}{2a}-\frac{7c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 91/2*c^4*arccsc(a*x)/a-7*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+64*c^4*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)+68/3*c^4*(1-1/a^2/x^2)^(1/2)/a+1/3*c^4*(1-1/a^2/x^2
)^(1/2)/a^3/x^2-7/2*c^4*(1-1/a^2/x^2)^(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6312, 1819, 1821, 1823, 858, 222, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{7c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^4x\sqrt{1-\frac{1}{a^2x^2}}+\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}+\frac{91c^4\csc^{-1}(ax)}{2a}$$

```
[In] Int[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (68*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) + (64*c^4*(a - x^(-1)))/(a^2*Sqrt[1 -
1/(a^2*x^2)]) + (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*x^2) - (7*c^4*Sqrt[1 - 1
/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*x + (91*c^4*ArcCsc[a*x])
/(2*a) - (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^7+\frac{7c^7x}{a}+\frac{42c^7x^2}{a^2}-\frac{22c^7x^3}{a^3}+\frac{7c^7x^4}{a^4}-\frac{c^7x^5}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{7c^7}{a}-\frac{42c^7x}{a^2}+\frac{22c^7x^2}{a^3}-\frac{7c^7x^3}{a^4}+\frac{c^7x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\text{Subst}\left(\int \frac{\frac{21c^7}{a^3}+\frac{126c^7x}{a^4}-\frac{68c^7x^2}{a^5}+\frac{21c^7x^3}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^3}$$

$$
\begin{aligned}
&= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} \\
&\quad + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^4\text{Subst}\left(\int \frac{-\frac{42c^7}{a^5}-\frac{273c^7x}{a^6}+\frac{136c^7x^2}{a^7}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6c^3} \\
&= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} \\
&\quad + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^6\text{Subst}\left(\int \frac{\frac{42c^7}{a^7}+\frac{273c^7x}{a^8}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6c^3} \\
&= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x \\
&\quad + \frac{(91c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(7c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} \\
&\quad + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{91c^4\csc^{-1}(ax)}{2a} + \frac{(7c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a} \\
&= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x \\
&\quad + \frac{91c^4\csc^{-1}(ax)}{2a} - (7ac^4)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) \\
&= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} \\
&\quad + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{91c^4\csc^{-1}(ax)}{2a} - \frac{7c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.33 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.46

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4\left(2772\sqrt{2}a^3x^3(-1+ax)^3(1+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+1980\sqrt{2}a^2x^2(-1+ax)^4(1-\right.}{}$$

```
[In] Integrate[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(2772*Sqrt[2]*a^3*x^3*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/
2, 7/2, (1 - 1/(a*x))/2] + 1980*Sqrt[2]*a^2*x^2*(-1 + a*x)^4*(1 + a*x)*Hype
rgeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] + 35*(-198*a^2*Sqrt[1 + 1/(a*
x)]*x^2 + 1716*a^3*Sqrt[1 + 1/(a*x)]*x^3 - 7425*a^4*Sqrt[1 + 1/(a*x)]*x^4 +
 26268*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 29403*a^6*Sqrt[1 + 1/(a*x)]*x^6 - 50160*
a^7*Sqrt[1 + 1/(a*x)]*x^7 + 396*a^8*Sqrt[1 + 1/(a*x)]*x^8 + 66726*a^6*Sqrt[
1 - 1/(a*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 66726*a^7*Sqrt[1 - 1/(
a*x)]*x^7*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1980*a^6*Sqrt[1 - 1/(a*x)]*x^
6*ArcSin[1/(a*x)] - 1980*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[1/(a*x)] - 2772*a
^7*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]] + 44*Sqrt[2]*a*x*(-1 + a*x)^5*(1 + a*x)*Hypergeometric2F1[3/2, 9/2, 11/2
, (1 - 1/(a*x))/2] + 36*Sqrt[2]*(-1 + a*x)^6*(1 + a*x)*Hypergeometric2F1[3/
2, 11/2, 13/2, (1 - 1/(a*x))/2])))/(13860*a^7*Sqrt[1 - 1/(a*x)]*x^6*(1 + a*
x))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(ax+1)(136a^2x^2-21ax+2)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{7a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{91a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}+\frac{64a^2\sqrt{a^2\left(x+\frac{1}{a}\right.}}{x+}\right.}{a^4(ax-1)}$
default	$-\frac{\left(-138\sqrt{a^2x^2-1}\sqrt{a^2}a^6x^6+96\sqrt{(ax-1)(ax+1)}\sqrt{a^2}a^5x^5+138(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^4x^4-549\sqrt{a^2x^2-1}\sqrt{a^2}a^5x^5-273\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{}$

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x+1)*(136*a^2*x^2-21*a*x+2)/x^3*c^4/a^4*((a*x-1)/(a*x+1))^(1/2)+(-7*
a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+91/2*a^3*arctan(1/(
```

```
a^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2)+64*a^2/(x+1/a)*(a^2*(x+1/a)^2
-2*a*(x+1/a))^(1/2))*c^4/a^4/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+
1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{546\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^4x^4+5}{6\,a^4x^3}$$

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(546*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 42*a^3*c^4*x^3*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
 1)) - 1) - (6*a^4*c^4*x^4 + 526*a^3*c^4*x^3 + 115*a^2*c^4*x^2 - 19*a*c^4*x
 + 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{5a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{10a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{10a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(\right.}{a^4}$$

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Int
egral(5*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(
-10*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(1
0*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-5*a**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{1}{3}\left(\frac{273\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{192\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{153\,c\right.$$

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(273*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 21*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 21*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
192*c^4*sqrt((a*x - 1)/(a*x + 1))/a^2 + (153*c^4*((a*x - 1)/(a*x + 1))^(7/2
) + 91*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 169*c^4*((a*x - 1)/(a*x + 1))^(3/2
) - 123*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x
- 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \int \left(c-\frac{c}{ax}\right)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{41\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{169\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{91\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-51\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} + \frac{64\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{91\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,14{}\mathrm{i}}{a}$$

```
[In] int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (41*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (169*c^4*((a*x - 1)/(a*x + 1))^(3/2))
/3 - (91*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 51*c^4*((a*x - 1)/(a*x + 1))^
(7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (64*c^4*((a*x - 1)/(a*x + 1))^(1/2))/a - (91*c^
4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^4*atan(((a*x - 1)/(a*x + 1))^(1
/2)*1i)*14i)/a
```

3.430 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}x+\frac{33c^3\csc^{-1}(ax)}{2a}-\frac{6c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 33/2*c^3*arccsc(a*x)/a-6*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)+6*c^3*(1-1/a^2/x^2)^(1/2)/a-1/2*c^3*(1-1/a^2/x^2)^(
1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec), antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.409$, Rules used = {6312, 1819, 1821, 1823, 858, 222, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{6c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{33c^3\csc^{-1}(ax)}{2a}$$

```
[In] Int[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (6*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^
2*x^2)]) - (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*Sqrt[1 - 1/(a^2*x^2)
]*x + (33*c^3*ArcCsc[a*x])/(2*a) - (6*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^6+\frac{6c^6x}{a}+\frac{16c^6x^2}{a^2}-\frac{6c^6x^3}{a^3}+\frac{c^6x^4}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{6c^6}{a}-\frac{16c^6x}{a^2}+\frac{6c^6x^2}{a^3}-\frac{c^6x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\text{Subst}\left(\int \frac{\frac{12c^6}{a^3}+\frac{33c^6x}{a^4}-\frac{12c^6x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^4\text{Subst}\left(\int \frac{-\frac{12c^6}{a^5}-\frac{33c^6x}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{(33c^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(6c^3)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{33c^3\csc^{-1}(ax)}{2a} + \frac{(3c^3)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{33c^3\csc^{-1}(ax)}{2a} - (6ac^3)\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{33c^3\csc^{-1}(ax)}{2a} - \frac{6c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.91

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$

$$= \frac{c^3\left(420a^2\sqrt{1+\frac{1}{ax}}x^2 - 3465a^3\sqrt{1+\frac{1}{ax}}x^3 + 16800a^4\sqrt{1+\frac{1}{ax}}x^4 + 17955a^5\sqrt{1+\frac{1}{ax}}x^5 - 32340a^6\sqrt{1+\frac{1}{a}}\right.}{}$$

```
[In] Integrate[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(420*a^2*Sqrt[1 + 1/(a*x)]*x^2 - 3465*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 1680
0*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 17955*a^5*Sqrt[1 + 1/(a*x)]*x^5 - 32340*a^6*S
qrt[1 + 1/(a*x)]*x^6 + 630*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 44730*a^5*Sqrt[1 - 1
```

```
/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 44730*a^6*Sqrt[1 - 1/(a*x)]
*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2520*a^5*Sqrt[1 - 1/(a*x)]*x^5*Arc
Sin[1/(a*x)] - 2520*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 3780*a^6*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] +
126*Sqrt[2]*a^2*x^2*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/2, 7/2,
 (1 - 1/(a*x))/2] + 90*Sqrt[2]*a*x*(-1 + a*x)^4*(1 + a*x)*Hypergeometric2F1
[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] - 70*Sqrt[2]*Hypergeometric2F1[3/2, 9/2, 1
1/2, (1 - 1/(a*x))/2] + 280*Sqrt[2]*a*x*Hypergeometric2F1[3/2, 9/2, 11/2, (
1 - 1/(a*x))/2] - 350*Sqrt[2]*a^2*x^2*Hypergeometric2F1[3/2, 9/2, 11/2, (1
- 1/(a*x))/2] + 350*Sqrt[2]*a^4*x^4*Hypergeometric2F1[3/2, 9/2, 11/2, (1 -
1/(a*x))/2] - 280*Sqrt[2]*a^5*x^5*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/
(a*x))/2] + 70*Sqrt[2]*a^6*x^6*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/(a*
x))/2]))/(630*a^6*Sqrt[1 - 1/(a*x)]*x^5*(1 + a*x))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(ax+1)(12ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(-\frac{6a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{33a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^2\sqrt{(ax-1)(ax+1)}+\frac{32a\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right.}}{x+\frac{1}{a}}\right.}{a^3(ax-1)}$
default	$-\frac{\left(-12\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+12(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-57\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4-33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^4x^4+12\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.\right.}{}$

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x+1)*(12*a*x-1)/x^2*c^3/a^3*((a*x-1)/(a*x+1))^(1/2)+(-6*a^3*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+33/2*a^2*arctan(1/(a^2*x^2-1)^(
1/2))+a^2*((a*x-1)*(a*x+1))^(1/2)+32*a/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2))*c^3/a^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\frac{66\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^3x^3+78}{2\,a^3x^2}$$

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(66*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2*c^3*x^2*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c^3)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^4x}{}\right.}{a^3}$$

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Inte
gral(-4*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(
6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-4*a**
3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(\frac{33\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{32\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right.}{\frac{(ax-1)a^2}{ax+1}}\right.$$

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(33*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 6*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 - 6*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 32*c^3*
sqrt((a*x - 1)/(a*x + 1))/a^2 + (11*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 6*c^3
*((a*x - 1)/(a*x + 1))^(3/2) - 13*c^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)
*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3
+ a^2))*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\int\left(c-\frac{c}{ax}\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{13\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{33\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,12{}\mathrm{i}}{a}$$

```
[In] int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] (13*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 1
1*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (32*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/a - (33*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^3*atan(
((a*x - 1)/(a*x + 1))^(1/2)*1i)*12i)/a
```

3.431 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{5c^2\csc^{-1}(ax)}{a}-\frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 5*c^2*arccsc(a*x)/a-5*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2
/(1-1/a^2/x^2)^(1/2)+c^2*(1-1/a^2/x^2)^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6312, 1819, 1821, 1823, 858, 222, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx=-\frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{5c^2\csc^{-1}(ax)}{a}$$

```
[In] Int[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/a + (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*
x^2)]) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x + (5*c^2*ArcCsc[a*x])/a - (5*c^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^5+\frac{5c^5x}{a}+\frac{5c^5x^2}{a^2}-\frac{c^5x^3}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{5c^5}{a}-\frac{5c^5x}{a^2}+\frac{c^5x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\text{Subst}\left(\int \frac{\frac{5c^5}{a^3}+\frac{5c^5x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}x + \frac{5c^2\csc^{-1}(ax)}{a} + \frac{(5c^2)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}x + \frac{5c^2\csc^{-1}(ax)}{a} - (5ac^2)\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}x + \frac{5c^2\csc^{-1}(ax)}{a} - \frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.04

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$

$$= \frac{c^2\left(-35a^2\sqrt{1+\frac{1}{ax}}x^2 + 315a^3\sqrt{1+\frac{1}{ax}}x^3 + 280a^4\sqrt{1+\frac{1}{ax}}x^4 - 595a^5\sqrt{1+\frac{1}{ax}}x^5 + 35a^6\sqrt{1+\frac{1}{ax}}x^6 + 910a\right.}{}$$

```
[In] Integrate[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-35*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 315*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 280*a
^4*Sqrt[1 + 1/(a*x)]*x^4 - 595*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 35*a^6*Sqrt[1 +
1/(a*x)]*x^6 + 910*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[
2]] + 910*a^5*Sqrt[1 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 105
*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[1/(a*x)] - 105*a^5*Sqrt[1 - 1/(a*x)]*x^5*
ArcSin[1/(a*x)] - 175*a^5*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^5*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]] + 7*Sqrt[2]*a*x*(-1 + a*x)^3*(1 + a*x)*Hypergeome
tric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 5*Sqrt[2]*(-1 + a*x)^4*(1 + a*x)*
Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2]))/(35*a^5*Sqrt[1 - 1/(a*x
)]*x^4*(1 + a*x))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.61

method	result
risch	$\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{5a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a\sqrt{(ax-1)(ax+1)}+5a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)c^2\sqrt{\frac{ax-}{ax-}}}{a^2(ax-1)}$
default	$-\frac{\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-7\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-5a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{}$

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+1)/x*c^2/a^2*((a*x-1)/(a*x+1))^(1/2)+(-5*a^2*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2-1)^(1/2))/(a^2)^(1/2)+a*((a*x-1)*(a*x+1))^(1/2)+5*a*arctan(1/(a^2*x^2-
1)^(1/2))+16/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*c^2/a^2/(a*x-1)*((a
*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2dx = -\frac{10\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+18\,ac^2x+c}{a^2x}$$

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(10*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 5*a*c^2*x*log(sqrt((a*x -
1)/(a*x + 1)) + 1) - 5*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^
2*x^2 + 18*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2dx = \frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Int
egral(3*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-3*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{10\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{16\,c^2\sqrt{\frac{ax-}{ax+}}}{a^2}\right.$$

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 10*
c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 5*c^2*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 - 5*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 16*c^2*sqrt(
(a*x - 1)/(a*x + 1))/a^2)*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \int \left(c-\frac{c}{ax}\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}} - \frac{10\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,10{}\mathrm{i}}{a}$$

```
[In] int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (16*c^2*((a*x - 1)/(a*x + 1))^(1/2))/a + (4*c^2*((a*x - 1)/(a*x + 1))^(1/2)
)/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (10*c^2*atan(((a*x - 1)/(a*x + 1))^(1
/2)))/a + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*10i)/a
```

3.432 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}-\frac{4c\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a-4*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2
/x^2)^(1/2)+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6312, 1819, 1821, 858, 222, 272, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{4c\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

```
[In] Int[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + c*Sqrt[1 - 1/(a^2*x^2)]*x
+ (c*ArcCsc[a*x])/a - (4*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^4+\frac{4c^4x}{a}+\frac{c^4x^2}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{4c^4}{a}-\frac{c^4x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{(4c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} + \frac{(2c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} - (4ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} - \frac{4c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.12

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$$
$$=\frac{5a^2cx^2\left((1+ax)\left(\sqrt{1+\frac{1}{ax}}(2-3ax+a^2x^2)+6a\sqrt{1-\frac{1}{ax}}x\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2a\sqrt{1-\frac{1}{ax}}x\arcsin\left(\frac{1}{ax}\right)\right.\right.}{5a^4\sqrt{}}$$

```
[In] Integrate[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*a^2*c*x^2*((1 + a*x)*(Sqrt[1 + 1/(a*x)]*(2 - 3*a*x + a^2*x^2) + 6*a*Sqrt
[1 - 1/(a*x)]*x*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*a*Sqrt[1 - 1/(a*x)]*x
*ArcSin[1/(a*x)]) - 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]]) + Sqrt[2]*c*(-1 + a*x)^3*(1 + a*x)*Hypergeometri
c2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2])/(5*a^4*Sqrt[1 - 1/(a*x)]*x^3*(1 + a*x
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(69) = 138.

Time = 0.14 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.95

method	result
default	$\frac{\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-4\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+4\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+2\sqrt{a^2x^2}\right.}{}$

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^
2-1)^(1/2))-4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a
^3*x^2+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+2*(a^2*x^2-1)^(1/2)*(a
^2)^(1/2)*a*x+2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-8*ln((a^2*x+(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-4*((a*x-1)*(a*x+1))^(3
/2)*(a^2)^(1/2)+8*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-4*a*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/a*c*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(acx+9\,c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 4*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a*c*x + 9*c)*sqrt((a*x -
1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^2x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a}$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-
2*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 4*c*sqrt((a*x - 1)/(a*x +
 1))/a^2)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,8\mathrm{i}}{a}$$

```
[In] int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*atan
(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*
8i)/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

3.433 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -2*arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(
1-1/a^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6312, 1819, 821, 272, 65, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}+\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/
c - (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int\frac{-c^2+\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c}+\frac{2\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c}+\frac{\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c}-\frac{(2a)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c}-\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int\frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(3+ax)-2(1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a(c+acx)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(3 + a*x) - 2*(1 + a*x)*Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x])/(a*(c + a*c*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(66) = 132.

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.90

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a\sqrt{a^2}}+\frac{2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^3\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-2\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+4\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a}}{a\sqrt{a^2}\,c\sqrt{(ax-1)(ax+1)}\,(ax-}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-2/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1
)^(1/2))/(a^2)^(1/2)+2/a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a/c*(
(a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{(ax+3)\sqrt{\frac{ax-1}{ax+1}}-2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] ((a*x + 3)*sqrt((a*x - 1)/(a*x + 1)) - 2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 2*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\left(\int\left(-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx+\int\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx\right)}{c}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(-x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Inte
gral(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{c-\frac{c}{ax}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (2*((a*
x - 1)/(a*x + 1))^(1/2))/(a*c) - (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*
c)
```

3.434 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c^2-(a-1/x)*x/a/c^2/(1-1/a^2/x^2)^(1/2)+2*x
*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6312, 837, 821, 272, 65, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}-\frac{x\left(a-\frac{1}{x}\right)}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - ((a - x^(-1))*x)/(a*c^2*Sqrt[1 - 1/(a^2*x
^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{c-\frac{cx}{a}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a^2\text{Subst}\left(\int \frac{\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-2+ax+a^2x^2-a\sqrt{1-\frac{1}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
[Out] (-2 + a*x + a^2*x^2 - a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(68) = 136.

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.86

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^2(ax-1)}$
default	$-\frac{\left(-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,ax+4\ln\left(\right.\right.}{2a\sqrt{a^2}\,c^2\sqrt{(ax-1)(ax+1)}\,(ax-}$

[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)

[Out] 1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+1/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^2/c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac^2}$$

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")

[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c^2)

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx\right)}{c^2}$$

[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)

[Out] a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x + 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x + 1), x))/c**2

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^2) + l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^2) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^2,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + ((a
*x - 1)/(a*x + 1))^(1/2)/(a*c^2) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(
a*c^2)
```

3.435 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -2/a^2/c^3/x/(1-1/a^2/x^2)^(1/2)+x/c^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6312, 277, 197}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -2/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x) + x/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2\text{Subst}\left(\int \frac{1}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^2c^3}$$

$$= -\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-2+a^2x^2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
[Out] (-2 + a^2*x^2)/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

method	result	size
trager	$\frac{(a^2x^2-2)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^3(ax-1)}$	41
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a^2x^2-2)(ax+1)}{a(ax-1)^2c^3}$	44
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a^3x^3+a^2x^2-2ax-2)}{a(ax-1)^2c^3}$	50
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3(ax-1)}$	59

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
[Out] 1/a/c^3*(a^2*x^2-2)/(a*x-1)*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{(a^2x^2-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3x-ac^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
[Out] (a^2*x^2 - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{a^3\left(\int\left(-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx+\int\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx\right)}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
[Out] a**3*(Integral(-x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*
x**3 + 2*a*x - 1), x) + Integral(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(41) = 82.

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{1}{2}\,a\left(\frac{\frac{5\,(ax-1)}{ax+1}-1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
[Out] -1/2*a*((5*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{\left(\frac{\sqrt{a^2x^2-1}}{c^3}-\frac{1}{\sqrt{a^2x^2-1}c^3}\right)\mathrm{sgn}(ax+1)}{a}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
[Out] (sqrt(a^2*x^2 - 1)/c^3 - 1/(sqrt(a^2*x^2 - 1)*c^3))*sgn(a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^2\,x^2-2}{\left(x\,a^2\,c^3+a\,c^3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^3,x)
[Out] (a^2*x^2 - 2)/((a*c^3 + a^2*c^3*x)*((a*x - 1)/(a*x + 1))^(1/2))
```

3.436 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a/c^4-1/3*a*x/c^4/(a-1/x)/(1-1/a^2/x^2)^(1/2)-
1/3*(4*a+3/x)*x/a/c^4/(1-1/a^2/x^2)^(1/2)+8/3*x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 871, 837, 821, 272, 65, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}-\frac{x\left(4a+\frac{3}{x}\right)}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*c^4) - (a*x)/(3*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a
 - x^(-1))) - ((4*a + 3/x)*x)/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) + ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]/(a*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
```

```
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{4c}{a^2}-\frac{3cx}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a^4\text{Subst}\left(\int \frac{-\frac{8c}{a^4}-\frac{3cx}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{8-5ax-7a^2x^2+3a^3x^3+3a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(97) = 194$.

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{4a^6\left(x+\frac{1}{a}\right)}-\frac{19\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{12a^6\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{6a^7\left(x-\frac{1}{a}\right)^2}\right)a^4\sqrt{\frac{ax-1}{ax+1}}}{c^4(ax-1)}$
default	$\frac{\left(24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4-21((ax-1)(ax+1))\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)+1/4/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1
9/12/a^6/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/6/a^7/(x-1/a)^2*((x-1/
a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^4*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-
1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-7\,a^2x^2-5\,ax+8}{3\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx\right)}{c^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*
x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x**5*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*
x**2 - 3*a*x + 1), x))/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^4*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^4) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4, x)
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^4}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^4) - ((17*(a*x - 1))/(3*(a*x + 1)) - (14
*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 4*a
*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^4)
```

3.437 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5}+\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

```
[Out] -2/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(5/2)+1/15*(-10*a-13/x)/a^2/c^5/(1-1/a^2
/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c^5+1/15*(-30*a-41/x)/a^2/c^5/
(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^5}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-2*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(5/2)) - (10*a + 13/x)/(15*a
^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) - (30*a + 41/x)/(15*a^2*c^5*Sqrt[1 - 1/(a^2
*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^5 + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
)/(a*c^5)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7} \\
&= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^2-\frac{10c^2x}{a}-\frac{8c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7} \\
&= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^2+\frac{30c^2x}{a}+\frac{26c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7} \\
&= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} \\
&\quad - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^2-\frac{30c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7} \\
&= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} \\
&\quad + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^5}
\end{aligned}
$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} + \frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*a*Sqrt[1 - 1/(a^2
*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^5*Sqrt[1 -
1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(122) = 244$.

Time = 0.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.88

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^5}+\frac{\left(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^5\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{10a^9\left(x-\frac{1}{a}\right)^3}-\frac{41\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{60a^8\left(x-\frac{1}{a}\right)^2}-\frac{383\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{120a^7\left(x-\frac{1}{a}\right)}+\frac{\sqrt{a^2(x-}}{8}\right.}{c^5(ax-1)}$
default	$-\frac{\left(-75\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^6x^6-60\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^7x^6+45((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+150\sqrt{(ax-1)(ax+1)}\,\sqrt{a}\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^5*((a*x-1)/(a*x+1))^(1/2)+(2/a^5*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/10/a^9/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2
)-41/60/a^8/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-383/120/a^7/(x-1/a)
*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)+1/8/a^7/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/
a))^(1/2))*a^5/c^5/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^4x}{15\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 76*a^3*x^3 + 32*a^2*x^2 + 82*a*x - 56)*sqrt((a*x - 1)/(a*
x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{a^5\left(\int \left(-\frac{x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int \frac{ax^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx\right)}{c^5}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**5,x)
[Out] a**5*(Integral(-x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*
x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x**6*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*
x**2 + 4*a*x - 1), x))/c**5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{1}{120}\,a\left(\frac{\frac{32\,(ax-1)}{ax+1}+\frac{310\,(ax-1)^2}{(ax+1)^2}-\frac{585\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^5}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^5}+\frac{15\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
[Out] 1/120*a*((32*(a*x - 1)/(a*x + 1) + 310*(a*x - 1)^2/(a*x + 1)^2 - 585*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^5*((a*x
 - 1)/(a*x + 1))^(5/2)) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^5)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^5) + 15*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*c^5))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{\sqrt{\frac{ax-1}{ax+1}}}{8ac^5} - \frac{\frac{62(ax-1)^2}{3(ax+1)^2} - \frac{39(ax-1)^3}{(ax+1)^3} + \frac{32(ax-1)}{15(ax+1)} + \frac{1}{5}}{8ac^5\left(\frac{ax-1}{ax+1}\right)^{5/2} - 8ac^5\left(\frac{ax-1}{ax+1}\right)^{7/2}} + \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{ac^5}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^5,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(8*a*c^5) - ((62*(a*x - 1)^2)/(3*(a*x + 1)^2) -
 (39*(a*x - 1)^3)/(a*x + 1)^3 + (32*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(8*a*c
^5*((a*x - 1)/(a*x + 1))^(5/2) - 8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2)) + (4*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^5)
```

3.438 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 235

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2} dx = \frac{173c^5\sqrt{1-\frac{1}{a^2x^2}}}{105a\sqrt{c-\frac{c}{ax}}}+\frac{227c^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{105a} + \frac{59c^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35a}+\frac{9c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7a} + c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{7c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -7*c^(9/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+59/35*c^3
*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+9/7*c^2*(c-c/a/x)^(5/2)*(1-1/a^2/x^2
)^(1/2)/a+c*(c-c/a/x)^(7/2)*x*(1-1/a^2/x^2)^(1/2)+173/105*c^5*(1-1/a^2/x^2)
^(1/2)/a/(c-c/a/x)^(1/2)+227/105*c^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{6317, 6314, 99, 158, 152, 65, 214\}$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{x\left(a-\frac{1}{x}\right)^4\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(400a-\frac{227}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{7\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] ((400*a - 227/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(105*a^2*(1 - 1/(a*
x))^(9/2)) + (59*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(35*
a^3*(1 - 1/(a*x))^(9/2)) + (9*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x)
)^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c
 - c/(a*x))^(9/2)*x)/(a^4*(1 - 1/(a*x))^(9/2)) - (7*(c - c/(a*x))^(9/2)*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
```

```
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}\\
&= \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{Subst}\left(\int\frac{\left(-\frac{7}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}
\end{aligned}$$

$$= \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{49}{4a^2}-\frac{59x}{4a^3}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(4a^2\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{245}{8a^3}-\frac{227x}{8a^4}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{35\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(7\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(7\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(400a - \frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{7\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-30+162ax-356a^2x^2+292a^3x^3+105a^4x^4)-735a^3x^3\operatorname{arctanh}\left(\sqrt{\right.\right.}{105a^4\sqrt{1-\frac{1}{ax}}x^3}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-30 + 162*a*x - 356*a^2*x^2 + 29
2*a^3*x^3 + 105*a^4*x^4) - 735*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(105*a^
4*Sqrt[1 - 1/(a*x)]*x^3)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(210a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4+584a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-735\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4-712a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+324a^{\frac{3}{2}}x\sqrt{(a\right.}{210\sqrt{\frac{ax-1}{ax+1}}\,x^3a^{\frac{9}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(105a^5x^5+397a^4x^4-64a^3x^3-194a^2x^2+132ax-30)c^4\sqrt{\frac{c(ax-1)}{ax}}}{105x^3a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)} - \frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^4*(210*a^(9/2)*((a*x+
1)*x)^(1/2)*x^4+584*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-735*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-712*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+
324*a^(3/2)*x*((a*x+1)*x)^(1/2)-60*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/2)/(
(a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.86

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \left[\frac{735\,(a^4c^4x^4-a^3c^4x^3)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(105\,a^5c}{420\,(a^5x^4-a^4x^3}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(105*a^5*c^4*x^5 + 397*a^4*c^4*x^4 -
64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x - 30*c^4)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/210*(735*(a^4*c^4*x
^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(105*a^5
*c^4*x^5 + 397*a^4*c^4*x^4 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x
 - 30*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^
4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.439 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{49c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}}+\frac{31c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a}+\frac{7c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{5c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -5*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+7/5*c^2*(
c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+c*(c-c/a/x)^(5/2)*x*(1-1/a^2/x^2)^(1/2
)+49/15*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+31/15*c^3*(1-1/a^2/x^2)^(
1/2)*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6317, 6314, 99, 158, 152, 65, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{5\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] ((80*a - 31/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2*(1 - 1/(a*x))
^(7/2)) + (7*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(
1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)
*x)/(a^3*(1 - 1/(a*x))^(7/2)) - (5*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{Subst}\left(\int\frac{\left(-\frac{5}{2a}-\frac{7x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= \frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}\\
&\quad-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int\frac{\left(-\frac{25}{4a^2}-\frac{31x}{4a^3}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= \frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}\\
&\quad+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(5\left(c-\frac{c}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{7/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} \\
&\quad+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}} \\
&\quad+\frac{\left(5\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}} \\
&= \frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} \\
&\quad+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{7/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52

$$
\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(6-28ax+56a^2x^2+15a^3x^3)-75a^2x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}x^2}
$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^3
*x^3) - 75*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}+112a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-75\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3-56a^{\frac{3}{2}}x\sqrt{(ax+1)x}+12\sqrt{(ax+1)x}\,\sqrt{a}\right)}{30\sqrt{\frac{ax-1}{ax+1}}\,x^2a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(15a^4x^4+71a^3x^3+28a^2x^2-22ax+6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/30/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+112*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-75*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3-56*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/((a*x+1)*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.12

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{75\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(15\,a^4c^3x^4\cdots}{60\,(a^4x^3-a^3x^2)}\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")

[Out] [1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{7/2} dx = \int \frac{\left(c - \frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}} dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{7/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{7/2} dx = \int \frac{\left(c - \frac{c}{a\,x}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}} dx$$

```
[In] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.440 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -2/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^4*(1-1/a^2/x^2)^(3/2)*x/(c
-c/a/x)^(3/2)-3*c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
)/a+3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6317, 6314, 91, 81, 52, 65, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{3\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(a*(1 - 1/(a*x))^(5/2)) - (2*(1 +
 1/(a*x))^(3/2)*(c - c/(a*x))^(5/2))/(3*a*(1 - 1/(a*x))^(5/2)) + ((1 + 1/(a
*x))^(3/2)*(c - c/(a*x))^(5/2)*x)/(1 - 1/(a*x))^(5/2) - (3*(c - c/(a*x))^(5
/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{\left(-\frac{3}{2a}+\frac{x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&\quad+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&\quad+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&\quad+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}
\end{aligned}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+10ax+3a^2x^2)-9ax\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(-6a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+9\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2-20a^{\frac{3}{2}}x\sqrt{(ax+1)x}+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,x\,a^{\frac{5}{2}}\sqrt{(ax+1)x}}$	132
risch	$\frac{(3a^3x^3+13a^2x^2+8ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)} - \frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^2\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	168

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-6*a^(5/2
)*x^2*((a*x+1)*x)^(1/2)+9*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1
/2))*a^2*x^2-20*a^(3/2)*x*((a*x+1)*x)^(1/2)+4*((a*x+1)*x)^(1/2)*a^(1/2))/((
a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.43

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{9\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3+1\right.}{12\left(a^3x^2-a^2x\right)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x
- 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*
x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 2*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x - 2*c^2)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.441 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)-c^(3/2)*arctanh(c^(1/2)*(1-1/a^2/
x^2)^(1/2)/(c-c/a/x)^(1/2))/a+c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6312, 893, 879, 889, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2))^
(3/2)*x)/(c - c/(a*x))^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c^2\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}
\end{aligned}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c^3\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+ax)-\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + a*x) - ArcTanh[Sqrt[1 + 1/(a*x
)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}\sqrt{(ax+1)x}}$	105
risch	$\frac{(a^2x^2+3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)} - \frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-2*a^(3/2)*x*
((a*x+1)*x)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x
-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.68

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\left[\frac{(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2+3\,acx+2}{4\,(a^2x-a)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*((a*c*x - c)*sqrt(-c)*arctan(2*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*
c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.442 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6312, 877, 889, 214}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)x}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\right.$$
$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.443 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 3*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-2*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)+x*(1-1/a
/x)^(1/2)*(1+1/a/x)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6317, 6314, 101, 162, 65, 214, 212}

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{3\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}+\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - c/(a*x)] + (3*Sqrt[1 - 1/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) - (2*Sqrt[2]*Sqrt[1
 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c - c/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-\frac{1}{ax}}\text{Subst}\left(\int \frac{\frac{3}{2a}+\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\sqrt{1-\frac{1}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(\sqrt{1+\frac{1}{ax}}x+\frac{3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
[Out] (Sqrt[1 - 1/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
- (2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[c - c/(a*x)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}c\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{(ax+1)c\cdots}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(3/2)*(1/a)^(1/2)-2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3
*a*x+1)/(a*x-1))*a^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a*(1/a)^(1/2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.40

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$
$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\frac{2\sqrt{2}\cdots}{\cdots}}{4\,(a^2cx-ac)}\cdots\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c*x - a*c), 1/2*(2*sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2 - 2*a*x - 1)) - 3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)

[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))), x)

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.444 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 215

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 5*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-7/2*(1-1/a/x)^
(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*(1-1
/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a*(1-1/a/x)^(3/2)*x*(1+
1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6317, 6314, 101, 156, 162, 65, 214, 212}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{5\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*(c - c/(a*x))^(3/2
)) + (a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*x)
)^(3/2)) + (5*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*
x))^(3/2)) - (7*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sq
rt[2]*a*(c - c/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\\
&\quad+\frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{-\frac{5}{a^2}-\frac{2x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\left(c-\frac{c}{ax}\right)^{3/2}}
\end{aligned}
$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{3/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\left(1-\frac{1}{ax}\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-2+ax)+10(-1+ax)\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-7\sqrt{2}(-1+ax)\text{a}\right.}{2ac\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-2 + a*x) + 10*(-1 + a*x)*ArcT
anh[Sqrt[1 + 1/(a*x)]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqr
t[2]]))/(2*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-7a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x-8\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+10\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)c}{4\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)a^{\frac{3}{2}}c^{2}\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{4}c\left(x-\frac{1}{a}\right)}-\frac{7\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}}}{x-\frac{1}{a}}\right.}{4a^{3}\sqrt{c}}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}$

[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x-7*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-8*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+7*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/c^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.76

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{7\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")

[Out] [1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/4*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*

```
sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2
*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 4*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.445 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 277

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{7\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{79\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{8\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 7*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-79/16*(1-1/a/x
)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*
a*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-23/8*(1-1/a/x)^
(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^2*(1-1/a/x)^(5/2)*x*(1+1/a/
x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{6317, 6314, 101, 156, 162, 65, 214, 212\}$

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{7\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{79\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{8\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))^2*(c - c/(a*x)
)^(5/2)) - (23*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(8*(a - x^(-1))*(c -
c/(a*x))^(5/2)) + (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1
))^2*(c - c/(a*x))^(5/2)) + (7*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)
]])/(a*(c - c/(a*x))^(5/2)) - (79*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]/Sqrt[2]])/(8*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}
\end{aligned}$$

$$=-\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{-\frac{14}{a^2}-\frac{9x}{a^3}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(a^2\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{\frac{28}{a^3}+\frac{23x}{2a^4}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(79\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(7\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(7\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(79\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{8a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{79\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{8\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(23-35ax+8a^2x^2)+112(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-79\right.}{16ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(23 - 35*a*x + 8*a^2*x^2) + 112
*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 79*Sqrt[2]*(-1 + a*x)^2*ArcTanh[
Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(16*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.18

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{2a^6c\left(x-\frac{1}{a}\right)^2}-\frac{19\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{8a^5c\left(x-\frac{1}{a}\right)}-\frac{79\sqrt{2}\,\mathrm{l}}{}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(ax-}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-79a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2-140\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+112\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{}}{2\sqrt{}}\right.\right.}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/c^2/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(7/2/a^3*ln((
1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/2/a
^6/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-19/8/a^5/c/(x-1/a)
*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-79/32/a^4/c^(1/2)*2^(1/2)*ln((4*
c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)
)/(x-1/a)))*a^2/c^2/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)
*((a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.41

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{79\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\cdots}}{a^3x^3-3a^2x^2+3ax-1}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*
a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(8*a^4*x^4 - 27*a^
3*x^3 - 12*a^2*x^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), 1/32*(79*sqrt(2)*
(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*
a^2*c*x^2 - a*c*x - c)) + 4*(8*a^4*x^4 - 27*a^3*x^3 - 12*a^2*x^2 + 23*a*x)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3
*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.446 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 143

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{5c^{9/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 5/3*c^3*(c-c/a/x)^(3/2)/a+c^2*(c-c/a/x)^(5/2)/a+5/7*c*(c-c/a/x)^(7/2)/a+(c-
c/a/x)^(9/2)*x-5*c^(9/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+5*c^4*(c-c/a/x)
^(1/2)/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=-\frac{5c^{9/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+x\left(c-\frac{c}{ax}\right)^{9/2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (5*c^4*Sqrt[c - c/(a*x)])/a + (5*c^3*(c - c/(a*x))^(3/2))/(3*a) + (c^2*(c -
 c/(a*x))^(5/2))/a + (5*c*(c - c/(a*x))^(7/2))/(7*a) + (c - c/(a*x))^(9/2)*
x - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{9/2}x+\frac{(5c)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{\left(5c^3\right)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} \\
&\quad + \left(c-\frac{c}{ax}\right)^{9/2} x + \frac{(5c^4)\operatorname{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{5c^4\sqrt{c-\frac{c}{ax}}}{a} + \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} \\
&\quad + \left(c-\frac{c}{ax}\right)^{9/2} x + \frac{(5c^5)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{5c^4\sqrt{c-\frac{c}{ax}}}{a} + \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} \\
&\quad + \left(c-\frac{c}{ax}\right)^{9/2} x - (5c^4)\operatorname{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) \\
&= \frac{5c^4\sqrt{c-\frac{c}{ax}}}{a} + \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} \\
&\quad + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2} x - \frac{5c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2} dx = \frac{c^4\sqrt{c-\frac{c}{ax}}(6-18ax+4a^2x^2+92a^3x^3+21a^4x^4)}{21a^4x^3} - \frac{5c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
[Out] (c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))/
(21*a^4*x^3) - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\left(21 a^{5} x^{5}+71 a^{4} x^{4}-88 a^{3} x^{3}-22 a^{2} x^{2}+24 a x -6\right) c^{4} \sqrt{\frac{c \left(a x -1\right)}{a x}}}{21 x^{3} a^{4} \left(a x -1\right)}-\frac{5 \ln \left(\frac{-\frac{1}{2} a c +a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-a c x}\right) c^{4} \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \sqrt{c \left(a x -1\right) a x}}{2 \sqrt{a^{2} c}\, \left(a x -1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a x -1\right)}{a x}}\, c^{4} \left(-210 a^{\frac{9}{2}} \sqrt{a \,x^{2}-x}\, x^{5}+105 \ln \left(\frac{2 \sqrt{a \,x^{2}-x}\, \sqrt{a}+2 a x -1}{2 \sqrt{a}}\right) a^{4} x^{5}+168 a^{\frac{7}{2}} \left(a \,x^{2}-x \right)^{\frac{3}{2}} x^{3}-16 a^{\frac{5}{2}} \left(a \,x^{2}-x \right)^{\frac{3}{2}} x^{2}-24 a^{\frac{3}{2}} \left(a \,x^{2}-x \right)\right.}{42 x^{4} \sqrt{\left(a x -1\right) x}\, a^{\frac{9}{2}}}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/21*(21*a^5*x^5+71*a^4*x^4-88*a^3*x^3-22*a^2*x^2+24*a*x-6)/x^3*c^4/a^4/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)*c^4/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.64

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \left[\frac{105\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(21\,a^4c^4x^4+92\,a^3c^4x^3+4\,a^2c^4x^2-18\,ac\right.}{42\,a^4x^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")

[Out] [1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), 1/21*(105*a^3*sqrt(-c)*c^4*x^3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3)]

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.54 (sec) , antiderivative size = 2222, normalized size of antiderivative = 15.54

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(9/2),x)
```

```
[Out] c**4*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*
x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1)
+ I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x
+ 1)), True)) + 2*c**4*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt
(-c) + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a + 2*c**
4*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5
/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(
5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) -
15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a*x - 1)/(15*a**(7/2)*x**(
7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sqrt(a*x - 1)/(15*a**(7/2)*
x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*sqrt(a*x - 1)/(15*a**(7/2
)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*a**(11/2)*sqrt(c)*x*
*(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x
**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*I*a**5*sqrt(c)*x*
*3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 2*I*a**4*
sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) -
 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(
5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)
*x**(5/2)), True))/a**3 - c**4*Piecewise((-16*a**(19/2)*sqrt(c)*x**(13/2)/(
105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) -
 105*a**(7/2)*x**(7/2)) + 48*a**(17/2)*sqrt(c)*x**(11/2)/(105*a**(13/2)*x**
(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**
(7/2)) - 48*a**(15/2)*sqrt(c)*x**(9/2)/(105*a**(13/2)*x**(13/2) - 315*a**(1
1/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 16*a**(13
/2)*sqrt(c)*x**(7/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 3
15*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 16*a**9*sqrt(c)*x**6*sqrt(a
*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x
**(9/2) - 105*a**(7/2)*x**(7/2)) - 40*a**8*sqrt(c)*x**5*sqrt(a*x - 1)/(105*
a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105
*a**(7/2)*x**(7/2)) + 30*a**7*sqrt(c)*x**4*sqrt(a*x - 1)/(105*a**(13/2)*x**
(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**
(7/2)) - 40*a**6*sqrt(c)*x**3*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*
a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 100*
a**5*sqrt(c)*x**2*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x*
*(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) - 96*a**4*sqrt(c)*
```

```
x*sqrt(a*x - 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a*
*(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 30*a**3*sqrt(c)*sqrt(a*x - 1)/(1
05*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) -
105*a**(7/2)*x**(7/2)), Abs(a*x) > 1), (-16*a**(19/2)*sqrt(c)*x**(13/2)/(10
5*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 1
05*a**(7/2)*x**(7/2)) + 48*a**(17/2)*sqrt(c)*x**(11/2)/(105*a**(13/2)*x**(1
3/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7
/2)) - 48*a**(15/2)*sqrt(c)*x**(9/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/
2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 16*a**(13/2
)*sqrt(c)*x**(7/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315
*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 16*I*a**9*sqrt(c)*x**6*sqrt(-
a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*
x**(9/2) - 105*a**(7/2)*x**(7/2)) - 40*I*a**8*sqrt(c)*x**5*sqrt(-a*x + 1)/(
105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) -
 105*a**(7/2)*x**(7/2)) + 30*I*a**7*sqrt(c)*x**4*sqrt(-a*x + 1)/(105*a**(13
/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7
/2)*x**(7/2)) - 40*I*a**6*sqrt(c)*x**3*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13
/2) - 315*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/
2)) + 100*I*a**5*sqrt(c)*x**2*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315
*a**(11/2)*x**(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) - 96*
I*a**4*sqrt(c)*x*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x*
*(11/2) + 315*a**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)) + 30*I*a**3*sqrt(c
)*sqrt(-a*x + 1)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + 315*a
**(9/2)*x**(9/2) - 105*a**(7/2)*x**(7/2)), True))/a**4
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1), x)
```

3.447 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-3*c^(7/2)*a
rctanh((c-c/a/x)^(1/2)/c^(1/2))/a+3*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=-\frac{3c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (3*c^3*Sqrt[c - c/(a*x)])/a + (c^2*(c - c/(a*x))^(3/2))/a + (3*c*(c - c/(a*
x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a
*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$$

$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{x}\,dx}{a}$$

$$= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c^2)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= \frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c^3)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$
\begin{aligned}
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} \\
&\quad + \left(c-\frac{c}{ax}\right)^{7/2} x + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} \\
&\quad + \left(c-\frac{c}{ax}\right)^{7/2} x - (3c^3)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) \\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2} x - \frac{3c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2} dx = \frac{c^3\sqrt{c-\frac{c}{ax}}(-2+4ax+8a^2x^2+5a^3x^3)}{5a^3x^2} - \frac{3c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
[Out] (c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) -
(3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(-30\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x^4+20a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2+15\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4+4a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x-4\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\right)}{10x^3\sqrt{(ax-1)x}\,a^{\frac{7}{2}}}$
risch	$\frac{(5a^4x^4+3a^3x^3-4a^2x^2-6ax+2)c^3\sqrt{\frac{c(ax-1)}{ax}}}{5x^2a^3(ax-1)}-\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^2c}\,(ax-1)}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^(
5/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*a^3*x^4+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1
)*x)^(1/2)/a^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.80

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\left[\frac{15\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(5\,a^3c^3x^3+8\,a^2c^3x^2+4\,ac^3x-2\,c^3)\sqrt{\frac{acx-c}{ax}}}{10\,a^3x^2}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*
x - c)/(a*x)))/(a^3*x^2), 1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3
)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.54 (sec) , antiderivative size = 740, normalized size of antiderivative = 6.27

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=c^3\left(\begin{cases}-\frac{\sqrt{c}\,\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for } |ax|>1\\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\,\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\frac{c^3\left(\begin{cases}\frac{2c\,\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}} & \text{for } \frac{c}{a}\neq 0\\ -\sqrt{c}\log(x) & \text{otherwise}\end{cases}\right)}{a}-\frac{c^3\left(\begin{cases}0 & \text{for } c=0\\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^2}$$

$$+\frac{c^3\left(\begin{cases}-\frac{4a^{\frac{11}{2}}\sqrt{c}x^{\frac{7}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4a^{\frac{9}{2}}\sqrt{c}x^{\frac{5}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4a^5\sqrt{c}x^3\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{2a^4\sqrt{c}x^2\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{8a^3\sqrt{c}x\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{6a^2\sqrt{c}\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}\\ -\frac{4a^{\frac{11}{2}}\sqrt{c}x^{\frac{7}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4a^{\frac{9}{2}}\sqrt{c}x^{\frac{5}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4ia^5\sqrt{c}x^3\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{2ia^4\sqrt{c}x^2\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{8ia^3\sqrt{c}x\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{6ia^2\sqrt{c}\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x}\end{cases}\right.}{a^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(7/2),x)
```

```
[Out] c**3*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*
x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1)
+ I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x
+ 1)), True)) + c**3*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-
c) + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a - c**3*Pi
ecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)/(3*c), True))/a**2 + c**3*
Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2
)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/
2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15
*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/
2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sqrt(a*x - 1)/(15*a**(7/2)*x*
*(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*sqrt(a*x - 1)/(15*a**(7/2)*
x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*a**(11/2)*sqrt(c)*x**(
7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**
(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*I*a**5*sqrt(c)*x**3
*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 2*I*a**4*sq
rt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8
*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/
2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x
**(5/2)), True))/a**3
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.448 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-c^(5/2)*arctanh((c-c/a/x)^(1/2)/c
^(1/2))/a+c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x))^(
5/2)*x - (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$

$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{x}\,dx}{a}$$

$$= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{c^2\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{c^3\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x - c^2\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x - \frac{c^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx = \frac{c^2\sqrt{c-\frac{c}{ax}}(2-2ax+3a^2x^2) - 3ac^{5/2}x\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 2*a*x + 3*a^2*x^2) - 3*a*c^(5/2)*x*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(-6\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x^3+3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^3+4\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^2\sqrt{(ax-1)x}\,a^{\frac{5}{2}}}$	108
risch	$\frac{(3a^3x^3-5a^2x^2+4ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)} - \frac{\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)c^2\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^2c}\,(ax-1)}$	139

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/2
*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^3+4*(a*x^2-x)^(3/2)*a^(
1/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx = \left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^2c^2x^2-2\,ac^2x+2\,c^2\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^2x}, \frac{3\,a\sqrt{-c}c^2x}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c) + 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)
, 1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (3*a
^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = c^2\left(\begin{cases} -\frac{\sqrt{c}\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for } |ax|>1 \\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) - \frac{c^2\left(\begin{cases} 0 & \text{for } c=0 \\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right)}{a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(5/2),x)
```

```
[Out] c**2*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*
x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1)
+ I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x
+ 1)), True)) - c**2*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)/(3*
c), True))/a**2
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.449 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] (c-c/a/x)^(3/2)*x+c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-c*(c-c/a/x)^(1
/2)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x + (c^(3/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\sqrt{c-\frac{cx}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+c\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}(-2+ax)+c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]

[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) + c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/a

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\left(-2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}x^{2}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}\right)}{2x\sqrt{(ax-1)x}\,a^{\frac{3}{2}}}$	103
risch	$\frac{\left(a^{2}x^{2}-3ax+2\right)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	122

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^(
3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*x^2)/((a
*x-1)*x)^(1/2)/a^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\ -\frac{\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2
*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(sqrt(-c)*c*arctan(sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.47

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=c\left(\begin{cases}-\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for }|ax|>1\\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)-\frac{c\left(\begin{cases}\frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}} & \text{for }\frac{c}{a}\neq 0\\ -\sqrt{c}\log(x) & \text{otherwise}\end{cases}\right)}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(3/2),x)
```

```
[Out] c*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*x -
 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1) + I
*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x + 1
)), True)) - c*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 2
*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/(a*x - 1), x)

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}{a\,x-1}\,dx$$

[In] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1),x)

[Out] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1), x)

3.450 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 382, 79, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+x\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \sqrt{c-\frac{c}{ax}}x-\frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \sqrt{c-\frac{c}{ax}}x+3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= \sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

method	result	size
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,\sqrt{a}-4\sqrt{(ax-1)x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)x}\,\sqrt{a}}$	120

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*(a*x-1)/a/x)^(1/2)+3/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-
a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c
)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{3\sqrt{c}\log\left(|a||c|\right)\operatorname{sgn}(x)}{2a}-\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{2a\operatorname{sgn}(x)}+\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(a^
2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.451 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)-5/a/(c-c/a/x)^(1/2)+x/(c-c/a/x
)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5}{a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] -5/(a*Sqrt[c - c/(a*x)]) + x/Sqrt[c - c/(a*x)] + (5*ArcTanh[Sqrt[c - c/(a*x
)]/Sqrt[c]])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
```

```
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx\\
&= -\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}x}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{(5c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{c}\\
&= -\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{ax-5\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
[Out] (a*x - 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*x)]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(60) = 120.

Time = 0.54 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.14

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a\sqrt{a^2c}}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{a^3c\left(x-\frac{1}{a}\right)}\right)\sqrt{c(ax-1)ax}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(10a^{\frac{5}{2}}\sqrt{(ax-1)x}\,x^2+5\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^2-8a^{\frac{3}{2}}((ax-1)x)^{\frac{3}{2}}-20\sqrt{(ax-1)x}\,a^{\frac{3}{2}}x-10\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax\right.}{2\sqrt{a}}\right.}{2\sqrt{(ax-1)x}\,c(ax-1)^2\sqrt{a}}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x-1)/(c*(a*x-1)/a/x)^(1/2)+(5/2/a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2
)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/a^3/c/(x-1/a)*(a^2*c*(x-1/a)^2+(
x-1/a)*a*c)^(1/2))/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{5\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(a^2x^2-5\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^2cx-ac)}, -\frac{5\,(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(a^2x^2-5\,ax)\sqrt{\frac{acx-c}{ax}}}{a^2cx-ac}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] [1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c), -(
5*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x^2
- 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(1/2),x)
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a*x))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(60) = 120$.

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{5\,\log\left(c^2|a|\right)\mathrm{sgn}(x)}{6\,a\sqrt{c}} - \frac{5\,\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3\sqrt{c}|a|-5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^2ac+4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-a\cdots}\right.\right.\right.}{6\,a\sqrt{c}} + \frac{\sqrt{a^2cx^2-acx}|a|\mathrm{sgn}(x)}{a^2c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 5/6*log(c^2*abs(a))*sgn(x)/(a*sqrt(c)) - 5/6*log(abs(2*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - a*c*x))^3*sqrt(c)*abs(a) - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- a*c*x))^2*a*c + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(3/2)*abs(a
) - a*c^2))*sgn(x)/(a*sqrt(c)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2
*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{a\,x+1}{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)), x)
```

3.452 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -7/3/a/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(3/2)+7*arctanh((c-c/a/x)^(1/2)/c^(1/2))
/a/c^(3/2)-7/a/c/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{7\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -7/(3*a*(c - c/(a*x))^(3/2)) - 7/(a*c*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^
(3/2) + (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
```

```
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx\\
&= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}x}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(7c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac}
\end{aligned}$$

$$= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{x\left(3ax - 7\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, 1-\frac{1}{ax}\right)\right)}{3c\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
[Out] (x*(3*a*x - 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c -
 c/(a*x)]*(-1 + a*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(81) = 162$.

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.12

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}} + \frac{\left(\frac{7\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^2\sqrt{a^2c}} - \frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{3a^5c\left(x-\frac{1}{a}\right)^2} - \frac{22\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{3a^4c\left(x-\frac{1}{a}\right)}\right)a\sqrt{c(ax-1)ax}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(42a^{\frac{7}{2}}\sqrt{(ax-1)x}\,x^3+21\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^3-36a^{\frac{5}{2}}\left((ax-1)x\right)^{\frac{3}{2}}x-126a^{\frac{5}{2}}\sqrt{(ax-1)x}\,x^2-63\ln\left(\frac{2\sqrt{(ax-1)x}\ldots}{2\sqrt{\ldots}}\right.\ldots}{6\sqrt{(ax-\ldots}}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x-1)/c/(c*(a*x-1)/a/x)^(1/2)+(7/2/a^2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^
(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/3/a^5/c/(x-1/a)^2*(a^2*c*(x-
1/a)^2+(x-1/a)*a*c)^(1/2)-22/3/a^4/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^
(1/2))*a/c/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^3x^3-28\,a^2x^2+21\right.}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right.$$

$$\left.-\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c
)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(21*(a^2*x^2 - 2*a*x +
1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 28*a^
2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2
)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\left(ax-1\right)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{ax+1}{\left(ax-1\right)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(81) = 162.

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{7\,\log\left(c^2|a||c|\right)\operatorname{sgn}(x)}{10\,ac^{\frac{3}{2}}}$$

$$-\frac{7\,\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5\sqrt{c}|a|-9\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4ac+16\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-ac\right.\right.\right.}{10\,ac^{\frac{3}{2}}}$$

$$+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] 7/10*log(c^2*abs(a)*abs(c))*sgn(x)/(a*c^(3/2)) - 7/10*log(abs(2*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - a*c*x))^5*sqrt(c)*abs(a) - 9*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - a*c*x))^4*a*c + 16*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*c
^(3/2)*abs(a) - 14*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^2 + 6*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(5/2)*abs(a) - a*c^3))*sgn(x)/(a*
c^(3/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)), x)
```

3.453 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{9\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

```
[Out] -9/5/a/(c-c/a/x)^(5/2)-3/a/c/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(5/2)+9*arctanh((c
-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)-9/a/c^2/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{9\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] -9/(5*a*(c - c/(a*x))^(5/2)) - 3/(a*c*(c - c/(a*x))^(3/2)) - 9/(a*c^2*Sqrt[
c - c/(a*x)]) + x/(c - c/(a*x))^(5/2) + (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/(a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\operatorname{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{(9c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, 1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
[Out] x/(c - c/(a*x))^(5/2) - (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)])/(
5*a*(c - c/(a*x))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(102) = 204.

Time = 0.54 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.09

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^3\sqrt{a^2c}}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{5a^7c\left(x-\frac{1}{a}\right)^3}-\frac{18\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{5a^6c\left(x-\frac{1}{a}\right)^2}-\frac{54\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\right.}}{5a^5c\left(x-\frac{1}{a}\right)}\right.}{c^2x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(90a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^4+45\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^4-80a^{\frac{7}{2}}((ax-1)x)^{\frac{3}{2}}x^2-360a^{\frac{7}{2}}\sqrt{(ax-1)x}\,x^3-180\ln\left(\frac{2\sqrt{(ax-}}{}\right.\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/a*(a*x-1)/c^2/(c*(a*x-1)/a/x)^(1/2)+(9/2/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c
)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/5/a^7/c/(x-1/a)^3*(a^2*c*(
x-1/a)^2+(x-1/a)*a*c)^(1/2)-18/5/a^6/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+(x-1/a)*a
c)^(1/2)-54/5/a^5/c/(x-1/a)(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2))*a^2/c^2/x
/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{45\,(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(5\,a^4x^4-69\,a}{10\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}\right.$$
$$\left.-\frac{45\,(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(5\,a^4x^4-69\,a^3x^3+105\,a^2x^2-45\,ax)\sqrt{\frac{acx-c}{ax}}}{5\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}\right]$$

[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")

[Out] [1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^
2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c
^3*x - a*c^3), -1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^2
- 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)]

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(5/2),x)

[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(5/2)), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(102) = 204$.

Time = 0.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{9\log\left(c^4|a|\right)\operatorname{sgn}(x)}{14\,ac^{\frac{5}{2}}}$$

$$-\frac{9\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7\sqrt{c}|a|-13\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6ac+36\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-ac\right.\right.\right.}{}$$

$$+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")

[Out] 9/14*log(c^4*abs(a))*sgn(x)/(a*c^(5/2)) - 9/14*log(abs(2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*sqrt(c)*abs(a) - 13*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a*c + 36*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^(3/2)*abs(a) - 55*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^2 + 50*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*c^(5/2)*abs(a) - 27*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^3 + 8*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(7/2)*abs(a) - a*c^4))*sgn(x)/(a*c^(5/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2*c^3)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)), x)
```

3.454 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

```
[Out] -11/7/a/(c-c/a/x)^(7/2)-11/5/a/c/(c-c/a/x)^(5/2)-11/3/a/c^2/(c-c/a/x)^(3/2)
+x/(c-c/a/x)^(7/2)+11*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(7/2)-11/a/c^3/(
c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{11\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] -11/(7*a*(c - c/(a*x))^(7/2)) - 11/(5*a*c*(c - c/(a*x))^(5/2)) - 11/(3*a*c^
2*(c - c/(a*x))^(3/2)) - 11/(a*c^3*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^(7/
2) + (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{9/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{(11c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^3}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^4}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{7x - \frac{11\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\frac{1}{ax}\right)}{a}}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
[Out] (7*x - (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*x
))^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(123) = 246.

Time = 0.54 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.01

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^4\sqrt{a^2c}}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{7a^9c\left(x-\frac{1}{a}\right)^4}-\frac{102\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{35a^8c\left(x-\frac{1}{a}\right)^3}-\frac{712\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\cdots\right.}}{105a^7c\left(x-\frac{1}{a}\right)^2}\cdots\right.}{c^3x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2310a^{\frac{11}{2}}\sqrt{(ax-1)x}\,x^5+1155\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^5x^5-2100a^{\frac{9}{2}}((ax-1)x)^{\frac{3}{2}}x^3-11550a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^4-5775\ln\cdots\right.}{\cdots}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^3/(c*(a*x-1)/a/x)^(1/2)+(11/2/a^4*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/7/a^9/c/(x-1/a)^4*(a^2*c*
(x-1/a)^2+(x-1/a)*a*c)^(1/2)-102/35/a^8/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+(x-1/a
)*a*c)^(1/2)-712/105/a^7/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-15
16/105/a^6/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2))/c^3*a^3/x/(c*(a*x
-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.39

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\left[\frac{1155\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(10\cdots}{210\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\cdots}\right.$$

$$-\frac{1155\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(105\,a^5x^5-1936\,a^4x^4+4466\,a^3x^3-\cdots}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*a
*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(105*a^5*x^5 - 1936*a
^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(
a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/105*
(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sqrt(-c
```

```
)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3*x^3 -
 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*a^4*c^4
*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(7/2),x)
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(123) = 246.

Time = 0.63 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{11\,\log\left(c^4|a||c|\right)\operatorname{sgn}(x)}{18\,ac^{\frac{7}{2}}}$$
$$-\frac{11\,\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^9\sqrt{c}|a|-17\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8ac+64\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\cdots}\right.\right.\right.}{\cdots}$$
$$+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] 11/18*log(c^4*abs(a)*abs(c))*sgn(x)/(a*c^(7/2)) - 11/18*log(abs(2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*sqrt(c)*abs(a) - 17*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - a*c*x))^8*a*c + 64*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^
```

```
7*c^(3/2)*abs(a) - 140*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a*c^2 +
196*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^(5/2)*abs(a) - 182*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^3 + 112*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - a*c*x))^3*c^(7/2)*abs(a) - 44*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*
c*x))^2*a*c^4 + 10*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(9/2)*abs(a)
 - a*c^5))*sgn(x)/(a*c^(7/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2*
c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(a\,x-1\right)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)), x)
```

3.455 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 268

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] 3/35*(28*a-17/x)*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^2/(1-1/a/x)^(9/2)+9/7*(a
-1/x)^2*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^3/(1-1/a/x)^(9/2)+(a-1/x)^3*(1+1/
a/x)^(3/2)*(c-c/a/x)^(9/2)*x/a^3/(1-1/a/x)^(9/2)-3*(c-c/a/x)^(9/2)*arctanh(
(1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)/a/(1-1
/a/x)^(9/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6317, 6314, 99, 158, 152, 52, 65, 214\}$

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{9\left(a-\frac{1}{x}\right)^2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{3\mathrm{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(a*(1 - 1/(a*x))^(9/2)) + (3*(28*
a - 17/x)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(35*a^2*(1 - 1/(a*x))^(9
/2)) + (9*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(7*a^3*(1
 - 1/(a*x))^(9/2)) + ((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2
)*x)/(a^3*(1 - 1/(a*x))^(9/2)) - (3*(c - c/(a*x))^(9/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{9/2} \operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2} x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(c-\frac{c}{ax}\right)^{9/2} \operatorname{Subst}\left(\int \frac{\left(-\frac{3}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2} x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right) \operatorname{Subst}\left(\int \frac{\left(-\frac{21}{4a^2}-\frac{51x}{4a^3}\right)\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}{x}\,dx, x, \frac{1}{x}\right)}{7\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2} x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(3\left(c-\frac{c}{ax}\right)^{9/2}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x}\,dx, x, \frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2} x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(3\left(c-\frac{c}{ax}\right)^{9/2}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2} x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(3\left(c-\frac{c}{ax}\right)^{9/2}\right) \operatorname{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}$$
$$+ \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$
$$+ \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{3\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2} dx = \frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(10-26ax-12a^2x^2+164a^3x^3+35a^4x^4)-105a^3x^3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{a\ldots}}\right.\right.}{35a^4\sqrt{1-\frac{1}{ax}}x^3}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(10 - 26*a*x - 12*a^2*x^2 + 164*a
^3*x^3 + 35*a^4*x^4) - 105*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(35*a^4*Sqr
t[1 - 1/(a*x)]*x^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(70a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4+328a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-105\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4-24a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-52a^{\frac{3}{2}}x\ldots\right.}{70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3a^{\frac{9}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(35a^5x^5+199a^4x^4+152a^3x^3-38a^2x^2-16ax+10)c^4\sqrt{\frac{c(ax-1)}{ax}}}{35x^3a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^4*(70*
a^(9/2)*((a*x+1)*x)^(1/2)*x^4+328*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/2*
(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-24*a^(5/2)*x^2*((a*x
+1)*x)^(1/2)-52*a^(3/2)*x*((a*x+1)*x)^(1/2)+20*((a*x+1)*x)^(1/2)*a^(1/2))/x
^3/a^(9/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.63

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \left[\frac{105\,(a^4c^4x^4-a^3c^4x^3)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(35\,a^5c^4 x}{140\,(a^5x^4-a^4x^3)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^5*c^4*x^5 + 199*a^4*c^4*x^4 + 1
52*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10*c^4)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/70*(105*(a^4*c^4*x^4
- a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(35*a^5*c^4
*x^5 + 199*a^4*c^4*x^4 + 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10
*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3
)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.456 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] 1/3*(1+1/a/x)^(3/2)*(c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)-2/5*(1+1/a/x)^(5/2)*(
c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)+(1+1/a/x)^(5/2)*(c-c/a/x)^(7/2)*x/(1-1/a/x
)^(7/2)-(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+(c-c/a/x
)^(7/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(7/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 91, 81, 52, 65, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(
a*x))^(3/2)*(c - c/(a*x))^(7/2))/(3*a*(1 - 1/(a*x))^(7/2)) - (2*(1 + 1/(a*x
))^(5/2)*(c - c/(a*x))^(7/2))/(5*a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(a*x))^(5
/2)*(c - c/(a*x))^(7/2)*x)/(1 - 1/(a*x))^(7/2) - ((c - c/(a*x))^(7/2)*ArcTa
nh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= \frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(-\frac{1}{2a}+\frac{x}{a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= -\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&\quad+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}\\
&\quad+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{7/2}}
\end{aligned}
$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-6+8ax+44a^2x^2+15a^3x^3)-15a^2x^2\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}x^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-6 + 8*a*x + 44*a^2*x^2 + 15*a^3
*x^3) - 15*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^3 \left(30a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}+88a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-15\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+16a^{\frac{3}{2}}x\sqrt{(ax+1)x}-12\sqrt{(ax+1)x}\right.}{30\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^2a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(15a^4x^4+59a^3x^3+52a^2x^2+2ax-6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)+88*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-15*ln(1/2*(2
*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+16*a^(3/2)*x*((a*x+1)*
x)^(1/2)-12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.75

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\left[\frac{15\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15\,a^4c^3x\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 + 52*a
^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 +
52*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.457 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -1/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^5*(1-1/a^2/x^2)^(5/2)*x/(c
-c/a/x)^(5/2)+c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/
a-c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6312, 893, 879, 889, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^5x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -1/3*(c^4*(1 - 1/(a^2*x^2))^(3/2))/(a*(c - c/(a*x))^(3/2)) - (c^3*Sqrt[1 -
1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^5*(1 - 1/(a^2*x^2))^(5/2)*x)/(c -
c/(a*x))^(5/2) + (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c -
c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{c^4\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{c^3\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{c^2\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{c^4\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+2ax+3a^2x^2)+3ax\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + 2*a*x + 3*a^2*x^2) + 3*a*x*A
rcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^2\left(6a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2+4a^{\frac{3}{2}}x\sqrt{(ax+1)x}+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\,a^{\frac{5}{2}}\sqrt{(ax+1)x}}$	144
risch	$\frac{(3a^3x^3+5a^2x^2+4ax+2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^2\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	168

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(
5/2)*(6*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a^2*x^2+4*a^(3/2)*x*((a*x+1)*x)^(1/2)+4*((a*x+1)*x)^(1/2
)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.44

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{3\,(a^2c^2x^2-ac^2x)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^3c^2x^3+\dots}{12\,(a^3x^2-a^2x)}\right.$$

$$-\frac{3\,(a^2c^2x^2-ac^2x)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(3\,a^3c^2x^3+5\,a^2c^2x^2+4\,ac^2x+2\,c^2)\sqrt{\frac{ax-\dots}{ax+\dots}}\dots}{6\,(a^3x^2-a^2x)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x +
 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x
), -1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x + 2*c^2)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.458 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)+3*c^(3/2)*arctanh(c^(1/2)*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a-3*c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6312, 877, 879, 889, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2
))^(3/2)*x)/(c - c/(a*x))^(3/2) + (3*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3 \text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2 \left(c-\frac{cx}{a}\right)^{3/2}} \, dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}} \, dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(3c)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(3c^3)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= -\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = -\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{1}{ax}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(1 + 1/(a*x))^(5/2)*(c - c/(a*x))^(3/2)*Hypergeometric2F1[2, 5/2, 7/2,
1 + 1/(a*x)])/(5*a*(1 - 1/(a*x))^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}\sqrt{(ax+1)x}}$	118
risch	$\frac{(a^2x^2-ax-2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)
*(2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*a*x-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{3\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-2\,c)}{4\,(a^2x-a)}\right.$$

$$\left.-\frac{3\,(acx-c)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2cx^2-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), -1/2*(3*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) - 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.459 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1/a
/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 100, 162, 65, 214, 212}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{5\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$
$$- \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$- \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} + \frac{5\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x+\frac{5\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (5*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
- (4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[1 - 1/(a*x)]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax-}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*
x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a
*x+1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.37

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(ax-1)\sqrt{c}\log\left(-\right.}{4(a^2x-a)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*
x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*
x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.460 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 7*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-5*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-3*(1-1/a
/x)^(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)+a*x*(1-1/a/x)^(1/2)*(1+1/
a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6317, 6314, 100, 156, 162, 65, 214, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{7\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}+\frac{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (-3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - c/(a*x)]) +
 (a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*Sqrt[c - c/(a*x)])
 + (7*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) -
 (5*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c
 - c/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{-\frac{7}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(a\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{3x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(5\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} - \frac{\left(7\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(7\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(10\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{7\sqrt{1-\frac{1}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.53

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(a\sqrt{1+\frac{1}{ax}}x(-3+ax)+7(-1+ax)\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-5\sqrt{2}(-1+ax)\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x*(-3 + a*x) + 7*(-1 + a*x)*ArcTanh
[Sqrt[1 + 1/(a*x)]] - 5*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2
]]))/(a*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-5a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x-6\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}c\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{2\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{3}c\left(x-\frac{1}{a}\right)}-\frac{5\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}}}{x-\frac{1}{a}}\right)}{2a^{2}\sqrt{c}}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x-5*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-6*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+5*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$=\left[\frac{7\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{3}x^{3}-2\,a^{2}x^{2}-3\,ax\right)\sqrt{}}{4\left(a^{3}cx^{2}-2\,a^{2}cx+\right.}\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] [1/4*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)

```
/(a*x)) - c)/(a*x - 1)) + 4*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)) + 5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*log(
-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)
*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 -
3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c), 1/2*(5*sqrt
(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2 - 2*
a*x - 1)) - 7*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
- c)) + 2*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.461 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 275

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{51\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 9*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-51/8*(1-1/a/x)
^(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*a*(
1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)-15/4*(1-1/a/x)^(3/
2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a^2*(1-1/a/x)^(3/2)*x*(1+1/a/x)^
(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{6317, 6314, 100, 156, 162, 65, 214, 212\}$

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{51\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))^2*(c - c/(a*x))^
(3/2)) - (15*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))*(c - c/
(a*x))^(3/2)) + (a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))
^2*(c - c/(a*x))^(3/2)) + (9*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]]
)/(a*(c - c/(a*x))^(3/2)) - (51*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]/Sqrt[2]])/(4*Sqrt[2]*a*(c - c/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}
\end{aligned}$$

$$=-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{\frac{18}{a^2}+\frac{12x}{a^3}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$=-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(a^2\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{-\frac{36}{a^3}-\frac{15x}{a^4}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$=-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(51\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(9\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$=-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(9\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(51\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{4a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$=-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{51\left(1-\frac{1}{ax}\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(15-23ax+4a^2x^2)+72(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-51\sqrt{}\right.}{8ac\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(15 - 23*a*x + 4*a^2*x^2) + 72*
(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh[S
qrt[1 + 1/(a*x)]/Sqrt[2]]))/(8*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.19

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{2a^2\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{a^5c\left(x-\frac{1}{a}\right)^2}-\frac{15\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^4c\left(x-\frac{1}{a}\right)}-\frac{51\sqrt{2}\ln}{}\right.}{c\sqrt{\frac{ax-1}{ax+1}}(ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-51a^{\frac{5}{2}}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2-92\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+72\ln\left(\frac{2\sqrt{(ax+1)x}\sqrt{a}}{2\sqrt{a}}\right.\right.}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/c/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(9/2/a^2*ln((1/
2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/a^5/c
/(x-1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-15/4/a^4/c/(x-1/a)*(a^
2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-51/16/a^3/c^(1/2)*2^(1/2)*ln((4*c+3*
(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2))/(x
-1/a)))*a/c/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)*((a*x+1
)*a*c*x)^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{51\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{c}{c}}}{a^3x^3-3a^2x^2+3ax-1}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*a
^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^4*x^4 - 19*a^3
*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/16*(51*sqrt(2)*(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*
a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 4*(4*a^4*x^4 - 19*a^3*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2
 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.462 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 335

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 11*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-249/32*(1-1/a
/x)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-5/
3*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)-29/12*a*(1-
1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-73/16*(1-1/a/x)^(5/2
)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^3*(1-1/a/x)^(5/2)*x*(1+1/a/x)^(
1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6317, 6314, 100, 156, 162, 65, 214, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{a^3x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-5*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(3*(a - x^(-1))^3*(c - c/(a*
x))^(5/2)) - (29*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(12*(a - x^(-1))^
2*(c - c/(a*x))^(5/2)) - (73*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(16*(a
- x^(-1))*(c - c/(a*x))^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]
*x)/((a - x^(-1))^3*(c - c/(a*x))^(5/2)) + (11*(1 - 1/(a*x))^(5/2)*ArcTanh[
Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - (249*(1 - 1/(a*x))^(5/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^4} \, dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2} \text{Subst}\left(\int \frac{-\frac{11}{2a}-\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right) \text{Subst}\left(\int \frac{\frac{33}{a^2}+\frac{25x}{a^3}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{6\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2 \left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{5/2}\right) \text{Subst}\left(\int \frac{-\frac{132}{a^3}-\frac{87x}{a^4}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{24\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right) \left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a^3\left(1-\frac{1}{ax}\right)^{5/2}\right) \text{Subst}\left(\int \frac{\frac{264}{a^4}+\frac{219x}{2a^5}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{48\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right) \left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2} \sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3 \left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(249\left(1-\frac{1}{ax}\right)^{5/2}\right) \text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{32a^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(11\left(1-\frac{1}{ax}\right)^{5/2}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(11\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(249\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{16a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{249\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.43

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-219+554ax-415a^2x^2+48a^3x^3)+1056(-1+ax)^3\text{arctanh}\left(\right.\right.}{96ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)^3}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-219 + 554*a*x - 415*a^2*x^2 +
 48*a^3*x^3) + 1056*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]] - 747*Sqrt[2]*(
-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(96*a*c^2*Sqrt[c - c/(a*x)
]*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.12

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{3}\sqrt{a^{2}c}}-\frac{2\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{3a^{7}c\left(x-\frac{1}{a}\right)^{3}}-\frac{11\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^{6}c\left(x-\frac{1}{a}\right)^{2}}-\frac{271\sqrt{}}{}\right.}{c^{2}\sqrt{\frac{ax-}{ax+}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(192\sqrt{(ax+1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^{3}-747a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{3}-1660\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}+1056\ln\left(\frac{2\sqrt{(ax}}{}\right.\right.}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/c^2/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(11/2/a^3*ln(
(1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-2/3/
a^7/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-11/4/a^6/c/(x-1/a
)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-271/48/a^5/c/(x-1/a)*(a^2*c*(
x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-249/64/a^4/c^(1/2)*2^(1/2)*ln((4*c+3*(x-1
/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2))/(x-1/a
)))*a^2/c^2/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)*((a*x+1
)*a*c*x)^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.20

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{747\sqrt{2}(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,a}\right.}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*l
og(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x
^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 -
4*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 8*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219*a*x)*s
```

```
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), 1/192*(747*sqrt(2)*(a^4*x^4 - 4
*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^
3*x^3 + 335*a^2*x^2 - 219*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.463 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -9*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)-1/5*(80*a-7/x
)*(c-c/a/x)^(7/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+3/5*(a-1/x)^2*(c-c/a/
x)^(7/2)*(1+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)+(a-1/x)^3*(c-c/a/x)^(7/2)*x*(1
+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

$= \{6317, 6314, 100, 158, 152, 65, 214\}$

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{3\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{9\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/5*((80*a - 7/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a^2*(1 - 1/(a*x)
)^(7/2)) + (3*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*
(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2
)*x)/(a^3*(1 - 1/(a*x))^(7/2)) - (9*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
```

```
3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}\\
&= \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}
\end{aligned}$$

$$= \frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{\left(\frac{45}{4a^2}-\frac{21x}{4a^3}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(9\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(9\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{9\left(c-\frac{c}{ax}\right)^{7/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+16ax-92a^2x^2+5a^3x^3)-45a^2x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{5a^3\sqrt{1-\frac{1}{ax}}x^2}$$

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^3
*x^3) - 45*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(5*a^3*Sqrt[1 - 1/(a*x)]*x^
2)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(10a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-184a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-45\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+32a^{\frac{3}{2}}x\sqrt{(ax+1)x}-4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{10x^2a^{\frac{7}{2}}(ax-1)\sqrt{(ax+1)x}}$
risch	$\frac{(5a^4x^4-87a^3x^3-76a^2x^2+14ax-2)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{5x^2a^3(ax-1)}-\frac{9\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(10*a^(7/2)*
x^3*((a*x+1)*x)^(1/2)-184*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-45*ln(1/2*(2*((a*x+
1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+32*a^(3/2)*x*((a*x+1)*x)^(1/2
)-4*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.88

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \left[\frac{45\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(5\,a^4c^3x^4}{20\,(a^4x^3-a^3x^2)}\right.$$

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 76*a^
2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/10*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 7
6*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{a\,x}\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.464 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -7*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)-1/3*(16*a+1/x
)*(c-c/a/x)^(5/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)+(a-1/x)^2*(c-c/a/x)^(
5/2)*x*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 100, 152, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{x\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/3*((16*a + x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(a^2*(1 - 1/(a
*x))^(5/2)) + ((a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)*x)/(a^2
```

```
*(1 - 1/(a*x))^(5/2)) - (7*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/
(a*(1 - 1/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{7}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(7\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(7\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{7\left(c-\frac{c}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2-22ax+3a^2x^2)-21ax\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 - 22*a*x + 3*a^2*x^2) - 21*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-44a^{\frac{3}{2}}x\sqrt{(ax+1)x}-21\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{6x\,a^{\frac{5}{2}}(ax-1)\sqrt{(ax+1)x}}$	144
risch	$\frac{(3a^3x^3-19a^2x^2-20ax+2)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	168

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)-21*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2+4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(
5/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.37

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\left[\frac{21\,(a^2c^2x^2-ac^2x)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^3c^2x^3}{12\,(a^3x^2-a^2x)}\right.$$

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*
x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^
2*x), 1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*x + 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int \left(c-\frac{c}{a\,x}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.465 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -5*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)-2*(c-c/a/x)^(
3/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(3/2)+(c-c/a/x)^(3/2)*x*(1+1/a/x)^(1/2)/(1
-1/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 91, 81, 65, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{5\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{x\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Int[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))/(a*(1 - 1/(a*x))^(3/2)) + (Sqrt[
1 + 1/(a*x)]*(c - c/(a*x))^(3/2)*x)/(1 - 1/(a*x))^(3/2) - (5*(c - c/(a*x))^
(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{-\frac{5}{2a}+\frac{x}{a^2}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{5\left(c-\frac{c}{ax}\right)^{3/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+ax)-5\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 5*ArcTanh[Sqrt[1 + 1/(
a*x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{2a^{\frac{3}{2}}(ax-1)\sqrt{(ax+1)x}}$	118
risch	$\frac{(a^2x^2-ax-2)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	151

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-2*a^
(3/2)*x*((a*x+1)*x)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*a*x+4*((a*x+1)*x)^(1/2)*a^(1/2))/(a*x-1)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{5\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-2}{4\,(a^2x-a)}\right.$$

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(5*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^
2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\left(c-\frac{c}{a\,x}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.466 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/
a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6312, 893, 889, 214}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3} \\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x-\frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	101
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	139

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\ 3\,(a\right.$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.467 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] -arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+x*(1-1/a^2/
x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6312, 887, 889, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2ac}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\sqrt{1+\frac{1}{ax}}x-\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - ArcTanh[Sqrt[1 + 1/(a*x)]]/a))/Sq
rt[c - c/(a*x)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{a}\,c(ax-1)\sqrt{(ax+1)x}}$	102
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{2a\sqrt{a^2c}\,\sqrt{\frac{c(ax-1)}{ax}}\,x}$	133

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(-2*
((a*x+1)*x)^(1/2)*a^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1
/2)))/(a*x-1)/((a*x+1)*x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.83

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$
$$= \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2cx-ac)}, \frac{(ax-}{}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*c*x - a*c), 1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2), x)
```

3.468 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-(1-1/a/x)^(3/2)*
arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)+(1-1/a/x)^(3
/2)*x*(1+1/a/x)^(1/2)/(c-c/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6317, 6314, 105, 21, 85, 65, 214, 212}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] ((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(c - c/(a*x))^(3/2) + ((1 - 1/(a*
x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(3/2)) - (Sqrt[2]*(1
 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*(c - c/(a*x))^(3/2
))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int\frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{1}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{-\frac{1}{2a}-\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(a\sqrt{1+\frac{1}{ax}}\,x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
[Out] ((1 - 1/(a*x))^(3/2)*(a*Sqrt[1 + 1/(a*x)]*x + ArcTanh[Sqrt[1 + 1/(a*x)]] - 
Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*(c - c/(a*x))^(3/2))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}c^{2}(ax-1)\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{2a^{3}\sqrt{c}}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(2*
((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(a*x-1)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.46

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\cdots}}{4\,(a^2c^2x-\cdots}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)) + sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2
)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c^2*x
- a*c^2), 1/2*(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a
*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2
 - 2*a*x - 1)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2
*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*c^
2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2), x)
```

3.469 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}} dx = -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{2\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 3*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-9/4*(1-1/a/x)^
(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*(1
-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a*(1-1/a/x)^(5/2)*x*(
1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6317, 6314, 105, 21, 101, 162, 65, 214, 212}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}} dx = \frac{3\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{2\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))*(c - c/(a*x))^(5
/2)) + (a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*
x))^(5/2)) + (3*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(
a*x))^(5/2)) - (9*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(
2*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int\frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{1}{x^2\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{-\frac{3}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}
\end{aligned}$$

$$=-\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{-1-\frac{x}{2a}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(9\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4a^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(9\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$=-\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{2\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-3+2ax)+12(-1+ax)\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-9\sqrt{2}(-1+ax)\right.}{4ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-3 + 2*a*x) + 12*(-1 + a*x)*Ar
cTanh[Sqrt[1 + 1/(a*x)]] - 9*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/S
qrt[2]]))/(4*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(8\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-9a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x-12\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+12\ln\left(\frac{2\sqrt{(a}}{}\right.\right.}{8a^{\frac{3}{2}}c^3(ax-1)^2\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{9\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{8a^4\sqrt{c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+}}{2a^5c(}\right.}{c^2x\sqrt{\frac{c(ax-1)}{ax}}}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(8*((a*x+1)*x)^
(1/2)*a^(5/2)*(1/a)^(1/2)*x-9*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-12*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/
2)+12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)
*x-12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+9
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^
(1/2))/a^(3/2)/c^3/(a*x-1)^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.72

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{9\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
- 1)) + 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), 1
/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2
- 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 4*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2), x)
```

3.470 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 277

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{115\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] 5*(1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-115/32*(1-1/a/
x)^(7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)-5/4
*a*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)-35/16*(1-1/a/x
)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(7/2)+a^2*(1-1/a/x)^(7/2)*x*(1+1/
a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules

used = {6317, 6314, 105, 21, 101, 156, 162, 65, 214, 212}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{115\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{5a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (-5*a*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))^2*(c - c/(a*x)
)^(7/2)) - (35*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(16*(a - x^(-1))*(c -
 c/(a*x))^(7/2)) + (a^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-
1))^2*(c - c/(a*x))^(7/2)) + (5*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]])/(a*(c - c/(a*x))^(7/2)) - (115*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int\frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{1}{x^2\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{-\frac{5}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}$$
$$+\frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{-2-\frac{3x}{2a}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}$$
$$+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{\frac{4}{a}+\frac{7x}{4a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}$$
$$-\frac{\left(115\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{32a^2\left(c-\frac{c}{ax}\right)^{7/2}}$$
$$-\frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(115\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{2-x^2}dx, x, \sqrt{1+\frac{1}{ax}}\right)}{16a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{7/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{115\left(1-\frac{1}{ax}\right)^{7/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(35-55ax+16a^2x^2)+160(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-115\right.}{32ac^3\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(35 - 55*a*x + 16*a^2*x^2) + 16
0*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcTan
h[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(32*a*c^3*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{3}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{4}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^{7}c\left(x-\frac{1}{a}\right)^{2}}-\frac{23\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{16a^{6}c\left(x-\frac{1}{a}\right)}-\frac{115\sqrt{2}\,\ln\left(\frac{4c+}{}\right.}{}\right.}{c^{3}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(64\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}-115a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}-220\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+160\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)+(5/2/a^4*ln((
1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/4/a
^7/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-23/16/a^6/c/(x-1/a
)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-115/64/a^5/c^(1/2)*2^(1/2)*ln((
4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/
2))/(x-1/a)))/c^3*a^3*((a*x-1)/(a*x+1))^(1/2)/x/(c*(a*x-1)/a/x)^(1/2)*((a*x
+1)*a*c*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\left[\frac{115\sqrt{2}(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right.}{}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a
^2*x^2 + 3*a*x - 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(
8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(16*a^4*x^4 - 39
*a^3*x^3 - 20*a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/64*(115*sqrt
(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2
```

```
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) + 4*(16*a^4*x^4 - 39*a^3*x^3 - 20*a^2*x^2 + 35*
a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^
3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2), x)
```

3.471 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{11c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -5/3*c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-11*c^(
7/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+32*c^(7/2)*arctanh(1/2*(c-c/a/x)^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-21*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 100, 159, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\frac{11c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[In] Int[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-21*c^3*Sqrt[c - c/(a*x)])/a - (5*c^2*(c - c/(a*x))^(3/2))/(3*a) + (3*c*(c
 - c/(a*x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (11*c^(7/2)*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$$

$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{9/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \left(c-\frac{c}{ax}\right)^{7/2}x + \frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{11c^2}{2}+\frac{3c^2x}{2a}\right)}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2}x + \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{55c^3}{4}-\frac{25c^3x}{4a}\right)}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= -\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2}x + \frac{4\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\left(\frac{165c^4}{8}-\frac{315c^4x}{8a}\right)}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{15c}$$

$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a} - \frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2}x + \frac{8\text{Subst}\left(\int \frac{\frac{165c^5}{16}-\frac{795c^5x}{16a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{15c}$$

$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a} - \frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2}x + \frac{(11c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{(32c^4)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a} - \frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2}x - (11c^3)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) + (64c^3)\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a} - \frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a} + \left(c-\frac{c}{ax}\right)^{7/2}x - \frac{11c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\,(-6+52ax-376a^2x^2+15a^3x^3)}{15a^3x^2}-\frac{11c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-6 + 52*a*x - 376*a^2*x^2 + 15*a^3*x^3))/(15*a^3*x^
2) - (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2
)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.37

method	result
risch	$\frac{(15a^4x^4-391a^3x^3+428a^2x^2-58ax+6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{11a^3\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{16a^2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\cdots}}{\cdots}\right)}{\sqrt{c}}\cdots\right.}{a^3(ax-1)\cdots}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(480\sqrt{(ax-1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4-1110a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^4-480a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)x^4+660a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,(a\,x^2-\cdots\right.}{30x^3a^{\frac{7}{2}}\cdots}$

```
[In] int((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(15*a^4*x^4-391*a^3*x^3+428*a^2*x^2-58*a*x+6)/x^2*c^3/a^3/(a*x-1)*(c*(
a*x-1)/a/x)^(1/2)+(-11/2*a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2
-a*c*x)^(1/2))/(a^2*c)^(1/2)-16*a^2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2
*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*c^3/a
^3/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \left[\frac{480\sqrt{2}a^2c^{\frac{7}{2}}x^2\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+165\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+\right.}{30\,a^3x^2}\right.$$
$$-\frac{480\sqrt{2}a^2\sqrt{-c}c^3x^2\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-165\,a^2\sqrt{-c}c^3x^2\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(15\,a^3c^3x^3-376\,a^2c^3}{15\,a^3x^2}$$

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x +
2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(15*a^3*c^3*x^3 - 376*a^2*c^
3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/15*(480*
sqrt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))
/c) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x -
 c)/(a*x)))/(a^3*x^2)]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] integrate((a*x - 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x-1)}{a\,x+1}\,dx$$

[In] int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1),x)

[Out] int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1), x)

3.472 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] 1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-9*c^(5/2)*arctanh((c-c/a/x)^(1/2)
/c^(1/2))/a+16*c^(5/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)
/a-7*c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 100, 159, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{9c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[In] Int[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x)
)^(5/2)*x - (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (16*Sqrt[2]*
c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$$

$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \left(c-\frac{c}{ax}\right)^{5/2}x+\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{9c^2}{2}+\frac{c^2x}{2a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\left(\frac{27c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{3c}$$

$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{4\text{Subst}\left(\int \frac{\frac{27c^4}{8}-\frac{69c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{3c}$$

$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x + \frac{(9c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}-\frac{(16c^3)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x-(9c^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right) + (32c^2)\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)$$

$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}(2-26ax+3a^2x^2)-27ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+48\sqrt{2}ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 26*a*x + 3*a^2*x^2) - 27*a*c^(5/2)*x*ArcTanh[Sq
rt[c - c/(a*x)]/Sqrt[c]] + 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x)]
/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(3a^3x^3-29a^2x^2+28ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}+\frac{\left(-\frac{9a^2\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{8a\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3}}{x+\frac{1}{a}}\right)}{\sqrt{c}}\right.}{a^2(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(48\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3-90\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3+48a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+45\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^2x^3-48a\right.}{6x^2a^{\frac{5}{2}}\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}}$

```
[In] int((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*a^3*x^3-29*a^2*x^2+28*a*x-2)/x*c^2/a^2/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)
+(-9/2*a^2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^
2*c)^(1/2)-8*a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2
*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*c^2/a^2/(a*x-1)*(c*(a*x-1)
/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.07

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{48\sqrt{2}ac^{\frac{5}{2}}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+27\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+}{6\,a^2x}\right.$$

$$-\frac{48\sqrt{2}a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-27\,a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(3\,a^2c^2x^2-26\,ac^2x+2\,c^2)}{3\,a^2x}$$

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(48*sqrt(2)*a*c^(5/2)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + 3*a*c*x - c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x), -1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arctan(1/
```

```
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*arctan(
sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*
sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)**(5/2)*(a*x-1)/(a*x+1),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.473 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] (c-c/a/x)^(3/2)*x-7*c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+8*c^(3/2)*ar
ctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-c*(c-c/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 100, 159, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{7c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[In] Int[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x - (7*c^(3/2)*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/a + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$

$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{7c^2}{2}-\frac{c^2x}{2a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{2\text{Subst}\left(\int\frac{\frac{7c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{(7c^2)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}-\frac{(8c^2)\,\text{Subst}\left(\int\frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-(7c)\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)+(16c)\text{Subst}\left(\int\frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}(-2+ax)-7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) - 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[
c]] + 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(94) = 188$.

Time = 0.50 (sec), antiderivative size = 193, normalized size of antiderivative = 1.71

method	result
risch	$\frac{(a^2x^2-3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{7a\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{4\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}}\right)}{a(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\left(8\sqrt{(ax-1)x}\sqrt{\frac{1}{a}}a^{\frac{3}{2}}x^2-10\sqrt{\frac{1}{a}}\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}x^2+4(a\,x^2-x)^{\frac{3}{2}}\sqrt{a}\sqrt{\frac{1}{a}}+5\sqrt{\frac{1}{a}}\ln\left(\frac{2\sqrt{a\,x^2-x}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^2-8\ln\left(\frac{2\sqrt{2}\sqrt{}}{}\right.\right.}{2x\,a^{\frac{3}{2}}\sqrt{(ax-1)x}\sqrt{\frac{1}{a}}}$

```
[In] int((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] (a^2*x^2-3*a*x+2)*c/a*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)+(-7/2*a*ln((-1/2*a*c+a^
2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4*2^(1/2)/c^(1/
2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2
*c)^(1/2))/(x+1/a)))*c/a*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)/(a*x-1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec), antiderivative size = 235, normalized size of antiderivative = 2.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{8\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-}{2\,a},\ -\frac{8\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-7\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(8*sqrt(2)*c^(3/2)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 + 3*a*c*x - c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a
```

```
*c*x - c)/(a*x)) + c) + 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(8*sqr
t(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 7*
sqrt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt(
(a*c*x - c)/(a*x)))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)**(3/2)*(a*x-1)/(a*x+1),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.474 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 100, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+x\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}x + \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}x + \frac{(5c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a} - \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \sqrt{c-\frac{c}{ax}}x - 5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right) + 8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)$$

$$= \sqrt{c-\frac{c}{ax}}x - \frac{5\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(75) = 150.

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c}}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}-6\,\mathrm{l}}{2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*(a*x-1)/a/x)^(1/2)+(-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^
2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2
^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)
*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.38

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ ax\sqrt{\cdots}\right.$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.475 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)+2*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)+x*(c-c/a/x)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 101, 162, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}+\frac{x\sqrt{c-\frac{c}{ax}}}{c}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
```

```
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= -\int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}(1+ax)}\,dx$$

$$= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}+\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c} + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{2\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c} + \frac{4\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}x}{c}-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right)}{2\sqrt{(ax-1)x}\,c\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{c(ax-1)ax}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-2*2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)
-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2))/((a
*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+2\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+3\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac}, -\frac{2\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right)-ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{ac}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x
*sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + 3*sqrt(c)*log(-2
*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), -(2*sqrt(2)*c*s
qrt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1))
- a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)
/(a*x))/c))/(a*c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)), x)
```

3.476 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)+arctanh(1/2*(c-c/a/x)^(1/2)*2^(
1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)+x*(c-c/a/x)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 105, 21, 85, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}+\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx \\
&= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx \\
&= \frac{a\int \frac{x}{\sqrt{c-\frac{c}{ax}}(1+ax)}\,dx}{c} \\
&= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}\,dx}{c} \\
&= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c} \\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c^2}+\frac{\text{Subst}\left(\int \frac{\frac{c}{2}-\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c^2} \\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c^2}+\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{2c^2} \\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c^2}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac}-\frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{ac}
\end{aligned}
$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c^2} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} + \frac{2\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c^2} - \frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}x}{c^2} - \frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{(ax-1)x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}\sqrt{(ax-1)x}\,c^2\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^2\sqrt{a^2c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{2a^3\sqrt{c}}\right)a\sqrt{c(ax-1)ax}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/
2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))
/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\,\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{2\,ac^2}\right.$$
$$\left.-\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right)-ax\sqrt{\frac{acx-c}{ax}}-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{ac^2}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x*s
qrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + sqrt(c)*log(-2*a*c
*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), -(sqrt(2)*c*sqrt(
-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) - a*
x*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x)
)/c))/(a*c^2)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)), x)
```

3.477 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

```
[Out] arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)+1/2*arctanh(1/2*(c-c/a/x)^(1/2)*
2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-2/a/c^2/(c-c/a/x)^(1/2)+x/c^2/(c-c/a/x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 105, 157, 162, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] -2/(a*c^2*Sqrt[c - c/(a*x)]) + x/(c^2*Sqrt[c - c/(a*x)]) + ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c]
)]/(Sqrt[2]*a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx \\
&= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx \\
&= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx}{c} \\
&= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{c} \\
&= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{c}{2}-\frac{3cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}
\end{aligned}
$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\frac{c^2}{2}+\frac{c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{ax - \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right) - \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{ac^2\sqrt{c-\frac{c}{ax}}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
[Out] (a*x - Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] - Hypergeometric
2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(98) = 196.

Time = 0.50 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{3}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{4a^{4}\sqrt{c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+(}}{a^{5}c\left(x-\frac{1}{a}\right.}\right.}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-8\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^{2}+4((ax-1)x)^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,a^{\frac{5}{2}}+\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)a^{\frac{5}{2}}x^{2}-2\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{(ax-1)x}}{2\sqrt{}}\right.\right.}{}$

[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/a*(a*x-1)/c^2/(c*(a*x-1)/a/x)^(1/2)+(1/2/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c
)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/4/a^4*2^(1/2)/c^(1/2)*ln((
4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/
2))/(x+1/a))-1/a^5/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2))*a^2/c^2/x
/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.47

$$\int\frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\right.}{4\,(a^{2}c^{3}x-ac^{3})}\right.$$

$$-\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+2\,(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\,(a^{2}x^{2}-2\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}c^{3}x-ac^{3})}$$

[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a
*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c^3*x - a*c^3), -1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*arctan
(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*x - c)
/(a*x)))/(a^2*c^3*x - a*c^3)]

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(5/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{5/2}(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)), x)
```

3.478 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

```
[Out] -4/3/a/c^2/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(3/2)+3*arctanh((c-c/a/x)^(1/2)/
c^(1/2))/a/c^(7/2)+1/4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(7/
2)*2^(1/2)-7/2/a/c^3/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 105, 157, 162, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] -4/(3*a*c^2*(c - c/(a*x))^(3/2)) - 7/(2*a*c^3*Sqrt[c - c/(a*x)]) + x/(c^2*(
c - c/(a*x))^(3/2)) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) +
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx\\
&= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx\\
&= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx}{c}\\
&= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{c}\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}
\end{aligned}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}-\frac{5cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}+\frac{6c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{9c^3}{2}-\frac{21c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^6}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4ac^3} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^3}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{2c^4} + \frac{3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^4}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.54

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{x\left(3ax - \text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \frac{a-\frac{1}{x}}{2a}\right) - 3\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, 1 -\right.\right.}{3c^3\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (x*(3*a*x - Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] - 3*Hyperg
eometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 + a*
x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(120) = 240$.

Time = 0.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.84

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^4\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{3a^7c\left(x-\frac{1}{a}\right)^2}-\frac{17\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{6a^6c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+\right.}{}\right.}{}\right.}{c^3x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-84\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a^{\frac{9}{2}}x^3+60\sqrt{\frac{1}{a}}\,((ax-1)x)^{\frac{3}{2}}a^{\frac{7}{2}}x-36\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^3+3\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1}}{ax+1}\right.\right.}{}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^3/(c*(a*x-1)/a/x)^(1/2)+(3/2/a^4*ln((-1/2*a*c+a^2*c*x)/(a^2*c
)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/3/a^7/c/(x-1/a)^2*(a^2*c*(
x-1/a)^2+(x-1/a)*a*c)^(1/2)-17/6/a^6/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c
)^(1/2)-1/8/a^5*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^
2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/c^3*a^3/x/(c*(a*x-1)/a/x)
^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.44

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+36\,(a^2x^2-2\,ax+1)\sqrt{c}\,\mathrm{lo}}{24\,(a^3c^4x^2-2\,a^2c^4x}\right.$$

$$-\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+36\,(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\,(}{12\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x + 1
)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(6*
```

```
a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 29*a^2*x
^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(7/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)), x)
```

3.479 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

```
[Out] -6/5/a/c^2/(c-c/a/x)^(5/2)-11/6/a/c^3/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(5/2)
+5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(9/2)+1/8*arctanh(1/2*(c-c/a/x)^(1/
2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-21/4/a/c^4/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 105, 157, 162, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] -6/(5*a*c^2*(c - c/(a*x))^(5/2)) - 11/(6*a*c^3*(c - c/(a*x))^(3/2)) - 21/(4
*a*c^4*Sqrt[c - c/(a*x)]) + x/(c^2*(c - c/(a*x))^(5/2)) + (5*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt
[c])]/(4*Sqrt[2]*a*c^(9/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx \\
&= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}\,dx \\
&= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx}{c} \\
&= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{c}
\end{aligned}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{5c}{2}-\frac{7cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\frac{25c^2}{2}+\frac{15c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}dx, x, \frac{1}{x}\right)}{5c^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{75c^3}{2}-\frac{165c^3x}{4a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{15c^6}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\frac{75c^4}{2}+\frac{315c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{15c^8}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{8ac^4} - \frac{5\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{2ac^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right)}{4c^5} + \frac{5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^5}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.48

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \frac{ax^2\left(5ax - \text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{a-\frac{1}{x}}{2a}\right) - 5\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1\right.\right.}{5c^4\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] (a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*Hy
pergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-1
+ a*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(141) = 282.

Time = 0.49 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.83

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^5\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{5a^9c\left(x-\frac{1}{a}\right)^3}-\frac{37\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{30a^8c\left(x-\frac{1}{a}\right)^2}-\frac{317\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right.}}{60a^7c\left(x-\frac{1}{a}\right)}\right.}{c^4x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-1260\sqrt{(ax-1)x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^4+1020((ax-1)x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^2-600\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^5x^4+15a^{\frac{9}{2}}\sqrt{2}\,\ln\left(\right.\right.}{}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^4/(c*(a*x-1)/a/x)^(1/2)+(5/2/a^5*ln((-1/2*a*c+a^2*c*x)/(a^2*c
)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/5/a^9/c/(x-1/a)^3*(a^2*c*(
x-1/a)^2+(x-1/a)*a*c)^(1/2)-37/30/a^8/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+(x-1/a)*
a*c)^(1/2)-317/60/a^7/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-1/16/a^
6*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-
3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*a^4/c^4/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x
-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.51

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \left[\frac{15\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+600\,(a^3x^3-3\,a^2\cdots}{240\,(a^4c^5\cdots}\right.$$

$$-\frac{15\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+600\,(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\arctan\cdots}{120\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\cdots}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(2*sqrt(2
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 600*(a^3*
x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c
*x - c)/(a*x)) + c) + 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*
sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5
), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*x^2 + 3
*a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(60*a^4*x
^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5
*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(9/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(9/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(9/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{9/2}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)), x)
```

3.480 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 335

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{15\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] -15*(c-c/a/x)^(9/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+10*(a-1/x)^4
*(c-c/a/x)^(9/2)/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+(a-1/x)^5*(c-c/a/x)^(9
/2)*x/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+5/7*(304*a-65/x)*(c-c/a/x)^(9/2)*
(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(9/2)+135/7*(a-1/x)^2*(c-c/a/x)^(9/2)*(1+1/a/
x)^(1/2)/a^3/(1-1/a/x)^(9/2)+65/7*(a-1/x)^3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)
/a^4/(1-1/a/x)^(9/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6317, 6314, 100, 155, 158, 152, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{x\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{65\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{15\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[In] Int[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^4*(c - c/(a*x))^(9/2))/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1
/(a*x)]) + (5*(304*a - 65/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(7*a^2*
(1 - 1/(a*x))^(9/2)) + (135*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(9/2))/(7*a^3*(1 - 1/(a*x))^(9/2)) + (65*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(
c - c/(a*x))^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^5*(c - c/(a
*x))^(9/2)*x)/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]) - (15*(c - c/(a*x
))^(9/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^6}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(\frac{15}{2a}+\frac{5x}{2a^2}\right)\left(1-\frac{x}{a}\right)^4}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{15}{4a^2}-\frac{65x}{4a^3}\right)\left(1-\frac{x}{a}\right)^3}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(4a^2\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{105}{8a^3}-\frac{675x}{8a^4}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(8a^3\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{525}{16a^4}-\frac{4875x}{16a^5}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{35\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(15\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(15\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{15\left(c-\frac{c}{ax}\right)^{9/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{c^4\sqrt{c-\frac{c}{ax}}\left(-2+20ax-110a^2x^2+720a^3x^3+1685a^4x^4+7a^5x^5-35a^4\sqrt{1+\frac{1}{ax}}x^4\text{arctanh}\right.}{7a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

[Out] (c^4*Sqrt[c - c/(a*x)]*(-2 + 20*a*x - 110*a^2*x^2 + 720*a^3*x^3 + 1685*a^4*x^4 + 7*a^5*x^5 - 35*a^4*Sqrt[1 + 1/(a*x)]*x^4*ArcTanh[Sqrt[1 + 1/(a*x)]] + 70*a^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(7*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.68

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(14\sqrt{(ax+1)x}\,a^{\frac{11}{2}}x^5+3510a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4+1440a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-105\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a}{14(ax-1)^2x^3a^{\frac{9}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(7a^5x^5+859a^4x^4+720a^3x^3-110a^2x^2+20ax-2)c^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{7x^3a^4(ax-1)}+\frac{\left(-\frac{15a^4\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{2\sqrt{a^2c}}+\frac{128a^2\sqrt{a^2c\left(x+\frac{1}{a}\right)^2}}{c\left(x+\frac{1}{a}\right)}\right.}{a^4(ax-1)}$

[In] int((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/14*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^4*(14*((a*x+1)*x)^(1/2)*a^(11/2)*x^5+3510*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+1440*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^5*x^5-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-220*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+40*a^(3/2)*x*((a*x+1)*x)^(1/2)-4*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/2)/((a*x+1)*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(7a^5c^4x\right.}{28\left(a^5x^4-a^4x^3\right)}\right.$$

[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/28*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt((a*x - 1)/(a*x +

```
 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/14*(105*(a^4*c^4*x^4 -
 a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(7*a^5*c^4*x
^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*
c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)
]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int \left(c-\frac{c}{a\,x}\right)^{9/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.481 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 277

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{13\left(c-\frac{c}{ax}\right)^{7/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -13*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+10*(a-1/x)^3
*(c-c/a/x)^(7/2)/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+(a-1/x)^4*(c-c/a/x)^(7
/2)*x/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+1/15*(1360*a-311/x)*(c-c/a/x)^(7/
2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+47/5*(a-1/x)^2*(c-c/a/x)^(7/2)*(1+1/
a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6317, 6314, 100, 155, 158, 152, 65, 214\}$

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{x\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} + \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} + \frac{47\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{13\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[In] Int[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^3*(c - c/(a*x))^(7/2))/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1
/(a*x)]) + ((1360*a - 311/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2
*(1 - 1/(a*x))^(7/2)) + (47*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(7/2))/(5*a^3*(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^4*(c - c/(a*x))^(7/2)*x)
/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - (13*(c - c/(a*x))^(7/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
```

```
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^5}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{\left(\frac{13}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\left(-\frac{13}{4a^2}-\frac{47x}{4a^3}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(4a^2\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\left(-\frac{65}{8a^3}-\frac{311x}{8a^4}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(13\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(13\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{13\left(c-\frac{c}{ax}\right)^{7/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{c^3\sqrt{c-\frac{c}{ax}}\left(6-62ax+548a^2x^2+1441a^3x^3+15a^4x^4-45a^3\sqrt{1+\frac{1}{ax}}x^3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right.}{15a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(6 - 62*a*x + 548*a^2*x^2 + 1441*a^3*x^3 + 15*a^4*x^
4 - 45*a^3*Sqrt[1 + 1/(a*x)]*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]] + 150*a^3*x^3*H
ypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(15*a^4*Sqrt[1 - 1/(a^2*x^2)]
*x^3)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^{3}\left(30a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^{4}+3182a^{\frac{7}{2}}x^{3}\sqrt{(ax+1)x}-195\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{4}x^{4}+1096a^{\frac{5}{2}}x^{2}\sqrt{(ax+1}\right.}{30(ax-1)^{2}x^{2}a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(15a^4x^4+631a^3x^3+548a^2x^2-62ax+6)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{13a^3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{64a\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{c\left(x+\frac{1}{a}\right)}\right)}{a^3(ax-1)}$

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^3*(3
0*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+3182*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-195*ln(1
/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4+1096*a^(5/2)*x^2*
((a*x+1)*x)^(1/2)-195*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*a^3*x^3-124*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/
a^(7/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{195\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(15\,a^4c^3\cdots}{60\,(a^4x^3-a^3x^2)}\right.$$

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 5
48*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqr
t(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 1591*a^3*c^
3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{a\,x}\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

```
[In] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.482 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -11*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)+10*(a-1/x)^2
*(c-c/a/x)^(5/2)/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+(a-1/x)^3*(c-c/a/x)^(5
/2)*x/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+1/3*(112*a-29/x)*(c-c/a/x)^(5/2)*
(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 100, 155, 152, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{x\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{11\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^2*(c - c/(a*x))^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1
/(a*x)]) + ((112*a - 29/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1
 - 1/(a*x))^(5/2)) + ((a - x^(-1))^3*(c - c/(a*x))^(5/2)*x)/(a^3*(1 - 1/(a*
x))^(5/2)*Sqrt[1 + 1/(a*x)]) - (11*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{5/2}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}\,\text{Subst}\left(\int\frac{\left(\frac{11}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}\\
&\quad-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{11}{4a^2}-\frac{29x}{4a^3}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}
\end{aligned}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(11\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(11\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{11\left(c-\frac{c}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{c^2\sqrt{c-\frac{c}{ax}}\left(-2+32ax+103a^2x^2+3a^3x^3-3a^2\sqrt{1+\frac{1}{ax}}x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)+30a^2x^2\,\text{Hy}\right.}{3a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 103*a^2*x^2 + 3*a^3*x^3 - 3*a^2*Sqrt[
1 + 1/(a*x)]*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 30*a^2*x^2*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + 1/(a*x)]))/(3*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^{2}\left(6a^{\frac{7}{2}}x^{3}\sqrt{(ax+1)x}+266a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}-33\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{3}x^{3}+64a^{\frac{3}{2}}x\sqrt{(ax+1)x}-33\ln\right.}{6(ax-1)^{2}x\,a^{\frac{5}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(3a^{3}x^{3}+37a^{2}x^{2}+32ax-2)c^{2}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}(ax-1)}+\frac{\left(-\frac{11a^{2}\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{32\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{c\left(x+\frac{1}{a}\right)}\right)c^{2}\sqrt{\frac{ax-1}{ax+1}}\sqrt{}}{a^{2}(ax-1)}$

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)+266*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-33*ln(1/2*(
2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+64*a^(3/2)*x*((a*x+1)
*x)^(1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-
4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(5/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.74

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{33\left(a^{2}c^{2}x^{2}-ac^{2}x\right)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3\,a^{3}c^{2}x^{3}+\right.}{12\left(a^{3}x^{2}-a^{2}x\right)}\right.$$

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2
*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a
^2*x), 1/6*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\left(c-\frac{c}{a\,x}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

```
[In] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.483 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -9*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)+(21*a+1/x)*(c
-c/a/x)^(3/2)/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+(a-1/x)^2*(c-c/a/x)^(3/2)
*x/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 100, 151, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{x\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{9\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Int[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((21*a + x^(-1))*(c - c/(a*x))^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(
a*x)]) + ((a - x^(-1))^2*(c - c/(a*x))^(3/2)*x)/(a^2*(1 - 1/(a*x))^(3/2)*Sq
rt[1 + 1/(a*x)]) - (9*(c - c/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1
 - 1/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
```

```
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}-\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{9\left(c-\frac{c}{ax}\right)^{3/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.45

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(2+10ax+a^2x^2+9ax\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{1}{ax}\right)\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 + 10*a*x + a^2*x^2 + 9*a*x*Hypergeometric2F1[-1/2,
1, 1/2, 1 + 1/(a*x)]))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\,(2a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+38a^{\frac{3}{2}}x\sqrt{(ax+1)x}-9\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2-9\ln(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+}{2\sqrt{a}}}{2(ax-1)^2a^{\frac{3}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(a^2x^2+3ax+2)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{9a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{16\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(ax}}{a(ax-1)}$

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^
(5/2)*x^2*((a*x+1)*x)^(1/2)+38*a^(3/2)*x*((a*x+1)*x)^(1/2)-9*ln(1/2*(2*((a*
x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-9*ln(1/2*(2*((a*x+1)*x)^(1/
2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+4*((a*x+1)*x)^(1/2)*a^(1/2))/a^(3/2)/((a*x
+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.99

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \left[\frac{9\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2+19\,acx+2}{4\,(a^2x-a)}\right.$$

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(9*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(9*(a*c*x - c)*sqrt(-c)*arctan(2*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \int \left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2} dx = \int \left(c-\frac{c}{a\,x}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} dx$$

```
[In] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.484 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^(
1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 91, 79, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=-\frac{7\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{7\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)x}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{(ax+1)x}\,\sqrt{a}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)^2\sqrt{a}\,\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^2c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.14

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\ \frac{7\ldots}{\ldots}\right.$$

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,dx$$

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.485 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] -5*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+5*(c-c/a/
x)^(1/2)/a/c/(1-1/a^2/x^2)^(1/2)+x*(c-c/a/x)^(1/2)/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6312, 893, 883, 889, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=-\frac{5\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}+\frac{x\sqrt{c-\frac{c}{ax}}}{c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (5*Sqrt[c - c/(a*x)])/(a*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a*x)]*x)/(
c*Sqrt[1 - 1/(a^2*x^2)]) - (5*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[
c - c/(a*x)]])/(a*Sqrt[c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 883

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*(p + 1)*(e*f + d*g))), x] + Dist[e^2*g*((m - n - 2)/(c*(p +
1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(5c)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{5\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(ax+5\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{1}{ax}\right)\right)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/
(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-10\sqrt{(ax+1)x}\,\sqrt{a}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+\right.\right.}{2\sqrt{a}}}{2(ax-1)^2\sqrt{a}\,c\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a\sqrt{a^2c}}+\frac{4\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^3c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1
/2)/c*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
```

```
+2*a*x+1)/a^(1/2))*a*x-10*((a*x+1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{5\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+5\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2cx-ac)}, \frac{5\,(a\cdots}{\cdots}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c*x - a*c), 1/2*(5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 2*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2), x)
```

3.486 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/2)+3*x*(1-1/
a^2/x^2)^(1/2)/c/(c-c/a/x)^(1/2)-2*x*(c-c/a/x)^(1/2)/c^2/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6312, 883, 887, 889, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}-\frac{2x\sqrt{c-\frac{c}{ax}}}{c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (3*Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a*x)]) - (2*Sqrt[c - c/(a*x)]*x)
/(c^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/S
qrt[c - c/(a*x)]])/(a*c^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 883

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*(p + 1)*(e*f + d*g))), x] + Dist[e^2*g*((m - n - 2)/(c*(p +
1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$=\frac{3\sqrt{1-\frac{1}{a^2x^2}}x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{2ac^2}$$

$$=\frac{3\sqrt{1-\frac{1}{a^2x^2}}x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=\frac{3\sqrt{1-\frac{1}{a^2x^2}}x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1+\frac{1}{ax}\right)}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
[Out] (2*(1 - 1/(a*x))^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, 1 + 1/(a*x)])/(a*Sqr
t[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-6\sqrt{(ax+1)x}\,\sqrt{a}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right.}{2(ax-1)^2\sqrt{a}\,c^2\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^2\sqrt{a^2c}}+\frac{2\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1
/2)/c^2*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
```

```
2)+2*a*x+1)/a^(1/2))*a*x-6*((a*x+1)*x)^(1/2)*a^(1/2)+3*ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.66

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+3\,ax}{4\,(a^2c^2x-ac^2)}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c^2*x - a*c^2), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2
+ a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*
x^2 - a*c*x - c)) + 2*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

3.487 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] -(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-1/2*(1-1/a/x)^(
5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)+2*(1-1/
a/x)^(5/2)/a/(c-c/a/x)^(5/2)/(1+1/a/x)^(1/2)+(1-1/a/x)^(5/2)*x/(c-c/a/x)^(5
/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6317, 6314, 105, 157, 162, 65, 214, 212}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{x\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (2*(1 - 1/(a*x))^(5/2))/(a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) + ((1 - 1
/(a*x))^(5/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int\frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{1}{x^2\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{\frac{1}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}\\
&= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}\\
&\quad+\frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int\frac{\frac{1}{2a^2}-\frac{x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}
\end{aligned}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(ax + \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right) + \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1\right.\right.}{ac^2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + Hypergeometric2F1[-1/2, 1, 1/2, (a + x^(-1))/(2*a
)] + Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(a*c^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - c/(a*x)])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.30

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{3}\sqrt{a^{2}c}}+\frac{\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{5}c\left(x+\frac{1}{a}\right)}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right.}}{x-\frac{1}{a}}\right.}{4a^{4}\sqrt{c}}\right.}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x+8\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\ln\left(\frac{2\sqrt{(ax-}}{}\right.\right.}{4(ax-1)^{2}a^{\frac{3}{2}}c^{3}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)}}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)+(-1/2/a^3*ln(
(1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)+1/a^
5/c/(x+1/a)*(a^2*c*(x+1/a)^2-(x+1/a)*a*c)^(1/2)-1/4/a^4/c^(1/2)*2^(1/2)*ln(
(4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1
/2))/(x-1/a)))*a^2/c^2*((a*x-1)/(a*x+1))^(1/2)/x/(c*(a*x-1)/a/x)^(1/2)*((a*
x+1)*a*c*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.63

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}\left(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+2}{8}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x
 - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1)
)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a*x
- 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) +
 8*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*c^3*x - a*c^3), 1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c
*x^2 - 2*a*c*x - c)) + 2*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
```

```
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 4*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

3.488 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] (1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-11/8*(1-1/a/x)^(
7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)+7/4*(1-
1/a/x)^(7/2)/a/(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)-3/2*(1-1/a/x)^(7/2)/(a-1/x)/
(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)+a*(1-1/a/x)^(7/2)*x/(a-1/x)/(c-c/a/x)^(7/2)
/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {6317, 6314, 105, 156, 157, 162, 65, 214, 212}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{ax\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (7*(1 - 1/(a*x))^(7/2))/(4*a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) - (3*(1
 - 1/(a*x))^(7/2))/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) +
 (a*(1 - 1/(a*x))^(7/2)*x)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7
/2)) + ((1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(7
/2)) - (11*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(4*Sqrt[
2]*a*(c - c/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{\frac{1}{a^2}+\frac{9x}{2a^3}}{x\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{\frac{1}{a^3}+\frac{7x}{4a^4}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(11\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(11\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{4a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2ax(-3+2ax)+11(-1+ax)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+(4-4a\right.}{4ac^3\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*x*(-3 + 2*a*x) + 11*(-1 + a*x)*Hypergeometric2F1[-1
/2, 1, 1/2, (a + x^(-1))/(2*a)] + (4 - 4*a*x)*Hypergeometric2F1[-1/2, 1, 1/
2, 1 + 1/(a*x)]))/(4*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-11a^{\frac{5}{2}}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2+4\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+8\ln\left(2\right.\right.}{16(ax-1)^3a^{\frac{3}{2}}c}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^4\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^6c\left(x-\frac{1}{a}\right)}-\frac{11\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x\right.}}{x-\frac{1}{a}}\right.}{16a^5\sqrt{c}}\right.}{c^3x\sqrt{\frac{c(ax-1)}{ax}}}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(c*(a*x-1)/a/x)^(1/2)*x*(16*
((a*x+1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-11*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+4*((a*x+1)*x)^(1/2)*a
^(5/2)*(1/a)^(1/2)*x+8*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2-28*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-8*ln(1/2*(2*
((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+11*2^(1/2)*ln((2*
2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/
c^4/(1/a)^(1/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{11\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{ac\cdots}}{a^3x^3-3\,a^2x^2+3\,ax-1}\cdots\right.}{\cdots}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/32*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 8*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1
/16*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 8*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 4*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

3.489 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] x^(1+m)*hypergeom([-1/2, -1-m],[-m],-1/a/x)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6317, 6316, 66}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-1,-m,-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{ax}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^m\,dx}{\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Maple [F]

$$\int \frac{x^m \sqrt{c - \frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}}\, x^m \,dx = \int \frac{\sqrt{c - \frac{c}{ax}}\, x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a
*x - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}}\, x^m \,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}}\, x^m \,dx = \int \frac{\sqrt{c - \frac{c}{ax}}\, x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int\frac{x^m\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.490 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=-\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] 1/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3-1/8*c*
x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)+1/12*c*x^2*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6313, 877, 887, 889, 214}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -1/8*(c*Sqrt[1 - 1/(a^2*x^2)]*x)/(a^2*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a
^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*S
qrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
 - c/(a*x)]])/(8*a^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^4\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^5}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3+2ax+8a^2x^2)}{-1+ax} - 3\sqrt{c}\log(1-ax) + 3\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-ax\right.}{48a^3}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3 + 2*a*x + 8*a^2*x^2
))/(-1 + a*x) - 3*Sqrt[c]*Log[1 - a*x] + 3*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1
 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]])/(48*a^3)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(16a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}+4a^{\frac{3}{2}}x\sqrt{(ax+1)x}-6\sqrt{(ax+1)x}\,\sqrt{a}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{48\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{5}{2}}\sqrt{(ax+1)x}}$	121
risch	$\frac{(8a^{2}x^{2}+2ax-3)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{16a^{2}\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	148

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(5/2)*(16*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)+4*a^(3/2)*x*((a*x+1)*x)^(1/2)-6*((a*x+1)*x)^(1/2)*a^(1/
2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax)}{96\,(a^4x-a^3)}\right.$$

$$-\frac{3\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{48\,(a^4x-a^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(3*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x
^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3
)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a/x)**(1/2),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.491 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] -1/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2+1/4*c
*x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-c
/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6313, 877, 887, 889, 214}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}+\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^
2)]*x^2)/(2*Sqrt[c - c/(a*x)]) - (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{4a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}
\end{aligned}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(1+2ax) + \sqrt{c}(-1+ax)\log(1-ax) + \sqrt{c}(1-ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\right.}{8a^2(-1+ax)}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(1 + 2*a*x) + Sqrt[c]*(-
1 + a*x)*Log[1 - a*x] + Sqrt[c]*(1 - a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]])/(8*a^2*(-1 + a*x)
)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4a^{\frac{3}{2}}x\sqrt{(ax+1)x}-2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}\sqrt{(ax+1)x}}$	102
risch	$\frac{(2ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}} - \frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{8a\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	140

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-4*a^(3/2)*x*
((a*x+1)*x)^(1/2)-2*((a*x+1)*x)^(1/2)*a^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a
^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.56

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{a}}{16\,(a^3x-a^2)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.492 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6312, 877, 889, 214}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)x}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\right.$$
$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.493 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)-2*c*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6313, 879, 889, 214}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\frac{-2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x+\sqrt{c}(1-ax)\log(1-ax)+\sqrt{c}(-1+ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-\right.}{-1+ax}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
 a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
 c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-2\sqrt{(ax+1)x}\sqrt{a}\right)}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)x}\sqrt{a}}$	88
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(ax+1)acx}}{\sqrt{a^2c}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$	127

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x)^(1/2
)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-2*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.494 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6313, 663}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
[Out] (-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\ &= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1+ax)}{-3+3ax}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + a*x))/(-3 + 3*a*x)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
gosper	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(a^2x^2+2ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x}$	56

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -2/3*(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
)/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x\,(a\,x-1)}$$

```
[In] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] -(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*x*(a*x
- 1))
```

3.495 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -2/15*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+2/5*a^2*c*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6313, 809, 663}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) + (2*a^2*c*(1
 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}} - \frac{1}{5}(ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-3-ax+2a^2x^2\right)}{15x(-1+ax)}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-3 - a*x + 2*a^2*x^2))/(15*x*
(-1 + a*x))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}\sqrt{\frac{ax-1}{ax+1}}}$	47
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(2a^{3}x^{3}+a^{2}x^{2}-4ax-3)}{15\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^{2}}$	64

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 2/15*(a*x+1)*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\left(2\,a^3x^3+a^2x^2-4\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
[Out] 2/15*(2*a^3*x^3 + a^2*x^2 - 4*a*x - 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^2\,(2\,a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,x^2\,(a\,x-1)}$$

```
[In] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))
/(15*x^2*(a*x - 1))
```

3.496 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

```
[Out] 8/105*a^3*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/7*a*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)/x^2-8/35*a^3*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6313, 885, 809, 663}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7x^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (8*a^3*c^2*(1 - 1/(a^2*x^2))^(3/2))/(105*(c - c/(a*x))^(3/2)) - (8*a^3*c*(1
 - 1/(a^2*x^2))^(3/2))/(35*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^2))^
(3/2))/(7*(c - c/(a*x))^(3/2)*x^2)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 885

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
p + 1)/(c*(m - n - 1))), x] - Dist[n*((e*f + d*g)/(e*(m - n - 1))), Int[(d
+ e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
&& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Int
egerQ[n])
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2} + \frac{1}{7}(4ac)\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2} + \frac{1}{35}\left(4a^2c\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(15+3ax-4a^2x^2+8a^3x^3)}{105x^2(-1+ax)}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(15 + 3*a*x - 4*a^2*x^2 + 8*a
^3*x^3))/(105*x^2*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(ax+1)(8a^2x^2-12ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$	55
default	$-\frac{2(ax+1)(8a^2x^2-12ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$	55
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(8a^4x^4+4a^3x^3-a^2x^2+18ax+15)}{105\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^3}$	73

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(a*x+1)*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/((a*x-1)/(a*
x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\,(8\,a^4x^4+4\,a^3x^3-a^2x^2+18\,ax+15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\,(ax^4-x^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] -2/105*(8*a^4*x^4 + 4*a^3*x^3 - a^2*x^2 + 18*a*x + 15)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,(8\,a^3\,x^3+12\,a^2\,x^2+11\,a\,x+29)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3} - \frac{88\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,(a\,x-1)}$$

```
[In] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*(11*a*x + 12*a^2*x^2 + 8*a^3*x^3 + 29)*((c
*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (88*((a*x - 1)/(a*x + 1))^(1/2)*((c*(
a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.497 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3}+\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

```
[Out] -16/315*a^4*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/9*a*c^2*(1-1/a^2/x^2)
^(3/2)/(c-c/a/x)^(3/2)/x^3+4/21*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
/x^2+16/105*a^4*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6313, 885, 809, 663}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9x^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-16*a^4*c^2*(1 - 1/(a^2*x^2))^(3/2))/(315*(c - c/(a*x))^(3/2)) + (16*a^4*c
*(1 - 1/(a^2*x^2))^(3/2))/(105*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^
```

```
2))^(3/2))/(9*(c - c/(a*x))^(3/2)*x^3) + (4*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)
)/(21*(c - c/(a*x))^(3/2)*x^2)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 885

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
p + 1)/(c*(m - n - 1))), x] - Dist[n*((e*f + d*g)/(e*(m - n - 1))), Int[(d
+ e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
&& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Int
egerQ[n])
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{1}{3}(2ac)\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{21}\left(8a^2c\right)\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{105}\left(8a^3c\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

$$\int\frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-35-5ax+6a^2x^2-8a^3x^3+16a^4x^4\right)}{315x^3(-1+ax)}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-35 - 5*a*x + 6*a^2*x^2 - 8*a
^3*x^3 + 16*a^4*x^4))/(315*x^3*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)\left(16a^3x^3-24a^2x^2+30ax-35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
default	$\frac{2(ax+1)\left(16a^3x^3-24a^2x^2+30ax-35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\left(16a^5x^5+8a^4x^4-2a^3x^3+a^2x^2-40ax-35\right)}{315\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^4}$	80

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
[Out] 2/315*(a*x+1)*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4/(
(a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\left(16\,a^5x^5+8\,a^4x^4-2\,a^3x^3+a^2x^2-40\,ax-35\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^5-x^4\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] 2/315*(16*a^5*x^5 + 8*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 40*a*x - 35)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}\,(16\,a^4\,x^4+24\,a^3\,x^3+22\,a^2\,x^2+23\,a\,x-17)}{315\,x^4} - \frac{104\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{315\,x^4\,(a\,x-1)}$$

```
[In] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(23*a*x + 22*a^2
*x^2 + 24*a^3*x^3 + 16*a^4*x^4 - 17))/(315*x^4) - (104*((a*x - 1)/(a*x + 1)
)^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(315*x^4*(a*x - 1))
```

3.498 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{75\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

```
[Out] 75/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4+75/64*x*(c-c/a/x)^(1/2)/
a^3+25/32*x^2*(c-c/a/x)^(1/2)/a^2+5/8*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/
x)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 79, 44, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{75\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (75*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) + (
5*Sqrt[c - c/(a*x)]*x^3)/(8*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (75*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{x^2(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^5\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{(15c)\text{Subst}\left(\int \frac{1}{x^4\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{(25c)\text{Subst}\left(\int \frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^2}$$

$$= \frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2} + \frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{(75c)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{64a^3}$$

$$= \frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2} + \frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{(75c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{128a^4}$$

$$= \frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2} + \frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{75\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{64a^3}$$

$$= \frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2} + \frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{75\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(a^4x^4 + 15\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\frac{1}{ax}\right)\right)}{4a^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x)
]))/(4*a^4)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(16a^3x^3+40a^2x^2+50ax+75)x\sqrt{\frac{c(ax-1)}{ax}}}{64a^3} + \frac{75\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{c(ax-1)ax}}{128a^3\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32x(ax^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}+112(ax^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+212\sqrt{ax^2-x}\,a^{\frac{5}{2}}x-106\sqrt{ax^2-x}\,a^{\frac{3}{2}}+256a^{\frac{3}{2}}\sqrt{(ax-1)x}+128a\ln\left(\frac{2\sqrt{(ax-1)x}\sqrt{}}{2\sqrt{a}}\right.\right.}{128\sqrt{(ax-1)x}\,a^{\frac{9}{2}}}$

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(16*a^3*x^3+40*a^2*x^2+50*a*x+75)/a^3*x*(c*(a*x-1)/a/x)^(1/2)+75/128/a
^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/
2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$= \left[\frac{2\,(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax)\sqrt{\frac{acx-c}{ax}}+75\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},\ \frac{(16\,a^4x^4+4\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] [1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)/
(a*x)) + 75*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c))/a^4, 1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x
- c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a/x)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^3}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^3/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\frac{1}{64}\sqrt{a^2cx^2-acx}\left(2\left(4x\left(\frac{2x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{5|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{25|a|}{a^4\mathrm{sgn}\,(x)}\right)x+\frac{75|a|}{a^5\mathrm{sgn}\,(x)}\right)$$
$$+\frac{75\sqrt{c}\log\left(|a||c|\right)\mathrm{sgn}(x)}{128\,a^4}-\frac{75\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{128\,a^4\mathrm{sgn}\,(x)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(a
^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) + 75/128*
sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^4 - 75/128*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.499 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{11\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

```
[Out] 11/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+11/8*x*(c-c/a/x)^(1/2)/a^
2+11/12*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 79, 44, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (11*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 + (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{x(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^4\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(11c)\text{Subst}\left(\int \frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(11c)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(11c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{11\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{8a^2}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{11\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(a^3x^3 + 11\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{1}{ax}\right)\right)}{3a^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
[Out] (Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x)
]))/(3*a^3)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(8a^2x^2+22ax+33)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2} + \frac{11\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{16a^2\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16(a\,x^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+60\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-30\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+96a^{\frac{3}{2}}\sqrt{(ax-1)x}+48a\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-15\ln\left(\frac{2\sqrt{a\,x^2}}{}\right.\right.}{48\sqrt{(ax-1)x}\,a^{\frac{7}{2}}}$

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(8*a^2*x^2+22*a*x+33)/a^2*x*(c*(a*x-1)/a/x)^(1/2)+11/16/a^2*ln((-1/2*a
*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c
*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{2\left(8a^3x^3+22a^2x^2+33ax\right)\sqrt{\frac{acx-c}{ax}}+33\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48a^3},\frac{(8a^3x^3+22a^2x^2+33a}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) + 33*sqr
t(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, 1/24*(
(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*arc
tan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^2}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{1}{24}\sqrt{a^2cx^2-acx}\left(2\,x\left(\frac{4\,x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{11\,|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{33\,|a|}{a^4\mathrm{sgn}\,(x)}\right) + \frac{11\,\sqrt{c}\log\left(|a||c|\right)\mathrm{sgn}(x)}{16\,a^3} - \frac{11\,\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{16\,a^3\mathrm{sgn}\,(x)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^3
*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) + 11/16*sqrt(c)*log(abs(a)*abs(c))*sgn(
x)/a^3 - 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))
*sqrt(c)*abs(a) + a*c))/(a^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.500 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{7\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

```
[Out] 7/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2+7/4*x*(c-c/a/x)^(1/2)/a+1/
2*x^2*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6302, 6268, 25, 445, 457, 79, 44, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (7*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (7*Sqrt[c]*ArcT
anh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 - \frac{(7c)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{7\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 - \frac{(7c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{7\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{7\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{4a}$$

$$= \frac{7\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{7\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a\sqrt{1-\frac{1}{ax}}x(7+2ax)+7\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

method	result	si
risch	$\frac{(2ax+7)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{7\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{8a\sqrt{a^2c}\,(ax-1)}$	11
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+16a^{\frac{3}{2}}\sqrt{(ax-1)x}+8a\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\right)}{8\sqrt{(ax-1)x}\,a^{\frac{5}{2}}}$	13

```
[In] int(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*a*x+7)/a*x*(c*(a*x-1)/a/x)^(1/2)+7/8/a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)
^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)
*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{2\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}+7\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},\frac{\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{-c}\,\text{arc}}{4\,a^2}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) + 7*sqrt(c)*log(-2*a*c*
x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, 1/4*((2*a^2*x^2 + 7*a*
x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a
*x))/c))/a^2]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)**(1/2),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=&\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2x|a|}{a^2\mathrm{sgn}(x)}+\frac{7|a|}{a^3\mathrm{sgn}(x)}\right)\\&+\frac{7\sqrt{c}\log(|a||c|)\,\mathrm{sgn}(x)}{8a^2}\\&-\frac{7\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{8a^2\mathrm{sgn}(x)}\end{aligned}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x)
)) + 7/8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^2 - 7/8*sqrt(c)*log(abs(-2*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.501 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 382, 79, 65, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+x\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \sqrt{c-\frac{c}{ax}}x-\frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \sqrt{c-\frac{c}{ax}}x+3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= \sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

method	result	size
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,\sqrt{a}-4\sqrt{(ax-1)x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)x}\,\sqrt{a}}$	120

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*(a*x-1)/a/x)^(1/2)+3/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-
a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c
)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx&=\frac{3\sqrt{c}\log\left(|a||c|\right)\operatorname{sgn}(x)}{2\,a}\\&-\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{2\,a\operatorname{sgn}(x)}\\&+\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}\end{aligned}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(a^
2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.502 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

```
[Out] 2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)+2*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6268, 25, 528, 457, 81, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+2\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx \\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x(1-ax)}\,dx \\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^2}\,dx}{a} \\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a} \\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= 2\sqrt{c-\frac{c}{ax}} - c\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right) \\
&= 2\sqrt{c-\frac{c}{ax}} + (2a)\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) \\
&= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result	size
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{a\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(-2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}x^{2}+2\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}-\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}\right)}{x\sqrt{(ax-1)x}\,\sqrt{a}}$	99

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*(a*x-1)/a/x)^(1/2)+a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*
c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\left[\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\sqrt{\frac{acx-c}{ax}},\right.$$
$$\left.-2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+2\sqrt{\frac{acx-c}{ax}}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*sqrt
((a*c*x - c)/(a*x)), -2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c)
 + 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(34) = 68.

Time = 4.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\begin{cases} -\frac{2a\left(\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c} & \text{for } \frac{c}{a}\neq 0\\ -\frac{3a\sqrt{c}\left(\frac{\log\left(\frac{2}{x}\right)}{a}-\frac{\log\left(2a-\frac{2}{x}\right)}{a}\right)}{2}+\frac{\sqrt{c}\log\left(\frac{a}{x}-\frac{1}{x^2}\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Piecewise((-2*a*(c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - c*sqr
t(c - c/(a*x))/a)/c, Ne(c/a, 0)), (-3*a*sqrt(c)*(log(2/x)/a - log(2*a - 2/x
)/a)/2 + sqrt(c)*log(a/x - 1/x**2)/2, True))
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.503 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

```
[Out] -2/3*a*(c-c/a/x)^(3/2)/c+4*a*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6268, 25, 528, 455, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
[Out] 4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^2(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^3}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}\,dx}{a}$$

$$
= -\frac{c\text{Subst}\left(\int \frac{a+x}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}
$$

$$
= -\frac{c\text{Subst}\left(\int \left(\frac{2a}{\sqrt{c-\frac{cx}{a}}} - \frac{a\sqrt{c-\frac{cx}{a}}}{c}\right)\,dx, x, \frac{1}{x}\right)}{a}
$$

$$
= 4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(1+5ax)}{3x}
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result
gosper	$\frac{2(5ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x}$
trager	$\frac{2(5ax+1)\sqrt{-\frac{-acx+c}{ax}}}{3x}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2-4ax-1)}{3(ax-1)x}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-6\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x^3-6a^{\frac{5}{2}}\sqrt{(ax-1)x}\,x^3+12a^{\frac{3}{2}}(a\,x^2-x)^{\frac{3}{2}}x+3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^3-3\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax\cdots}{2\sqrt{a}}\right.\right.}{3x^2\sqrt{(ax-1)x}\,\sqrt{a}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 2/3*(5*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
[Out] 2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^2\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**2,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^2}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{6,[2,1,5]%%%}+%%%{-6,[1,1,4]%%%}+%%%{-6,[0,1,3]%%%},[4]%%
%}+%%%
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,(5\,a\,x+1)}{3\,x}$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
[Out] (2*(c - c/(a*x))^(1/2)*(5*a*x + 1))/(3*x)
```

3.504 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = 4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

```
[Out] -2*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2+4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6268, 25, 528, 457, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c} + 4a^2\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/c + (2*a^2*(c - c/(a*
x))^(5/2))/(5*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^3(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^4}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}\,dx}{a}
\end{aligned}
$$

$$= -\frac{c\text{Subst}\left(\int \frac{x(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c\text{Subst}\left(\int \left(\frac{2a^2}{\sqrt{c-\frac{cx}{a}}} - \frac{3a^2\sqrt{c-\frac{cx}{a}}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{a}$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(1+3ax+6a^2x^2)}{5x^2}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{2(6a^2x^2+3ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{5x^2}$
trager	$\frac{2(6a^2x^2+3ax+1)\sqrt{-\frac{-acx+c}{ax}}}{5x^2}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(6a^3x^3-3a^2x^2-2ax-1)}{5(ax-1)x^2}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-10\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x^4-10a^{\frac{7}{2}}\sqrt{(ax-1)x}\,x^4+20a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2+5\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4-5\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}}{2\sqrt{a}}\right.\right.}{5x^3\sqrt{(ax-1)x}\,\sqrt{a}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 2/5*(6*a^2*x^2+3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\,(6\,a^2x^2+3\,ax+1)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
[Out] 2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^3\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**3,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^3}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{%%{[-5,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,5]%%%},[6]%%%}+%%%{%
%{[%%%
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c-\frac{c}{a\,x}}\,(6\,a^2\,x^2+3\,a\,x+1)}{5\,x^2}$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
[Out] (2*(c - c/(a*x))^(1/2)*(3*a*x + 6*a^2*x^2 + 1))/(5*x^2)
```

3.505 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 96

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = 4a^3\sqrt{c-\frac{c}{ax}}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

```
[Out] -10/3*a^3*(c-c/a/x)^(3/2)/c+8/5*a^3*(c-c/a/x)^(5/2)/c^2-2/7*a^3*(c-c/a/x)^(
7/2)/c^3+4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6268, 25, 528, 457, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] - (10*a^3*(c - c/(a*x))^(3/2))/(3*c) + (8*a^3*(c -
c/(a*x))^(5/2))/(5*c^2) - (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
```

```
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^4(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^5}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^4}dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{x^2(a+x)}{\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c\text{Subst}\left(\int \left(\frac{2a^3}{\sqrt{c-\frac{cx}{a}}} - \frac{5a^3\sqrt{c-\frac{cx}{a}}}{c} + \frac{4a^3\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}\right)dx, x, \frac{1}{x}\right)}{a}$$

$$= 4a^3\sqrt{c-\frac{c}{ax}} - \frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx = \frac{2\sqrt{c-\frac{c}{ax}}(15+39ax+52a^2x^2+104a^3x^3)}{105x^3}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
[Out] (2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

method	result
gosper	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(104a^3x^3+52a^2x^2+39ax+15)}{105x^3}$
trager	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{-\frac{-acx+c}{ax}}}{105x^3}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(104a^4x^4-52a^3x^3-13a^2x^2-24ax-15)}{105(ax-1)x^3}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-210a^{\frac{9}{2}}\sqrt{a\,x^2-x}\,x^5-210a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^5+420a^{\frac{7}{2}}(a\,x^2-x)^{\frac{3}{2}}x^3+105\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^5-105\ln\left(\frac{2\sqrt{(ax-}{}\right.\right.}{105x^4\sqrt{(ax-1)x}\,\sqrt{a}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 2/105*(c*(a*x-1)/a/x)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
[Out] 2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**4,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^4}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{210,[2,1,9]%%%}+%%%{-210,[1,1,8]%%%}+%%%{-210,[0,1,7]%%%}
,[8]%%
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{208\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{105}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{7\,x^3}+\frac{26\,a\,\sqrt{c-\frac{c}{a\,x}}}{35\,x^2}+\frac{104\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x}$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] (208*a^3*(c - c/(a*x))^(1/2))/105 + (2*(c - c/(a*x))^(1/2))/(7*x^3) + (26*a
*(c - c/(a*x))^(1/2))/(35*x^2) + (104*a^2*(c - c/(a*x))^(1/2))/(105*x)
```

3.506 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = 4a^4\sqrt{c-\frac{c}{ax}} - \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

```
[Out] -14/3*a^4*(c-c/a/x)^(3/2)/c+18/5*a^4*(c-c/a/x)^(5/2)/c^2-10/7*a^4*(c-c/a/x)
^(7/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4+4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6268, 25, 528, 457, 78}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + 4a^4\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] - (14*a^4*(c - c/(a*x))^(3/2))/(3*c) + (18*a^4*(c -
 c/(a*x))^(5/2))/(5*c^2) - (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c
 - c/(a*x))^(9/2))/(9*c^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx\\ &= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^5(1-ax)}\,dx\end{aligned}$$

$$= \frac{c \int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^6} dx}{a}$$

$$= \frac{c \int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^5} dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{x^3(a+x)}{\sqrt{c-\frac{cx}{a}}} dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c\text{Subst}\left(\int \left(\frac{2a^4}{\sqrt{c-\frac{cx}{a}}} - \frac{7a^4\sqrt{c-\frac{cx}{a}}}{c} + \frac{9a^4\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{5a^4\left(c-\frac{cx}{a}\right)^{5/2}}{c^3} + \frac{a^4\left(c-\frac{cx}{a}\right)^{7/2}}{c^4}\right) dx, x, \frac{1}{x}\right)}{a}$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} - \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5} dx = \frac{2\sqrt{c-\frac{c}{ax}}(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315x^4}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
[Out] (2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^4
))/(315*x^4)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

method	result
gosper	$\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4}$
trager	$\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{-\frac{-acx+c}{ax}}}{315x^4}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(272a^5x^5-136a^4x^4-34a^3x^3-17a^2x^2-50ax-35)}{315(ax-1)x^4}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-630\sqrt{a\,x^2-x}\,a^{\frac{11}{2}}x^6-630a^{\frac{11}{2}}\sqrt{(ax-1)x}\,x^6+1260(a\,x^2-x)^{\frac{3}{2}}a^{\frac{9}{2}}x^4+315\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^5x^6-315\ln\left(\frac{2\sqrt{}}{}\right.\right.}{315x^5\sqrt{(ax-1)x}\,\sqrt{}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)*(c*(a*x-1)/a/x)^(1/2)
/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
[Out] 2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x -
 c)/(a*x))/x^4
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^5\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**5,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^5}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^5), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{%%{[-315,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9]%%%},[10]%%%}+%%
%{%%{[
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{544\,a^4\,\sqrt{c-\frac{c}{a\,x}}}{315}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{9\,x^4}+\frac{34\,a\,\sqrt{c-\frac{c}{a\,x}}}{63\,x^3}+\frac{68\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x^2}+\frac{272\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{315\,x}$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] (544*a^4*(c - c/(a*x))^(1/2))/315 + (2*(c - c/(a*x))^(1/2))/(9*x^4) + (34*a
*(c - c/(a*x))^(1/2))/(63*x^3) + (68*a^2*(c - c/(a*x))^(1/2))/(105*x^2) + (
272*a^3*(c - c/(a*x))^(1/2))/(315*x)
```

3.507 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 313

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}}+\frac{363\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 363/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-4*arcta
nh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)
+149/64*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+107/96*x^2*(1
+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+17/24*x^3*(1+1/a/x)^(1/2)
*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/4*x^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/
(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6317, 6315, 100, 156, 162, 65, 214, 212}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{363\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{a^4\sqrt{1-\frac{1}{ax}}} + \frac{149x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{x^4\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-\frac{1}{ax}}} + \frac{17x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{24a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (149*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) + (1
07*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) + (1
7*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]) + (Sqrt
[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]) + (363*Sqrt[c -
c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2
]*Sqrt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^4*Sqrt[1 - 1/(a*
x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^5\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{17}{2a}-\frac{15x}{2a^2}}{x^4\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\frac{107}{4a^2}+\frac{85x}{4a^3}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{12\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{447}{8a^3}-\frac{321x}{8a^4}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{24\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\frac{1089}{16a^4}+\frac{447x}{16a^5}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{24\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^5\sqrt{1-\frac{1}{ax}}} - \frac{\left(363\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{128a^4\sqrt{1-\frac{1}{ax}}}$$

$$
= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{a^4\sqrt{1-\frac{1}{ax}}} - \frac{\left(363\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{64a^3\sqrt{1-\frac{1}{ax}}}
$$

$$
= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} + \frac{363\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{64a^4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}
$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.81

$$
\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(447+214ax+136a^2x^2+48a^3x^3)}{-1+ax} - 1089\sqrt{c}\log(1-ax) + 768\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + 1089\sqrt{c}\,\mathrm{l}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(447 + 214*a*x + 136*a^
2*x^2 + 48*a^3*x^3))/(-1 + a*x) - 1089*Sqrt[c]*Log[1 - a*x] + 768*Sqrt[2]*S
qrt[c]*Log[(-1 + a*x)^2] + 1089*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 768*Sqrt[2]*Sqrt[c
]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c
*(-1 - 2*a*x + 3*a^2*x^2)])/(384*a^4)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.72

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(96\sqrt{(ax+1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^{3}+272\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}+428\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+894\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-768\sqrt{2}\,\mathrm{l}\right.}{384\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{9}{2}}\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(48a^{3}x^{3}+136a^{2}x^{2}+214ax+447)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{363\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{128a^{3}\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\cdots}}{x-\frac{1}{a}}\right.}{a^{4}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/384/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(96*((a*x+1)*x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3+272*((a*x+1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2+428*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x+894*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+1089*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(9/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.81

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^{3}\,dx$$

$$=\left[\frac{768\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+1089\,(ax-1)\sqrt{c}\cdots}{\cdots}\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] [1/768*(768*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1089*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x

```
)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4), 1/384*(
768*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)
) - 1089*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(48*a^5
*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.508 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 261

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}+\frac{45\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 45/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)-4*arctanh
(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+1
9/8*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+13/12*x^2*(1+1/a/
x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/3*x^3*(1+1/a/x)^(1/2)*(c-c/a/x
)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used

= {6317, 6315, 100, 156, 162, 65, 214, 212}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{45\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}} + \frac{19x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{13x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (19*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) + (13*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]) + (45*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^3*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{13}{2a}-\frac{11x}{2a^2}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\frac{57}{4a^2}+\frac{39x}{4a^3}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{135}{8a^3}-\frac{57x}{8a^4}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^4\sqrt{1-\frac{1}{ax}}}-\frac{\left(45\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}-\frac{\left(45\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{45\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{8a^3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(57+26ax+8a^2x^2)}{-1+ax} - 135\sqrt{c}\log(1-ax) + 96\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + 135\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1 \ldots}\right.}{\ldots}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(57 + 26*a*x + 8*a^2*x^
2))/(-1 + a*x) - 135*Sqrt[c]*Log[1 - a*x] + 96*Sqrt[2]*Sqrt[c]*Log[(-1 + a*
x)^2] + 135*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x
)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 96*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*S
qrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*
x^2)])/(48*a^3)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.77

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2+52\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+114\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-96\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\ldots\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{7}{2}}\sqrt{(ax+1)x}\sqrt{\frac{1}{a}}}$
risch	$\frac{(8a^2x^2+26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{45\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^3\sqrt{c}}\right)\ldots}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*((
a*x+1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2+52*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a)^
(1/2)*x+114*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-96*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+135*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(7/2)/((a*x+1)*x)
^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.11

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{96\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+135\,(ax-1)\sqrt{c}\log}{96\,(a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(96*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 135
*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 4*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(96*sqrt(2)*(a*x - 1)*s
qrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^
2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x -
 a^3)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a/x)**(1/2),x)

[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")

[Out] integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.509 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}}+\frac{23\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 23/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-4*arctanh
(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+9
/4*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/2*x^2*(1+1/a/x)^(1
/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{6317, 6315, 100, 156, 162, 65, 214, 212\}$

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \frac{23\mathrm{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}} + \frac{x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}} + \frac{9x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (9*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]) + (23*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^2*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\frac{23}{4a^2}+\frac{9x}{4a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^3\sqrt{1-\frac{1}{ax}}} - \frac{\left(23\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{a^2\sqrt{1-\frac{1}{ax}}} - \frac{\left(23\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{4a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}} + \frac{23\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(9+2ax)}{-1+ax}-23\sqrt{c}\log(1-ax)+16\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+23\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\right.}{8a}$$

[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]

[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(9 + 2*a*x))/(-1 + a*x) - 23*Sqrt[c]*Log[1 - a*x] + 16*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + 23*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 16*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)])/(8*a^2)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+18\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-16\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}+23\ln\left(\frac{2\sqrt{(ax+1)x}\sqrt{a}+}{2\sqrt{a}}\right.\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{5}{2}}\sqrt{(ax+1)x}\sqrt{\frac{1}{a}}}$
risch	$\frac{(2ax+9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{23\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{8a\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^2\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x+18*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-16*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+23*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(5/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.56

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \left[\frac{16\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+23\,(ax-1)\sqrt{c}\log}{16\,(a^3x-a}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(16*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 23*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 4*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2
*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.510 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1/a
/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 100, 162, 65, 214, 212}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{5\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} + \frac{5\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x+\frac{5\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (5*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
- (4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[1 - 1/(a*x)]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*
x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a
*x+1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.37

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(ax-1)\sqrt{c}\log\left(\right.}{4(a^2x-a)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*
x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*
x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.511 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(1/2*(1
+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*(1+1/a/x)^
(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6317, 6315, 86, 162, 65, 214, 212}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)] - (4*Sqrt[2]*Sqrt[c - c
/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.49

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x}{-1+ax} - \sqrt{c}\log(1-ax) + 2\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + \sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c\left(-1-ax+2a^2x^2\right)\right) - 2\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c\left(-1-2ax+3a^2x^2\right)\right)$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x)/(-1 + a*x) - Sqrt[c]*Log[1
- a*x] + 2*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 2*S
qrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(
a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,ax-2\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x+2\sqrt{(ax+1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*
((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*(1/a)^(1/2)*a*x-2*a^(1/2)*2^(1/
2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+2*((a*
x+1)*x)^(1/2)*a^(1/2)*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.36

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\left[\frac{2\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+(ax-1)\sqrt{c}\log\left(-\right.}{2\,(ax-1)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a*x
- 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) +
 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1),
(2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c))
 - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*x + 1)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.512 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 125

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-4*a*arctanh(1/2*c^(1/2)*(1-1/
a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a*c*(1-1/a^2/x^2)
^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6313, 679, 675, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-4\sqrt{2}a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)+\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a*c*Sqrt[1 -
 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 675

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4c)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \frac{(8c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a\sqrt{c}\text{carctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= \frac{2a\left(\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1+7ax)+3\sqrt{2}\sqrt{c}(-1+ax)\log\left((-1+ax)^2\right)-3\sqrt{2}\sqrt{c}(-1+ax)\log\left(2\sqrt{2}a^2\sqrt{\right.\right.}{-3+3ax}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + 7*a*x) + 3*Sqrt[2]*Sqrt[
c]*(-1 + a*x)*Log[(-1 + a*x)^2] - 3*Sqrt[2]*Sqrt[c]*(-1 + a*x)*Log[2*Sqrt[2
]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x +
 3*a^2*x^2)]))/(-3 + 3*a*x)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-3a\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2+7x\sqrt{(ax+1)x}\,a\sqrt{\frac{1}{a}}+\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$	140
risch	$\frac{2(7a^2x^2+8ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)} - \frac{2a\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	180

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-3*a*2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+7*
x*((a*x+1)*x)^(1/2)*a*(1/a)^(1/2)+((a*x+1)*x)^(1/2)*(1/a)^(1/2))/x/((a*x+1)
*x)^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.82

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= \left[\frac{3\sqrt{2}(a^2x^2-ax)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(7\,a^2x^2+8\,ax+\cdots}{3\,(ax^2-x)}\right.$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x), 2/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.513 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2/5*a^2*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^2*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)-4*a^2*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/
(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6313, 809, 679, 675, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right) + \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + (2*a^2*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a^2*c*Sqrt[1 - 1/(a^2*x
^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 675

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} - (ac^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - (2ac^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4ac)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + (8c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3+11ax+38a^2x^2)}{15x(-1+ax)} + 2\sqrt{2}a^2\sqrt{c}\log\left((-1+ax)^2\right) - 2\sqrt{2}a^2\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-2ax+3a^2x^2)\right)$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 11*a*x + 38*a^2*x^2))/(15
*x*(-1 + a*x)) + 2*Sqrt[2]*a^2*Sqrt[c]*Log[(-1 + a*x)^2] - 2*Sqrt[2]*a^2*Sq
rt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2
 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
default	$-\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(15a^2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^3-38a^2\sqrt{\frac{1}{a}}\,x^2\sqrt{(ax+1)x}-11x\sqrt{(ax+1)x}\,a\sqrt{\frac{1}{a}}-3\sqrt{(ax+1)x}\,\sqrt{}\right.}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^2\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\left(38a^3x^3+49a^2x^2+14ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)ac}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2*(15
*a^2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1)
)*x^3-38*a^2*(1/a)^(1/2)*x^2*((a*x+1)*x)^(1/2)-11*x*((a*x+1)*x)^(1/2)*a*(1/
a)^(1/2)-3*((a*x+1)*x)^(1/2)*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$=\left[\frac{15\sqrt{2}(a^3x^3-a^2x^2)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(38\,a^3x^3+}{15\,(ax^3-x^2)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(38*a^3*x^3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
```

```
qrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2), 2/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)
*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (38*a^3*x^3 + 49*
a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a
*x^3 - x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.514 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 4/7*a^3*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^3*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)-2/7*a^3*c^2*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(3/2)-4*a^3*
arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^
(1/2)+4*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6313, 1653, 809, 679, 675, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-4\sqrt{2}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)+\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (4*a^3*c^3*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(5/2)) + (2*a^3*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) - (2*a^3*c^2*(1 - 1/(a^2*x^2
))^(5/2))/(7*(c - c/(a*x))^(3/2)) + (4*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
- c/(a*x)] - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 675

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{1}{7}\left(2a^4c\right)\text{Subst}\left(\int \frac{\left(\frac{3c^2}{2a^2}-\frac{5c^2x}{a^3}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - \left(a^2c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2a^2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \left(4a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \left(8ac^2\right)\text{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3+9ax+16a^2x^2+52a^3x^3)}{21x^2(-1+ax)} + 2\sqrt{2}a^3\sqrt{c}\log\left((-1+ax)^2\right) - 2\sqrt{2}a^3\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c\left(-1-2ax+3a^2x^2\right)\right)$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 9*a*x + 16*a^2*x^2 + 52*a
^3*x^3))/(21*x^2*(-1 + a*x)) + 2*Sqrt[2]*a^3*Sqrt[c]*Log[(-1 + a*x)^2] - 2*
Sqrt[2]*a^3*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
- c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89

method	result
default	$-\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(21a^3\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^4-52a^3\sqrt{\frac{1}{a}}\,x^3\sqrt{(ax+1)x}-16a^2\sqrt{\frac{1}{a}}\,x^2\sqrt{(ax+1)x}-9x\sqrt{(ax+1)x}}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\left(52a^4x^4+68a^3x^3+25a^2x^2+12ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^3\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^3*(21
*a^3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1)
)*x^4-52*a^3*(1/a)^(1/2)*x^3*((a*x+1)*x)^(1/2)-16*a^2*(1/a)^(1/2)*x^2*((a*x
+1)*x)^(1/2)-9*x*((a*x+1)*x)^(1/2)*a*(1/a)^(1/2)-3*((a*x+1)*x)^(1/2)*(1/a)^
(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.90

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \left[\frac{21\sqrt{2}(a^4x^4-a^3x^3)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(52\,a^4x^4+6}{21\,(ax^4-x^3)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] [1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3), 2/21*(21*sqrt(2)*(a^4*x
^4 - a^3*x^3)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (52*
a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.515 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 303

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^4\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2/5*a^4*(1+1/a/x)^(
5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-2/7*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)+2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4
*a^4*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)
^(1/2)+4*a^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{6317, 6315, 90, 52, 65, 212\}$

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{4\sqrt{2}a^4\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{4a^4\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (4*a^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*a^4*(1 +
 1/(a*x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a
*x))^(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) - (2*a^4*(1 + 1/(a*x))^
(7/2)*Sqrt[c - c/(a*x)])/(7*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(9/2)
*Sqrt[c - c/(a*x)])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*a^4*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \left(-a^3\left(1+\frac{x}{a}\right)^{3/2}+\frac{a^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}+a^3\left(1+\frac{x}{a}\right)^{5/2}-a^3\left(1+\frac{x}{a}\right)^{7/2}\right)dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(a^3\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a^3\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{1-\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(4a^3\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(8a^4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^4\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.59

$$
\begin{aligned}
&\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx \\
&= \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(35+95ax+138a^2x^2+236a^3x^3+788a^4x^4)}{315x^3(-1+ax)} \\
&\quad + 2\sqrt{2}a^4\sqrt{c}\log\left((-1+ax)^2\right) \\
&\quad - 2\sqrt{2}a^4\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c\left(-1-2ax+3a^2x^2\right)\right)
\end{aligned}
$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(35 + 95*a*x + 138*a^2*x^2 + 2
36*a^3*x^3 + 788*a^4*x^4))/(315*x^3*(-1 + a*x)) + 2*Sqrt[2]*a^4*Sqrt[c]*Log
[(-1 + a*x)^2] - 2*Sqrt[2]*a^4*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

method	result
risch	$\frac{2\left(788a^5x^5+1024a^4x^4+374a^3x^3+233a^2x^2+130ax+35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^4\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-315a^4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^5+788x^4\sqrt{(ax+1)x}\,a^4\sqrt{\frac{1}{a}}+236a^3\sqrt{\frac{1}{a}}\,x^3\sqrt{(ax+1)x}+138a^2\sqrt{\frac{1}{a}}\,x^{}$
	$315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^4\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(788*a^5*x^5+1024*a^4*x^4+374*a^3*x^3+233*a^2*x^2+130*a*x+35)/x^4/((a
*x-1)/(a*x+1))^(1/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)-2*a^4*2^(1/2)/c^(1/2)*ln
((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(
1/2))/(x-1/a))/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((a*x+
1)*a*c*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.36

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \left[\frac{315\sqrt{2}(a^5x^5-a^4x^4)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\,(788\,a^5x^5}{315\,(ax^5-x^4)}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 2*(788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2 + 130*a*
x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^4), 2
/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) + (788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2
 + 130*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5
- x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.516 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

Optimal result

Integrand size = 27, antiderivative size = 126

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}-\frac{(3+4m)\sqrt{c-\frac{c}{ax}}x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -1/2*(3+4*m)*x^m*hypergeom([1/2, -m],[1-m],-1/a/x)*(c-c/a/x)^(1/2)/a/m/(1+m
)/(1-1/a/x)^(1/2)+x^(1+m)*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6317, 6316, 80, 66}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\sqrt{\frac{1}{ax}+1}x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)\sqrt{1-\frac{1}{ax}}}-\frac{(4m+3)x^m\sqrt{c-\frac{c}{ax}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a*x)])
 - ((3 + 4*m)*Sqrt[c - c/(a*x)]*x^m*Hypergeometric2F1[1/2, -m, 1 - m, -(1/(
a*x))])/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^m\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}} + \frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a(1+m)\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}-\frac{(3+4m)\sqrt{c-\frac{c}{ax}}x^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$$
$$=\frac{\sqrt{c-\frac{c}{ax}}x^m\left(2am\sqrt{1+\frac{1}{ax}}x-(3+4m)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a*x)]*x^m*(2*a*m*Sqrt[1 + 1/(a*x)]*x - (3 + 4*m)*Hypergeometri
c2F1[1/2, -m, 1 - m, -(1/(a*x))]))/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Maple [F]

$$\int x^m\sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}dx$$

```
[In] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int\sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \int\sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \int\sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \int x^m\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.517 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] -11/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3+11/8
*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)-11/12*c*x^2*(1-1/a^2/x^2)^(1/2
)/a/(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6313, 893, 887, 889, 214}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=-\frac{11cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}+\frac{11cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (11*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) - (11*c*Sqrt[1 - 1
/(a^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(
3*Sqrt[c - c/(a*x)]) - (11*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
Sqrt[c - c/(a*x)]])/(8*a^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{11\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= -\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{11\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} - \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{11\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} - \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{(11c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^5}$$

$$= \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} - \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{11\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(33-22ax+8a^2x^2)}{-1+ax} + 33\sqrt{c}\log(1-ax) - 33\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-\right.}{48a^3}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(33 - 22*a*x + 8*a^2*x^
2))/(-1 + a*x) + 33*Sqrt[c]*Log[1 - a*x] - 33*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a
^3)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}-44a^{\frac{3}{2}}x\sqrt{(ax+1)x}+66\sqrt{(ax+1)x}\,\sqrt{a}-33\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{48a^{\frac{5}{2}}(ax-1)\sqrt{(ax+1)x}}$	133
risch	$\frac{(8a^{2}x^{2}-22ax+33)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{24a^{2}(ax-1)}-\frac{11\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{16a^{2}\sqrt{a^{2}c}\,(ax-1)}$	160

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)+66*((a*x+1)*x)^(1/2)*a^(
1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(a*x
-1)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^{2}\,dx$$
$$=\left[\frac{33\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^{4}x^{4}-14\,a^{3}x^{3}+11\,a^{2}x^{2}+3}{96\,(a^{4}x-a^{3})}\right.$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(33*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(8*a^4*x^4 - 14*a^3*x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(33*(a*x - 1)*
sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 14*a^3*x^3 +
11*a^2*x^2 + 33*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
4*x - a^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\sqrt{c-\frac{c}{ax}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.518 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] 7/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2-7/4*c*
x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-c/
a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6313, 893, 887, 889, 214}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}+\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}-\frac{7cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(2*Sqrt[c - c/(a*x)]) + (7*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} + \frac{7\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} - \frac{7\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}} - \frac{(7c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}$$

$$= -\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}} + \frac{7\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-7+2ax)}{-4+4ax} - \frac{7\sqrt{c}\log(1-ax)}{8a^2} + \frac{7\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-7 + 2*a*x))/(-4 + 4*a*x) - (
7*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (7*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{3}{2}}x\sqrt{(ax+1)x}-14\sqrt{(ax+1)x}\,\sqrt{a}+7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}(ax-1)\sqrt{(ax+1)x}}$	116
risch	$\frac{(2ax-7)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{8a\sqrt{a^2c}\,(ax-1)}$	152

```
[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(4*a^(3
/2)*x*((a*x+1)*x)^(1/2)-14*((a*x+1)*x)^(1/2)*a^(1/2)+7*ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3-5\,a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\cdots}{16\,(a^3x-a^2)}, -\frac{7\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(2\,a^3x^3-5\,a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\,(a^3x-a^2)}\right]$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*
sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(7*(a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.519 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/
a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6312, 893, 889, 214}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x-\frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	101
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	139

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\ \frac{3\,(a\right.$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.520 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)+2*c*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6313, 895, 889, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 895

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2} \\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x+\sqrt{c}(1-ax)\log(1-ax)+\sqrt{c}(-1+ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1\right.}{-1+ax}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+2\sqrt{(ax+1)x}\,\sqrt{a}\right)}{(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	100
risch	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{a^2c}\,(ax-1)}$	139

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*((a*x+1)*x)^(1/2)*a^(1/2))/(a*x-1)/
((a*x+1)*x)^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)-2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c - \frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c - \frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c - \frac{c}{a\,x}}\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
[Out] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.521 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}$$

```
[Out] -8/3*a*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-2/3*a*(1-1/a^2/x^2)^(1/2)*(c-c
/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6313, 671, 663}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (-8*a*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (2*a*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)])/3
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{4}{3}\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}} - \frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-1+5ax)}{-3+3ax}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 5*a*x))/(-3 + 3*a*x)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)x}$	54
default	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)x}$	54
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2+4ax-1)}{3(ax-1)x}$	57

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(a*x+1)*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\,(5\,a^2x^2+4\,ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,(ax^2-x)}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x))/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^2\,x^2+4\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] -(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + 5*a^2*x^2 - 1)
)/(3*x*(a*x - 1))
```

3.522 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

```
[Out] 2/5*a^2*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/c+8/5*a^2*c*(1-1/a^2/x^2)^(1/2)
/(c-c/a/x)^(1/2)+2/5*a^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6313, 809, 671, 663}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (8*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(5*Sqrt[c - c/(a*x)]) + (2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)])/5 + (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))
^(3/2))/(5*c)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{(3a)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{1}{5}(4a)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}} + \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1-3ax+6a^2x^2)}{5x(-1+ax)}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 - 3*a*x + 6*a^2*x^2))/(5*x*
(-1 + a*x))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{2(ax+1)(6a^2x^2-3ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5(ax-1)x^2}$	62
default	$\frac{2(ax+1)(6a^2x^2-3ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5(ax-1)x^2}$	62
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(6a^3x^3+3a^2x^2-2ax+1)}{5(ax-1)x^2}$	65

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(a*x+1)*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/
2)/(a*x-1)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\,(6\,a^3x^3+3\,a^2x^2-2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{5\,(ax^3-x^2)}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] 2/5*(6*a^3*x^3 + 3*a^2*x^2 - 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima"
)
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(6\,a^3\,x^3+3\,a^2\,x^2-2\,a\,x+1\right)}{5\,x^2\,\left(a\,x-1\right)}$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^3,x)
[Out] (2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 2*a*x + 6*a
^3*x^3 + 1))/(5*x^2*(a*x - 1))
```

3.523 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 149

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3}-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2}$$

```
[Out] -104/105*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2/7*c*(1-1/a^2/x^2)^(1/2
)/x^3/(c-c/a/x)^(1/2)-26/35*a*c*(1-1/a^2/x^2)^(1/2)/x^2/(c-c/a/x)^(1/2)-104
/105*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6313, 895, 885, 809, 663}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35x^2\sqrt{c-\frac{c}{ax}}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] (-104*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) - (104*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (2*c*Sqrt[1 - 1/(a^2*x^2)])/(7*Sq
```

```
rt[c - c/(a*x)]*x^3) - (26*a*c*Sqrt[1 - 1/(a^2*x^2)])/(35*Sqrt[c - c/(a*x)]
*x^2)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 885

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
p + 1)/(c*(m - n - 1))), x] - Dist[n*((e*f + d*g)/(e*(m - n - 1))), Int[(d
+ e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
&& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Int
egerQ[n])
```

Rule 895

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{13}{7}\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2} + \frac{1}{35}(52a)\text{Subst}\left(\int \frac{x\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2} - \frac{1}{105}\left(52a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} - \frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-15+39ax-52a^2x^2+104a^3x^3)}{105x^2(-1+ax)}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 39*a*x - 52*a^2*x^2 + 104*a^3*x^3))/(105*x^2*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(ax+1)\left(104a^{3}x^{3}-52a^{2}x^{2}+39ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^{3}}$	70
default	$-\frac{2(ax+1)\left(104a^{3}x^{3}-52a^{2}x^{2}+39ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^{3}}$	70
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\left(104a^{4}x^{4}+52a^{3}x^{3}-13a^{2}x^{2}+24ax-15\right)}{105(ax-1)x^{3}}$	73

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(a*x+1)*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*((a
*x-1)/(a*x+1))^(1/2)/(a*x-1)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\left(104\,a^4x^4+52\,a^3x^3-13\,a^2x^2+24\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] -2/105*(104*a^4*x^4 + 52*a^3*x^3 - 13*a^2*x^2 + 24*a*x - 15)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(104\,a^3\,x^3+156\,a^2\,x^2+143\,a\,x+167\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3}-\frac{304\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^4,x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*(143*a*x + 156*a^2*x^2 + 104*a^3*x^3 + 167
)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (304*((a*x - 1)/(a*x + 1))^(1/2)
*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.524 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=-\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{363\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] 363/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4-4*arctanh(1/2*(c-c/a/x)
^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4-149/64*x*(c-c/a/x)^(1/2)/a^3+10
7/96*x^2*(c-c/a/x)^(1/2)/a^2-17/24*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6302, 6268, 25, 528, 457, 100, 156, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{363\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-149*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2)
- (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (363*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) - (4*Sqrt[2]*Sqrt[c]*ArcTa
nh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^3(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^5(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{\text{Subst}\left(\int \frac{\frac{17c^2}{2}-\frac{15c^2x}{2a}}{x^4(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4c}$$

$$= -\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{\text{Subst}\left(\int \frac{\frac{107c^3}{4}-\frac{85c^3x}{4a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{12ac^2}$$

$$= \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{\text{Subst}\left(\int \frac{\frac{447c^4}{8}-\frac{321c^4x}{8a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{24a^2c^3}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{\text{Subst}\left(\int \frac{\frac{1089c^5}{16}-\frac{447c^5x}{16a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{24a^3c^4}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{(363c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{128a^4} + \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a^4}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{363\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{64a^3} - \frac{8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{a^3}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{363\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4} - \frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.67

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$=\frac{a\sqrt{c-\frac{c}{ax}}x(-447+214ax-136a^2x^2+48a^3x^3)+1089\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-768\sqrt{2}\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right.}{192a^4}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-447 + 214*a*x - 136*a^2*x^2 + 48*a^3*x^3) + 1089*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 768*Sqrt[2]*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(48a^3x^3-136a^2x^2+214ax-447)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^3}+\frac{\left(\frac{363\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-acx}\right)}{128a^3\sqrt{a^2c}}+\frac{2\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3}}{x+\frac{1}{a}}\right.}{a^4\sqrt{c}}\right.}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-96x(ax^2-x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}+176(ax^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-252\sqrt{ax^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+768\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+126\sqrt{ax^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\right.}{384\sqrt{(ax-1)x}}$

```
[In] int(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(48*a^3*x^3-136*a^2*x^2+214*a*x-447)/a^3*x*(c*(a*x-1)/a/x)^(1/2)+(363
/128/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*
c)^(1/2)+2/a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2
*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1
/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.58

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \left[\frac{768\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\left(48\,a^4x^4-136\,a^3x^3+214\,a^2x^2-447\,ax\right)\sqrt{\frac{acx-c}{ax}}+1089\sqrt{c}}{384\,a^4}\right.$$

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] [1/384*(768*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2 -
 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, 1/192*(768*sqrt(2)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*x^3 +
 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arctan(sqrt
(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^3}{ax+1}\,dx$$

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)

[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)

3.525 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{45\sqrt{c}\,\text{carctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\,\text{carctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] -45/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+4*arctanh(1/2*(c-c/a/x)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3+19/8*x*(c-c/a/x)^(1/2)/a^2-13/12
*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6302, 6268, 25, 528, 457, 100, 156, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=-\frac{45\sqrt{c}\,\text{carctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\,\text{carctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (19*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 - (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^3
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{\text{Subst}\left(\int \frac{\frac{13c^2}{2}-\frac{11c^2x}{2a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{\text{Subst}\left(\int \frac{\frac{57c^3}{4}-\frac{39c^3x}{4a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6ac^2}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{\text{Subst}\left(\int \frac{\frac{135c^4}{8}-\frac{57c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a^2c^3}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{(45c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3} - \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a^3}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{45\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{8a^2} + \frac{8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{a^2}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{45\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3} + \frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= \frac{a\sqrt{c-\frac{c}{ax}}x(57-26ax+8a^2x^2) - 135\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) + 96\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(57 - 26*a*x + 8*a^2*x^2) - 135*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]] + 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt
[2]*Sqrt[c])])/(24*a^3)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.29

method	result
risch	$\frac{(8a^2x^2-26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2}+\frac{\Big(-\frac{45\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\Big(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+\cdots}}{x+\frac{1}{a}}}{a^3\sqrt{c}}\cdots}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\Big(16(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-36\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+96\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+18\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-96a^{\frac{3}{2}}\sqrt{2}\ln\Big(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax-\cdots}}{ax+\cdots}}{48\sqrt{(ax-1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}}$

```
[In] int(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*a^2*x^2-26*a*x+57)/a^2*x*(c*(a*x-1)/a/x)^(1/2)+(-45/16/a^2*ln((-1/2
*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a^3*2^
(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x
+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*
x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$=\left[\frac{96\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2\,acx+\cdots\right.}{48\,a^3}\right.$$

$$-\frac{96\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}-135\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\cdots\right.}{24\,a^3}$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
)) + 3*a*c*x - c)/(a*x + 1)) + 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*
c*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)
/(a*x)) + c))/a^3, -1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*s
```

```
qrt((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c*x -
 c)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^2}{ax+1}\,dx$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.526 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] 23/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2-4*arctanh(1/2*(c-c/a/x)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2-9/4*x*(c-c/a/x)^(1/2)/a+1/2*x^2*(
c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6302, 6268, 25, 528, 457, 100, 156, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-9*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (23*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}-\frac{7c^2x}{2a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 - \frac{\text{Subst}\left(\int \frac{\frac{23c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 - \frac{(23c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2} + \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{23\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{4a} - \frac{8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{a}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{23\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2} - \frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \frac{a\sqrt{c-\frac{c}{ax}}x(-9+2ax) + 23\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 16\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-9 + 2*a*x) + 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]] - 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])
/(4*a^2)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(2ax-9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{\left(\frac{23\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{8a\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a^2\sqrt{c}}\right)\sqrt{\frac{c(ax}{a}}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+16\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+2\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-16a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)-24a^2\right.}{8\sqrt{(ax-1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}}$

```
[In] int(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*a*x-9)/a*x*(c*(a*x-1)/a/x)^(1/2)+(23/8/a*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)+2/a^2*2^(1/2)/c^(1/2)*ln((4
*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2
))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.96

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{16\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+2\left(2a^2x^2-9ax\right)\sqrt{\frac{acx-c}{ax}}+23\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{8a^2}\right.$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 - 3*a*c*x + c)/(a*x + 1)) + 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x))
+ 23*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2
, 1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x}{ax+1}\,dx$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.527 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 100, 162, 65, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+x\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}x + \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}x + \frac{(5c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \sqrt{c-\frac{c}{ax}}x - 5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) + 8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= \sqrt{c-\frac{c}{ax}}x - \frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(75) = 150.

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\cdots}}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}-6\cdots\right.}{2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*(a*x-1)/a/x)^(1/2)+(-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^
2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2
^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)
*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.38

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a}, \frac{ax\sqrt{\ldots}}{\ldots}\right.$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)

[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)

3.528 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(c-c/a/x)^(1/2)*2^
(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 445, 457, 86, 162, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+2\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= 2\sqrt{c-\frac{c}{ax}} - \frac{a\text{Subst}\left(\int \frac{c^2-\frac{3c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= 2\sqrt{c-\frac{c}{ax}} - c\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right) + (4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 2\sqrt{c-\frac{c}{ax}} + (2a)\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) - (8a)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(69) = 138.

Time = 0.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

method	result
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(\frac{a\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{\sqrt{a^{2}c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}}\right)\sqrt{c(ax-1)ax}\,\sqrt{}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{3}{2}}x^{2}-4\sqrt{\frac{1}{a}}\,\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}x^{2}-3\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}-2\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right.\right.}{x\sqrt{(ax-1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*(a*x-1)/a/x)^(1/2)+(a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a
*c*x)^(1/2))/(a^2*c)^(1/2)+2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2
)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*(c*(a*x-1)*a
*x)^(1/2)*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.36

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[2\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right) + \sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right) + 2\sqrt{\frac{acx-c}{ax}}, 4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right) - 2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right) + 2\sqrt{\frac{acx-c}{ax}}\right]$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a
*c*x + c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sq
rt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*sqrt(-c)*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(70) = 140.

Time = 4.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.64

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \begin{cases} -\frac{2a\left(\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c} & \text{for}\ \frac{c}{a}\neq 0 \\ -\frac{3a\sqrt{c}\left(\frac{\log\left(-\frac{2}{x}\right)}{a}-\frac{\log\left(2a+\frac{2}{x}\right)}{a}\right)}{2} + \frac{\sqrt{c}\log\left(\frac{a}{x}+\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Piecewise((-2*a*(c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 2*sqr
t(2)*c**2*atan(sqrt(2)*sqrt(c - c/(a*x))/(2*sqrt(-c)))/(a*sqrt(-c)) - c*sqr
t(c - c/(a*x))/a)/c, Ne(c/a, 0)), (-3*a*sqrt(c)*(log(-2/x)/a - log(2*a + 2/
x)/a)/2 + sqrt(c)*log(a/x + x**(-2))/2, True))
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.529 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2/3*a*(c-c/a/x)^(3/2)/c+4*a*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)-4*a*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6268, 25, 528, 455, 52, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = 4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c) + 4*Sqrt[2]*a*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^2(1+ax)}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x(1+ax)}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}\,dx}{c}\\
&= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-(2a)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)\\
&= -4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-(4ac)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\left(8a^2\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= -4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4\sqrt{2}a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(1-7ax)}{3x}+4\sqrt{2}a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
[Out] (2*Sqrt[c - c/(a*x)]*(1 - 7*a*x))/(3*x) + 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(67) = 134$.

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{2\left(7a^2x^2-8ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}-\frac{2a\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{\sqrt{c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(6\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3-18\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3+12a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+9\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^2x^3-6a^{\frac{3}{2}}\sqrt{2}\,\ln\right.}{3x^2\sqrt{(ax-1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(7*a^2*x^2-8*a*x+1)/x/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)-2*a*2^(1/2)/c^(1/2
)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*
c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.96

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \left[\frac{2\left(3\sqrt{2}a\sqrt{c}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x},\ -\frac{2\left(6\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x}\right]$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a*sqrt(c)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + 3*a*c*x - c)/(a*x + 1)) - (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x, -2
/3*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^2\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.530 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = 4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-4\sqrt{2}a^2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2/3*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2-4*a^2*arctanh(1/2*(c-
c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 81, 52, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -4\sqrt{2}a^2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^2\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/(3*c) + (2*a^2*(c - c
/(a*x))^(5/2))/(5*c^2) - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx \\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^3(1+ax)}\,dx \\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^2(1+ax)}\,dx}{c} \\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}\,dx}{c} \\
&= -\frac{a\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{a^2\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right) \\
&= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} \\
&\quad + \left(4a^2c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= 4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\left(8a^3\right)\text{Subst}\left(\int\frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)$$

$$= 4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

$$\int\frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}\left(3-11ax+38a^2x^2\right)}{15x^2}-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) - 4*Sqrt[2]*a^2*Sq
rt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2\left(38a^3x^3-49a^2x^2+14ax-3\right)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}+\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)a}}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(30\sqrt{(ax-1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4-90a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^4+60a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\left(a\,x^2-x\right)^{\frac{3}{2}}x^2+45\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4-3}{15x^3\sqrt{(ax}}$

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
[Out] 2/15*(38*a^3*x^3-49*a^2*x^2+14*a*x-3)/x^2/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)+2*a
^2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2
-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1
)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.60

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \left[\frac{2\left(15\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}, \frac{2\left(30\sqrt{2}a^2\sqrt{-c}x^2\operatorname{arcta}\right.}{}\right.$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x
 - c)/(a*x)))/x^2, 2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x -
c)/(a*x)))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^3\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(94) = 188.

Time = 0.64 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{4\sqrt{2}a^3c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)}$$

$$+\frac{2\left(60\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3a^4c^{\frac{3}{2}}|a|+35\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\cdots}\right.\right.}{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^2|a|\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^3*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/15*(60*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^4
*c^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a
^2*abs(a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.531 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2/3*a^3*(c-c/a/x)^(3/2)/c-2/7*a^3*(c-c/a/x)^(7/2)/c^3+4*a^3*arctanh(1/2*(c
-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 90, 52, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = 4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/(3*c) - (2*a^3*(c -
c/(a*x))^(7/2))/(7*c^3) + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^4(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^3(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\text{Subst}\left(\int \left(\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a\left(c-\frac{cx}{a}\right)^{5/2}}{c}\right)\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{a^3\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(2a^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$
$$- \left(4a^3c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + (8a^4)\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + 4\sqrt{2}a^3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(3-9ax+16a^2x^2-52a^3x^3)}{21x^3} + 4\sqrt{2}a^3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 9*a*x + 16*a^2*x^2 - 52*a^3*x^3))/(21*x^3) + 4*Sq
rt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{2\left(52a^4x^4-68a^3x^3+25a^2x^2-12ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3(ax-1)} - \frac{2a^3\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(42\sqrt{(ax-1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^5-126\sqrt{a\,x^2-x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^5+84\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^3+63\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^4x^5-42a\right.}{}$

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
[Out] -2/21*(52*a^4*x^4-68*a^3*x^3+25*a^2*x^2-12*a*x+3)/x^3/(a*x-1)*(c*(a*x-1)/a/
x)^(1/2)-2*a^3*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2
*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/
2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \left[\frac{2\left(21\sqrt{2}a^3\sqrt{c}x^3\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(52\,a^3x^3-16\,a^2x^2+9\,ax-3)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3},\right.$$
$$\left.-\frac{2\left(42\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(52\,a^3x^3-16\,a^2x^2+9\,ax-3)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}\right]$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x - 3
)*sqrt((a*c*x - c)/(a*x)))/x^3, -2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1
/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x^2 +
 9*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^4\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(94) = 188.

Time = 0.74 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{4\sqrt{2}a^4c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)} - \frac{2\left(84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^6c^{\frac{3}{2}}|a|+112\left(\sqrt{a^2c}x-\sqrt{a^2cx^2}\right.\right.}{21}$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
[Out] 4*sqrt(2)*a^4*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/21*(84*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*
a^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^7*c^3
 - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) + 3*a^7*
c^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.532 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = 4a^4\sqrt{c-\frac{c}{ax}}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2/3*a^4*(c-c/a/x)^(3/2)/c+2/5*a^4*(c-c/a/x)^(5/2)/c^2-2/7*a^4*(c-c/a/x)^(7/
2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4-4*a^4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2
)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 90, 52, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] + (2*a^4*(c - c/(a*x))^(3/2))/(3*c) + (2*a^4*(c - c
/(a*x))^(5/2))/(5*c^2) - (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c -
c/(a*x))^(9/2))/(9*c^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^5(1+ax)}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^4(1+ax)}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}\,dx}{c}\\
&= -\frac{a\text{Subst}\left(\int \frac{x^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{a\text{Subst}\left(\int \left(a^2\left(c-\frac{cx}{a}\right)^{3/2}-\frac{a^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}-\frac{a^2\left(c-\frac{cx}{a}\right)^{5/2}}{c}+\frac{a^2\left(c-\frac{cx}{a}\right)^{7/2}}{c^2}\right)dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{a^4\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}
\end{aligned}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \left(2a^4\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \left(4a^4c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \left(8a^5\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - 4\sqrt{2}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}\left(35-95ax+138a^2x^2-236a^3x^3+788a^4x^4\right)}{315x^4} - 4\sqrt{2}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
[Out] (2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^4
))/(315*x^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqr
t[c])]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

method	result
risch	$\frac{2(788a^5x^5-1024a^4x^4+374a^3x^3-233a^2x^2+130ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4(ax-1)}+\frac{2a^4\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(630\sqrt{(ax-1)x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6-1890\sqrt{a\,x^2-x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6+1260\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^4+945\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a}{}$

[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)

[Out] 2/315*(788*a^5*x^5-1024*a^4*x^4+374*a^3*x^3-233*a^2*x^2+130*a*x-35)/x^4/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)+2*a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \left[\frac{2\left(315\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4}\right.$$

[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")

[Out] [2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, 2/315*(630*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (788*a^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4]

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^5(ax+1)}\,dx$$

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^5}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(136) = 272$.

Time = 0.88 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.66

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{4\sqrt{2}a^5c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}(x)} + \frac{2\left(1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7a^8c^{\frac{3}{2}}|a|+2100\left(\sqrt{a^2c}x-\sqrt{\right.\right.}{}$$

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
[Out] -4*sqrt(2)*a^5*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/315*(12
60*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*
c*x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))
^4*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^(7/2)*a
bs(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 - 315*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*c^5)/((sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.533 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 303

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = -\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{1115\sqrt{c-\frac{c}{ax}}x}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{223\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1115\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1115/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-1115/6
4*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1115/192*x*(c-c/a/x)^
(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+223/96*x^2*(c-c/a/x)^(1/2)/a^2/(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-25/24*x^3*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2)+1/4*x^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used

= {6317, 6315, 91, 79, 44, 53, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{1115\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}} - \frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{1115x\sqrt{c-\frac{c}{ax}}}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{223x^2\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{x^4\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{25x^3\sqrt{c-\frac{c}{ax}}}{24a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-1115*Sqrt[c - c/(a*x)])/(64*a^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (1
115*Sqrt[c - c/(a*x)]*x)/(192*a^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2
23*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2
5*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt
[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1115*Sqrt[c -
 c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{-\frac{25}{2a}+\frac{4x}{a^2}}{x^4\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(223\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{48a^2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{223\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1115\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{192a^3\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{1115\sqrt{c-\frac{c}{ax}}x}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{223\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(1115\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{128a^4\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{1115\sqrt{c-\frac{c}{ax}}x}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{223\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(1115\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{128a^4\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{1115\sqrt{c-\frac{c}{ax}}x}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{223\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(1115\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{64a^3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{1115\sqrt{c-\frac{c}{ax}}x}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{223\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3345-1115ax+446a^2x^2-200a^3x^3+48a^4x^4)}{-1+a^2x^2} - 3345\sqrt{c}\log(1-ax) + 3345\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\right.}{384a^4}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3345 - 1115*a*x + 446
*a^2*x^2 - 200*a^3*x^3 + 48*a^4*x^4))/(-1 + a^2*x^2) - 3345*Sqrt[c]*Log[1 -
 a*x] + 3345*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*
x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(384*a^4)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.65

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(96a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4-400a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}+892a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-2230a^{\frac{3}{2}}x\sqrt{(ax+1)x}+3345\ln\left(\frac{2\sqrt{(}\right.\right.}{384(ax-1)^2a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(48a^3x^3-248a^2x^2+694ax-1809)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{192a^3(ax-1)} + \frac{\left(\frac{1115\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{128a^3\sqrt{a^2c}} - \frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^5c\left(x+\frac{1}{a}\right)}\right)}{ax-1}$

```
[In] int(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(96
*a^(9/2)*((a*x+1)*x)^(1/2)*x^4-400*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+892*a^(5/2
)*x^2*((a*x+1)*x)^(1/2)-2230*a^(3/2)*x*((a*x+1)*x)^(1/2)+3345*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-6690*((a*x+1)*x)^(1/2)*a^(1/2)
+3345*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(7/2)/((a*x+
1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.17

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$= \left[\frac{3345\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(48\,a^5x^5-200\,a^4x^4+446\,a^3x}{768\,(a^5x-a^4)}\right.$$
$$-\frac{3345\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2x^2}{384\,(a^5x-a^4)}$$

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3345*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 +
 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2
 - 3345*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^
4), -1/384*(3345*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2
*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\sqrt{c-\frac{c}{ax}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int x^3\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

```
[In] int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.534 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 251

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{c-\frac{c}{ax}}x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{119\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -119/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+119/8*(
c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+119/24*x*(c-c/a/x)^(1/2)
/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-19/12*x^2*(c-c/a/x)^(1/2)/a/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)+1/3*x^3*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used

$= \{6317, 6315, 91, 79, 44, 53, 65, 214\}$

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = -\frac{119\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}} + \frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{119x\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} + \frac{x^3\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{19x^2\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (119*Sqrt[c - c/(a*x)])/(8*a^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (119*
Sqrt[c - c/(a*x)]*x)/(24*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (19*Sqr
t[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c -
c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (119*Sqrt[c - c/(a*
x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{-\frac{19}{2a}+\frac{3x}{a^2}}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{24a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{119\sqrt{c-\frac{c}{ax}}x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{16a^3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{119\sqrt{c-\frac{c}{ax}}x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{119\sqrt{c-\frac{c}{ax}}x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{119\sqrt{c-\frac{c}{ax}}x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{119\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.63

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(357+119ax-38a^2x^2+8a^3x^3)}{-1+a^2x^2}+357\sqrt{c}\log(1-ax)-357\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+\right.}{48a^3}$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(357 + 119*a*x - 38*a^2
*x^2 + 8*a^3*x^3))/(-1 + a^2*x^2) + 357*Sqrt[c]*Log[1 - a*x] - 357*Sqrt[c]*
Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x
 + 2*a^2*x^2)])/(48*a^3)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-76a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+238a^{\frac{3}{2}}x\sqrt{(ax+1)x}-357\ln\left(\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+714\sqrt{}\right.}{48(ax-1)^2a^{\frac{5}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(8a^2x^2-46ax+165)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{24a^2(ax-1)}+\frac{\left(-\frac{119\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{16a^2\sqrt{a^2c}}+\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax}{ax}}}{ax-1}$

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(16*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)-76*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+238*a^(3/2)*
x*((a*x+1)*x)^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2
))*a*x+714*((a*x+1)*x)^(1/2)*a^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2
)+2*a*x+1)/a^(1/2)))/a^(5/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.34

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{357\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2}{96\,(a^4x-a^3)}\right.$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(357*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(357*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 38*a^3*x^
3 + 119*a^2*x^2 + 357*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*x - a^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\sqrt{c-\frac{c}{ax}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int x^2\,\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.535 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{47\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 47/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-47/4*(c-c
/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-13/4*x*(c-c/a/x)^(1/2)/a/(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1/2*x^2*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/
a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6317, 6315, 91, 79, 53, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{47\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{x^2\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{13x\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-47*Sqrt[c - c/(a*x)])/(4*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (13*S
qrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (47*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{13}{2a}+\frac{2x}{a^2}}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$
$$+ \frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$
\begin{aligned}
&= -\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} \\
&\quad - \frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{4a\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} \\
&\quad + \frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{47\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.76

$$
\begin{aligned}
&\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx \\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-47-13ax+2a^2x^2)}{-4+4a^2x^2} - \frac{47\sqrt{c}\log(1-ax)}{8a^2} \\
&\quad + \frac{47\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-ax+2a^2x^2)\right)}{8a^2}
\end{aligned}
$$

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-47 - 13*a*x + 2*a^2*x^2))/(-
4 + 4*a^2*x^2) - (47*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (47*Sqrt[c]*Log[2*a^2*
Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x
^2)])/(8*a^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-26a^{\frac{3}{2}}x\sqrt{(ax+1)x}+47\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-94\sqrt{(ax+1)x}\,\sqrt{a}+47\ln\left(2\right.\right.}{8(ax-1)^2a^{\frac{3}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(2ax-15)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{\left(\frac{47\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{8a\sqrt{a^2c}}-\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^3c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(ax+}}{ax-1}$

[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-26*a^(3/2)*x*((a*x+1)*x)^(1/2)+47*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-94*((a*x+1)*x)^(1/2)*a^(1/2)+47*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(3/2)/((a*x+1)*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{47\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3-13\,a^2x^2-47\,ax)\sqrt{\frac{a}{a}}}{16\,(a^3x-a^2)}\right.$$
$$\left.-\frac{47\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(2\,a^3x^3-13\,a^2x^2-47\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\,(a^3x-a^2)}\right]$$

[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/16*(47*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(47*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Timed out}$$

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\sqrt{c-\frac{c}{a\,x}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.536 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^(
1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 91, 79, 65, 214}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=-\frac{7\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{7\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)x}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{(ax+1)x}\,\sqrt{a}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)^{2}\sqrt{a}\,\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.14

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)}, \frac{7\,(a\right.$$

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.537 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/
x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6317, 6315, 89, 65, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-8*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2*Sqrt[1 +
1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(a*x)]*ArcTan
h[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \left(-\frac{4}{a\left(1+\frac{x}{a}\right)^{3/2}}+\frac{1}{a\sqrt{1+\frac{x}{a}}}+\frac{1}{x\sqrt{1+\frac{x}{a}}}\right)\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x(1+5ax)}{-1+a^2x^2}-\sqrt{c}\log(1-ax)+\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c\left(-1-ax+2a^2x^2\right)\right)$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x*(1 + 5*a*x))/(-1 + a^2*x^2)
 - Sqrt[c]*Log[1 - a*x] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*S
qrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{\frac{3}{2}}x\sqrt{(ax+1)x}-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}x^{2}-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+2\sqrt{(ax+1)x}\,\sqrt{}\right.}{(ax-1)^{2}\sqrt{a}\,\sqrt{(ax+1)x}}$
risch	$-\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{\sqrt{a^{2}c}}-\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*(10*a^(3/2
)*x*((a*x+1)*x)^(1/2)-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*a^2*x^2-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*((a*x+
1)*x)^(1/2)*a^(1/2))/a^(1/2)/((a*x+1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},\right.$$
$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
)/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(
5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
[Out] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.538 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -16/3*a*(c-c/a/x)^(3/2)/c/(1-1/a^2/x^2)^(1/2)-2/3*a*(c-c/a/x)^(5/2)/c^2/(1-1/a^2/x^2)^(1/2)+64/3*a*(c-c/a/x)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6313, 671, 663}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (64*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a^2*x^2)]) - (16*a*(c - c/(a*x))^(3/2))/(3*c*Sqrt[1 - 1/(a^2*x^2)]) - (2*a*(c - c/(a*x))^(5/2))/(3*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
```

```
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}\\
&= -\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c^2}\\
&= -\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{32\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c}\\
&= \frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-1+10ax+23a^2x^2)}{-3+3a^2x^2}$$

[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]

[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(-3 + 3*a^2*x^2)

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
default	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
risch	$\frac{2(11a^2x^2+10ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}+\frac{8a^2x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	101

[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] 2/3*(a*x+1)*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x/(a*x-1)^2

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\,(23\,a^2x^2+10\,ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,(ax^2-x)}$$

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")

[Out] 2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^2 - x)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(23\,a^2\,x^2+10\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(10*a*x + 23*a^2*x^2 - 1
))/(3*x*(a*x - 1))
```

3.539 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -a^2*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)-7/5*a^2*(c-c/a/x)^(3/2)*(1-1/a
^2/x^2)^(1/2)/c-224/15*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-56/15*a^2*
(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6313, 803, 671, 663}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-224*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(15*Sqrt[c - c/(a*x)]) - (56*a^2*Sqrt[1
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/15 - (7*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/
(a*x))^(3/2))/(5*c) - (a^2*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(7a)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(28a)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= -\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{15}(112a)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}} - \frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3-16ax+79a^2x^2+158a^3x^3)}{15x\left(-1+a^2x^2\right)}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 15
8*a^3*x^3))/(15*x*(-1 + a^2*x^2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(ax+1)\left(158a^3x^3+79a^2x^2-16ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	70
default	$-\frac{2(ax+1)\left(158a^3x^3+79a^2x^2-16ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	70
risch	$-\frac{2\left(98a^3x^3+79a^2x^2-16ax+3\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}-\frac{8a^3x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	109

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(a*x+1)*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*((a*x
-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2\left(158\,a^3x^3+79\,a^2x^2-16\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] -2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(158\,a^3\,x^3+79\,a^2\,x^2-16\,a\,x+3\right)}{15\,x^2\,\left(a\,x-1\right)}$$

[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)

[Out] -(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a^2*x^2 - 16*a*x + 158*a^3*x^3 + 3))/(15*x^2*(a*x - 1))

3.540 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] a^3*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)+59/35*a^3*(c-c/a/x)^(3/2)*(1-1/
a^2/x^2)^(1/2)/c+2/7*a^3*(c-c/a/x)^(5/2)*(1-1/a^2/x^2)^(1/2)/c^2+1888/105*a
^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+472/105*a^3*(1-1/a^2/x^2)^(1/2)*(c
-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

= {6313, 1649, 809, 671, 663}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (1888*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) + (472*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (59*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c
- c/(a*x))^(3/2))/(35*c) + (2*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5/2)
)/(7*c^2) + (a^3*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
```

```
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{7a^2}{2}-ax\right)\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(59a^2)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{14c^2}$$

$$= \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(236a^2)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35c}$$

$$= \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{105}(944a^2)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} + \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-15+66ax-167a^2x^2+668a^3x^3+1336a^4x^4)}{105x^2(-1+a^2x^2)}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 + 668*a^3*x^3 + 1336*a^4*x^4))/(105*x^2*(-1 + a^2*x^2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(ax+1)(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
default	$\frac{2(ax+1)(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
risch	$\frac{2(916a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{105x^3(ax-1)} + \frac{8a^4x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	117

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(a*x+1)*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\,(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\,(ax^4-x^3)}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] 2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(1336\,a^3\,x^3+2004\,a^2\,x^2+1837\,a\,x+1903\right)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3} + \frac{3776\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2)*(1837*a*x + 2004*a^2*x^2 + 1336*a^3*x^3 + 19
03)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) + (3776*((a*x - 1)/(a*x + 1))^(1
/2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.541 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 289

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 50/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-38/5*a^4*(1+1/a/x)
^(5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)-2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8
*a^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32*a^4*(1+1/a/x)^(1/2)
*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

= {6317, 6315, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{38a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} + \frac{50a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} - \frac{32a^4\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*a^4*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (32*a^4*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (50*a^4*(1 + 1/(a*
x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) - (38*a^4*(1 + 1/(a*x))^
(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(7/2)
*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] - (2*a^4*(1 + 1/(a*x))^(9/2)*Sqrt[c -
 c/(a*x)])/(9*Sqrt[1 - 1/(a*x)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int\frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int\left(-\frac{4a^3}{\left(1+\frac{x}{a}\right)^{3/2}}+\frac{16a^3}{\sqrt{1+\frac{x}{a}}}-25a^3\sqrt{1+\frac{x}{a}}+19a^3\left(1+\frac{x}{a}\right)^{3/2}-7a^3\left(1+\frac{x}{a}\right)^{5/2}+a^3(1\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.30

$$\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(5-20ax+41a^2x^2-82a^3x^3+328a^4x^4+656a^5x^5)}{45x^3\left(-1+a^2x^2\right)}$$

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 82
*a^3*x^3 + 328*a^4*x^4 + 656*a^5*x^5))/(45*x^3*(-1 + a^2*x^2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.30

method	result	size
gosper	$-\frac{2(ax+1)\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}(ax-1)^{2}}$	86
default	$-\frac{2(ax+1)\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}(ax-1)^{2}}$	86
risch	$-\frac{2\left(476a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{45x^{4}(ax-1)}-\frac{8a^{5}x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	125

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -2/45*(a*x+1)*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*(
a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.29

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{2\left(656\,a^{5}x^{5}+328\,a^{4}x^{4}-82\,a^{3}x^{3}+41\,a^{2}x^{2}-20\,ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{45\left(ax^{5}-x^{4}\right)}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas"
)
[Out] -2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}\,\left(656\,a^4\,x^4+984\,a^3\,x^3+902\,a^2\,x^2+943\,a\,x+923\right)}{45\,x^4} - \frac{1856\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{45\,x^4\,\left(a\,x-1\right)}$$

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(943*a*x + 902
*a^2*x^2 + 984*a^3*x^3 + 656*a^4*x^4 + 923))/(45*x^4) - (1856*((a*x - 1)/(a
*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(45*x^4*(a*x - 1))
```

3.542 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x - \frac{2c(1-n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an} - \frac{2^{n/2}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

```
[Out] c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x-2*c*(1-n)*(1+1/a/x)^(1/2*n)*hyper
geom([1, 1/2*n],[1+1/2*n],(a+1/x)/(a-1/x))/a/n/((1-1/a/x)^(1/2*n))-2^(1/2*n
)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/
a)/a/(2-n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{6314, 130, 71, 98, 133\}$

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$= -\frac{c2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$
$$-\frac{2c(1-n)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an}$$
$$+cx\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x - (2*c*(1 - n)*(1 + 1/(a*x)
)^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a
*n*(1 - 1/(a*x))^(n/2)) - (2^(n/2)*c*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric
2F1[1 - n/2, 1 - n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(a*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 130

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_Symbol] :> Dist[b*(d/f^2), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)
, x], x] + Dist[(b*e - a*f)*((d*e - c*f)/f^2), Int[(a + b*x)^(m - 1)*((c +
d*x)^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right) \\
&= -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right) \\
&\quad + \frac{c\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}\,dx, x, \frac{1}{x}\right)}{a^2} \\
&= c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x \\
&\quad - \frac{2^{n/2}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2}, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)} \\
&\quad + \frac{(c(1-n))\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{a} \\
&= c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x \\
&\quad - \frac{2c(1-n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1, \frac{n}{2}, \frac{2+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an} \\
&\quad - \frac{2^{n/2}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2}, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{ce^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}(-1+n)n\,\text{Hy}\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] (c*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2,
 2 + n/2, -E^(2*ArcCoth[a*x])]) + E^(2*ArcCoth[a*x])*(-1 + n)*n*Hypergeomet
ric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n*x + Hypergeo
metric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])] + (-1 + n)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*n*(2 + n))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)dx$$

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="fricas")
[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*x), x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{ax}\right) dx = \frac{c\left(\int ae^{n \operatorname{acoth}(ax)}\, dx + \int \left(-\frac{e^{n \operatorname{acoth}(ax)}}{x}\right) dx\right)}{a}$$

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x),x)
[Out] c*(Integral(a*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x, x))/a
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{ax}\right) dx = \int \left(c - \frac{c}{ax}\right) \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{ax}\right) dx = \int \left(c - \frac{c}{ax}\right) \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="giac")
[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{ax}\right) dx = \int \mathrm{e}^{n \operatorname{acoth}(a\,x)} \left(c - \frac{c}{a\,x}\right) dx$$

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x)),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x)), x)
```

3.543 $\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}} dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x}{c} - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] (1+1/a/x)^(1+1/2*n)*x/c/((1-1/a/x)^(1/2*n))-2*(1+n)*(1+1/a/x)^(1/2*n)*hyper
geom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6314, 98, 133}

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}} dx = \frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c} - \frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((1 + 1/(a*x))^((2 + n)/2)*x)/(c*(1 - 1/(a*x))^(n/2)) - (2*(1 + n)*(1 + 1/(
a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(a + x^(-1))
])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{(1+n)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac} \\
&= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} \\
&\quad - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$=\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(1+n)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+\right.\right.}{acn(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(1 + n)*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + (1 + n)*Hypergeom
etric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{c-\frac{c}{ax}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x),x)
```

```
[Out] a*Integral(x*exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{ax}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{ax}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a\,x}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x)),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x)), x)
```

3.544 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 166

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)+(1-1/a/x)^(-1-1
/2*n)*(1+1/a/x)^(1+1/2*n)*x/c^2-2*(2+n)*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/
2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c^2/n/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6314, 105, 160, 12, 133}

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx = -\frac{2(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n} - \frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{c^2}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 +
n))) + ((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x)/c^2 - (2*(2 +
 n)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/
(a + x^(-1))])/(a*c^2*n*(1 - 1/(a*x))^(n/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{2+n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{a\text{Subst}\left(\int \frac{(2+n)^2\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{a^2x}\,dx, x, \frac{1}{x}\right)}{c^2(2+n)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{(2+n)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(n(1+ax)(-3+2ax+n(-1+ax))-2(2+n)^2(-1+ax)\text{Hypergeometric2F1}\right.}{ac^2n(2+n)(-1+ax)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(n*(1 + a*x)*(-3 + 2*a*x + n*(-1 + a*x)) - 2*(2 + n)^2
*(-1 + a*x)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (-1 + a*x)/(1 + a*x)]))/(
a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x))
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)

[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="fricas")

[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x + c^2), x)

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**2,x)

[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="maxima")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="giac")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^2,x)

[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^2, x)

3.545 $\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{3/2} dx = -\frac{2^{\frac{5}{2}-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} \left(c-\frac{c}{ax}\right)^{3/2} \text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(-3+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -2^(5/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^(3/2)*AppellF1(1+1/2*n,-3/2+1/
2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(1-1/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6317, 6314, 141}

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{3/2} dx = -\frac{2^{\frac{5}{2}-\frac{n}{2}} \left(c-\frac{c}{ax}\right)^{3/2} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \text{AppellF1}\left(\frac{n+2}{2},\frac{n-3}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
+ n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(1 - 1/(a*x))^(3/2)))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}\\
&= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}\\
&= -\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(-3+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}
\end{aligned}$$

Mathematica [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\$\text{Aborted}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
[Out] $Aborted
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}dx$$

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/
(a*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\left(c-\frac{c}{a\,x}\right)^{3/2}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2), x)
```

3.546 $\int e^{n \coth^{-1}(ax)} \sqrt{c-\frac{c}{ax}}\, dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int e^{n \coth^{-1}(ax)} \sqrt{c-\frac{c}{ax}}\, dx = -\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}\,\text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(-1+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -2^(3/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,-1/2+1/2*n,2,2+1/2*n,1/
2*(a+1/x)/a,1+1/a/x)*(c-c/a/x)^(1/2)/a/(2+n)/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6317, 6314, 141}

$$\int e^{n \coth^{-1}(ax)} \sqrt{c-\frac{c}{ax}}\, dx = -\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n-1}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 +
n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n
)*Sqrt[1 - 1/(a*x)]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{ax}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}\,\text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(-1+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\$\text{Aborted}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{-c\left(-1+\frac{1}{ax}\right)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}} \, dx = \int \sqrt{c - \frac{c}{ax}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}} \, dx = \int \sqrt{c - \frac{c}{ax}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}} \, dx = \int e^{n \operatorname{acoth}(a x)} \sqrt{c - \frac{c}{a x}} \, dx$$

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2), x)
```

3.547 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\operatorname{AppellF1}\left(\frac{2+n}{2},\frac{1+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -2^(1/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,1/2+1/2*n,2,2+1/2*n,1/2
*(a+1/x)/a,1+1/a/x)*(1-1/a/x)^(1/2)/a/(2+n)/(c-c/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6317, 6314, 141}

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\operatorname{AppellF1}\left(\frac{n+2}{2},\frac{n+1}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] -((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)
*Sqrt[c - c/(a*x)]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
```

```
a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}} \\
&= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}} \\
&= -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\text{AppellF1}\left(\frac{2+n}{2},\frac{1+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}
\end{aligned}$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \$\text{Aborted}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)

[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x - c), x)

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{e^{n \operatorname{acoth}{(ax)}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(1/2),x)

[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2), x)
```

3.548 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx = -\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} \text{AppellF1}\left(\frac{2+n}{2},\frac{3+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -2^(-1/2-1/2*n)*(1-1/a/x)^(3/2)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,3/2+1/
2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(c-c/a/x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6317, 6314, 141}

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx = -\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \text{AppellF1}\left(\frac{n+2}{2},\frac{n+3}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2
 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(c - c/(a*x))^(3/2)))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
```

```
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}\\
&= -\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\text{AppellF1}\left(\frac{2+n}{2},\frac{3+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}
\end{aligned}$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \$\text{Aborted}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
[Out] $Aborted
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}dx$$

[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)

[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")

[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^2
*c^2*x^2 - 2*a*c^2*x + c^2), x)

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(3/2),x)

[Out] Integral(exp(n*acoth(a*x))/(-c*(-1 + 1/(a*x)))**(3/2), x)

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2),x)

[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2), x)

3.549 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^p \text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(n-2p),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}$$

```
[Out] -2^(1-1/2*n+p)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^p*AppellF1(1+1/2*n,1/2*n-p,2,2
+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/((1-1/a/x)^p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6317, 6314, 141}

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p \text{AppellF1}\left(\frac{n+2}{2},\frac{1}{2}(n-2p),2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*(1 - 1/(a*x))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^{p}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^{p}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^{p}\text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(n-2p),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}$$

Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}\,dx = \int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}\,dx$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p, x]
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx$$

[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)

[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int\left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")

[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**p,x)

[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int\left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")

[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.550 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}$$

```
[Out] -(1+1/a/x)^(p+1)*(c-c/a/x)^p*hypergeom([2, p+1],[2+p],1+1/a/x)/a/(p+1)/((1-
1/a/x)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6317, 6314, 67}

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(2,p+1,p+2,1+\frac{1}{ax}\right)}{a(p+1)}$$

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^{p}\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{p}dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^{p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{p}}{x^{2}}dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^{p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx = -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^{p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}$$

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Maple [F]

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

Fricas [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{2p\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \mathrm{e}^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

```
[In] int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.551 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p\operatorname{AppellF1}\left(1-p,-2p,2,2-p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}$$

```
[Out] -4^p*(1+1/a/x)^(1-p)*(c-c/a/x)^p*AppellF1(1-p,-2*p,2,2-p,1/2*(a+1/x)/a,1+1/
a/x)/a/(1-p)/((1-1/a/x)^p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6317, 6314, 141}

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}\operatorname{AppellF1}\left(1-p,-2p,2,2-p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

```
[In] Int[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] -((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 - p
, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
```

```
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p\,dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}\left(1+\frac{x}{a}\right)^{-p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(1-p,-2p,2,2-p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}\end{aligned}$$

Mathematica [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

```
[In] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
[Out] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]), x]
```

Maple [F]

$$\int \left(c-\frac{c}{ax}\right)^p \mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}dx$$

[In] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)

[Out] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)

Fricas [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")

[Out] integral(((a*c*x - c)/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)

Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{-2p\,\mathrm{acoth}\,(ax)}\,dx$$

[In] integrate((c-c/a/x)**p/exp(2*p*acoth(a*x)),x)

[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)

Maxima [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")

[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)

Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \mathrm{e}^{-2\,p\,\mathrm{acoth}\left(a\,x\right)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

```
[In] int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.552 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)}{ap}$$

```
[Out] (c-c/a/x)^p*x+(2-p)*(c-c/a/x)^p*hypergeom([1, p],[p+1],1-1/a/x)/a/p
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6302, 6268, 25, 528, 382, 79, 67}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)}{ap}+x\left(c-\frac{c}{ax}\right)^p$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] (c - c/(a*x))^p*x + ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p,
 1 - 1/(a*x)])/(a*p)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
```

```
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{-1+p}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{-1+p}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{-1+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^p x-\frac{(c(2-p))\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)}{ap}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{\left(c-\frac{c}{ax}\right)^p\left(apx-(-2+p)\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)\right)}{ap}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] ((c - c/(a*x))^p*(a*p*x - (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*
x)]))/(a*p)
```

Maple [F]

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}dx$$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)

[Out] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="fricas")

[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=a\left(\begin{cases}\frac{0^px}{a}+\frac{0^p\log(ax-1)}{a^2}-\frac{a^{-p}c^ppx^{2-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\left.\begin{matrix}1-p,2-p\\3-p\end{matrix}\right|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^px}{a}+\frac{0^p\log(-ax+1)}{a^2}-\frac{a^{-p}c^ppx^{2-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\left.\begin{matrix}1-p,2-p\\3-p\end{matrix}\right|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right)$$
$$+\begin{cases}\frac{0^p\log(ax-1)}{a}-\frac{a^{-p}c^ppx^{1-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\left.\begin{matrix}1-p,1-p\\2-p\end{matrix}\right|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^p\log(-ax+1)}{a}-\frac{a^{-p}c^ppx^{1-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\left.\begin{matrix}1-p,1-p\\2-p\end{matrix}\right|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{otherwise}\end{cases}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**p,x)

[Out] a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - c**p*p*x**(2 - p)*exp(I*pi
*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(a**p*gamma(

```
3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a**2 -
c**p*p*x**(2 - p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (
3 - p,), a*x)/(a**p*gamma(3 - p)*gamma(p + 1)), True)) + Piecewise((0**p*lo
g(a*x - 1)/a - c**p*p*x**(1 - p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1
 - p, 1 - p), (2 - p,), a*x)/(a**p*gamma(2 - p)*gamma(p + 1)), Abs(a*x) > 1
), (0**p*log(-a*x + 1)/a - c**p*p*x**(1 - p)*exp(I*pi*p)*gamma(p)*gamma(1 -
 p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(a**p*gamma(2 - p)*gamma(p + 1)),
True))
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1), x)
```

3.553 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}$$

```
[Out] -1/3*2^(1/2+p)*(1+1/a/x)^(3/2)*(c-c/a/x)^p*AppellF1(3/2,1/2-p,2,5/2,1/2*(a+
1/x)/a,1+1/a/x)/a/((1-1/a/x)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6317, 6314, 141}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] -1/3*(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 - p
, 2, 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
```

```
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p\,dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}\end{aligned}$$

Mathematica [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
[Out] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p, x]
```

Maple [F]

$$\int \frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1
), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.554 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-p,2,\frac{3}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}$$

```
[Out] -2^(3/2+p)*(c-c/a/x)^p*AppellF1(1/2,-1/2-p,2,3/2,1/2*(a+1/x)/a,1+1/a/x)*(1+
1/a/x)^(1/2)/a/((1-1/a/x)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6317, 6314, 141}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p-\frac{1}{2},2,\frac{3}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

```
[In] Int[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

```
[Out] -((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2,
 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
```

```
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^{p}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{p}\,dx\\
&= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^{p}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{p}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-p,2,\frac{3}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}
\end{aligned}$$

Mathematica [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx$$

```
[In] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
[Out] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x], x]
```

Maple [F]

$$\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}dx$$

```
[In] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral(((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \sqrt{\frac{ax-1}{ax+1}}\left(-c\left(-1+\frac{1}{ax}\right)\right)^p\,dx$$

```
[In] integrate((c-c/a/x)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(-1 + 1/(a*x)))**p, x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{a\,x}\right)^p\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.555 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}-\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,1-\frac{1}{ax}\right)}{ac^2}$$

```
[Out] (c-c/a/x)^(2+p)*x/c^2+1/2*(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1/2*(a-1
/x)/a)/a/c^2/(2+p)-(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1-1/a/x)/a/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6302, 6268, 25, 528, 382, 105, 162, 67, 70}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{ac^2}+\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

```
[In] Int[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^(2 + p)*x)/c^2 + ((c - c/(a*x))^(2 + p)*Hypergeometric2F1[1,
 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) - ((c - c/(a*x))^(2 +
 p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{1+ax}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}x}{1+ax}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}}{a+\frac{1}{x}}\,dx}{c}\\
&= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}\left(c(2+p)+\frac{c(1+p)x}{a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c^2}\\
&= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}-\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{a+x}\,dx,x,\frac{1}{x}\right)}{ac}+\frac{(2+p)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x}\,dx,x,\frac{1}{x}\right)}{ac}\\
&= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}\\
&\quad-\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,1-\frac{1}{ax}\right)}{ac^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=\frac{\left(c-\frac{c}{ax}\right)^p(-1+ax)^2\left(\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)+2(2+p)\left(ax-\text{Hypergeometric2F1}\left(\right.\right.\right.}{2a^3(2+p)x^2}$$

```
[In] Integrate[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(-
1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x
)])))/(2*a^3*(2 + p)*x^2)
```

Maple [F]

$$\int \frac{\left(c-\frac{c}{ax}\right)^p(ax-1)}{ax+1}dx$$

```
[In] int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

```
[Out] int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1), x)
```

3.556 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 393

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/}$$

```
[Out] 5/72*a^6*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-7/72*a^7*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(11/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(11/2)*x^9+3
5/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+35/384*a*c^4*(1+1/a/x)
^(3/2)*x^2*(1-1/a/x)^(1/2)+7/192*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2
)+1/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/144*a^4*c^4*(1+1/a/x)^
(9/2)*x^5*(1-1/a/x)^(1/2)-5/144*a^5*c^4*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2
)+35/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{11/2}$$
$$-\frac{7}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{5}{72}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{5}{144}a^5c^4x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (35*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(5/2)*x^3)/192 + (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/6
4 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/144 - (5*a^5*c^4*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/144 + (5*a^6*c^4*(1 - 1/(a*x))^(3
/2)*(1 + 1/(a*x))^(11/2)*x^7)/72 - (7*a^7*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a
*x))^(11/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(11/2)*x^9
)/9 + (35*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(a^8c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4 x^8\,dx$$

$$= -\left(\left(a^8c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^{10}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9+\frac{1}{9}\left(7a^7c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^9}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9-\frac{1}{72}\left(35a^6c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7 - \frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9+\frac{1}{24}\left(5a^5c^4\right)\text{Subst}\left(\int\right.$$

$$= -\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7 - \frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9-\frac{1}{144}\left(5a^4c^4\right)\text{Subst}\left(\right.$$

$$= \frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 + \frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}$$

$$= \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 + \frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}$$

$$= \frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$
$$+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5$$
$$-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1\right.$$

$$= \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$= \frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$= \frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$= \frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(128-837ax-512a^2x^2+978a^3x^3+768a^4x^4-600a^5x^5-512a^6x^6+144a^7x^7+128a^8x^8\cdots\right.}{1152a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3
+ 768*a^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) + 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-512a^2x^2-837ax+128)(ax-1)c^4}{1152a\sqrt{\frac{ax-1}{ax+1}}}+\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax\cdots}}{128\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+\cdots}$
default	$-\frac{(ax-1)c^4\left(-128(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-144(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+384(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+456(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-384(a^2x\cdots\right.}{1152a\sqrt{\cdots}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1152*(128*a^8*x^8+144*a^7*x^7-512*a^6*x^6-600*a^5*x^5+768*a^4*x^4+978*a^3
*x^3-512*a^2*x^2-837*a*x+128)*(a*x-1)/a*c^4/((a*x-1)/(a*x+1))^(1/2)+35/128*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/((a*x-1)/(a*x+1))^(
1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.43

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(128\,a^9c^4x^9+272\,a^8c^4x^8-368\,a^7c^4x^7-1112\,a^6c^4x^6\cdots\right.}{1152\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (128*a^9*c^4*x^9 + 272*a^8*c^4*x^8 - 368*a^7*c^4*x^7
 - 1112*a^6*c^4*x^6 + 168*a^5*c^4*x^5 + 1746*a^4*c^4*x^4 + 466*a^3*c^4*x^3
- 1349*a^2*c^4*x^2 - 709*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\left(\int\left(-\frac{4a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^8x^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(6*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**6
/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8/sqrt(a*x/(a*x +
 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-1)}{(ax+1)}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 27
30*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/2
) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 32768*c^4*((a*x - 1)/(a*x + 1)
```

```
)^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*x
 + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x - 1
)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2 +
 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a*
x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7*
a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1)
^9 - a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.55

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=-\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{128\,|a|\mathrm{sgn}\,(ax+1)}$$
$$-\frac{1}{1152}\sqrt{a^2x^2-1}\left(\left(\frac{837\,c^4}{\mathrm{sgn}\,(ax+1)}+2\left(\frac{256\,ac^4}{\mathrm{sgn}\,(ax+1)}-\left(\frac{489\,a^2c^4}{\mathrm{sgn}\,(ax+1)}+4\left(\frac{96\,a^3c^4}{\mathrm{sgn}\,(ax+1)}-\left(\frac{75\,a^4c^4}{\mathrm{sgn}\,(ax+\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) -
 1/1152*sqrt(a^2*x^2 - 1)*((837*c^4/sgn(a*x + 1) + 2*(256*a*c^4/sgn(a*x + 1
) - (489*a^2*c^4/sgn(a*x + 1) + 4*(96*a^3*c^4/sgn(a*x + 1) - (75*a^4*c^4/sg
n(a*x + 1) + 2*(32*a^5*c^4/sgn(a*x + 1) - (8*a^7*c^4*x/sgn(a*x + 1) + 9*a^6
*c^4/sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x - 128*c^4/(a*sgn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{\frac{455\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}-\frac{35\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{581\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{1289\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{32}+\frac{512\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{9}-\frac{1289\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{32}+\frac{581\,c^4\,(\cdots}{\cdots}}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{36\,a\,(a\,x\cdots}{(a\,x+\cdots}}$$
$$+\frac{35\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

```
[In] int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((455*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1))^(
1/2))/64 - (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (1289*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 - (1289*c^4*
```

```
((a*x - 1)/(a*x + 1))^(11/2))/32 + (581*c^4*((a*x - 1)/(a*x + 1))^(13/2))/3
2 - (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (35*c^4*((a*x - 1)/(a*x + 1
))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a*x + 1
)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^4 - (1
26*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*a*(a*x
 - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9)/(a*x
 + 1)^9) + (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.557 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 313

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx = \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 +\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 -\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}$$

```
[Out] 5/42*a^5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(9/2)*x^7+5/16*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5
/48*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/24*a^2*c^3*(1+1/a/x)^(5/2)*
x^3*(1-1/a/x)^(1/2)+1/56*a^3*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-1/14*a
^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+5/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/
a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx = -\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2} +\frac{5}{42}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{14}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{56}a^3c^3x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (5*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/48 + (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/56 - (a^
4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/14 + (5*a^5*c^3*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/42 - (a^6*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1
/(a*x))^(9/2)*x^7)/7 + (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])
/(16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left((a^6c^3)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= (a^6c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{7}(5a^5c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^7}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 + \frac{1}{14}(5a^4c^3)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{14}(a^3c^3)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{7/2}}{x^5\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{8}(a^2c^3)\,\text{Subst}\left(\int \frac{(1}{x^4}\right.$$

$$= \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{24}(5ac^3)\,\text{Subst}\left(\int \frac{(}{x}\right.$$

$$= \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\right.$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a\ldots\right.$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a\ldots\right.$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a\ldots\right.$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(48-231ax-144a^2x^2+182a^3x^3+144a^4x^4-56a^5x^5-48a^6x^6)+105\log\left(\left(1+\sqrt{1-\ldots\right.\right.\right.}{336a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 - 231*a*x - 144*a^2*x^2 + 182*a^3*x^3 +
 144*a^4*x^4 - 56*a^5*x^5 - 48*a^6*x^6) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{\left(48 a^{6} x^{6}+56 a^{5} x^{5}-144 a^{4} x^{4}-182 a^{3} x^{3}+144 a^{2} x^{2}+231 a x -48\right) \left(a x -1\right) c^{3}}{336 a \sqrt{\frac{a x -1}{a x +1}}}+\frac{5 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{3} \sqrt{\left(a x -1\right) \left(a x +1\right)}}{16 \sqrt{a^{2}}\, \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$
default	$\frac{\left(a x -1\right) c^{3} \left(-48 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{4} x^{4}-56 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{3} x^{3}+96 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}+126 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x +64 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a}}{336 a \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/336*(48*a^6*x^6+56*a^5*x^5-144*a^4*x^4-182*a^3*x^3+144*a^2*x^2+231*a*x-4
8)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2)+5/16*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(48\,a^7c^3x^7+104\,a^6c^3x^6-88\,a^5c^3x^5-326\,a^4c^3x^4-\right.}{336\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (48*a^7*c^3*x^7 + 104*a^6*c^3*x^6 - 88*a^5*c^3*x^5 -
326*a^4*c^3*x^4 - 38*a^3*c^3*x^3 + 375*a^2*c^3*x^2 + 183*a*c^3*x - 48*c^3)*
sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-c^3\left(\int\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6/s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\cdots\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(ax\cdots}{(ax\cdots}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 700
*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) -
3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2)
 + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1)))
/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1)
^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(a
*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 - a
^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{16\,|a|\mathrm{sgn}\left(ax+1\right)}-\frac{1}{336}\,\sqrt{a^2x^2-1}\left(\left(\left(2\left(\frac{72\,ac^3}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{91\,a^2c^3}{\mathrm{sgn}\left(ax+1\right)}+4\left(\frac{18\,a^3c^3}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{6\,a^5c^3x}{\mathrm{sgn}\left(ax+1\right)}+\frac{7\,a^4c^3}{\mathrm{sgn}\left(ax+1\right)}\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1
/336*sqrt(a^2*x^2 - 1)*((2*(72*a*c^3/sgn(a*x + 1) - (91*a^2*c^3/sgn(a*x + 1
) + 4*(18*a^3*c^3/sgn(a*x + 1) - (6*a^5*c^3*x/sgn(a*x + 1) + 7*a^4*c^3/sgn(
a*x + 1))*x)*x)*x)*x + 231*c^3/sgn(a*x + 1))*x - 48*c^3/(a*sgn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}+\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{24}+\frac{128\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{7}-\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{24}+\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{6}-\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}$$

```
[In] int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((5*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/8 - (25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 + (283*c^3*((a*x -
1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*x + 1))^(7/2))/7 - (283*c^
3*((a*x - 1)/(a*x + 1))^(9/2))/24 + (25*c^3*((a*x - 1)/(a*x + 1))^(11/2))/6
 - (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x + 1) +
 (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (35*a*(a
*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x - 1)^6)
/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

3.558 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 233

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}$$

```
[Out] 1/5*a^4*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+3/8*c^2*arctanh((1-1/a/x)^(
1/2)*(1+1/a/x)^(1/2))/a+1/8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/20*
a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-3/20*a^3*c^2*(1+1/a/x)^(7/2)*x^
4*(1-1/a/x)^(1/2)+3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2} +\frac{1}{20}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a}+\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{3}{8}c^2x$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 + (a*c^2*Sqrt[1 - 1/(a*x)]*
(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)
*x^3)/20 - (3*a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (a^4*
c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/5 + (3*c^2*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = (a^4c^2)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2 x^4\,dx$$

$$= -\left(\left(a^4c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{5}\left(3a^3c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{1}{20}\left(3a^2c^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{1}{4}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{1}{8}\left(3c^2\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a^2}$$

$$= \frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 + \frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8-25ax-16a^2x^2+10a^3x^3+8a^4x^4)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*a
^4*x^4) + 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8)(ax-1)c^2}{40a\sqrt{\frac{ax-1}{ax+1}}} + \frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$\frac{(ax-1)c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-45\sqrt{a^2x^2-1}\sqrt{a^2}\,ax-40((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+45\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(8*a^4*x^4+10*a^3*x^3-16*a^2*x^2-25*a*x+8)*(a*x-1)/a*c^2/((a*x-1)/(a*x
+1))^(1/2)+3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/((a*
x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(8\,a^5c^2x^5+18\,a^4c^2x^4-6\,a^3c^2x^3-41\,a^2c^2x^2-17\,ac^2x\right.}{40\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) + (8*a^5*c^2*x^5 + 18*a^4*c^2*x^4 - 6*a^3*c^2*x^3 - 41*a^2
*c^2*x^2 - 17*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\left(\int\left(-\frac{2a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(-2*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{1}{40}a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+128\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}\right.}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-5\,(}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^2
*((a*x - 1)/(a*x + 1))^(7/2) + 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^2
*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
 + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2
))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{40}\sqrt{a^2x^2-1}\left(\left(2\left(\left(\frac{4\,a^3c^2x}{\mathrm{sgn}\left(ax+1\right)}+\frac{5\,a^2c^2}{\mathrm{sgn}\left(ax+1\right)}\right)x-\frac{8\,ac^2}{\mathrm{sgn}\left(ax+1\right)}\right)x-\frac{25\,c^2}{\mathrm{sgn}\left(ax+1\right)}\right)x+\frac{8\,c^2}{a\mathrm{sgn}\left(ax+\right.}$$
$$-\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/40*sqrt(a^2*x^2 - 1)*((2*((4*a^3*c^2*x/sgn(a*x + 1) + 5*a^2*c^2/sgn(a*x +
 1))*x - 8*a*c^2/sgn(a*x + 1))*x - 25*c^2/sgn(a*x + 1))*x + 8*c^2/(a*sgn(a*
x + 1))) - 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x
+ 1))
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}-\frac{3\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{32\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{5}-\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{2}+\frac{3\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}$$
$$+\frac{3\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((7*c^2*((a*x - 1)/(a*x + 1))^(3/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(1/2)
)/4 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 - (7*c^2*((a*x - 1)/(a*x + 1))
^(7/2))/2 + (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (3*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.559 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

```
[Out] 1/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/6*a*c*(1+1/a/x)^(3/2)*x^
2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/2*c*x*(1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx=-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}+\frac{1}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 + 1
/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^3)/3
+ (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^2c\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^4}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{3}(ac)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{2}c\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{c\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a^2}$$

$$= \frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(2-3ax-2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{(2a^2x^2+3ax-2)(ax-1)c}{6a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	108
default	$-\frac{(ax-1)c\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	119

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*a^2*x^2+3*a*x-2)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x/(
a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3+5\,a^2cx^2+acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x +
 1)) - 1) - (2*a^3*c*x^3 + 5*a^2*c*x^2 + a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(\int\frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/
sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) - 8*c*((a*x - 1)/(a*x + 1))^(3/2
) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)
^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,acx}{\operatorname{sgn}(ax+1)}+\frac{3\,c}{\operatorname{sgn}(ax+1)}\right)x-\frac{2\,c}{a\operatorname{sgn}(ax+1)}\right) - \frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a^2*x^2 - 1)*((2*a*c*x/sgn(a*x + 1) + 3*c/sgn(a*x + 1))*x - 2*c/(
a*sgn(a*x + 1))) - 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sg
n(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}$$

```
[In] int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2) +
 (8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(5/2))/(a -
(3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)
/(a*x + 1)^3)
```

3.560 $\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\coth^{-1}(ax)}}{ac}$$

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6318}

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\coth^{-1}(ax)}}{ac}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcCoth[a*x]/(a*c)
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{e^{\coth^{-1}(ax)}}{ac}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\coth^{-1}(ax)}}{ac}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
[Out] E^ArcCoth[a*x]/(a*c)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

method	result	size
gosper	$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,ac}$	23
default	$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,ac}$	23
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac(ax-1)}$	37

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x +
1) - 1/(a*x + 1))), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/(a*c*sqrt((a*x - 1)/(a*x + 1)))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] 1/(a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.561 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx = \frac{2e^{\coth^{-1}(ax)}}{3ac^2} - \frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)/a/c^2-1/3/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/
c^2/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6320, 6318}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx = \frac{2e^{\coth^{-1}(ax)}}{3ac^2} - \frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcCoth[a*x])/(3*a*c^2) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*c^2*(1 - a
^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2\,(1-a^2x^2)}+\frac{2\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}\\ &= \frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2\,(1-a^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1-2ax+2a^2x^2)}{3c^2(-1+ax)^2(1+ax)}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^2*(-1 + a*x)^2*(1 +
 a*x))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

method	result	size
trager	$\frac{(2a^2x^2-2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)^2}$	47
gosper	$\frac{2a^2x^2-2ax-1}{3(a^2x^2-1)c^2\sqrt{\frac{ax-1}{ax+1}}\,a}$	49
default	$\frac{2a^2x^2-2ax-1}{3\sqrt{\frac{ax-1}{ax+1}}\,c^2(ax-1)a(ax+1)}$	52

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/3/a/c^2*(2*a^2*x^2-2*a*x-1)/(a*x-1)^2*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{(2\,a^2x^2-2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*
c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x**2*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{12}\,a\left(\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \int \frac{1}{(a^2cx^2-c)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 - c)^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^2-3\,a^3\,c^2\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*a*x - 2*a^2*x^2 + 1)/((3*a*c^2 - 3*a^3*c^2*x^2)*((a*x - 1)/(a*x + 1))^(1
/2))
```

3.562 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\frac{8e^{\coth^{-1}(ax)}}{15ac^3}-\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3(1-a^2x^2)}$$

```
[Out] 8/15/((a*x-1)/(a*x+1))^(1/2)/a/c^3-1/15/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/
a/c^3/(-a^2*x^2+1)^2-4/15/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^3/(-a^2*x^
2+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6320, 6318}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}-\frac{4(1-2ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)}+\frac{8e^{\coth^{-1}(ax)}}{15ac^3}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8*E^ArcCoth[a*x])/(15*a*c^3) - (E^ArcCoth[a*x]*(1 - 4*a*x))/(15*a*c^3*(1 -
 a^2*x^2)^2) - (4*E^ArcCoth[a*x]*(1 - 2*a*x))/(15*a*c^3*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4\int\frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{5c}\\ &= -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3\left(1-a^2x^2\right)}+\frac{8\int\frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2}\\ &= \frac{8e^{\coth^{-1}(ax)}}{15ac^3}-\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3\left(1-a^2x^2\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int\frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(3+12ax-12a^2x^2-8a^3x^3+8a^4x^4)}{15c^3(-1+ax)^3(1+ax)^2}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4))
/(15*c^3*(-1 + a*x)^3*(1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15(a^2x^2-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}\,a}$	65
default	$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15\sqrt{\frac{ax-1}{ax+1}}\,c^3(ax-1)^2a(ax+1)^2}$	68
trager	$\frac{(8a^4x^4-8a^3x^3-12a^2x^2+12ax+3)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax+1)(ax-1)^3}$	70

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/(a^2*x^2-1)^2/c^3/((a*x-1)/(
a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{(8\,a^4x^4-8\,a^3x^3-12\,a^2x^2+12\,ax+3)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] 1/15*(8*a^4*x^4 - 8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
+ 1))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\int \frac{1}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
[Out] -Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{240}\,a\left(\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-12\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -1/240*a*(5*(((a*x - 1)/(a*x + 1))^(3/2) - 12*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c^3) + (20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a^2*c^
3*((a*x - 1)/(a*x + 1))^(5/2)))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{1}{(a^2cx^2-c)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{8\,a^4\,x^4-8\,a^3\,x^3-12\,a^2\,x^2+12\,a\,x+3}{15\,a\,c^3\,(a\,x+1)^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4 + 3)/(15*a*c^3*(a*x + 1)^4*((a
*x - 1)/(a*x + 1))^(5/2))
```

3.563 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 119

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{16e^{\coth^{-1}(ax)}}{35ac^4} - \frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4(1-a^2x^2)^3} - \frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4(1-a^2x^2)^2} - \frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4(1-a^2x^2)}$$

```
[Out] 16/35/((a*x-1)/(a*x+1))^(1/2)/a/c^4-1/35/((a*x-1)/(a*x+1))^(1/2)*(-6*a*x+1)
/a/c^4/(-a^2*x^2+1)^3-2/35/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/a/c^4/(-a^2*x
^2+1)^2-8/35/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^4/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6320, 6318}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3} - \frac{8(1-2ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)} - \frac{2(1-4ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^2} + \frac{16e^{\coth^{-1}(ax)}}{35ac^4}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (16*E^ArcCoth[a*x])/(35*a*c^4) - (E^ArcCoth[a*x]*(1 - 6*a*x))/(35*a*c^4*(1
- a^2*x^2)^3) - (2*E^ArcCoth[a*x]*(1 - 4*a*x))/(35*a*c^4*(1 - a^2*x^2)^2) -
 (8*E^ArcCoth[a*x]*(1 - 2*a*x))/(35*a*c^4*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{6\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{7c}\\
&= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}+\frac{24\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{35c^2}\\
&= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)}+\frac{16\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^3}\\
&= \frac{16e^{\coth^{-1}(ax)}}{35ac^4}-\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(-5-30ax+30a^2x^2+40a^3x^3-40a^4x^4-16a^5x^5+16a^6x^6)}{35c^4(-1+ax)^4(1+ax)^3}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 - 30*a*x + 30*a^2*x^2 + 40*a^3*x^3 - 40*a^4*x^
4 - 16*a^5*x^5 + 16*a^6*x^6))/(35*c^4*(-1 + a*x)^4*(1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5}{35(a^2x^2-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	81
default	$\frac{16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5}{35\sqrt{\frac{ax-1}{ax+1}}\,c^4(ax-1)^3(ax+1)^3a}$	84
trager	$\frac{(16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^4(ax+1)^2(ax-1)^4}$	86

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
[Out] 1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/(a^2
*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{(16\,a^6x^6-16\,a^5x^5-40\,a^4x^4+40\,a^3x^3+30\,a^2x^2-30\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
[Out] 1/35*(16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*x^2 - 30*a
*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^
4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\int \frac{1}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
[Out] Integral(1/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x**6*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$=\frac{1}{2240}\,a\left(\frac{7\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+75\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{42\,(ax-1)}{ax+1}-\frac{175\,(ax-1)^2}{(ax+1)^2}+\frac{700\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*(7*(((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) +
 75*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + (42*(a*x - 1)/(a*x + 1) - 175*(a
*x - 1)^2/(a*x + 1)^2 + 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(7/2)))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=\int \frac{1}{(a^2cx^2-c)^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{32\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}-\frac{\frac{5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{20\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{6\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((a*x - 1)/(a*x + 1))^(3/2)/(
32*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - ((5*(a*x - 1)^2)/(a*x
 + 1)^2 - (20*(a*x - 1)^3)/(a*x + 1)^3 - (6*(a*x - 1))/(5*(a*x + 1)) + 1/7)
/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.564 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

```
[Out] -16/7*c^5*(a*x+1)^7/a+4*c^5*(a*x+1)^8/a-8/3*c^5*(a*x+1)^9/a+4/5*c^5*(a*x+1)
^10/a-1/11*c^5*(a*x+1)^11/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(ax+1)^{11}}{11a}+\frac{4c^5(ax+1)^{10}}{5a}-\frac{8c^5(ax+1)^9}{3a}+\frac{4c^5(ax+1)^8}{a}-\frac{16c^5(ax+1)^7}{7a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (-16*c^5*(1 + a*x)^7)/(7*a) + (4*c^5*(1 + a*x)^8)/a - (8*c^5*(1 + a*x)^9)/(
3*a) + (4*c^5*(1 + a*x)^10)/(5*a) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\left(c-a^2cx^2\right)^5\,dx \\
&= -\left(c^5\int (1-ax)^4(1+ax)^6\,dx\right) \\
&= -\left(c^5\int \left(16(1+ax)^6-32(1+ax)^7+24(1+ax)^8-8(1+ax)^9+(1+ax)^{10}\right)\,dx\right) \\
&= -\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.56

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(1+ax)^7\left(281-812ax+938a^2x^2-504a^3x^3+105a^4x^4\right)}{1155a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
[Out] -1/1155*(c^5*(1 + a*x)^7*(281 - 812*a*x + 938*a^2*x^2 - 504*a^3*x^3 + 105*a
^4*x^4))/a
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

method	result
gosper	$-\frac{c^{5} x \left(105 a^{10} x^{10}+231 a^{9} x^{9}-385 a^{8} x^{8}-1155 a^{7} x^{7}+330 a^{6} x^{6}+2310 a^{5} x^{5}+462 a^{4} x^{4}-2310 a^{3} x^{3}-1155 a^{2} x^{2}+1155 a x +1155\right)}{1155}$
default	$c^{5} \left(-\frac{1}{11} a^{10} x^{11}-\frac{1}{5} a^{9} x^{10}+\frac{1}{3} a^{8} x^{9}+a^{7} x^{8}-\frac{2}{7} a^{6} x^{7}-2 a^{5} x^{6}-\frac{2}{5} a^{4} x^{5}+2 a^{3} x^{4}+a^{2} x^{3}-a \,x^{2}-x \right)$
norman	$a^{7} c^{5} x^{8}+c^{5} a^{2} x^{3}-c^{5} x -a \,c^{5} x^{2}+2 a^{3} c^{5} x^{4}-\frac{2}{5} a^{4} c^{5} x^{5}-2 a^{5} c^{5} x^{6}-\frac{2}{7} a^{6} c^{5} x^{7}+\frac{1}{3} a^{8} c^{5} x^{9}-\frac{1}{5} a^{9} c^{5}$
risch	$a^{7} c^{5} x^{8}+c^{5} a^{2} x^{3}-c^{5} x -a \,c^{5} x^{2}+2 a^{3} c^{5} x^{4}-\frac{2}{5} a^{4} c^{5} x^{5}-2 a^{5} c^{5} x^{6}-\frac{2}{7} a^{6} c^{5} x^{7}+\frac{1}{3} a^{8} c^{5} x^{9}-\frac{1}{5} a^{9} c^{5}$
parallelrisch	$a^{7} c^{5} x^{8}+c^{5} a^{2} x^{3}-c^{5} x -a \,c^{5} x^{2}+2 a^{3} c^{5} x^{4}-\frac{2}{5} a^{4} c^{5} x^{5}-2 a^{5} c^{5} x^{6}-\frac{2}{7} a^{6} c^{5} x^{7}+\frac{1}{3} a^{8} c^{5} x^{9}-\frac{1}{5} a^{9} c^{5}$
meijerg	$\frac{c^{5} \left(-\frac{x a \left(2520 a^{10} x^{10}+2772 a^{9} x^{9}+3080 a^{8} x^{8}+3465 a^{7} x^{7}+3960 a^{6} x^{6}+4620 a^{5} x^{5}+5544 a^{4} x^{4}+6930 a^{3} x^{3}+9240 a^{2} x^{2}+13860 a x +27720\right)}{27720}-\ln(-a x}{a}$

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)

[Out] -1/1155*c^5*x*(105*a^10*x^10+231*a^9*x^9-385*a^8*x^8-1155*a^7*x^7+330*a^6*x
^6+2310*a^5*x^5+462*a^4*x^4-2310*a^3*x^3-1155*a^2*x^2+1155*a*x+1155)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="fricas")

[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}c^5x^{11}}{11}-\frac{a^9c^5x^{10}}{5}+\frac{a^8c^5x^9}{3}+a^7c^5x^8-\frac{2a^6c^5x^7}{7}-2a^5c^5x^6-\frac{2a^4c^5x^5}{5}+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - a**9*c**5*x**10/5 + a**8*c**5*x**9/3 + a**7*c**5*x**
8 - 2*a**6*c**5*x**7/7 - 2*a**5*c**5*x**6 - 2*a**4*c**5*x**5/5 + 2*a**3*c**
5*x**4 + a**2*c**5*x**3 - a*c**5*x**2 - c**5*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{a^9\,c^5\,x^{10}}{5}+\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8-\frac{2\,a^6\,c^5\,x^7}{7}-2\,a^5\,c^5\,x^6-\frac{2\,a^4\,c^5\,x^5}{5}+2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3-a\,c^5\,x^2-c^5\,x$$

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] a^2*c^5*x^3 - a*c^5*x^2 - c^5*x + 2*a^3*c^5*x^4 - (2*a^4*c^5*x^5)/5 - 2*a^5
*c^5*x^6 - (2*a^6*c^5*x^7)/7 + a^7*c^5*x^8 + (a^8*c^5*x^9)/3 - (a^9*c^5*x^1
0)/5 - (a^10*c^5*x^11)/11
```

3.565 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=-\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}$$

```
[Out] -4/3*c^4*(a*x+1)^6/a+12/7*c^4*(a*x+1)^7/a-3/4*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+
1)^9/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=\frac{c^4(ax+1)^9}{9a}-\frac{3c^4(ax+1)^8}{4a}+\frac{12c^4(ax+1)^7}{7a}-\frac{4c^4(ax+1)^6}{3a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-4*c^4*(1 + a*x)^6)/(3*a) + (12*c^4*(1 + a*x)^7)/(7*a) - (3*c^4*(1 + a*x)^
8)/(4*a) + (c^4*(1 + a*x)^9)/(9*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^4\,dx\\ &= -\left(c^4\int (1-ax)^3(1+ax)^5\,dx\right)\\ &= -\left(c^4\int \left(8(1+ax)^5-12(1+ax)^6+6(1+ax)^7-(1+ax)^8\right)\,dx\right)\\ &= -\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(1+ax)^6\left(-65+138ax-105a^2x^2+28a^3x^3\right)}{252a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
[Out] (c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{c^4x(28a^8x^8+63a^7x^7-72a^6x^6-252a^5x^5+378a^3x^3+168a^2x^2-252ax-252)}{252}$
default	$c^4\left(\frac{1}{9}a^8x^9+\frac{1}{4}a^7x^8-\frac{2}{7}a^6x^7-a^5x^6+\frac{3}{2}a^3x^4+\frac{2}{3}a^2x^3-a\,x^2-x\right)$
norman	$-c^4x-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
risch	$-c^4x-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
parallelrisch	$-c^4x-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
meijerg	$-\frac{c^4\left(-\frac{ax\left(280a^8x^8+315a^7x^7+360a^6x^6+420a^5x^5+504a^4x^4+630a^3x^3+840a^2x^2+1260ax+2520\right)}{2520}-\ln(-ax+1)\right)}{a}+\frac{4c^4\left(-\frac{ax\left(120a^6\right.}{}\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/252*c^4*x*(28*a^8*x^8+63*a^7*x^7-72*a^6*x^6-252*a^5*x^5+378*a^3*x^3+168*a^2*x^2-252*a*x-252)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")

[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}-a^5c^4x^6+\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}-ac^4x^2-c^4x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**4,x)

[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 - a**5*c**4*x**6 + 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 - a*c**4*x**2 - c**4*x

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}-a^5\,c^4\,x^6+\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}-a\,c^4\,x^2-c^4\,x$$

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*a^2*c^4*x^3)/3 - a*c^4*x^2 - c^4*x + (3*a^3*c^4*x^4)/2 - a^5*c^4*x^6 - (
2*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

3.566 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

```
[Out] -4/5*c^3*(a*x+1)^5/a+2/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(ax+1)^7}{7a}+\frac{2c^3(ax+1)^6}{3a}-\frac{4c^3(ax+1)^5}{5a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a) + (2*c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/
(7*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^3\,dx\\
&= -\left(c^3\int(1-ax)^2(1+ax)^4\,dx\right)\\
&= -\left(c^3\int\left(4(1+ax)^4-4(1+ax)^5+(1+ax)^6\right)\,dx\right)\\
&= -\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(1+ax)^5\left(29-40ax+15a^2x^2\right)}{105a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
[Out] -1/105*(c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/a
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result
gosper	$-\frac{c^{3} x \left(15 a^{6} x^{6}+35 a^{5} x^{5}-21 a^{4} x^{4}-105 a^{3} x^{3}-35 a^{2} x^{2}+105 a x +105\right)}{105}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}-\frac{1}{3} a^{5} x^{6}+\frac{1}{5} a^{4} x^{5}+a^{3} x^{4}+\frac{1}{3} a^{2} x^{3}-a \,x^{2}-x \right)$
norman	$a^{3} c^{3} x^{4}-c^{3} x -a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}+\frac{1}{5} a^{4} c^{3} x^{5}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
risch	$a^{3} c^{3} x^{4}-c^{3} x -a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}+\frac{1}{5} a^{4} c^{3} x^{5}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
parallelrisch	$a^{3} c^{3} x^{4}-c^{3} x -a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}+\frac{1}{5} a^{4} c^{3} x^{5}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
meijerg	$\frac{c^{3} \left(-\frac{a x \left(120 a^{6} x^{6}+140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}+280 a^{2} x^{2}+420 a x +840\right)}{840}-\ln \left(-a x +1\right)\right)}{a}-\frac{3 c^{3} \left(-\frac{a x \left(12 a^{4} x^{4}+15 a^{3} x^{3}+20 a^{2} x^{2}+30 a}{60}\right.}{a}$

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/105*c^3*x*(15*a^6*x^6+35*a^5*x^5-21*a^4*x^4-105*a^3*x^3-35*a^2*x^2+105*a*x+105)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")

[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 - a*c^3*x^2 - c^3*x

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}-\frac{a^5c^3x^6}{3}+\frac{a^4c^3x^5}{5}+a^3c^3x^4+\frac{a^2c^3x^3}{3}-ac^3x^2-c^3x$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**3,x)

[Out] -a**6*c**3*x**7/7 - a**5*c**3*x**6/3 + a**4*c**3*x**5/5 + a**3*c**3*x**4 + a**2*c**3*x**3/3 - a*c**3*x**2 - c**3*x

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}a^6c^3x^7-\frac{1}{3}a^5c^3x^6+\frac{1}{5}a^4c^3x^5+a^3c^3x^4+\frac{1}{3}a^2c^3x^3-ac^3x^2-c^3x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}a^6c^3x^7-\frac{1}{3}a^5c^3x^6+\frac{1}{5}a^4c^3x^5+a^3c^3x^4+\frac{1}{3}a^2c^3x^3-ac^3x^2-c^3x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}-\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5}+a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}-a\,c^3\,x^2-c^3\,x$$

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1))/(a*x - 1),x)
[Out] (a^2*c^3*x^3)/3 - a*c^3*x^2 - c^3*x + a^3*c^3*x^4 + (a^4*c^3*x^5)/5 - (a^5*
c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

3.567 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=-\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}$$

```
[Out] -1/2*c^2*(a*x+1)^4/a+1/5*c^2*(a*x+1)^5/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=\frac{c^2(ax+1)^5}{5a}-\frac{c^2(ax+1)^4}{2a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/2*(c^2*(1 + a*x)^4)/a + (c^2*(1 + a*x)^5)/(5*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
```

```
c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^2\,dx\\
&= -\left(c^2\int (1-ax)(1+ax)^3\,dx\right)\\
&= -\left(c^2\int \left(2(1+ax)^3-(1+ax)^4\right)\,dx\right)\\
&= -\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2(1+ax)^4(-3+2ax)}{10a}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
[Out] (c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{c^2 x(2a^4x^4+5a^3x^3-10ax-10)}{10}$
default	$c^2\left(\frac{1}{5}a^4x^5+\frac{1}{2}a^3x^4-a\,x^2-x\right)$
norman	$-c^2x-a\,c^2x^2+\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
risch	$-c^2x-a\,c^2x^2+\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
parallelrisch	$-c^2x-a\,c^2x^2+\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
meijerg	$-\frac{c^2\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln(-ax+1)\right)}{a}+\frac{2c^2\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c^2(-ax-\ln(-}{a}$

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/10*c^2*x*(2*a^4*x^4+5*a^3*x^3-10*a*x-10)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}+\frac{a^3c^2x^4}{2}-ac^2x^2-c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**2,x)
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 - c**2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}+\frac{a^3\,c^2\,x^4}{2}-a\,c^2\,x^2-c^2\,x$$

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1))/(a*x - 1),x)
[Out] (a^3*c^2*x^4)/2 - a*c^2*x^2 - c^2*x + (a^4*c^2*x^5)/5
```

3.568 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(1+ax)^3}{3a}$$

```
[Out] -1/3*c*(a*x+1)^3/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6302, 6275, 32}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(ax+1)^3}{3a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -1/3*(c*(1 + a*x)^3)/a
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)\,dx\\ &= -\left(c\int(1+ax)^2\,dx\right)\\ &= -\frac{c(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(x+ax^2+\frac{a^2x^3}{3}\right)$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -(c*(x + a*x^2 + (a^2*x^3)/3))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{c(ax+1)^3}{3a}$	14
gosper	$-\frac{cx\left(a^2x^2+3ax+3\right)}{3}$	18
norman	$-cx-ac\,x^2-\frac{1}{3}a^2c\,x^3$	22
parallelrisch	$-cx-ac\,x^2-\frac{1}{3}a^2c\,x^3$	22
risch	$-\frac{a^2c\,x^3}{3}-ac\,x^2-cx-\frac{c}{3a}$	28
meijerg	$\frac{c\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c(-ax-\ln(-ax+1))}{a}-\frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c\ln(-ax+1)}{a}$	91

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/3*c*(a*x+1)^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}-acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - a*c*x**2 - c*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

```
[In] int(((c - a^2*c*x^2)*(a*x + 1))/(a*x - 1),x)
[Out] -(c*x*(3*a*x + a^2*x^2 + 3))/3
```

3.569 $\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{1}{ac(1-ax)}$$

```
[Out] -1/a/c/(-a*x+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6302, 6275, 32}

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{1}{ac(1-ax)}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] -(1/(a*c*(1 - a*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{c-a^2cx^2}\,dx \\ &= -\frac{\int \frac{1}{(1-ax)^2}\,dx}{c} \\ &= -\frac{1}{ac(1-ax)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{2\coth^{-1}(ax)}}{2ac}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
[Out] E^(2*ArcCoth[a*x])/(2*a*c)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{x}{c(ax-1)}$	13
parallelrisch	$\frac{x}{c(ax-1)}$	13
gosper	$\frac{1}{ac(ax-1)}$	15
default	$\frac{1}{ac(ax-1)}$	15
risch	$\frac{1}{ac(ax-1)}$	15

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] x/c/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c),x)
[Out] 1/(a**2*c*x - a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/(a^2*c*x - a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{(ax-1)ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{1}{a\,\left(c-a\,c\,x\right)}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)*(a*x - 1)),x)
[Out] -1/(a*(c - a*c*x))
```

3.570 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^2}$$

```
[Out] -1/4/a/c^2/(-a*x+1)^2-1/4/a/c^2/(-a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{4ac^2}-\frac{1}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/4*1/(a*c^2*(1 - a*x)^2) - 1/(4*a*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^2}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^2} \\
&= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^2} \\
&= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2} \\
&= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\text{arctanh}(ax)}{4ac^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$
\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{-2+ax-(-1+ax)^2\text{arctanh}(ax)}{4ac^2(-1+ax)^2}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{(ax-1)^{2} c^{2}}+\frac{\ln(-ax+1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	51
default	$\frac{-\frac{\ln(ax+1)}{8a}-\frac{1}{4(ax-1)^{2} a}+\frac{1}{4a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^{2}}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^{2}}{2c}-\frac{a^{2} x^{3}}{4c}}{c(ax+1)(ax-1)^{2}}+\frac{\ln(ax-1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	77
parallelrisch	$\frac{a^{2} \ln(ax-1)x^{2}-a^{2} \ln(ax+1)x^{2}+4a^{2}x^{2}-2a\ln(ax-1)x+2a\ln(ax+1)x-6ax+\ln(ax-1)-\ln(ax+1)}{8c^{2}(ax-1)^{2} a}$	90

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] (1/4*x-1/2/a)/(a*x-1)^2/c^2+1/8*ln(-a*x+1)/a/c^2-1/8*ln(a*x+1)/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+(a^2x^2-2\,ax+1)\log(ax-1)-4}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**2,x)
[Out] (a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax-2}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)} - \frac{\log(ax+1)}{8\,ac^2} + \frac{\log(ax-1)}{8\,ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\log(|ax+1|)}{8\,ac^2} + \frac{\log(|ax-1|)}{8\,ac^2} + \frac{ax-2}{4\,(ax-1)^2ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^2}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^2*(a*x - 1)),x)
```

```
[Out] (x/4 - 1/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

3.571 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{12ac^3(1-ax)^3} - \frac{1}{8ac^3(1-ax)^2} - \frac{3}{16ac^3(1-ax)} + \frac{1}{16ac^3(1+ax)} - \frac{\operatorname{arctanh}(ax)}{4ac^3}$$

```
[Out] -1/12/a/c^3/(-a*x+1)^3-1/8/a/c^3/(-a*x+1)^2-3/16/a/c^3/(-a*x+1)+1/16/a/c^3/
(a*x+1)-1/4*arctanh(a*x)/a/c^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\operatorname{arctanh}(ax)}{4ac^3} - \frac{3}{16ac^3(1-ax)} + \frac{1}{16ac^3(ax+1)} - \frac{1}{8ac^3(1-ax)^2} - \frac{1}{12ac^3(1-ax)^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/12*1/(a*c^3*(1 - a*x)^3) - 1/(8*a*c^3*(1 - a*x)^2) - 3/(16*a*c^3*(1 - a*
x)) + 1/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{4(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^3}\\
&= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}\\
&= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\text{arctanh}(ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{4+ax-6a^2x^2+3a^3x^3-3(-1+ax)^3(1+ax)\text{arctanh}(ax)}{12ac^3(-1+ax)^3(1+ax)}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
[Out] (4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/(
12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{1}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{12a(ax-1)^3}-\frac{1}{8(ax-1)^2a}+\frac{3}{16a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^3}$
risch	$\frac{\frac{a^2x^3}{4}-\frac{a\,x^2}{2}+\frac{x}{12}+\frac{1}{3a}}{(ax-1)^2(a^2x^2-1)c^3}+\frac{\ln(-ax+1)}{8a\,c^3}-\frac{\ln(ax+1)}{8a\,c^3}$
norman	$\frac{\frac{3x}{4c}+\frac{a\,x^2}{4c}-\frac{11a^2x^3}{12c}-\frac{a^3x^4}{12c}+\frac{a^4x^5}{3c}}{(ax+1)^2(ax-1)^3c^2}+\frac{\ln(ax-1)}{8a\,c^3}-\frac{\ln(ax+1)}{8a\,c^3}$
parallelrisch	$\frac{3\ln(ax-1)x^4a^4-3\ln(ax+1)x^4a^4+8a^4x^4-6a^3\ln(ax-1)x^3+6a^3\ln(ax+1)x^3-10a^3x^3-12a^2x^2+6a\ln(ax-1)x-6a\ln(ax+1)x}{24c^3(ax-1)^2(a^2x^2-1)a}$

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(1/16/a/(a*x+1)-1/8*ln(a*x+1)/a+1/12/a/(a*x-1)^3-1/8/(a*x-1)^2/a+3/16
/a/(a*x-1)+1/8/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{6\,a^3x^3-12\,a^2x^2+2\,ax-3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax+1)+3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax}{24\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] 1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5*c
^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{-3a^3x^3+6a^2x^2-ax-4}{12a^5c^3x^4-24a^4c^3x^3+24a^2c^3x-12ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(-3*a**3*x**3 + 6*a**2*x**2 - a*x - 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*x
**3 + 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c
**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)} - \frac{\log(ax+1)}{8\,ac^3} + \frac{\log(ax-1)}{8\,ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\log(|ax+1|)}{8\,ac^3} + \frac{\log(|ax-1|)}{8\,ac^3} + \frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\frac{x}{12}-\frac{a\,x^2}{2}+\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^3*(a*x - 1)),x)
```

```
[Out] - (x/12 - (a*x^2)/2 + 1/(3*a) + (a^2*x^3)/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^3
*x^4 - 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

3.572 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1+ax)}-\frac{15\text{arctanh}(ax)}{64ac^4}$$

```
[Out] -1/32/a/c^4/(-a*x+1)^4-1/16/a/c^4/(-a*x+1)^3-3/32/a/c^4/(-a*x+1)^2-5/32/a/c
^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^
4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\text{arctanh}(ax)}{64ac^4}-\frac{5}{32ac^4(1-ax)}+\frac{5}{64ac^4(ax+1)}-\frac{3}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}-\frac{1}{16ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/32*1/(a*c^4*(1 - a*x)^4) - 1/(16*a*c^4*(1 - a*x)^3) - 3/(32*a*c^4*(1 - a
*x)^2) - 5/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) + 5/(64*a*c^4*(1
 + a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{(c-a^2cx^2)^4}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^5(1+ax)^3}\,dx}{c^4} \\
&= -\frac{\int \left(-\frac{1}{8(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{5}{32(-1+ax)^2}+\frac{1}{32(1+ax)^3}+\frac{5}{64(1+ax)^2}-\frac{15}{64(-1+a^2x^2)}\right)dx}{c^4} \\
&= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)} \\
&\quad+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1+ax)}+\frac{15\int \frac{1}{-1+a^2x^2}\,dx}{64c^4} \\
&= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2} \\
&\quad-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1+ax)}-\frac{15\operatorname{arctanh}(ax)}{64ac^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16+17ax-50a^2x^2+10a^3x^3+30a^4x^4-15a^5x^5+15(-1+ax)^4(1+ax)^2\text{arctanh}(ax)}{64ac^4(-1+ax)^4(1+ax)^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/64*(16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 15
*(-1 + a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\frac{15a^4x^5}{64}-\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}+\frac{25a\,x^2}{32}-\frac{17x}{64}-\frac{1}{4a}}{(ax-1)^2(a^2x^2-1)^2c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
default	$\frac{\frac{1}{64a(ax+1)^2}+\frac{5}{64a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{32a(ax-1)^4}+\frac{1}{16a(ax-1)^3}-\frac{3}{32(ax-1)^2a}+\frac{5}{32a(ax-1)}+\frac{15\ln(ax-1)}{128a}}{c^4}$
norman	$\frac{-\frac{49x}{64c}-\frac{15a\,x^2}{64c}+\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}-\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}+\frac{a^6x^7}{4c}}{(ax+1)^3c^3(ax-1)^4}+\frac{15\ln(ax-1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
parallelrisch	$30a\ln(ax+1)x+15a^2\ln(ax+1)x^2-34a^5x^5+108a^3x^3+30\ln(ax+1)x^5a^5-15\ln(ax+1)x^6a^6+15\ln(ax+1)x^4a^4+15\ln(ax-1)x^6a^6$

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/64*a^4*x^5-15/32*a^3*x^4-5/32*a^2*x^3+25/32*a*x^2-17/64*x-1/4/a)/(a*x-1
)^2/(a^2*x^2-1)^2/c^4+15/128*ln(-a*x+1)/a/c^4-15/128*ln(a*x+1)/a/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{30\,a^5x^5-60\,a^4x^4-20\,a^3x^3+100\,a^2x^2-34\,ax-15\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log}{128\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-c\right.}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1)
 + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3
- a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15a^5x^5 - 30a^4x^4 - 10a^3x^3 + 50a^2x^2 - 17ax - 16}{64a^7c^4x^6 - 128a^6c^4x^5 - 64a^5c^4x^4 + 256a^4c^4x^3 - 64a^3c^4x^2 - 128a^2c^4x + 64ac^4} + \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128} - \frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/(
64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\,a^5x^5 - 30\,a^4x^4 - 10\,a^3x^3 + 50\,a^2x^2 - 17\,ax - 16}{64\,(a^7c^4x^6 - 2\,a^6c^4x^5 - a^5c^4x^4 + 4\,a^4c^4x^3 - a^3c^4x^2 - 2\,a^2c^4x + ac^4)} - \frac{15\,\log(ax+1)}{128\,ac^4} + \frac{15\,\log(ax-1)}{128\,ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^7
*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4} + \frac{15\,\log\left(|ax-1|\right)}{128\,ac^4} + \frac{15\,a^5x^5 - 30\,a^4x^4 - 10\,a^3x^3 + 50\,a^2x^2 - 17\,ax - 16}{64\,(ax+1)^2(ax-1)^4ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x +
1)^2*(a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\frac{17\,x}{64} - \frac{25\,a\,x^2}{32} + \frac{1}{4\,a} + \frac{5\,a^2\,x^3}{32} + \frac{15\,a^3\,x^4}{32} - \frac{15\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6 + 2\,a^5\,c^4\,x^5 + a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + a^2\,c^4\,x^2 + 2\,a\,c^4\,x - c^4} - \frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^4*(a*x - 1)),x)
```

```
[Out] ((17*x)/64 - (25*a*x^2)/32 + 1/(4*a) + (5*a^2*x^3)/32 + (15*a^3*x^4)/32 - (
15*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 + 2*a^5*c^
4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

3.573 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 393

$$\begin{aligned}
&\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx \\
&= -\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 \\
&\quad -\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 \\
&\quad -\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008} \\
&\quad +\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)
\end{aligned}$$

```
[Out] -5/72*a^7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(13/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(13/2)*x^9-55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
)/a-55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-11/192*a^2*c^4*(1+1/a/
x)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/448*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(
1/2)-11/1008*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-5/1008*a^5*c^4*(1+
1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)+5/168*a^6*c^4*(1+1/a/x)^(13/2)*x^7*(1-1/a
/x)^(1/2)-55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{13/2}$$
$$-\frac{5}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{13/2}+\frac{5}{168}a^6c^4x^7\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5a^5c^4x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{1008}-$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (55*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x
^4)/448 - (11*a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/1008 - (5*
a^5*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/1008 + (5*a^6*c^4*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(13/2)*x^7)/168 - (5*a^7*c^4*(1 - 1/(a*x))^(3/2)
*(1 + 1/(a*x))^(13/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(13/2)*x^9)/9 - (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*
a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = (a^8c^4)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4 x^8\,dx$$

$$= -\left((a^8c^4)\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{1}{9}(5a^7c^4)\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^9}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9-\frac{1}{24}(5a^6c^4)\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{11/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{1}{168}(5a^5c^4)\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{11/2}}{x^7\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7 - \frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{(55a^4c^4)\,\text{Subst}\left(\int\frac{(\cdots}{x\cdots}\right.}{1008\cdots}$$

$$= -\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}$$
$$+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7$$
$$-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{1}{112}\left(11a^3c^4\right)\text{Subst}\Bigg($$

$$= -\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}$$
$$+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\Bigg(1-$$

$$= -\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}$$
$$+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\Bigg(1-$$

$$= -\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}$$
$$+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\Bigg(1-$$

$$= -\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}$$

$$+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\Bigg(1-$$

$$= -\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008} + \frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7 - \frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\bigl($$

$$= -\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008} - \frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008} + \frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7 - \frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\bigl($$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-3712+4599ax+10240a^2x^2+3066a^3x^3-8448a^4x^4-7224a^5x^5+1024a^6x^6+3024a\right.}{8064a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a^
3*x^3 - 8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a^8
*x^8) - 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-3712)(ax-1)c^4}{8064a\sqrt{\frac{ax-1}{ax+1}}} - \frac{55\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right.}{128\sqrt{a^2}\sqrt{\frac{ax}{ax}}}$
default	$\frac{(ax-1)^2c^4\left(896(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3024(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+1920(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-4200(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-6528(a^2x^2\right.}{8064a\left(\frac{ax-1}{ax+1}\right.}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/8064*(896*a^8*x^8+3024*a^7*x^7+1024*a^6*x^6-7224*a^5*x^5-8448*a^4*x^4+306
6*a^3*x^3+10240*a^2*x^2+4599*a*x-3712)*(a*x-1)/a*c^4/((a*x-1)/(a*x+1))^(1/2
)-55/128*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/((a*x-1)/(
a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx =$$
$$-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(896\,a^9c^4x^9+3920\,a^8c^4x^8+4048\,a^7c^4x^7-6200}{8064\,a}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")

[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1) - (896*a^9*c^4*x^9 + 3920*a^8*c^4*x^8 + 4048*a^7*c^
4*x^7 - 6200*a^6*c^4*x^6 - 15672*a^5*c^4*x^5 - 5382*a^4*c^4*x^4 + 13306*a^3
*c^4*x^3 + 14839*a^2*c^4*x^2 + 887*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x
+ 1)))/a

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\left(\int\left(-\frac{4a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{6a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{4a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^8x^8}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4*x**4
/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)
+ Integral(a**8*x**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = -\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}\right.}{\frac{9\,(ax-1)a}{ax+1}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2)
- 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))
^(13/2) - 255222*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 360448*c^4*((a*x - 1)/(
a*x + 1))^(9/2) - 334602*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
 - 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*s
qrt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(
a*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)
^4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*
(a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a
^2/(a*x + 1)^9 - a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.55

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{128\,|a|\mathrm{sgn}\left(ax+1\right)}$$
$$+\frac{1}{8064}\sqrt{a^2x^2-1}\left(\left(\frac{4599\,c^4}{\mathrm{sgn}\left(ax+1\right)}+2\left(\frac{5120\,ac^4}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{1533\,a^2c^4}{\mathrm{sgn}\left(ax+1\right)}-4\left(\frac{1056\,a^3c^4}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{903\,a^4c^4}{\mathrm{sgn}\left(ax+1\right.}\right.\right.\right.\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) +
1/8064*sqrt(a^2*x^2 - 1)*((4599*c^4/sgn(a*x + 1) + 2*(5120*a*c^4/sgn(a*x +
1) + (1533*a^2*c^4/sgn(a*x + 1) - 4*(1056*a^3*c^4/sgn(a*x + 1) + (903*a^4*c
^4/sgn(a*x + 1) - 2*(64*a^5*c^4/sgn(a*x + 1) + 7*(8*a^7*c^4*x/sgn(a*x + 1)
+ 27*a^6*c^4/sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x - 3712*c^4/(a*sgn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{\frac{55\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{715\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}+\frac{913\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}+\frac{18589\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{224}-\frac{5632\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{63}+\frac{14179\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{224}-\frac{913\,c^4}{\cdots}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,(a\,x}{(a\,x+1}}$$
$$-\frac{55\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

```
[In] int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

[Out] ((55*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (715*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/96 + (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (18589*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (14179
c^4((a*x - 1)/(a*x + 1))^(11/2))/224 - (913*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/32 + (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (55*c^4*((a*x - 1)/(a
*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^
4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*
a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9
)/(a*x + 1)^9) - (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)

3.574 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7$$

```
[Out] -1/7*a^6*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-9/16*c^3*arctanh((1-1/a/x
)^(1/2)*(1+1/a/x)^(1/2))/a-3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3
/40*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-9/280*a^3*c^3*(1+1/a/x)^(7/
2)*x^4*(1-1/a/x)^(1/2)-1/70*a^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+1/1
4*a^5*c^3*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)-9/16*c^3*x*(1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$$

$$=-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{14}a^5c^3x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{1}{70}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{9}{280}a^3c^3x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{40}a^2c^3x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{9c}{}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (3*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(5/2)*x^3)/40 - (9*a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/280
 - (a^4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/70 + (a^5*c^3*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(3/2)*(1
+ 1/(a*x))^(11/2)*x^7)/7 - (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]])/(16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^6c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{1}{7}\left(3a^5c^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}{x^7}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6$$
$$-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7+\frac{1}{14}\left(a^4c^3\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{9/2}}{x^6\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6$$
$$-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7+\frac{1}{70}\left(9a^3c^3\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{7/2}}{x^5\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6$$
$$-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7+\frac{1}{40}\left(9a^2c^3\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$
$$-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5$$
$$+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7+\frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int \frac{(1+}{x^3\sqrt{1}}\right.$$

$$= -\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right.$$

$$= -\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}$$

$$= -\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}$$

$$= -\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(368-245ax-656a^2x^2-350a^3x^3+208a^4x^4+280a^5x^5+80a^6x^6)+315\log\left(\left(1+\right.\right.\right.}{560a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 - 245*a*x - 656*a^2*x^2 - 350*a
^3*x^3 + 208*a^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 315*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax+368)(ax-1)c^3}{560a\sqrt{\frac{ax-1}{ax+1}}}-\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{16\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^3\left(80(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+280(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+288(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-70(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+192(a^2x^2-1)^{\frac{3}{2}}\right.}{560a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/560*(80*a^6*x^6+280*a^5*x^5+208*a^4*x^4-350*a^3*x^3-656*a^2*x^2-245*a*x+
368)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2)-9/16*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+
1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(80\,a^7c^3x^7+360\,a^6c^3x^6+488\,a^5c^3x^5-142\,a^4c^3x^4}{560\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (80*a^7*c^3*x^7 + 360*a^6*c^3*x^6 + 488*a^5*c^3*x^5
- 142*a^4*c^3*x^4 - 1006*a^3*c^3*x^3 - 901*a^2*c^3*x^2 + 123*a*c^3*x + 368*
c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -c^3\left(\int \frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{3a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int \frac{a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4*x**
4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1)), x) + Integral(a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+59\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 21
00*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 5943*c^3*((a*x - 1)/(a*x + 1))^(9/2)
- 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 8393*c^3*((a*x - 1)/(a*x + 1))^(5/
```

```
2) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2
/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7
- a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.57

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{16\,|a|\mathrm{sgn}\left(ax+1\right)}+\frac{1}{560}\sqrt{a^2x^2-1}\left(\left(2\left(\frac{328\,ac^3}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{175\,a^2c^3}{\mathrm{sgn}\left(ax+1\right)}-4\left(\frac{26\,a^3c^3}{\mathrm{sgn}\left(ax+1\right)}+5\left(\frac{2\,a^5c^3x}{\mathrm{sgn}\left(ax+1\right)}+\frac{7\,a^4c^3}{\mathrm{sgn}\left(ax+1\right)}\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1/
560*sqrt(a^2*x^2 - 1)*((2*(328*a*c^3/sgn(a*x + 1) + (175*a^2*c^3/sgn(a*x +
1) - 4*(26*a^3*c^3/sgn(a*x + 1) + 5*(2*a^5*c^3*x/sgn(a*x + 1) + 7*a^4*c^3/s
gn(a*x + 1))*x)*x)*x)*x + 245*c^3/sgn(a*x + 1))*x - 368*c^3/(a*sgn(a*x + 1)
))
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{\frac{15\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}-\frac{9\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{1199\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{1152\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{35}+\frac{849\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{40}-\frac{15\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{2}+\frac{9\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{8}}{a-\frac{7\,a\,(a\,x-1)}{a\,x+1}+\frac{21\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{35\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{35\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{21\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{7\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{a\,(a\,x-1)^7}{(a\,x+1)^7}}-\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

```
[In] int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - ((15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(1
/2))/8 + (1199*c^3*((a*x - 1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/35 + (849*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 - (15*c^3*((
a*x - 1)/(a*x + 1))^(11/2))/2 + (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a
- (7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x -
 1)^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a
*x + 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7) -
(9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

3.575 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 233

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx = -\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 -\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 +\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{7c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}$$

```
[Out] -7/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/24*a*c^2*(1+1/a/x)^(3
/2)*x^2*(1-1/a/x)^(1/2)-7/60*a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/
20*a^3*c^2*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/5*a^4*c^2*(1+1/a/x)^(9/2)*
x^5*(1-1/a/x)^(1/2)-7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used

$= \{6326, 6330, 96, 94, 214\}$

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{1}{5}a^4c^2x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2} - \frac{1}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2} - \frac{7}{60}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2} - \frac{7c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a} - \frac{7}{24}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2} - \frac{7}{8}c^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (-7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/24 - (7*a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/60 - (a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (
a^4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/5 - (7*c^2*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^4c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2 x^4\,dx\\
&= -\left(\left(a^4c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{5}\left(a^3c^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{7/2}}{x^5\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\
&\quad+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{20}\left(7a^2c^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\
&\quad+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{12}\left(7ac^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\
&\quad-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\
&\quad+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{8}\left(7c^2\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)
\end{aligned}$$

$$= -\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{(7c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= -\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 - \frac{(7c^2)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a^2}$$

$$= -\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 - \frac{7c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-136+15ax+112a^2x^2+90a^3x^3+24a^4x^4) - 105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3 +
 24*a^4*x^4) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136)(ax-1)c^2}{120a\sqrt{\frac{ax-1}{ax+1}}}-\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+90(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+105\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+120((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/120*(24*a^4*x^4+90*a^3*x^3+112*a^2*x^2+15*a*x-136)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)-7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx =$$
$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(24\,a^5c^2x^5+114\,a^4c^2x^4+202\,a^3c^2x^3+127\,a^2c^2x^2}{120\,a}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] -1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (24*a^5*c^2*x^5 + 114*a^4*c^2*x^4 + 202*a^3*c^2*x^3 + 127*a^2*c^2*x^2 - 121*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\left(\int\left(-\frac{2a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4*x**4/(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a
*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = -\frac{1}{120}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-490\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+896\,c\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1}{(ax+1}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 4
90*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) -
790*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5
*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*
a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)
^5 - a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(2\left(3\left(\frac{4\,a^3c^2x}{\operatorname{sgn}(ax+1)}+\frac{15\,a^2c^2}{\operatorname{sgn}(ax+1)}\right)x+\frac{56\,ac^2}{\operatorname{sgn}(ax+1)}\right)x+\frac{15\,c^2}{\operatorname{sgn}(ax+1)}\right)x-\frac{136\,c}{a\operatorname{sgn}(ax}\right.$$
$$+\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/120*sqrt(a^2*x^2 - 1)*((2*(3*(4*a^3*c^2*x/sgn(a*x + 1) + 15*a^2*c^2/sgn(a
*x + 1))*x + 56*a*c^2/sgn(a*x + 1))*x + 15*c^2/sgn(a*x + 1))*x - 136*c^2/(a
*sgn(a*x + 1))) + 7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*s
gn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{79\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{224\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{49\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}-\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}$$
$$-\frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((7*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 + (79*c^2*((a*x - 1)/(a*x + 1))^(3/2
))/6 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (49*c^2*((a*x - 1)/(a*x +
 1))^(7/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (7*c^2*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.576 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 -\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 -\frac{5c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

```
[Out] -5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/6*a*c*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-5/2*c*x*(1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2} -\frac{5c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a} -\frac{5}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 - (5*a*c*Sqrt[1 - 1/(a*x)]*(
1 + 1/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^
3)/3 - (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^2c\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{3}(5ac)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{2}(5c)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{(5c)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{(5c)\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a^2}$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{5c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(22+9ax+2a^2x^2)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -1/6*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 + 9*a*x + 2*a^2*x^2) + 15*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(2 a^{2} x^{2}+9 a x+22\right)\left(a x-1\right) c}{6 a \sqrt{\frac{a x-1}{a x+1}}}-\frac{5 \ln\left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c \sqrt{\left(a x-1\right)\left(a x+1\right)}}{2 \sqrt{a^{2}}\, \left(a x+1\right) \sqrt{\frac{a x-1}{a x+1}}}$
default	$-\frac{\left(a x-1\right)^{2} c \left(9 \sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a x+2 \left(\left(a x-1\right)\left(a x+1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}-9 \ln\left(\frac{a^{2} x+\sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}}{\sqrt{a^{2}}}\right) a+24 \sqrt{a^{2}}\, \sqrt{\left(a x-1\right)\left(a x+1\right)}+24 a \ln\left(\frac{a^{2} x+\sqrt{a}\right.}{}\right.}{6 \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}} \left(a x+1\right) \sqrt{\left(a x-1\right)\left(a x+1\right)}\, a \sqrt{a^{2}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*a^2*x^2+9*a*x+22)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)-5/2*ln(a^2*x/
(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3+11\,a^2cx^2+31\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (2*a^3*c*x^3 + 11*a^2*c*x^2 + 31*a*c*x + 22*c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(\int \frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int \left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{1}{6}\,a\left(\frac{2\left(15\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+33\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(15*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 33*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{1}{6}\,\sqrt{a^2x^2-1}\left(\left(\frac{2\,acx}{\mathrm{sgn}\,(ax+1)}+\frac{9\,c}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{22\,c}{a\mathrm{sgn}\,(ax+1)}\right) + \frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a^2*x^2 - 1)*((2*a*c*x/sgn(a*x + 1) + 9*c/sgn(a*x + 1))*x + 22*c/
(a*sgn(a*x + 1))) + 5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*s
gn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{11\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (11*c*((a*x - 1)/(a*x + 1))^(1/2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3
+ 5*c*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c*atanh(((a*x -
1)/(a*x + 1))^(1/2)))/a
```

3.577 $\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

```
[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6318}

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
gosper	$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}ac}$	24
default	$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}ac}$	24
trager	$\frac{(ax+1)^2\sqrt{-\frac{-ax+1}{ax+1}}}{3ac(ax-1)^2}$	40

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{(a^2x^2+2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3cx^2-2\,a^2cx+ac)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c*x^2 - 2*a^2*c*x
+ a*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{3\,ac\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3/(a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^
3*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{3\,a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

```
[In] int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] 1/(3*a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

3.578 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}+\frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}$$

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)/a/c^2+1/5/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/
a/c^2/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2*E^(3*ArcCoth[a*x]))/(15*a*c^2) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*
c^2*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)} - \frac{2\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c} \\ &= -\frac{2e^{3\coth^{-1}(ax)}}{15ac^2} + \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7-6ax+2a^2x^2)}{15c^2(-1+ax)^3}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^2*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2a^2x^2-6ax+7}{15(a^2x^2-1)c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	49
default	$-\frac{2a^2x^2-6ax+7}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)c^2(ax-1)a}$	52
trager	$-\frac{(ax+1)(2a^2x^2-6ax+7)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^2(ax-1)^3}$	52

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/15*(2*a^2*x^2-6*a*x+7)/(a^2*x^2-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{(2\,a^3x^3-4\,a^2x^2+ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*
x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**4*x**
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3}{60\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^2*((a*x
 - 1)/(a*x + 1))^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] -((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^2*((a
*x - 1)/(a*x + 1))^(5/2))
```

3.579 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{8e^{3\coth^{-1}(ax)}}{35ac^3} - \frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3(1-a^2x^2)^2} + \frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3(1-a^2x^2)}$$

```
[Out] -8/35/((a*x-1)/(a*x+1))^(3/2)/a/c^3-1/7/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/
a/c^3/(-a^2*x^2+1)^2+12/35/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^3/(-a^2*x
^2+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2} + \frac{12(3-2ax)e^{3\coth^{-1}(ax)}}{35ac^3(1-a^2x^2)} - \frac{8e^{3\coth^{-1}(ax)}}{35ac^3}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-8*E^(3*ArcCoth[a*x]))/(35*a*c^3) - (E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(7*a*
c^3*(1 - a^2*x^2)^2) + (12*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(35*a*c^3*(1 - a
^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\int\frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{7c}\\ &= -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}-\frac{24\int\frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}\\ &= -\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}-\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4)}{35c^3(-1+ax)^4(1+ax)}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a
^4*x^4))/(c^3*(-1 + a*x)^4*(1 + a*x))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

method	result	size
trager	$-\frac{\left(8a^{4}x^{4}-24a^{3}x^{3}+20a^{2}x^{2}+4ax-13\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^{3}\left(ax-1\right)^{4}}$	63
gosper	$-\frac{8a^{4}x^{4}-24a^{3}x^{3}+20a^{2}x^{2}+4ax-13}{35\left(a^{2}x^{2}-1\right)^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	65
default	$-\frac{8a^{4}x^{4}-24a^{3}x^{3}+20a^{2}x^{2}+4ax-13}{35\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)^{2}c^{3}a\left(ax-1\right)^{2}}$	68

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35/a/c^3*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^4*(-(-a*x+1)
/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\,\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx =$$

$$-\frac{\int \frac{1}{\frac{a^{7}x^{7}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^{6}x^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^{5}x^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\cdots}}{\cdots}}}{c^{3}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(1/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**6*x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3
*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + (28*(a*x - 1)/(a*x + 1)
- 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^3*((
a*x - 1)/(a*x + 1))^(7/2)))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{1}{(a^2cx^2-c)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^3\,(a\,x+1)^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

```
[In] int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] -(4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^3*(a*x + 1)^4*(
(a*x - 1)/(a*x + 1))^(7/2))
```

3.580 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16e^{3\coth^{-1}(ax)}}{63ac^4} - \frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4(1-a^2x^2)}$$

```
[Out] -16/63/((a*x-1)/(a*x+1))^(3/2)/a/c^4-1/9/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+1)
/a/c^4/(-a^2*x^2+1)^3-10/63/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/a/c^4/(-a^2*
x^2+1)^2+8/21/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^4/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{10(3-4ax)e^{3\coth^{-1}(ax)}}{63ac^4(1-a^2x^2)^2} + \frac{8(3-2ax)e^{3\coth^{-1}(ax)}}{21ac^4(1-a^2x^2)} - \frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3} - \frac{16e^{3\coth^{-1}(ax)}}{63ac^4}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-16*E^(3*ArcCoth[a*x]))/(63*a*c^4) - (E^(3*ArcCoth[a*x])*(1 - 2*a*x))/(9*a
*c^4*(1 - a^2*x^2)^3) - (10*E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(63*a*c^4*(1 -
a^2*x^2)^2) + (8*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(21*a*c^4*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{9c}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{40\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{21c^2}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4(1-a^2x^2)} - \frac{16\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{21c^3}$$

$$= -\frac{16e^{3\coth^{-1}(ax)}}{63ac^4} - \frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4(1-a^2x^2)}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(19+6ax-66a^2x^2+56a^3x^3+24a^4x^4-48a^5x^5+16a^6x^6)}{63c^4(-1+ax)^5(1+ax)^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
[Out] -1/63*(Sqrt[1 - 1/(a^2*x^2)]*x*(19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a
^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6))/(c^4*(-1 + a*x)^5*(1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{16a^{6}x^{6}-48a^{5}x^{5}+24a^{4}x^{4}+56a^{3}x^{3}-66a^{2}x^{2}+6ax+19}{63\left(a^{2}x^{2}-1\right)^{3}c^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	81
default	$-\frac{16a^{6}x^{6}-48a^{5}x^{5}+24a^{4}x^{4}+56a^{3}x^{3}-66a^{2}x^{2}+6ax+19}{63\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)^{3}c^{4}\left(ax-1\right)^{3}a}$	84
trager	$-\frac{\left(16a^{6}x^{6}-48a^{5}x^{5}+24a^{4}x^{4}+56a^{3}x^{3}-66a^{2}x^{2}+6ax+19\right)\sqrt{-\frac{-ax+1}{ax+1}}}{63a\,c^{4}\left(ax+1\right)\left(ax-1\right)^{5}}$	86

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
[Out] -1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/(a^
2*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx=-\frac{\left(16\,a^{6}x^{6}-48\,a^{5}x^{5}+24\,a^{4}x^{4}+56\,a^{3}x^{3}-66\,a^{2}x^{2}+6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\,\left(a^{7}c^{4}x^{6}-4\,a^{6}c^{4}x^{5}+5\,a^{5}c^{4}x^{4}-5\,a^{3}c^{4}x^{2}+4\,a^{2}c^{4}x-ac^{4}\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
[Out] -1/63*(16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a^2*x^2 + 6*a
*x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4
*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{1}{4032}a\left(\frac{21\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-18\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{54(ax-1)}{ax+1}-\frac{189(ax-1)^2}{(ax+1)^2}+\frac{420(ax-1)^3}{(ax+1)^3}-\frac{945(ax-1)^4}{(ax+1)^4}-7}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*(21*(((a*x - 1)/(a*x + 1))^(3/2) - 18*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*c^4) + (54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 420*(a*x
 - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/(a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{1}{(a^2cx^2-c)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16\,a^6\,x^6-48\,a^5\,x^5+24\,a^4\,x^4+56\,a^3\,x^3-66\,a^2\,x^2+6\,a\,x+19}{63\,a\,c^4\,(a\,x+1)^6\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

```
[In] int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -(6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6 +
19)/(63*a*c^4*(a*x + 1)^6*((a*x - 1)/(a*x + 1))^(9/2))
```

3.581 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=\frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

```
[Out] c^5*(a*x+1)^8/a-4/3*c^5*(a*x+1)^9/a+3/5*c^5*(a*x+1)^10/a-1/11*c^5*(a*x+1)^1
1/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^5\,dx\\
&= c^5\int (1-ax)^3(1+ax)^7\,dx\\
&= c^5\int \left(8(1+ax)^7-12(1+ax)^8+6(1+ax)^9-(1+ax)^{10}\right)\,dx\\
&= \frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(1+ax)^8\left(-29+67ax-54a^2x^2+15a^3x^3\right)}{165a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
[Out] -1/165*(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/a
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

method	result
default	$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{2}{5}a^9x^{10}-\frac{1}{3}a^8x^9+a^7x^8+2a^6x^7-\frac{14}{5}a^4x^5-2a^3x^4+a^2x^3+2a\,x^2+x\right)$
gosper	$-\frac{c^5x(15a^{10}x^{10}+66a^9x^9+55a^8x^8-165a^7x^7-330a^6x^6+462a^4x^4+330a^3x^3-165a^2x^2-330ax-165)}{165}$
risch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+c^5a^2x^3+2a\,c$
parallelrisch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+c^5a^2x^3+2a\,c$
norman	$\frac{-c^5x+a^7c^5x^8+c^5a^2x^3-a\,c^5x^2+3a^3c^5x^4+\frac{4}{5}a^4c^5x^5-\frac{14}{5}a^5c^5x^6-2a^6c^5x^7+\frac{4}{3}a^8c^5x^9+\frac{1}{15}a^9c^5x^{10}-\frac{17}{55}a^{10}c^5x^{11}-\frac{1}{11}a^{11}c^5x^{12}}{ax-1}$
meijerg	$c^5\left(-\frac{xa\left(-2730x^{11}a^{11}-3276a^{10}x^{10}-4004a^9x^9-5005a^8x^8-6435a^7x^7-8580a^6x^6-12012a^5x^5-18018a^4x^4-30030a^3x^3-60060a^2x^2-180180\right.}{30030(-ax+1)}\right.$ … over a

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
[Out] c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*a^8*x^9+a^7*x^8+2*a^6*x^7-14/5*a^4*x^
5-2*a^3*x^4+a^2*x^3+2*a*x^2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(54) = 108.

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.65

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*x
**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c**5
*x**3 + 2*a*c**5*x**2 + c**5*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{\left(15\,c^5+\frac{231\,c^5}{ax-1}+\frac{1540\,c^5}{(ax-1)^2}+\frac{5775\,c^5}{(ax-1)^3}+\frac{13200\,c^5}{(ax-1)^4}+\frac{18480\,c^5}{(ax-1)^5}+\frac{14784\,c^5}{(ax-1)^6}+\frac{5280\,c^5}{(ax-1)^7}\right)(ax-1)^{11}}{165\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/165*(15*c^5 + 231*c^5/(a*x - 1) + 1540*c^5/(a*x - 1)^2 + 5775*c^5/(a*x -
 1)^3 + 13200*c^5/(a*x - 1)^4 + 18480*c^5/(a*x - 1)^5 + 14784*c^5/(a*x - 1)
^6 + 5280*c^5/(a*x - 1)^7)*(a*x - 1)^11/a
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{2\,a^9\,c^5\,x^{10}}{5}-\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8+2\,a^6\,c^5\,x^7-\frac{14\,a^4\,c^5\,x^5}{5}-2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3+2\,a\,c^5\,x^2+c^5\,x$$

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^5*x + 2*a*c^5*x^2 + a^2*c^5*x^3 - 2*a^3*c^5*x^4 - (14*a^4*c^5*x^5)/5 + 2*
a^6*c^5*x^7 + a^7*c^5*x^8 - (a^8*c^5*x^9)/3 - (2*a^9*c^5*x^10)/5 - (a^10*c^
5*x^11)/11
```

3.582 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}$$

```
[Out] 4/7*c^4*(a*x+1)^7/a-1/2*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+1)^9/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6302, 6275, 45}

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^4\,dx\\ &= c^4\int (1-ax)^2(1+ax)^6\,dx\\ &= c^4\int \left(4(1+ax)^6-4(1+ax)^7+(1+ax)^8\right)\,dx\\ &= \frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(1+ax)^7\left(23-35ax+14a^2x^2\right)}{126a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
[Out] (c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result
gosper	$\frac{c^4x(14a^8x^8+63a^7x^7+72a^6x^6-84a^5x^5-252a^4x^4-126a^3x^3+168a^2x^2+252ax+126)}{126}$
default	$c^4\left(\frac{1}{9}a^8x^9+\frac{1}{2}a^7x^8+\frac{4}{7}a^6x^7-\frac{2}{3}a^5x^6-2a^4x^5-a^3x^4+\frac{4}{3}a^2x^3+2a\,x^2+x\right)$
risch	$\frac{1}{9}a^8c^4x^9+\frac{1}{2}a^7c^4x^8+\frac{4}{7}a^6c^4x^7-\frac{2}{3}a^5c^4x^6-2a^4c^4x^5-a^3c^4x^4+\frac{4}{3}a^2c^4x^3+2a\,c^4x^2+c^4x$
parallelrisch	$\frac{1}{9}a^8c^4x^9+\frac{1}{2}a^7c^4x^8+\frac{4}{7}a^6c^4x^7-\frac{2}{3}a^5c^4x^6-2a^4c^4x^5-a^3c^4x^4+\frac{4}{3}a^2c^4x^3+2a\,c^4x^2+c^4x$
norman	$\frac{-c^4x+a^4c^4x^5-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{7}{3}a^3c^4x^4-\frac{4}{3}a^5c^4x^6-\frac{26}{21}a^6c^4x^7+\frac{1}{14}a^7c^4x^8+\frac{7}{18}a^8c^4x^9+\frac{1}{9}a^9c^4x^{10}}{ax-1}$
meijerg	$-\frac{c^4\left(-\frac{xa\left(-308a^9x^9-385a^8x^8-495a^7x^7-660a^6x^6-924a^5x^5-1386a^4x^4-2310a^3x^3-4620a^2x^2-13860ax+27720\right)}{2772(-ax+1)}-10\ln(-ax+1)\right)}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/126*c^4*x*(14*a^8*x^8+63*a^7*x^7+72*a^6*x^6-84*a^5*x^5-252*a^4*x^4-126*a^
3*x^3+168*a^2*x^2+252*a*x+126)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.92

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{2}+\frac{4a^6c^4x^7}{7}-\frac{2a^5c^4x^6}{3}-2a^4c^4x^5-a^3c^4x^4+\frac{4a^2c^4x^3}{3}+2ac^4x^2+c^4x$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**6
/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x**2
 + c**4*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{\left(14\,c^4+\frac{189\,c^4}{ax-1}+\frac{1080\,c^4}{(ax-1)^2}+\frac{3360\,c^4}{(ax-1)^3}+\frac{6048\,c^4}{(ax-1)^4}+\frac{6048\,c^4}{(ax-1)^5}+\frac{2688\,c^4}{(ax-1)^6}\right)(ax-1)^9}{126\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/126*(14*c^4 + 189*c^4/(a*x - 1) + 1080*c^4/(a*x - 1)^2 + 3360*c^4/(a*x -
1)^3 + 6048*c^4/(a*x - 1)^4 + 6048*c^4/(a*x - 1)^5 + 2688*c^4/(a*x - 1)^6)*
(a*x - 1)^9/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{2}+\frac{4\,a^6\,c^4\,x^7}{7}-\frac{2\,a^5\,c^4\,x^6}{3}-2\,a^4\,c^4\,x^5-a^3\,c^4\,x^4+\frac{4\,a^2\,c^4\,x^3}{3}+2\,a\,c^4\,x^2+c^4\,x$$

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^4*x + 2*a*c^4*x^2 + (4*a^2*c^4*x^3)/3 - a^3*c^4*x^4 - 2*a^4*c^4*x^5 - (2*
a^5*c^4*x^6)/3 + (4*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/2 + (a^8*c^4*x^9)/9
```

3.583 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

```
[Out] 1/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
```

```
c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^3\,dx\\
&= c^3\int (1-ax)(1+ax)^5\,dx\\
&= c^3\int \left(2(1+ax)^5-(1+ax)^6\right)\,dx\\
&= \frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(1+ax)^6(-4+3ax)}{21a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
[Out] -1/21*(c^3*(1 + a*x)^6*(-4 + 3*a*x))/a
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

method	result
gosper	$-\frac{c^{3}x\left(3a^{6}x^{6}+14a^{5}x^{5}+21a^{4}x^{4}-35a^{2}x^{2}-42ax-21\right)}{21}$
default	$c^{3}\left(-\frac{1}{7}a^{6}x^{7}-\frac{2}{3}a^{5}x^{6}-a^{4}x^{5}+\frac{5}{3}a^{2}x^{3}+2a\,x^{2}+x\right)$
risch	$-\frac{1}{7}a^{6}c^{3}x^{7}-\frac{2}{3}a^{5}c^{3}x^{6}-a^{4}c^{3}x^{5}+\frac{5}{3}a^{2}c^{3}x^{3}+2a\,c^{3}x^{2}+c^{3}x$
parallelrisch	$-\frac{1}{7}a^{6}c^{3}x^{7}-\frac{2}{3}a^{5}c^{3}x^{6}-a^{4}c^{3}x^{5}+\frac{5}{3}a^{2}c^{3}x^{3}+2a\,c^{3}x^{2}+c^{3}x$
norman	$\frac{-c^{3}x+a^{4}c^{3}x^{5}-a\,c^{3}x^{2}+\frac{1}{3}a^{2}c^{3}x^{3}+\frac{5}{3}a^{3}c^{3}x^{4}-\frac{1}{3}a^{5}c^{3}x^{6}-\frac{11}{21}a^{6}c^{3}x^{7}-\frac{1}{7}a^{7}c^{3}x^{8}}{ax-1}$
meijerg	$\frac{c^{3}\left(-\frac{ax\left(-45a^{7}x^{7}-60a^{6}x^{6}-84a^{5}x^{5}-126a^{4}x^{4}-210a^{3}x^{3}-420a^{2}x^{2}-1260ax+2520\right)}{315(-ax+1)}-8\ln(-ax+1)\right)}{a}-\frac{2c^{3}\left(-\frac{ax\left(-14a^{5}x^{5}-21a^{4}x^{4}\right.}{}\right.}{}$

[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/21*c^3*x*(3*a^6*x^6+14*a^5*x^5+21*a^4*x^4-35*a^2*x^2-42*a*x-21)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{3}\,dx=-\frac{1}{7}\,a^{6}c^{3}x^{7}-\frac{2}{3}\,a^{5}c^{3}x^{6}-a^{4}c^{3}x^{5}+\frac{5}{3}\,a^{2}c^{3}x^{3}+2\,ac^{3}x^{2}+c^{3}x$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")

[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^3*x^2 + c^3*x

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int e^{4\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{3}\,dx=-\frac{a^{6}c^{3}x^{7}}{7}-\frac{2a^{5}c^{3}x^{6}}{3}-a^{4}c^{3}x^{5}+\frac{5a^{2}c^{3}x^{3}}{3}+2ac^{3}x^{2}+c^{3}x$$

[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**3,x)

[Out] -a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3/3 + 2*a*c**3*x**2 + c**3*x

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(31) = 62.

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{\left(3\,c^3+\frac{35\,c^3}{ax-1}+\frac{168\,c^3}{(ax-1)^2}+\frac{420\,c^3}{(ax-1)^3}+\frac{560\,c^3}{(ax-1)^4}+\frac{336\,c^3}{(ax-1)^5}\right)(ax-1)^7}{21\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/21*(3*c^3 + 35*c^3/(a*x - 1) + 168*c^3/(a*x - 1)^2 + 420*c^3/(a*x - 1)^3
 + 560*c^3/(a*x - 1)^4 + 336*c^3/(a*x - 1)^5)*(a*x - 1)^7/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}-\frac{2\,a^5\,c^3\,x^6}{3}-a^4\,c^3\,x^5+\frac{5\,a^2\,c^3\,x^3}{3}+2\,a\,c^3\,x^2+c^3\,x$$

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^3*x + 2*a*c^3*x^2 + (5*a^2*c^3*x^3)/3 - a^4*c^3*x^5 - (2*a^5*c^3*x^6)/3 -
 (a^6*c^3*x^7)/7
```

3.584 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=\frac{c^2(1+ax)^5}{5a}$$

```
[Out] 1/5*c^2*(a*x+1)^5/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 32}

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=\frac{c^2(ax+1)^5}{5a}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^2\,dx \\ &= c^2\int (1+ax)^4\,dx \\ &= \frac{c^2(1+ax)^5}{5a} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\left(x+2ax^2+2a^2x^3+a^3x^4+\frac{a^4x^5}{5}\right)$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
[Out] c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2(ax+1)^5}{5a}$
gosper	$\frac{c^2x\left(a^4x^4+5a^3x^3+10a^2x^2+10ax+5\right)}{5}$
parallelrisch	$\frac{1}{5}a^4c^2x^5+a^3c^2x^4+2a^2c^2x^3+2a\,c^2x^2+c^2x$
risch	$\frac{a^4c^2x^5}{5}+a^3c^2x^4+2a^2c^2x^3+2a\,c^2x^2+c^2x+\frac{c^2}{5a}$
norman	$\frac{-c^2x+a^3c^2x^4-a\,c^2x^2+\frac{4}{5}a^4c^2x^5+\frac{1}{5}a^5c^2x^6}{ax-1}$
meijerg	$-\frac{c^2\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70a^2x^2-210ax+420\right)}{70(-ax+1)}-6\ln(-ax+1)\right)}{a}+\frac{c^2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+\right.}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/5*c^2*(a*x+1)^5/a

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+a^3c^2x^4+2\,a^2c^2x^3+2\,ac^2x^2+c^2x$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}+a^3c^2x^4+2a^2c^2x^3+2ac^2x^2+c^2x$$

[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**2,x)

[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**2
*x

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+a^3c^2x^4+2\,a^2c^2x^3+2\,ac^2x^2+c^2x$$

[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{\left(c^2+\frac{10\,c^2}{ax-1}+\frac{40\,c^2}{(ax-1)^2}+\frac{80\,c^2}{(ax-1)^3}+\frac{80\,c^2}{(ax-1)^4}\right)(ax-1)^5}{5\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] 1/5*(c^2 + 10*c^2/(a*x - 1) + 40*c^2/(a*x - 1)^2 + 80*c^2/(a*x - 1)^3 + 80*
c^2/(a*x - 1)^4)*(a*x - 1)^5/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{a^4\,c^2\,x^5}{5}+a^3\,c^2\,x^4+2\,a^2\,c^2\,x^3+2\,a\,c^2\,x^2+c^2\,x$$

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

3.585 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}$$

```
[Out] -4*c*x-c*(a*x+1)^2/a-1/3*c*(a*x+1)^3/a-8*c*ln(-a*x+1)/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6302, 6275, 45}

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
```

```
c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)\,dx \\ &= c\int \frac{(1+ax)^3}{1-ax}\,dx \\ &= c\int \left(-4+\frac{8}{1-ax}-2(1+ax)-(1+ax)^2\right)dx \\ &= -4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(4+21ax+6a^2x^2+a^3x^3+24\log(1-ax))}{3a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -1/3*(c*(4 + 21*a*x + 6*a^2*x^2 + a^3*x^3 + 24*Log[1 - a*x]))/a
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result
default	$c\left(-\frac{a^2x^3}{3}-2a\,x^2-7x-\frac{8\ln(ax-1)}{a}\right)$
risch	$-\frac{a^2c\,x^3}{3}-2ac\,x^2-7cx-\frac{8c\ln(ax-1)}{a}$
parallelrisch	$-\frac{a^3c\,x^3+6a^2c\,x^2+21acx+24c\ln(ax-1)}{3a}$
norman	$\frac{7cx-5ac\,x^2-\frac{5}{3}a^2c\,x^3-\frac{1}{3}a^3c\,x^4}{ax-1}-\frac{8c\ln(ax-1)}{a}$
meijerg	$\frac{c\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}-\frac{2c\left(\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] c*(-1/3*a^2*x^3-2*a*x^2-7*x-8/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^3cx^3+6\,a^2cx^2+21\,acx+24\,c\log\left(ax-1\right)}{3\,a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}-2acx^2-7cx-\frac{8c\log\left(ax-1\right)}{a}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-2\,acx^2-7\,cx-\frac{8\,c\log\left(ax-1\right)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{(ax-1)^3\left(c+\frac{9\,c}{ax-1}+\frac{36\,c}{(ax-1)^2}\right)}{3\,a}+\frac{8\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/3*(a*x - 1)^3*(c + 9*c/(a*x - 1) + 36*c/(a*x - 1)^2)/a + 8*c*log(abs(a*x
 - 1)/((a*x - 1)^2*abs(a)))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-7\,c\,x-\frac{a^2\,c\,x^3}{3}-\frac{8\,c\,\ln\left(a\,x-1\right)}{a}-2\,a\,c\,x^2$$

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] - 7*c*x - (a^2*c*x^3)/3 - (8*c*log(a*x - 1))/a - 2*a*c*x^2
```

3.586 $\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c(1-ax)^2}$$

```
[Out] x/c/(-a*x+1)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 34}

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c(1-ax)^2}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] x/(c*(1 - a*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
```

```
c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{c-a^2cx^2}\,dx\\ &= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c}\\ &= \frac{x}{c(1-ax)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{(1+ax)^2}{4ac(1-ax)^2}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
[Out] (1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x}{c(ax-1)^2}$	13
norman	$\frac{x}{c(ax-1)^2}$	13
risch	$\frac{x}{c(ax-1)^2}$	13
parallelrisch	$\frac{x}{c(ax-1)^2}$	13
default	$\frac{\frac{1}{(ax-1)^2a}+\frac{1}{a(ax-1)}}{c}$	28

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] x/c/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = \frac{x}{a^2cx^2 - 2\,acx + c}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = \frac{x}{a^2cx^2 - 2acx + c}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c),x)
[Out] x/(a**2*c*x**2 - 2*a*c*x + c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = \frac{x}{a^2cx^2 - 2\,acx + c}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = \frac{\frac{1}{(ax-1)a} + \frac{1}{(ax-1)^2a}}{c}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] (1/((a*x - 1)*a) + 1/((a*x - 1)^2*a))/c
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = \frac{x}{c\,(a\,x-1)^2}$$

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)*(a*x - 1)^2),x)
[Out] x/(c*(a*x - 1)^2)
```

3.587 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx = \frac{1}{3ac^2(1-ax)^3}$$

```
[Out] 1/3/a/c^2/(-a*x+1)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 32}

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx = \frac{1}{3ac^2(1-ax)^3}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(3*a*c^2*(1 - a*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
```

```
c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{(c-a^2cx^2)^2}\,dx\\ &= \frac{\int \frac{1}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{1}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{3ac^2(1-ax)^3}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] 1/(3*a*c^2*(1 - a*x)^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{1}{3c^2(ax-1)^3a}$	16
default	$-\frac{1}{3c^2(ax-1)^3a}$	16
risch	$-\frac{1}{3c^2(ax-1)^3a}$	16
parallelrisch	$\frac{-a^2x^3+3a\,x^2-3x}{3(ax-1)^3c^2}$	31
norman	$\frac{-\frac{x}{c}-\frac{a^3x^4}{3c}+\frac{2a^2x^3}{3c}}{(ax-1)^3(ax+1)c}$	48

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/(a*x-1)^3/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**2,x)
[Out] -1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\,(ax-1)^3ac^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] -1/3/((a*x - 1)^3*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{-3\,a^4\,c^2\,x^3+9\,a^3\,c^2\,x^2-9\,a^2\,c^2\,x+3\,a\,c^2}$$

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^2*(a*x - 1)^2),x)
[Out] 1/(3*a*c^2 - 9*a^2*c^2*x + 9*a^3*c^2*x^2 - 3*a^4*c^2*x^3)
```

3.588 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\operatorname{arctanh}(ax)}{16ac^3}$$

```
[Out] 1/8/a/c^3/(-a*x+1)^4+1/12/a/c^3/(-a*x+1)^3+1/16/a/c^3/(-a*x+1)^2+1/16/a/c^3
/(-a*x+1)+1/16*arctanh(a*x)/a/c^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\operatorname{arctanh}(ax)}{16ac^3}+\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int \frac{e^{4\text{arctanh}(ax)}}{(c-a^2cx^2)^3}\,dx$$

$$= \frac{\int \frac{1}{(1-ax)^5(1+ax)}\,dx}{c^3}$$

$$= \frac{\int \left(-\frac{1}{2(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{1}{16(-1+ax)^2}-\frac{1}{16(-1+a^2x^2)}\right)dx}{c^3}$$

$$= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{16c^3}$$

$$= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\text{arctanh}(ax)}{16ac^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{16-19ax+12a^2x^2-3a^3x^3+3(-1+ax)^4\text{arctanh}(ax)}{48ac^3(-1+ax)^4}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
[Out] (16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a*
c^3*(-1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

method	result
risch	$\frac{-\frac{a^2x^3}{16}+\frac{a\,x^2}{4}-\frac{19x}{48}+\frac{1}{3a}}{c^3(ax-1)^4}+\frac{\ln(-ax-1)}{32a\,c^3}-\frac{\ln(ax-1)}{32a\,c^3}$
default	$\frac{\frac{\ln(ax+1)}{32a}+\frac{1}{8a(ax-1)^4}-\frac{1}{12a(ax-1)^3}+\frac{1}{16(ax-1)^2a}-\frac{1}{16a(ax-1)}-\frac{\ln(ax-1)}{32a}}{c^3}$
norman	$\frac{\frac{15x}{16c}+\frac{a\,x^2}{8c}-\frac{31a^2x^3}{24c}+\frac{11a^3x^4}{24c}+\frac{29a^4x^5}{48c}-\frac{a^5x^6}{3c}}{(ax+1)^2(ax-1)^4c^2}-\frac{\ln(ax-1)}{32a\,c^3}+\frac{\ln(ax+1)}{32a\,c^3}$
parallelrisch	$\frac{-3\ln(ax-1)x^4a^4+3\ln(ax+1)x^4a^4-32a^4x^4+12a^3\ln(ax-1)x^3-12a^3\ln(ax+1)x^3+122a^3x^3-18a^2\ln(ax-1)x^2+18a^2\ln(ax+}{96(ax-1)^4c^3a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] (-1/16*a^2*x^3+1/4*a*x^2-19/48*x+1/3/a)/c^3/(a*x-1)^4+1/32/a/c^3*ln(-a*x-1)
-1/32/a/c^3*ln(a*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(73) = 146.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{6\,a^3x^3-24\,a^2x^2+38\,ax-3\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log{(ax+1)}+3\,(a^4x^4-4\,a^3x^3+6\,a^2}{96\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] -1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2
 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1
```

```
)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c
^3*x + a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3} - \frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*c
**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1/a
)/32 - log(x + 1/a)/32)/(a*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)} + \frac{\log(ax+1)}{32\,ac^3} - \frac{\log(ax-1)}{32\,ac^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 +
 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*lo
g(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3} - \frac{\frac{3\,a^3c^9}{ax-1}-\frac{3\,a^3c^9}{(ax-1)^2}+\frac{4\,a^3c^9}{(ax-1)^3}-\frac{6\,a^3c^9}{(ax-1)^4}}{48\,a^4c^{12}}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(3*a^3*c^9/(a*x - 1) - 3*a^3
*c^9/(a*x - 1)^2 + 4*a^3*c^9/(a*x - 1)^3 - 6*a^3*c^9/(a*x - 1)^4)/(a^4*c^12
)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\operatorname{atanh}(a\,x)}{16\,a\,c^3} - \frac{\frac{19\,x}{48} - \frac{a\,x^2}{4} - \frac{1}{3\,a} + \frac{a^2\,x^3}{16}}{a^4\,c^3\,x^4 - 4\,a^3\,c^3\,x^3 + 6\,a^2\,c^3\,x^2 - 4\,a\,c^3\,x + c^3}$$

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^3*(a*x - 1)^2),x)
```

```
[Out] atanh(a*x)/(16*a*c^3) - ((19*x)/48 - (a*x^2)/4 - 1/(3*a) + (a^2*x^3)/16)/(c
^3 + 6*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x)
```

3.589 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{3\operatorname{arctanh}(ax)}{32ac^4}$$

```
[Out] 1/20/a/c^4/(-a*x+1)^5+1/16/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+1/16/a/c^
4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+3/32*arctanh(a*x)/a/c^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{3\operatorname{arctanh}(ax)}{32ac^4}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{(c-a^2cx^2)^4}\,dx\\
&= \frac{\int \frac{1}{(1-ax)^6(1+ax)^2}\,dx}{c^4}\\
&= \frac{\int \left(\frac{1}{4(-1+ax)^6}-\frac{1}{4(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{64(1+ax)^2}-\frac{3}{32(-1+a^2x^2)}\right)dx}{c^4}\\
&= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}\\
&\quad+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}-\frac{3\int \frac{1}{-1+a^2x^2}\,dx}{32c^4}\\
&= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}\\
&\quad+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{3\text{arctanh}(ax)}{32ac^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{-48+47ax+20a^2x^2-80a^3x^3+60a^4x^4-15a^5x^5+15(-1+ax)^5(1+ax)\operatorname{arctanh}(ax)}{160ac^4(-1+ax)^5(1+ax)}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

method	result
risch	$\frac{-\frac{3a^4x^5}{32}+\frac{3a^3x^4}{8}-\frac{a^2x^3}{2}+\frac{a\,x^2}{8}+\frac{47x}{160}-\frac{3}{10a}}{c^4(ax-1)^4(a^2x^2-1)}-\frac{3\ln(ax-1)}{64a\,c^4}+\frac{3\ln(-ax-1)}{64a\,c^4}$
default	$\frac{-\frac{1}{64a(ax+1)}+\frac{3\ln(ax+1)}{64a}-\frac{1}{20a(ax-1)^5}+\frac{1}{16a(ax-1)^4}-\frac{1}{16a(ax-1)^3}+\frac{1}{16(ax-1)^2a}-\frac{5}{64a(ax-1)}-\frac{3\ln(ax-1)}{64a}}{c^4}$
norman	$\frac{-\frac{a^3x^4}{2c}-\frac{29x}{32c}-\frac{3a\,x^2}{16c}+\frac{59a^2x^3}{32c}-\frac{263a^4x^5}{160c}+\frac{63a^5x^6}{80c}+\frac{81a^6x^7}{160c}-\frac{3a^7x^8}{10c}}{(ax-1)^5(ax+1)^3c^3}-\frac{3\ln(ax-1)}{64a\,c^4}+\frac{3\ln(ax+1)}{64a\,c^4}$
parallelrisch	$\frac{60a\ln(ax+1)x-75a^2\ln(ax+1)x^2+354a^5x^5-160a^3x^3-60\ln(ax+1)x^5a^5+15\ln(ax+1)x^6a^6+75\ln(ax+1)x^4a^4-15\ln(ax-1)x^6a}{320(a^2}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/32*a^4*x^5+3/8*a^3*x^4-1/2*a^2*x^3+1/8*a*x^2+47/160*x-3/10/a)/c^4/(a*x-
1)^4/(a^2*x^2-1)-3/64/a/c^4*ln(a*x-1)+3/64/a/c^4*ln(-a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(a\right.}{320\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2\right.}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*(
a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) + 96
)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*
x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax-48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4} + \frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (-15*a**5*x**5 + 60*a**4*x**4 - 80*a**3*x**3 + 20*a**2*x**2 + 47*a*x - 48)/
(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*c*
*4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x + 1
/a)/64)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)} + \frac{3\,\log\left(ax+1\right)}{64\,ac^4} - \frac{3\,\log\left(ax-1\right)}{64\,ac^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(a
^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x -
a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{3\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4} + \frac{1}{128\,ac^4\left(\frac{2}{ax-1}+1\right)} - \frac{\frac{25\,a^9c^{16}}{ax-1} - \frac{20\,a^9c^{16}}{(ax-1)^2} + \frac{20\,a^9c^{16}}{(ax-1)^3} - \frac{20\,a^9c^{16}}{(ax-1)^4} + \frac{16\,a^9c^{16}}{(ax-1)^5}}{320\,a^{10}c^{20}}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 3/64*log(abs(-2/(a*x - 1) - 1))/(a*c^4) + 1/128/(a*c^4*(2/(a*x - 1) + 1)) -
 1/320*(25*a^9*c^16/(a*x - 1) - 20*a^9*c^16/(a*x - 1)^2 + 20*a^9*c^16/(a*x
- 1)^3 - 20*a^9*c^16/(a*x - 1)^4 + 16*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{3\,\mathrm{atanh}(a\,x)}{32\,a\,c^4} - \frac{\frac{47\,x}{160} + \frac{a\,x^2}{8} - \frac{3}{10\,a} - \frac{a^2\,x^3}{2} + \frac{3\,a^3\,x^4}{8} - \frac{3\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6 + 4\,a^5\,c^4\,x^5 - 5\,a^4\,c^4\,x^4 + 5\,a^2\,c^4\,x^2 - 4\,a\,c^4\,x + c^4}$$

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^4*(a*x - 1)^2),x)
```

```
[Out] (3*atanh(a*x))/(32*a*c^4) - ((47*x)/160 + (a*x^2)/8 - 3/(10*a) - (a^2*x^3)/
2 + (3*a^3*x^4)/8 - (3*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 +
4*a^5*c^4*x^5 - a^6*c^4*x^6 - 4*a*c^4*x)
```

3.590 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 393

$$\begin{aligned}&\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx\\&=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\&\quad-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\&\quad+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}\end{aligned}$$

```
[Out] -5/48*a^5*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6+1/8*a^6*c^4*(1-1/a/x)^(5/
2)*(1+1/a/x)^(9/2)*x^7-1/8*a^7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x^8+1/9*
a^8*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(9/2)*x^9-35/128*c^4*arctanh((1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2))/a-35/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-7/1
92*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/64*a^3*c^4*(1+1/a/x)^(7/2)
*x^4*(1-1/a/x)^(1/2)+1/16*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-35/12
8*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{9/2}$$
$$-\frac{1}{8}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{8}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{5}{48}a^5c^4x^6\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9}$$

```
[In] Int[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

```
[Out] (-35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (35*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/
64 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/16 - (5*a^5*c^4*(1
 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/48 + (a^6*c^4*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(9/2)*x^7)/8 - (a^7*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
9/2)*x^8)/8 + (a^8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(9/2)*x^9)/9 - (35
*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = (a^8c^4)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4 x^8\,dx$$

$$= -\left((a^8c^4)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^{10}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9 + (a^7c^4)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^9}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9 - \frac{1}{8}(7a^6c^4)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9 + \frac{1}{8}(5a^5c^4)\,\text{Subst}\left(\int \frac{(1
$$

$$= -\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 + \frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+
$$

$$= \frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 - \frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 + \frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+
$$

$$= -\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5$$
$$-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\right.$$

$$= -\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$
$$-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5$$
$$-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\right.$$

$$= -\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a}\right.$$

$$= -\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a}\right.$$

$$= -\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a}\right.$$

$$= -\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a}\right.$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(128+837ax-512a^2x^2-978a^3x^3+768a^4x^4+600a^5x^5-512a^6x^6-144a^7x^7+128a^8x^8\right.}{1152a}$$

[In] Integrate[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]

[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3 + 768*a^4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) - 31 5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{1152a}-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{128\sqrt{}}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-128(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+144(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+384(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-456(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-\right.}{1}$

[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/1152*(128*a^8*x^8-144*a^7*x^7-512*a^6*x^6+600*a^5*x^5+768*a^4*x^4-978*a^3 *x^3-512*a^2*x^2+837*a*x+128)*(a*x+1)/a*c^4*((a*x-1)/(a*x+1))^(1/2)-35/128* ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.43

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=$$
$$-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(128\,a^9c^4x^9-16\,a^8c^4x^8-656\,a^7c^4x^7+88\,a^6c^4x^6}{1152\,a}$$

[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) - (128*a^9*c^4*x^9 - 16*a^8*c^4*x^8 - 656*a^7*c^4*x^7
 + 88*a^6*c^4*x^6 + 1368*a^5*c^4*x^5 - 210*a^4*c^4*x^4 - 1490*a^3*c^4*x^3 +
 325*a^2*c^4*x^2 + 965*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\left(\int\left(-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int 6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**6
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = -\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10\cdots\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-\cdots}{(ax+\cdots}}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 2
730*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/
```

```
2) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 32768*c^4*((a*x - 1)/(a*x + 1
))^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*
x + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x -
1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2
+ 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a
*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7
*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1
)^9 - a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|}+\frac{1}{1152}\sqrt{a^2x^2-1}\left(\frac{128\,c^4\operatorname{sgn}(ax+1)}{a}+\left(837\,c^4\operatorname{sgn}(ax+1)-2\left(256\,ac^4\operatorname{sgn}(ax+1)+\left(489\,a^2c^4\operatorname{sgn}(ax\right.\right.\right.\right.$$

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/
1152*sqrt(a^2*x^2 - 1)*(128*c^4*sgn(a*x + 1)/a + (837*c^4*sgn(a*x + 1) - 2*
(256*a*c^4*sgn(a*x + 1) + (489*a^2*c^4*sgn(a*x + 1) - 4*(96*a^3*c^4*sgn(a*x
 + 1) + (75*a^4*c^4*sgn(a*x + 1) - 2*(32*a^5*c^4*sgn(a*x + 1) - (8*a^7*c^4*
x*sgn(a*x + 1) - 9*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{\frac{35\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{455\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}+\frac{581\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}-\frac{1289\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{32}+\frac{512\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{9}+\frac{1289\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{32}-\frac{581\,c^4\,(}{}}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{36\,a\,(a\,x}{(a\,x+}}-\frac{35\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

```
[In] int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((35*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (455*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/96 + (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 - (1289*c^4*((a*x - 1)/
```

```
(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 + (1289*c^4*
((a*x - 1)/(a*x + 1))^(11/2))/32 - (581*c^4*((a*x - 1)/(a*x + 1))^(13/2))/3
2 + (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1
))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a*x + 1
)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^4 - (1
26*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*a*(a*x
 - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9)/(a*x
 + 1)^9) - (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.591 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 -\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 -\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}$$

```
[Out] -1/6*a^4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+1/6*a^5*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(7/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-5/16*
c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/48*a*c^3*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/24*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/8*a^3
*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-5/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2} +\frac{1}{6}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{6}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{8}a^3c^3x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-$$

```
[In] Int[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (-5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (5*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/48 - (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/8 - (a^
4*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/6 + (a^5*c^3*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/6 - (a^6*c^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(
a*x))^(7/2)*x^7)/7 - (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(
16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left((a^6c^3)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= (a^6c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7 - (a^5c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^7}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7 + \frac{1}{6}(5a^4c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 + \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7 - \frac{1}{2}(a^3c^3)\,\text{Subst}\left(\int \frac{\sqrt{1}\cdots}{\cdots}\right.$$

$$= \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 + \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7 + \frac{1}{8}(a^2c^3)\,\text{Subst}\left(\int \frac{(1-\cdots}{x^4\sqrt{\cdots}}\right.$$

$$= -\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 + \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\cdots\right.$$

$$= -\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 + \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\cdots\right.$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\right.$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\right.$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\right.$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$
$$=\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(48+231ax-144a^2x^2-182a^3x^3+144a^4x^4+56a^5x^5-48a^6x^6)-105\log\left(\left(1+\sqrt{1-\right.\right.\right.}{336a}$$

```
[In] Integrate[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 231*a*x - 144*a^2*x^2 - 182*a^3*x^3 +
 144*a^4*x^4 + 56*a^5*x^5 - 48*a^6*x^6) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{\left(48 a^{6} x^{6}-56 a^{5} x^{5}-144 a^{4} x^{4}+182 a^{3} x^{3}+144 a^{2} x^{2}-231 a x -48\right) \left(a x +1\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}}{336 a}-\frac{5 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) (a}{16 \sqrt{a^{2}}\, \left(a x -1\right)}$
default	$-\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) c^{3} \left(48 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{4} x^{4}-56 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{3} x^{3}-96 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}+126 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x -64 \left(a^{2} x^{2}\right.}{336 a \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/336*(48*a^6*x^6-56*a^5*x^5-144*a^4*x^4+182*a^3*x^3+144*a^2*x^2-231*a*x-4
8)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/2)-5/16*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)*c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/
(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(48\,a^7c^3x^7-8\,a^6c^3x^6-200\,a^5c^3x^5+38\,a^4c^3x^4+\right.}{336\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (48*a^7*c^3*x^7 - 8*a^6*c^3*x^6 - 200*a^5*c^3*x^5 +
38*a^4*c^3*x^4 + 326*a^3*c^3*x^3 - 87*a^2*c^3*x^2 - 279*a*c^3*x - 48*c^3)*s
qrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-c^3\left(\int 3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*
x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+35}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 70
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1
)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(
a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{16\,|a|}+\frac{1}{336}\sqrt{a^2x^2-1}\left(\frac{48\,c^3\operatorname{sgn}(ax+1)}{a}+\left(231\,c^3\operatorname{sgn}(ax+1)-2\left(72\,ac^3\operatorname{sgn}(ax+1)+\left(91\,a^2c^3\operatorname{sgn}(ax+1)\right.\right.\right.\right.$$

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/33
6*sqrt(a^2*x^2 - 1)*(48*c^3*sgn(a*x + 1)/a + (231*c^3*sgn(a*x + 1) - 2*(72*
a*c^3*sgn(a*x + 1) + (91*a^2*c^3*sgn(a*x + 1) - 4*(18*a^3*c^3*sgn(a*x + 1)
- (6*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{\frac{25\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{5\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{283\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{24}+\frac{128\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{7}+\frac{283\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{24}-\frac{25\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{6}+\frac{5\,c^3\left(\frac{a\,x-1}{a\,x+1}\right.}{8}}{a-\frac{7\,a\,(a\,x-1)}{a\,x+1}+\frac{21\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{35\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{35\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{21\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{7\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{a\,(a\,x-1)^7}{(a\,x+1)^7}}-\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

```
[In] int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - ((25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 - (5*c^3*((a*x - 1)/(a*x + 1))^(1
/2))/8 - (283*c^3*((a*x - 1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/7 + (283*c^3*((a*x - 1)/(a*x + 1))^(9/2))/24 - (25*c^3*((a*x
 - 1)/(a*x + 1))^(11/2))/6 + (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (
7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)
^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x
+ 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7) - (5*
c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

3.592 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 233

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5-\frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}$$

```
[Out] -1/4*a^3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(5/2
)*(1+1/a/x)^(5/2)*x^5-3/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-1/
8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/4*a^2*c^2*(1+1/a/x)^(5/2)*x^3
*(1-1/a/x)^(1/2)-3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{4}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{4}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a}-\frac{1}{8}\cdots$$

```
[In] Int[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (a*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
)*x^3)/4 - (a^3*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/4 + (a^4*c
^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/5 - (3*c^2*ArcTanh[Sqrt[1 -
 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(a^4c^2\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2 x^4\,dx$$

$$= -\left(\left(a^4c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \left(a^3c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 - \frac{1}{4}\left(3a^2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{1}{4}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{1}{8}\left(3c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{x^2\sqrt{1}}\right.$$

$$= -\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1}}\right.}{8a}$$

$$= -\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 - \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,\right.}{8a}$$

$$= -\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 - \frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{}\right.}{8a}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

```
[In] Integrate[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{40a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+45\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-40((ax-1)(ax+1))^{\frac{3}{2}}\right.}{120a\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/40*(8*a^4*x^4-10*a^3*x^3-16*a^2*x^2+25*a*x+8)*(a*x+1)/a*c^2*((a*x-1)/(a*x
+1))^(1/2)-3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2*((a*
x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = -\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(8\,a^5c^2x^5-2\,a^4c^2x^4-26\,a^3c^2x^3+9\,a^2c^2x^2+33\,ac^2}{40\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^2*x^5 - 2*a^4*c^2*x^4 - 26*a^3*c^2*x^3 + 9*a^2
*c^2*x^2 + 33*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=c^2\left(\int\left(-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\frac{1}{40}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^2\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^
2*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^
2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*
x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^
2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}$$
$$+\frac{1}{40}\sqrt{a^2x^2-1}\Big(\big(25\,c^2\operatorname{sgn}(ax+1)-2\,\big(8\,ac^2\operatorname{sgn}(ax+1)-\big(4\,a^3c^2x\operatorname{sgn}(ax+1)-5\,a^2c^2\operatorname{sgn}(ax+1)\big)x\big)$$

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/40*
sqrt(a^2*x^2 - 1)*((25*c^2*sgn(a*x + 1) - 2*(8*a*c^2*sgn(a*x + 1) - (4*a^3*
c^2*x*sgn(a*x + 1) - 5*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 8*c^2*sgn(a*x + 1)/a
)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{3\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}+\frac{32\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{5}+\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{2}-\frac{3\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}$$
$$-\frac{3\,c^2\,\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((3*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^2*((a*x - 1)/(a*x + 1))^(3/2)
)/2 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^2*((a*x - 1)/(a*x + 1))
^(7/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.593 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

```
[Out] -1/3*a^2*c*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-1/2*c*arctanh((1-1/a/x)^(1/2
)*(1+1/a/x)^(1/2))/a+1/2*a*c*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-1/2*c*x*(1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx=-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}+\frac{1}{2}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}$$

```
[In] Int[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (a*c*Sqrt[1 - 1/(a*x)]*(1
+ 1/(a*x))^(3/2)*x^2)/2 - (a^2*c*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x^
3)/3 - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^2c\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^4}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-(ac)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}{x^3}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3+\frac{1}{2}c\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3+\frac{c\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= -\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{c\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a^2}$$

$$= -\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(2+3ax-2a^2x^2)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

```
[In] Integrate[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{(2a^2x^2-3ax-2)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{6a}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$	108
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	119

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*a^2*x^2-3*a*x-2)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-1/2*ln(a^2*x/(
a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1
)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3-a^2cx^2-5\,acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (2*a^3*c*x^3 - a^2*c*x^2 - 5*a*c*x - 2*c)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(\int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sq
rt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) + 8*c*((a*x - 1)/(a*x + 1))^(3/2)
 - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^
2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}$$
$$-\frac{1}{6}\sqrt{a^2x^2-1}\left((2\,acx\operatorname{sgn}(ax+1)-3\,c\operatorname{sgn}(ax+1))x-\frac{2\,c\operatorname{sgn}(ax+1)}{a}\right)$$

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sqr
t(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 3*c*sgn(a*x + 1))*x - 2*c*sgn(a*x +
 1)/a)
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{\frac{8\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,\left(a\,x-1\right)^2}{\left(a\,x+1\right)^2}-\frac{a\,\left(a\,x-1\right)^3}{\left(a\,x+1\right)^3}}-\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - ((8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(1/2) + c*
((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1
)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c*atanh(((a*x - 1)/(a*x
+ 1))^(1/2)))/a
```

3.594 $\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2} \, dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

```
[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6318}

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2} \, dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
```

```
[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$	24
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$	24
trager	$-\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	26

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = -\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = \int -\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2cx^2 - c}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{c - a^2cx^2}\,dx = -\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2),x)
[Out] -((a*x - 1)/(a*x + 1))^(1/2)/(a*c)
```

3.595 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx = -\frac{2e^{-\coth^{-1}(ax)}}{3ac^2} + \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)}$$

```
[Out] -2/3/a/c^2*((a*x-1)/(a*x+1))^(1/2)+1/3*(2*a*x+1)/a/c^2*((a*x-1)/(a*x+1))^(1
/2)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx = \frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)} - \frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -2/(3*a*c^2*E^ArcCoth[a*x]) + (1 + 2*a*x)/(3*a*c^2*E^ArcCoth[a*x]*(1 - a^2*
x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)} + \frac{2\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}\\ &= -\frac{2e^{-\coth^{-1}(ax)}}{3ac^2} + \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+2ax+2a^2x^2)}{3(-1+ax)(c+acx)^2}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x + 2*a^2*x^2))/((-1 + a*x)*(c + a*
c*x)^2)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2+2ax-1)}{3(a^2x^2-1)a\,c^2}$	49
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2+2ax-1)}{3c^2(ax+1)(ax-1)a}$	52
trager	$-\frac{(2a^2x^2+2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)(ax+1)}$	54

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2+2*a*x-1)/(a^2*x^2-1)/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{(2\,a^2x^2+2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-ac^2)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/3*(2*a^2*x^2 + 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - a*c^2
)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x**2 + 1), x
)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{12}\,a\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-6\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}-\frac{3}{a^2c^2\sqrt{\frac{ax-1}{ax+1}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/12*a*((((a*x - 1)/(a*x + 1))^(3/2) - 6*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^
2) - 3/(a^2*c^2*sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(a^2cx^2-c)^2}\,dx$$

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^2, x)

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{(a\,x-1)^2}{(a\,x+1)^2}+3}{12\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^2,x)

[Out] -((6*(a*x - 1))/(a*x + 1) - (a*x - 1)^2/(a*x + 1)^2 + 3)/(12*a*c^2*((a*x - 1)/(a*x + 1))^(1/2))

3.596 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}+\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3(1-a^2x^2)}$$

```
[Out] -8/15/a/c^3*((a*x-1)/(a*x+1))^(1/2)+1/15*(4*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^
(1/2)/(-a^2*x^2+1)^2+4/15*(2*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2
+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{4(2ax+1)e^{-\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -8/(15*a*c^3*E^ArcCoth[a*x]) + (1 + 4*a*x)/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^
2*x^2)^2) + (4*(1 + 2*a*x))/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{5c} \\
&= \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3\left(1-a^2x^2\right)}+\frac{8\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2} \\
&= -\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}+\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(3-12ax-12a^2x^2+8a^3x^3+8a^4x^4\right)}{15(-1+ax)^2(c+acx)^3}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^4
*x^4))/((-1 + a*x)^2*(c + a*c*x)^3)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)}{15(a^2x^2-1)^2c^3a}$	65
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)}{15c^3(ax-1)^2a(ax+1)^2}$	68
trager	$-\frac{(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax-1)^2(ax+1)^2}$	70

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/(a^
2*x^2-1)^2/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\left(8\,a^4x^4+8\,a^3x^3-12\,a^2x^2-12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\,\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] -1/15*(8*a^4*x^4 + 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x**4 + 3*a*
*2*x**2 - 1), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{240}\,a\left(\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-20\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+90\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{5\left(\frac{12\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -1/240*a*((3*((a*x - 1)/(a*x + 1))^(5/2) - 20*((a*x - 1)/(a*x + 1))^(3/2) +
 90*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 5*(12*(a*x - 1)/(a*x + 1) - 1)/(
a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{\sqrt{\frac{ax-1}{ax+1}}}{(a^2cx^2-c)^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12\,a\,c^3}-\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}-\frac{\frac{4\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^3,x)
[Out] ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^3) - (3*((a*x - 1)/(a*x + 1))^(1/2))/(8
*a*c^3) - ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) - ((4*(a*x - 1))/(a*x + 1)
 - 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.597 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}+\frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3}+\frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2}+\frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)}$$

```
[Out] -16/35/a/c^4*((a*x-1)/(a*x+1))^(1/2)+1/35*(6*a*x+1)/a/c^4*((a*x-1)/(a*x+1))
^(1/2)/(-a^2*x^2+1)^3+2/35*(4*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^
2+1)^2+8/35*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{8(2ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)}+\frac{2(4ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^2}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -16/(35*a*c^4*E^ArcCoth[a*x]) + (1 + 6*a*x)/(35*a*c^4*E^ArcCoth[a*x]*(1 - a
^2*x^2)^3) + (2*(1 + 4*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (8
*(1 + 2*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{6\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{7c} \\
&= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{24\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{35c^2} \\
&= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)} + \frac{16\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^3} \\
&= -\frac{16e^{-\coth^{-1}(ax)}}{35ac^4} + \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5+30ax+30a^2x^2-40a^3x^3-40a^4x^4+16a^5x^5+16a^6x^6)}{35(-1+ax)^3(c+acx)^4}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]

[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40*
a^4*x^4 + 16*a^5*x^5 + 16*a^6*x^6))/((-1 + a*x)^3*(c + a*c*x)^4)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5)}{35(a^2x^2-1)^3c^4a}$	81
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5)}{35c^4(ax+1)^3(ax-1)^3a}$	84
trager	$-\frac{(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^4(ax-1)^3(ax+1)^3}$	86

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3+
30*a^2*x^2+30*a*x-5)/(a^2*x^2-1)^3/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x^2+30\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2*x^2 + 30*
a*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x**6 + 6*a**
4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{1}{2240}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+175\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-700\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{7\left(\frac{10\,(ax-1)}{ax+1}-\frac{75\,(ax-1)^2}{(ax+1)^2}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
[Out] 1/2240*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 42*((a*x - 1)/(a*x + 1))^(5/2) +
 175*((a*x - 1)/(a*x + 1))^(3/2) - 700*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4)
 + 7*(10*(a*x - 1)/(a*x + 1) - 75*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^4*((a
*x - 1)/(a*x + 1))^(5/2)))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(a^2cx^2-c)^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{64\,a\,c^4}-\frac{5\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{160\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{448\,a\,c^4}-\frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{a\,x+1}+\frac{1}{5}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^4,x)
[Out] (5*((a*x - 1)/(a*x + 1))^(3/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(1/2)
)/(16*a*c^4) - (3*((a*x - 1)/(a*x + 1))^(5/2))/(160*a*c^4) + ((a*x - 1)/(a*
x + 1))^(7/2)/(448*a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1)^2 - (2*(a*x - 1))/(
a*x + 1) + 1/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2))
```

3.598 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}$$

```
[Out] 4/3*c^4*(-a*x+1)^6/a-12/7*c^4*(-a*x+1)^7/a+3/4*c^4*(-a*x+1)^8/a-1/9*c^4*(-a
*x+1)^9/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=-\frac{c^4(1-ax)^9}{9a}+\frac{3c^4(1-ax)^8}{4a}-\frac{12c^4(1-ax)^7}{7a}+\frac{4c^4(1-ax)^6}{3a}$$

```
[In] Int[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^4*(1 - a*x)^6)/(3*a) - (12*c^4*(1 - a*x)^7)/(7*a) + (3*c^4*(1 - a*x)^8
)/(4*a) - (c^4*(1 - a*x)^9)/(9*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^4\,dx\\ &= -\left(c^4\int(1-ax)^5(1+ax)^3\,dx\right)\\ &= -\left(c^4\int\left(8(1-ax)^5-12(1-ax)^6+6(1-ax)^7-(1-ax)^8\right)\,dx\right)\\ &= \frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(-1+ax)^6\left(65+138ax+105a^2x^2+28a^3x^3\right)}{252a}$$

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
[Out] (c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{c^4x(28a^8x^8-63a^7x^7-72a^6x^6+252a^5x^5-378a^3x^3+168a^2x^2+252ax-252)}{252}$
default	$c^4\left(\frac{1}{9}a^8x^9-\frac{1}{4}a^7x^8-\frac{2}{7}a^6x^7+a^5x^6-\frac{3}{2}a^3x^4+\frac{2}{3}a^2x^3+a\,x^2-x\right)$
norman	$a\,c^4x^2+a^5c^4x^6-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
risch	$a\,c^4x^2+a^5c^4x^6-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
parallelrisch	$a\,c^4x^2+a^5c^4x^6-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
meijerg	$\frac{c^4\left(\frac{xa\left(280a^8x^8-315a^7x^7+360a^6x^6-420a^5x^5+504a^4x^4-630a^3x^3+840a^2x^2-1260ax+2520\right)}{2520}-\ln(ax+1)\right)}{a}-\frac{c^4\left(-\frac{ax\left(-315a^7x^7+36\right.}{}\right.}{}$

[In] int((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/252*c^4*x*(28*a^8*x^8-63*a^7*x^7-72*a^6*x^6+252*a^5*x^5-378*a^3*x^3+168*a^2*x^2+252*a*x-252)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}-\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}+a^5c^4x^6-\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}+ac^4x^2-c^4x$$

[In] integrate((-a**2*c*x**2+c)**4*(a*x-1)/(a*x+1),x)

[Out] a**8*c**4*x**9/9 - a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 + a**5*c**4*x**6 - 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 + a*c**4*x**2 - c**4*x

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}-\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}+a^5\,c^4\,x^6-\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}+a\,c^4\,x^2-c^4\,x$$

```
[In] int(((c - a^2*c*x^2)^4*(a*x - 1))/(a*x + 1),x)
[Out] a*c^4*x^2 - c^4*x + (2*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + a^5*c^4*x^6 - (
2*a^6*c^4*x^7)/7 - (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

3.599 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}$$

```
[Out] 4/5*c^3*(-a*x+1)^5/a-2/3*c^3*(-a*x+1)^6/a+1/7*c^3*(-a*x+1)^7/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{c^3(1-ax)^7}{7a}-\frac{2c^3(1-ax)^6}{3a}+\frac{4c^3(1-ax)^5}{5a}$$

```
[In] Int[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^3*(1 - a*x)^5)/(5*a) - (2*c^3*(1 - a*x)^6)/(3*a) + (c^3*(1 - a*x)^7)/(
7*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^3\,dx\\ &= -\left(c^3\int (1-ax)^4(1+ax)^2\,dx\right)\\ &= -\left(c^3\int \left(4(1-ax)^4-4(1-ax)^5+(1-ax)^6\right)\,dx\right)\\ &= \frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(-1+ax)^5\left(29+40ax+15a^2x^2\right)}{105a}$$

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
[Out] -1/105*(c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/a
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{c^{3} x \left(15 a^{6} x^{6}-35 a^{5} x^{5}-21 a^{4} x^{4}+105 a^{3} x^{3}-35 a^{2} x^{2}-105 a x +105\right)}{105}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}+\frac{1}{3} a^{5} x^{6}+\frac{1}{5} a^{4} x^{5}-a^{3} x^{4}+\frac{1}{3} a^{2} x^{3}+a \,x^{2}-x \right)$
norman	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{5} a^{4} c^{3} x^{5}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
risch	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{5} a^{4} c^{3} x^{5}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
parallelrisch	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{5} a^{4} c^{3} x^{5}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
meijerg	$-\frac{c^{3} \left(\frac{a x \left(120 a^{6} x^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}-210 a^{3} x^{3}+280 a^{2} x^{2}-420 a x +840\right)}{840}-\ln \left(a x +1\right)\right)}{a}+\frac{c^{3} \left(-\frac{a x \left(-70 a^{5} x^{5}+84 a^{4} x^{4}-105 a^{3} x^{3}+140\right.}{420}\right.}{a}$

[In] int((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] -1/105*c^3*x*(15*a^6*x^6-35*a^5*x^5-21*a^4*x^4+105*a^3*x^3-35*a^2*x^2-105*a*x+105)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + a*c^3*x^2 - c^3*x

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}+\frac{a^5c^3x^6}{3}+\frac{a^4c^3x^5}{5}-a^3c^3x^4+\frac{a^2c^3x^3}{3}+ac^3x^2-c^3x$$

[In] integrate((-a**2*c*x**2+c)**3*(a*x-1)/(a*x+1),x)

[Out] -a**6*c**3*x**7/7 + a**5*c**3*x**6/3 + a**4*c**3*x**5/5 - a**3*c**3*x**4 + a**2*c**3*x**3/3 + a*c**3*x**2 - c**3*x

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5 - a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5 - a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{a^6\,c^3\,x^7}{7}+\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5} - a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}+a\,c^3\,x^2-c^3\,x$$

```
[In] int(((c - a^2*c*x^2)^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] a*c^3*x^2 - c^3*x + (a^2*c^3*x^3)/3 - a^3*c^3*x^4 + (a^4*c^3*x^5)/5 + (a^5*
c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

3.600 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^2 \, dx = \frac{c^2(1-ax)^4}{2a} - \frac{c^2(1-ax)^5}{5a}$$

```
[Out] 1/2*c^2*(-a*x+1)^4/a-1/5*c^2*(-a*x+1)^5/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 45}

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^2 \, dx = \frac{c^2(1-ax)^4}{2a} - \frac{c^2(1-ax)^5}{5a}$$

```
[In] Int[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(1 - a*x)^4)/(2*a) - (c^2*(1 - a*x)^5)/(5*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
```

```
c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^2\,dx\\ &= -\left(c^2\int(1-ax)^3(1+ax)\,dx\right)\\ &= -\left(c^2\int\left(2(1-ax)^3-(1-ax)^4\right)\,dx\right)\\ &= \frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{10}c^2x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
[Out] (c^2*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{c^{2} x \left(2 a^{4} x^{4}-5 a^{3} x^{3}+10 a x -10\right)}{10}$
default	$c^{2} \left(\frac{1}{5} a^{4} x^{5}-\frac{1}{2} a^{3} x^{4}+a \,x^{2}-x \right)$
norman	$a \,c^{2} x^{2}-c^{2} x -\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
risch	$a \,c^{2} x^{2}-c^{2} x -\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
parallelrisch	$a \,c^{2} x^{2}-c^{2} x -\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
meijerg	$\frac{c^{2} \left(\frac{a x \left(12 a^{4} x^{4}-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}-\ln \left(a x +1\right)\right)}{a}-\frac{c^{2} \left(-\frac{a x \left(-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}+\ln \left(a x +1\right)\right)}{a}-\frac{2 c^{2} \left(\frac{a x \left(4 a\right.}{}\right.}{}$

[In] int((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/10*c^2*x*(2*a^4*x^4-5*a^3*x^3+10*a*x-10)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}-\frac{a^3c^2x^4}{2}+ac^2x^2-c^2x$$

[In] integrate((-a**2*c*x**2+c)**2*(a*x-1)/(a*x+1),x)

[Out] a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 - c**2*x

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}-\frac{a^3\,c^2\,x^4}{2}+a\,c^2\,x^2-c^2\,x$$

```
[In] int(((c - a^2*c*x^2)^2*(a*x - 1))/(a*x + 1),x)
[Out] a*c^2*x^2 - c^2*x - (a^3*c^2*x^4)/2 + (a^4*c^2*x^5)/5
```

3.601 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 16

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c(1-ax)^3}{3a}$$

```
[Out] 1/3*c*(-a*x+1)^3/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6302, 6275, 32}

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c(1-ax)^3}{3a}$$

```
[In] Int[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*(1 - a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)\,dx\\ &= -\left(c\int (1-ax)^2\,dx\right)\\ &= \frac{c(1-ax)^3}{3a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(x-ax^2+\frac{a^2x^3}{3}\right)$$

```
[In] Integrate[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
[Out] -(c*(x - a*x^2 + (a^2*x^3)/3))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{c(ax-1)^3}{3a}$	14
gosper	$-\frac{cx\left(a^2x^2-3ax+3\right)}{3}$	18
norman	$ac\,x^2-cx-\frac{1}{3}a^2c\,x^3$	21
parallelrisch	$ac\,x^2-cx-\frac{1}{3}a^2c\,x^3$	21
risch	$-\frac{a^2c\,x^3}{3}+ac\,x^2-cx+\frac{c}{3a}$	27
meijerg	$-\frac{c\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}+\frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}$	86

```
[In] int((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -1/3*c*(a*x-1)^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}+acx^2-cx$$

```
[In] integrate((-a**2*c*x**2+c)*(a*x-1)/(a*x+1),x)
[Out] -a**2*c*x**3/3 + a*c*x**2 - c*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c\,x\,\left(a^2\,x^2-3\,a\,x+3\right)}{3}$$

```
[In] int(((c - a^2*c*x^2)*(a*x - 1))/(a*x + 1),x)
[Out] -(c*x*(a^2*x^2 - 3*a*x + 3))/3
```

3.602 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{ac(1+ax)}$$

```
[Out] 1/a/c/(a*x+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 32}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{ac(ax+1)}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] 1/(a*c*(1 + a*x))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{c-a^2cx^2}\,dx \\ &= -\frac{\int \frac{1}{(1+ax)^2}\,dx}{c} \\ &= \frac{1}{ac(1+ax)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-2\coth^{-1}(ax)}}{2ac}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
[Out] -1/2*1/(a*c*E^(2*ArcCoth[a*x]))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
norman	$-\frac{x}{c(ax+1)}$	14
parallelrisch	$-\frac{x}{c(ax+1)}$	14
gosper	$\frac{1}{ac(ax+1)}$	15
default	$\frac{1}{ac(ax+1)}$	15
risch	$\frac{1}{ac(ax+1)}$	15

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -x/c/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx+ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/(a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx+ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c),x)
[Out] 1/(a**2*c*x + a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx+ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/(a^2*c*x + a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{(ax+1)ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/((a*x + 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a\,\left(c+a\,c\,x\right)}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)*(a*x + 1)),x)
[Out] 1/(a*(c + a*c*x))
```

3.603 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^2}$$

```
[Out] 1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=-\frac{\operatorname{arctanh}(ax)}{4ac^2}+\frac{1}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 1/(4*a*c^2*(1 + a*x)^2) + 1/(4*a*c^2*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^2}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)(1+ax)^3}\,dx}{c^2} \\
&= -\frac{\int \left(\frac{1}{2(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^2} \\
&= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2} \\
&= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\text{arctanh}(ax)}{4ac^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2+ax-(1+ax)^2\text{arctanh}(ax)}{4a(c+acx)^2}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
[Out] (2 + a*x - (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{2a}}{(ax+1)^2 c^2}+\frac{\ln(-ax+1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	51
default	$\frac{\frac{1}{4a(ax+1)^2}+\frac{1}{4a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{\ln(ax-1)}{8a}}{c^2}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^2}{2c}+\frac{a^2x^3}{4c}}{c(ax+1)^2(ax-1)}+\frac{\ln(ax-1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	77
parallelrisch	$\frac{a^2\ln(ax-1)x^2-a^2\ln(ax+1)x^2-4a^2x^2+2a\ln(ax-1)x-2a\ln(ax+1)x-6ax+\ln(ax-1)-\ln(ax+1)}{8c^2(ax+1)^2 a}$	90

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] (1/4*x+1/2/a)/(a*x+1)^2/c^2+1/8*ln(-a*x+1)/a/c^2-1/8*ln(a*x+1)/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2\,ax-(a^2x^2+2\,ax+1)\log(ax+1)+(a^2x^2+2\,ax+1)\log(ax-1)+4}{8\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**2,x)
[Out] (a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax+2}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)} - \frac{\log(ax+1)}{8\,ac^2} + \frac{\log(ax-1)}{8\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\log(|ax+1|)}{8\,ac^2} + \frac{\log(|ax-1|)}{8\,ac^2} + \frac{ax+2}{4\,(ax+1)^2ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x +
 2)/((a*x + 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\frac{x}{4}+\frac{1}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^2}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^2*(a*x + 1)),x)
```

```
[Out] (x/4 + 1/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

3.604 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3} dx = -\frac{1}{16ac^3(1-ax)} + \frac{1}{12ac^3(1+ax)^3} + \frac{1}{8ac^3(1+ax)^2} + \frac{3}{16ac^3(1+ax)} - \frac{\operatorname{arctanh}(ax)}{4ac^3}$$

```
[Out] -1/16/a/c^3/(-a*x+1)+1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+1)^2+3/16/a/c^3/(a
*x+1)-1/4*arctanh(a*x)/a/c^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3} dx = -\frac{\operatorname{arctanh}(ax)}{4ac^3} - \frac{1}{16ac^3(1-ax)} + \frac{3}{16ac^3(ax+1)} + \frac{1}{8ac^3(ax+1)^2} + \frac{1}{12ac^3(ax+1)^3}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/16*1/(a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) + 1/(8*a*c^3*(1 + a*x)
^2) + 3/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2(1+ax)^4}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{16(-1+ax)^2}+\frac{1}{4(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^3}\\
&= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}\\
&= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\text{arctanh}(ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{-4+ax+6a^2x^2+3a^3x^3-3(-1+ax)(1+ax)^3\text{arctanh}(ax)}{12a(-1+ax)(c+acx)^3}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
[Out] (-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])/
(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{12a(ax+1)^3}+\frac{1}{8a(ax+1)^2}+\frac{3}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{16a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^3}$
risch	$\frac{\frac{a^2x^3}{4}+\frac{a\,x^2}{2}+\frac{x}{12}-\frac{1}{3a}}{(ax+1)^2(a^2x^2-1)c^3}+\frac{\ln(-ax+1)}{8a\,c^3}-\frac{\ln(ax+1)}{8a\,c^3}$
norman	$\frac{-\frac{3x}{4c}+\frac{a\,x^2}{4c}+\frac{11a^2x^3}{12c}-\frac{a^3x^4}{12c}-\frac{a^4x^5}{3c}}{(ax+1)^3c^2(ax-1)^2}+\frac{\ln(ax-1)}{8a\,c^3}-\frac{\ln(ax+1)}{8a\,c^3}$
parallelrisch	$\frac{3\ln(ax-1)x^4a^4-3\ln(ax+1)x^4a^4-8a^4x^4+6a^3\ln(ax-1)x^3-6a^3\ln(ax+1)x^3-10a^3x^3+12a^2x^2-6a\ln(ax-1)x+6a\ln(ax+1)x}{24c^3(ax+1)^2(a^2x^2-1)a}$

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(1/12/a/(a*x+1)^3+1/8/a/(a*x+1)^2+3/16/a/(a*x+1)-1/8*ln(a*x+1)/a+1/16
/a/(a*x-1)+1/8/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$
$$= \frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log(ax+1)+3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log(ax}{24\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] 1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5*c
^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{-3a^3x^3-6a^2x^2-ax+4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(-3*a**3*x**3 - 6*a**2*x**2 - a*x + 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*x
**3 - 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c
**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)} - \frac{\log(ax+1)}{8\,ac^3} + \frac{\log(ax-1)}{8\,ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\log(|ax+1|)}{8\,ac^3} + \frac{\log(|ax-1|)}{8\,ac^3} + \frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(ax+1)^3(ax-1)ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 + 6*a^2*x^2 + a*x - 4)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\frac{x}{12}+\frac{a\,x^2}{2}-\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^3*(a*x + 1)),x)
[Out] - (x/12 + (a*x^2)/2 - 1/(3*a) + (a^2*x^3)/4)/(c^3 - 2*a^3*c^3*x^3 - a^4*c^3
*x^4 + 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

3.605 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=-\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1+ax)}-\frac{15\operatorname{arctanh}(ax)}{64ac^4}$$

```
[Out] -1/64/a/c^4/(-a*x+1)^2-5/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4+1/16/a/c^4/
(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=-\frac{15\operatorname{arctanh}(ax)}{64ac^4}-\frac{5}{64ac^4(1-ax)}+\frac{5}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{3}{32ac^4(ax+1)^2}+\frac{1}{16ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] -1/64*1/(a*c^4*(1 - a*x)^2) - 5/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1 + a*x
)^4) + 1/(16*a*c^4*(1 + a*x)^3) + 3/(32*a*c^4*(1 + a*x)^2) + 5/(32*a*c^4*(1
 + a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^4}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)^5}\,dx}{c^4} \\
&= -\frac{\int \left(-\frac{1}{32(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{8(1+ax)^5}+\frac{3}{16(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{5}{32(1+ax)^2}-\frac{15}{64(-1+a^2x^2)}\right)dx}{c^4} \\
&= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3} \\
&\quad +\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1+ax)}+\frac{15\int \frac{1}{-1+a^2x^2}\,dx}{64c^4} \\
&= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3} \\
&\quad +\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1+ax)}-\frac{15\text{arctanh}(ax)}{64ac^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$=\frac{16-17ax-50a^2x^2-10a^3x^3+30a^4x^4+15a^5x^5-15(-1+ax)^2(1+ax)^4\text{arctanh}(ax)}{64a(-1+ax)^2(c+acx)^4}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (16 - 17*a*x - 50*a^2*x^2 - 10*a^3*x^3 + 30*a^4*x^4 + 15*a^5*x^5 - 15*(-1 +
 a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\frac{15a^4x^5}{64}+\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}-\frac{25a\,x^2}{32}-\frac{17x}{64}+\frac{1}{4a}}{(ax+1)^2(a^2x^2-1)^2c^4}-\frac{15\ln(ax+1)}{128a\,c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}$
default	$\frac{\frac{1}{32a(ax+1)^4}+\frac{1}{16a(ax+1)^3}+\frac{3}{32a(ax+1)^2}+\frac{5}{32a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{64(ax-1)^2a}+\frac{5}{64a(ax-1)}+\frac{15\ln(ax-1)}{128a}}{c^4}$
norman	$\frac{\frac{49x}{64c}-\frac{15a\,x^2}{64c}-\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}+\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}-\frac{a^6x^7}{4c}}{(ax+1)^4c^3(ax-1)^3}+\frac{15\ln(ax-1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
parallelrisch	$\frac{-30a\ln(ax+1)x+15a^2\ln(ax+1)x^2-34a^5x^5+108a^3x^3-30\ln(ax+1)x^5a^5-15\ln(ax+1)x^6a^6+15\ln(ax+1)x^4a^4+15\ln(ax-1)x^6}{}$

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/64*a^4*x^5+15/32*a^3*x^4-5/32*a^2*x^3-25/32*a*x^2-17/64*x+1/4/a)/(a*x+1
)^2/(a^2*x^2-1)^2/c^4-15/128*ln(a*x+1)/a/c^4+15/128*ln(-a*x+1)/a/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$=\frac{30\,a^5x^5+60\,a^4x^4-20\,a^3x^3-100\,a^2x^2-34\,ax-15\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\log}{128\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a\right.}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3
- a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4} + \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/(
64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)} - \frac{15\,\log(ax+1)}{128\,ac^4} + \frac{15\,\log(ax-1)}{128\,ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^7
*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4} + \frac{15\,\log\left(|ax-1|\right)}{128\,ac^4} + \frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(ax+1)^4(ax-1)^2ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/((a*x +
1)^4*(a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{\frac{17\,x}{64}+\frac{25\,a\,x^2}{32}-\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}-\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4} - \frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^4*(a*x + 1)),x)
```

```
[Out] - ((17*x)/64 + (25*a*x^2)/32 - 1/(4*a) + (5*a^2*x^3)/32 - (15*a^3*x^4)/32 -
 (15*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 + 2*a^5*
c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

3.606 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 393

$$
\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= \frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x \\
&+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 \\
&+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 \\
&+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right.
\end{aligned}
$$

```
[Out] 11/48*a^4*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5-11/48*a^5*c^4*(1-1/a/x)^(
5/2)*(1+1/a/x)^(7/2)*x^6+11/56*a^6*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-
11/72*a^7*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(7/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(11
/2)*(1+1/a/x)^(7/2)*x^9+55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))
/a+55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+11/192*a^2*c^4*(1+1/a/x
)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/
2)+55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{7/2}$$
$$-\frac{11}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{56}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{48}a^5c^4x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right.$$

```
[In] Int[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (55*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^
4)/64 + (11*a^4*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/48 - (11*a
^5*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/48 + (11*a^6*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2)*x^7)/56 - (11*a^7*c^4*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(7/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^
(7/2)*x^9)/9 + (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a
)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(a^8c^4\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4 x^8\,dx$$

$$= -\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9+\frac{1}{9}\left(11a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^9}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9-\frac{1}{8}\left(11a^6c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7 - \frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9+\frac{1}{8}\left(11a^5c^4\right)\text{Subst}\left(\int\right.$$

$$= -\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 + \frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(\right.$$

$$= \frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 - \frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}$$

$$= -\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5$$
$$-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\Bigg($$

$$= \frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\Bigg($$

$$= \frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\Bigg($$

$$= \frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\Bigg($$

$$= \frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\Bigg($$

$$= \frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 - \frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\Bigg($$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-3712-4599ax+10240a^2x^2-3066a^3x^3-8448a^4x^4+7224a^5x^5+1024a^6x^6-3024a\right.}{8064a}$$

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a^
3*x^3 - 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a^8
*x^8) + 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(896a^8x^8-3024a^7x^7+1024a^6x^6+7224a^5x^5-8448a^4x^4-3066a^3x^3+10240a^2x^2-4599ax-3712)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{8064a}+\frac{55\ln\left(\frac{a^2x}{\sqrt{a^2}}+\right.}{}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(896(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-3024(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+1920(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+4200(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3\right.}{8}$

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8064*(896*a^8*x^8-3024*a^7*x^7+1024*a^6*x^6+7224*a^5*x^5-8448*a^4*x^4-306
6*a^3*x^3+10240*a^2*x^2-4599*a*x-3712)*(a*x+1)/a*c^4*((a*x-1)/(a*x+1))^(1/2
)+55/128*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4*((a*x-1)/(
a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.43

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(896\,a^9c^4x^9-2128\,a^8c^4x^8-2000\,a^7c^4x^7+8248\,a}{8064\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (896*a^9*c^4*x^9 - 2128*a^8*c^4*x^8 - 2000*a^7*c^4
*x^7 + 8248*a^6*c^4*x^6 - 1224*a^5*c^4*x^5 - 11514*a^4*c^4*x^4 + 7174*a^3*c
^4*x^3 + 5641*a^2*c^4*x^2 - 8311*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x +
1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\frac{4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\left(-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\left(-\frac{6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\frac{4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\left(-\frac{4a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\left(-\frac{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{a^9x^9\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(4*a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-6*a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**5*x**5*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(4*a**6*x**6*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**7*x**7*sqrt(a*x/(a*x + 1
```

```
) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**8*x**8*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1), x) + Integral(a**9*x**9*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+\right.}{\frac{9\,(ax-1)a^2}{ax+1}-}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2) -
 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))^
(13/2) + 334602*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 360448*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 255222*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
- 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*sq
rt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a
*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^
4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(
a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^
2/(a*x + 1)^9 - a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = -\frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|} - \frac{1}{8064}\sqrt{a^2x^2-1}\left(\frac{3712\,c^4\operatorname{sgn}(ax+1)}{a}+\left(4599\,c^4\operatorname{sgn}(ax+1)-2\left(5120\,ac^4\operatorname{sgn}(ax+1)-\left(1533\,a^2c^4\operatorname{sgn}\right.\right.\right.\right.$$

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1
/8064*sqrt(a^2*x^2 - 1)*(3712*c^4*sgn(a*x + 1)/a + (4599*c^4*sgn(a*x + 1) -
 2*(5120*a*c^4*sgn(a*x + 1) - (1533*a^2*c^4*sgn(a*x + 1) + 4*(1056*a^3*c^4*
sgn(a*x + 1) - (903*a^4*c^4*sgn(a*x + 1) + 2*(64*a^5*c^4*sgn(a*x + 1) + 7*(
8*a^7*c^4*x*sgn(a*x + 1) - 27*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{\frac{715\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}-\frac{55\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{913\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{14179\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224}-\frac{5632\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{63}+\frac{18589\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{224}+\frac{913\,c^4 \cdots}{}}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{36\,a\,(a\,x \cdots}{(a\,x+\cdots}}$$
$$+\frac{55\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

```
[In] int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((715*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (55*c^4*((a*x - 1)/(a*x + 1))^(
1/2))/64 - (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (14179*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (18589
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 + (913*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/32 - (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (55*c^4*((a*x - 1)/(a
*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^
4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*
a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9
)/(a*x + 1)^9) + (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.607 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4-\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)$$

```
[Out] 3/8*a^3*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4-3/10*a^4*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(5/2)*x^5+3/14*a^5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^6-1/7*
a^6*c^3*(1-1/a/x)^(9/2)*(1+1/a/x)^(5/2)*x^7+9/16*c^3*arctanh((1-1/a/x)^(1/2
)*(1+1/a/x)^(1/2))/a+3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3/8*a^2
*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+9/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{14}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{10}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{8}a^3c^3x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right.$$

```
[In] Int[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (3*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/8 + (3*a^3*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/8 -
 (3*a^4*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/10 + (3*a^5*c^3*(1
 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(9/2
)*(1 + 1/(a*x))^(5/2)*x^7)/7 + (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/(16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^6c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 - \frac{1}{7}\left(9a^5c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^7}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 + \frac{1}{2}\left(3a^4c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 - \frac{1}{2}\left(3a^3c^3\right)\text{Subst}\left(\int \right.$$

$$= \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}$$

$$= -\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}$$

$$= \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}$$

$$=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$

$$+\frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4-\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1\right.$$

$$=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$

$$+\frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4-\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1\right.$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx =$$

$$-\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(368+245ax-656a^2x^2+350a^3x^3+208a^4x^4-280a^5x^5+80a^6x^6)-315\log\left(\left(1+\sqrt{\right.\right.\right.}{560a}$$

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 + 245*a*x - 656*a^2*x^2 + 350*a
^3*x^3 + 208*a^4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 315*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{560a}+\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(a}}{16\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(80(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-280(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+288(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+70(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+192(a\right.}{560a(ax-1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/560*(80*a^6*x^6-280*a^5*x^5+208*a^4*x^4+350*a^3*x^3-656*a^2*x^2+245*a*x+
368)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/2)+9/16*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)*c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2
)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(80\,a^7c^3x^7-200\,a^6c^3x^6-72\,a^5c^3x^5+558\,a^4c^3x^4-}{560\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (80*a^7*c^3*x^7 - 200*a^6*c^3*x^6 - 72*a^5*c^3*x^5 +
558*a^4*c^3*x^4 - 306*a^3*c^3*x^3 - 411*a^2*c^3*x^2 + 613*a*c^3*x + 368*c^3
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
-a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*x**
```

```
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(3*a**3*x**3*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(3*a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**5*x**5*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**6*x**6*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x**7*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$
$$=\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-8393\,c\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(c}{(c}}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 210
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) - 8393*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 5943*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1)
))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/
(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
 a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{16\,|a|}$$
$$-\frac{1}{560}\sqrt{a^2x^2-1}\left(\frac{368\,c^3\mathrm{sgn}(ax+1)}{a}+\left(245\,c^3\mathrm{sgn}(ax+1)-2\left(328\,ac^3\mathrm{sgn}(ax+1)-\left(175\,a^2c^3\mathrm{sgn}(ax+1\right.\right.\right.\right.$$

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/5
60*sqrt(a^2*x^2 - 1)*(368*c^3*sgn(a*x + 1)/a + (245*c^3*sgn(a*x + 1) - 2*(3
28*a*c^3*sgn(a*x + 1) - (175*a^2*c^3*sgn(a*x + 1) + 4*(26*a^3*c^3*sgn(a*x +
 1) + 5*(2*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$
$$-\frac{\frac{9\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{849\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}-\frac{1152\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{35}+\frac{1199\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{40}+\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{2}-9\,c^3\,\left(\cdots\right.}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}$$

```
[In] int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((9*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/8 - (15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 + (849*c^3*((a*x -
1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(a*x + 1))^(7/2))/35 + (1199
*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 + (15*c^3*((a*x - 1)/(a*x + 1))^(11/2)
)/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x + 1
) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (35*a
*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x - 1)
^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

3.608 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 233

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x$$
$$-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3$$
$$-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{7c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}$$

```
[Out] 7/12*a^2*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-7/20*a^3*c^2*(1-1/a/x)^(5/
2)*(1+1/a/x)^(3/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^5+7/8*
c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/8*a*c^2*(1+1/a/x)^(3/2)*x^
2*(1-1/a/x)^(1/2)+7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}$$
$$-\frac{7}{20}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{12}a^2c^2x^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right.}{8a}$$

```
[In] Int[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(3/2)*x^2)/8 + (7*a^2*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))
^(3/2)*x^3)/12 - (7*a^3*c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^4)/20
 + (a^4*c^2*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^5)/5 + (7*c^2*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(a^4c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$= -\left(\left(a^4c^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{1}{5}\left(7a^3c^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5-\frac{1}{4}\left(7a^2c^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3 - \frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{1}{4}\left(7ac^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{1\ldots}}{\ldots}\right.$$

$$= -\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3 - \frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5-\frac{1}{8}\left(7c^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{1\ldots}}{x^2\sqrt{\ldots}}\right.$$

$$= \frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3 - \frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5-\frac{\left(7c^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{\ldots}}\right.}{8a}$$

$$= \frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3 - \frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{\left(7c^2\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx\ldots\right.}{8\ldots}$$

$$= \frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x - \frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3 - \frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4 + \frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5 + \frac{7c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\right.}{8a}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4)+105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{120a}+\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-90(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-105\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+120((ax-1)(ax+\right.}{120a(ax-1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(24*a^4*x^4-90*a^3*x^3+112*a^2*x^2-15*a*x-136)*(a*x+1)/a*c^2*((a*x-1)
/(a*x+1))^(1/2)+7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2
*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^2x^5-66\,a^4c^2x^4+22\,a^3c^2x^3+97\,a^2c^2x^2-151\right.}{120\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^2*x^5 - 66*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 9
7*a^2*c^2*x^2 - 151*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(2*a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**4*x**4*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x**5*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{120}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^2\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)}{(ax+1)}}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) + 79
0*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 4
90*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*
(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a
^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^
5 - a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{8\,|a|}$$
$$-\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(15\,c^2\mathrm{sgn}(ax+1)-2\left(56\,ac^2\mathrm{sgn}(ax+1)+3\left(4\,a^3c^2x\mathrm{sgn}(ax+1)-15\,a^2c^2\mathrm{sgn}(ax+\right.\right.\right.\right.$$

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/12
0*sqrt(a^2*x^2 - 1)*((15*c^2*sgn(a*x + 1) - 2*(56*a*c^2*sgn(a*x + 1) + 3*(4
*a^3*c^2*x*sgn(a*x + 1) - 15*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 136*c^2*sgn(a*
x + 1)/a)
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{49\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}$$
$$+\frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

```
[In] int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((49*c^2*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(1/2
))/4 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^2*((a*x - 1)/(a*x +
 1))^(7/2))/6 + (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*c^2*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.609 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 + \frac{5c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

```
[Out] 5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5/6*a*c*(1-1/a/x)^(3/2)*x^
2*(1+1/a/x)^(1/2)-1/3*a^2*c*(1-1/a/x)^(5/2)*x^3*(1+1/a/x)^(1/2)-5/2*c*x*(1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1} + \frac{5c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a} + \frac{5}{6}acx^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}$$

```
[In] Int[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (5*a*c*(1 - 1/(a*x))^(3/2)
*Sqrt[1 + 1/(a*x)]*x^2)/6 - (a^2*c*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^
3)/3 + (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^2c\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 - \frac{1}{3}(5ac)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 + \frac{1}{2}(5c)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 - \frac{(5c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 + \frac{(5c)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a^2}$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 + \frac{5c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-22+9ax-2a^2x^2)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

```
[In] Integrate[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-22 + 9*a*x - 2*a^2*x^2) + 15*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(2a^2x^2-9ax+22)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+24a\ln\left(\frac{a^2\cdots\right)\right)}{6(ax-1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$

[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/6*(2*a^2*x^2-9*a*x+22)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)+5/2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3cx^3-7\,a^2cx^2+13\,acx+22\,c)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (2*a^3*c*x^3 - 7*a^2*c*x^2 + 13*a*c*x + 22*c)*sqrt((a*x - 1)/(a*x + 1)))/a

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**2*x**2*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{1}{6}\,a\left(\frac{2\left(33\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(33*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 15*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) + 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|} - \frac{1}{6}\,\sqrt{a^2x^2-1}\left((2\,acx\operatorname{sgn}(ax+1)-9\,c\operatorname{sgn}(ax+1))x+\frac{22\,c\operatorname{sgn}(ax+1)}{a}\right)$$

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sq
rt(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 9*c*sgn(a*x + 1))*x + 22*c*sgn(a*x
 + 1)/a)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}$$

```
[In] int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (5*c*((a*x - 1)/(a*x + 1))^(1/
2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3 + 11*c*((a*x - 1)/(a*x + 1))^(5/2
))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x
 - 1)^3)/(a*x + 1)^3)
```

3.610 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

```
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6318}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
gosper	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$	24
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$	24
trager	$-\frac{(ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3ac(ax+1)}$	38

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2cx+ac)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx}{c}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x
- 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2
*x**2 - a*x - 1), x))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3\,ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/3*((a*x - 1)/(a*x + 1))^(3/2)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^3ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x + 1)^
3*a*c)
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2),x)
[Out] -((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c)
```

3.611 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2(1-a^2x^2)}$$

```
[Out] 2/15/a/c^2*((a*x-1)/(a*x+1))^(3/2)+1/5*(-2*a*x-3)/a/c^2*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 2/(15*a*c^2*E^(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*c^2*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2(1-a^2x^2)} - \frac{2\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c} \\ &= \frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7+6ax+2a^2x^2)}{15c^2(1+ax)^3}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(7 + 6*a*x + 2*a^2*x^2))/(15*c^2*(1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	size
trager	$\frac{(2a^2x^2+6ax+7)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^2(ax+1)^2}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2+6ax+7)}{15(a^2x^2-1)a\,c^2}$	49
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2+6ax+7)}{15(ax-1)c^2a(ax+1)}$	52

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/15/a/c^2*(2*a^2*x^2+6*a*x+7)/(a*x+1)^2*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\left(2\,a^2x^2+6\,ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\,\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/15*(2*a^2*x^2 + 6*a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 + 2*a^2
*c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**
3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x)
)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\sqrt{\frac{ax-1}{ax+1}}}{60\,ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/60*(3*((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 15*s
qrt((a*x - 1)/(a*x + 1)))/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^5ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x + 1)^5*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}-10\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{60\,a\,c^2}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^2,x)
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x
- 1)/(a*x + 1))^(5/2))/(60*a*c^2)
```

3.612 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}+\frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3(1-a^2x^2)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3(1-a^2x^2)}$$

```
[Out] 8/35/a/c^3*((a*x-1)/(a*x+1))^(3/2)+1/7*(4*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3
/2)/(-a^2*x^2+1)^2-12/35*(2*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+
1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=-\frac{12(2ax+3)e^{-3\coth^{-1}(ax)}}{35ac^3(1-a^2x^2)}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}+\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] 8/(35*a*c^3*E^(3*ArcCoth[a*x])) + (3 + 4*a*x)/(7*a*c^3*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2)^2) - (12*(3 + 2*a*x))/(35*a*c^3*E^(3*ArcCoth[a*x])*(1 - a^2*x^
2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\int\frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{7c}\\
&= \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}-\frac{24\int\frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}\\
&= \frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}+\frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13-4ax+20a^2x^2+24a^3x^3+8a^4x^4)}{35c^3(-1+ax)(1+ax)^4}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^3*(-1 + a*x)*(1 + a*x)^4)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(8a^{4}x^{4}+24a^{3}x^{3}+20a^{2}x^{2}-4ax-13\right)}{35\left(a^{2}x^{2}-1\right)^{2}c^{3}a}$	65
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(8a^{4}x^{4}+24a^{3}x^{3}+20a^{2}x^{2}-4ax-13\right)}{35(ax-1)^{2}c^{3}a(ax+1)^{2}}$	68
trager	$\frac{\left(8a^{4}x^{4}+24a^{3}x^{3}+20a^{2}x^{2}-4ax-13\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^{3}(ax-1)(ax+1)^{3}}$	70

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a^
2*x^2-1)^2/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{(8\,a^4x^4+24\,a^3x^3+20\,a^2x^2-4\,ax-13)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a*
*5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x) + Integral
(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a**5*x**5
 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x))/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{560}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-28\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-140\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{35}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -1/560*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 28*((a*x - 1)/(a*x + 1))^(5/2) +
 70*((a*x - 1)/(a*x + 1))^(3/2) - 140*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3)
- 35/(a^2*c^3*sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(a^2cx^2-c)^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{112\,a\,c^3}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^3,x)
[Out] 1/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2)) + ((a*x - 1)/(a*x + 1))^(1/2)/(4*a
*c^3) - ((a*x - 1)/(a*x + 1))^(3/2)/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)
/(20*a*c^3) - ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3)
```

3.613 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4} + \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} - \frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4(1-a^2x^2)}$$

```
[Out] 16/63/a/c^4*((a*x-1)/(a*x+1))^(3/2)+1/9*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)^3+10/63*(4*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2
+1)^2-8/21*(2*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3} - \frac{8(2ax+3)e^{-3\coth^{-1}(ax)}}{21ac^4(1-a^2x^2)} + \frac{10(4ax+3)e^{-3\coth^{-1}(ax)}}{63ac^4(1-a^2x^2)^2} + \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] 16/(63*a*c^4*E^(3*ArcCoth[a*x])) + (1 + 2*a*x)/(9*a*c^4*E^(3*ArcCoth[a*x])*
(1 - a^2*x^2)^3) + (10*(3 + 4*a*x))/(63*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x
^2)^2) - (8*(3 + 2*a*x))/(21*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{9c}\\
&= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{40\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{21c^2}\\
&= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4\left(1-a^2x^2\right)^2}\\
&\quad-\frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4\left(1-a^2x^2\right)}-\frac{16\int\frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{21c^3}\\
&= \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}+\frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4\left(1-a^2x^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$
\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(19-6ax-66a^2x^2-56a^3x^3+24a^4x^4+48a^5x^5+16a^6x^6)}{63c^4(-1+ax)^2(1+ax)^5}
$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^4
 + 48*a^5*x^5 + 16*a^6*x^6))/(63*c^4*(-1 + a*x)^2*(1 + a*x)^5)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(16a^{6}x^{6}+48a^{5}x^{5}+24a^{4}x^{4}-56a^{3}x^{3}-66a^{2}x^{2}-6ax+19\right)}{63\left(a^{2}x^{2}-1\right)^{3}c^{4}a}$	81
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(16a^{6}x^{6}+48a^{5}x^{5}+24a^{4}x^{4}-56a^{3}x^{3}-66a^{2}x^{2}-6ax+19\right)}{63\left(ax-1\right)^{3}c^{4}\left(ax+1\right)^{3}a}$	84
trager	$\frac{\left(16a^{6}x^{6}+48a^{5}x^{5}+24a^{4}x^{4}-56a^{3}x^{3}-66a^{2}x^{2}-6ax+19\right)\sqrt{-\frac{-ax+1}{ax+1}}}{63a\,c^{4}\left(ax-1\right)^{2}\left(ax+1\right)^{4}}$	86

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/63*((a*x-1)/(a*x+1))^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-6
6*a^2*x^2-6*a*x+19)/(a^2*x^2-1)^3/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{4}}\,dx=\frac{\left(16\,a^{6}x^{6}+48\,a^{5}x^{5}+24\,a^{4}x^{4}-56\,a^{3}x^{3}-66\,a^{2}x^{2}-6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^{7}c^{4}x^{6}+2\,a^{6}c^{4}x^{5}-a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}-a^{3}c^{4}x^{2}+2\,a^{2}c^{4}x+ac^{4}\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/63*(16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*a^2*x^2 - 6*a*
x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^
4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{4}}\,dx$$
$$=\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{9}x^{9}+a^{8}x^{8}-4a^{7}x^{7}-4a^{6}x^{6}+6a^{5}x^{5}+6a^{4}x^{4}-4a^{3}x^{3}-4a^{2}x^{2}+ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{9}x^{9}+a^{8}x^{8}-4a^{7}x^{7}-4a^{6}x^{6}+6a^{5}x^{5}+6a^{4}x^{4}-4a^{3}x^{3}-4a^{2}}}{c^{4}}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**9*x**9 + a**8*x**8 - 4*a**
7*x**7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a**3*x**3 - 4*a**2*x**
```

```
2 + a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**9*x**
9 + a**8*x**8 - 4*a**7*x**7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a
**3*x**3 - 4*a**2*x**2 + a*x + 1), x))/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{1}{4032}\,a\left(\frac{7\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}} - 54\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 189\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 420\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 945\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4} + \frac{21\left(\frac{18(ax-1)}{ax+1} - 1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*((7*((a*x - 1)/(a*x + 1))^(9/2) - 54*((a*x - 1)/(a*x + 1))^(7/2) +
 189*((a*x - 1)/(a*x + 1))^(5/2) - 420*((a*x - 1)/(a*x + 1))^(3/2) + 945*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + 21*(18*(a*x - 1)/(a*x + 1) - 1)/(a^2*c
^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(a^2cx^2-c)^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4} - \frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4} + \frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{64\,a\,c^4} - \frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4} + \frac{\frac{6\,(a\,x-1)}{a\,x+1} - \frac{1}{3}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^4,x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(3/2
))/(48*a*c^4) + (3*((a*x - 1)/(a*x + 1))^(5/2))/(64*a*c^4) - (3*((a*x - 1)/
(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4) + (
(6*(a*x - 1))/(a*x + 1) - 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3/2))
```

3.614 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 229

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] 8/3*(a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(a*x+1
)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(a*x+1)^8*(-a^2*c*x
^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(a*x+1)^9*(-a^2*c*x^2+c)^(9/2)
/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^10/(1-
1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(ax+1)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (8*(1 + a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c-a^2cx^2\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^4(1+ax)^5\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(1+ax)^5-32(1+ax)^6+24(1+ax)^7-8(1+ax)^8+(1+ax)^9\right)dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&\quad-\frac{8(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(1+ax)^6\sqrt{c-a^2cx^2}(193-528ax+588a^2x^2-308a^3x^3+63a^4x^4)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]

[Out] (c^4*(1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 - 528*a*x + 588*a^2*x^2 - 308*a^3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(63a^9x^9+70a^8x^8-315a^7x^7-360a^6x^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630\right)x\,c^4\sqrt{-c\left(a^2x^2-1\right)}}{630(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	113
gosper	$\frac{x\left(63a^9x^9+70a^8x^8-315a^7x^7-360a^6x^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{630(ax+1)^5(ax-1)^4\sqrt{\frac{ax-1}{ax+1}}}$	116

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/630*(63*a^9*x^9+70*a^8*x^8-315*a^7*x^7-360*a^6*x^6+630*a^5*x^5+756*a^4*x^4-630*a^3*x^3-840*a^2*x^2+315*a*x+630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(63\,a^9c^4x^{10}+70\,a^8c^4x^9-315\,a^7c^4x^8-360\,a^6c^4x^7+630\,a^5c^4x^6+756\,a^4c^4x^5-630\,a^3c^4x^4-}{630\,a}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")

```
[Out] 1/630*(63*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 - 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 + 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 - 840*a^2*c^4*x^3 +
315*a*c^4*x^2 + 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.615 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 183

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] -8/5*(a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(a*x+1)^6
*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(a*x+1)^7*(-a^2*c*x^2
+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^2+c)^(7/2)/a^
8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(ax+1)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-8*(1 + a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
```

```
(6*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{(c-a^2cx^2)^{7/2}\int(-1+ax)^3(1+ax)^4\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{(c-a^2cx^2)^{7/2}\int\left(-8(1+ax)^4+12(1+ax)^5-6(1+ax)^6+(1+ax)^7\right)\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{8(1+ax)^5\,(c-a^2cx^2)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\,(c-a^2cx^2)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&\quad-\frac{6(1+ax)^7\,(c-a^2cx^2)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\,(c-a^2cx^2)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(1+ax)^5\sqrt{c-a^2cx^2}(-93+185ax-135a^2x^2+35a^3x^3)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
[Out] -1/280*(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 +
35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{(35a^7x^7+40a^6x^6-140a^5x^5-168a^4x^4+210a^3x^3+280a^2x^2-140ax-280)\,x\,c^3\sqrt{-c(a^2x^2-1)}}{280(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	97
gosper	$\frac{x(35a^7x^7+40a^6x^6-140a^5x^5-168a^4x^4+210a^3x^3+280a^2x^2-140ax-280)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{280(ax-1)^3(ax+1)^4\sqrt{\frac{ax-1}{ax+1}}}$	100

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/280*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*x
^2-140*a*x-280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{(35\,a^7c^3x^8+40\,a^6c^3x^7-140\,a^5c^3x^6-168\,a^4c^3x^5+210\,a^3c^3x^4+280\,a^2c^3x^3-140\,ac^3x^2-280\,c^3x)\sqrt{-a^2c}}{280\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
[Out] -1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.616 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] (a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(a*x+1)^5*(-
a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)^6*(-a^2*c*x^2+c)
^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(c-a^2cx^2)^{5/2}\int(-1+ax)^2(1+ax)^3\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(c-a^2cx^2)^{5/2}\int\left(4(1+ax)^3-4(1+ax)^4+(1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(1+ax)^4\,(c-a^2cx^2)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\,(c-a^2cx^2)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\,(c-a^2cx^2)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(1+ax)^4\,(11-14ax+5a^2x^2)\,\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
[Out] (c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(5a^5x^5+6a^4x^4-15a^3x^3-20a^2x^2+15ax+30)x\,c^2\sqrt{-c(a^2x^2-1)}}{30(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	81
gosper	$\frac{x(5a^5x^5+6a^4x^4-15a^3x^3-20a^2x^2+15ax+30)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30(ax-1)^2(ax+1)^3\sqrt{\frac{ax-1}{ax+1}}}$	84

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/30*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*x*c^2*(-c*(a^2*x
^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.54

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
[Out] 1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.617 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=-\frac{2(1+ax)^3(c-a^2cx^2)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4(c-a^2cx^2)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -2/3*(a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(a*x+1)
^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\frac{(ax+1)^4(c-a^2cx^2)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^3(c-a^2cx^2)^{3/2}}{3a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c-a^2cx^2)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(c-a^2cx^2)^{3/2}\int(-1+ax)(1+ax)^2\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(c-a^2cx^2)^{3/2}\int\left(-2(1+ax)^2+(1+ax)^3\right)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= -\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c(1+ax)^3(-5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
[Out] -1/12*(c*(1 + a*x)^3*(-5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{(3a^3x^3+4a^2x^2-6ax-12)xc\sqrt{-c(a^2x^2-1)}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	63
gosper	$\frac{x(3a^3x^3+4a^2x^2-6ax-12)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{12(ax-1)(ax+1)^2\sqrt{\frac{ax-1}{ax+1}}}$	68

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*x*c*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a
*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{(3\,a^3cx^4+4\,a^2cx^3-6\,acx^2-12\,cx)\sqrt{-a^2c}}{12\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
[Out] -1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)} \left(c - a^2 c x^2\right)^{3/2} \, dx = \int \frac{\left(-a^2 c x^2 + c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}} \, dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Giac [F]

$$\int e^{\coth^{-1}(ax)} \left(c - a^2 c x^2\right)^{3/2} \, dx = \int \frac{\left(-a^2 c x^2 + c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}} \, dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)} \left(c - a^2 c x^2\right)^{3/2} \, dx = \int \frac{\left(c - a^2 \, c \, x^2\right)^{3/2}}{\sqrt{\frac{a \, x-1}{a \, x+1}}} \, dx$$

[In] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)

3.618 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6327, 6328}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c(a^2x^2-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2+2\,x)}{2\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.619 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] x*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6328, 31}

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}} \\ &= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{1}{-1+ax}\,dx}{\sqrt{c-a^2cx^2}} \\ &= \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\ln(ax-1)\sqrt{-c(a^2x^2-1)}}{ca(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	51

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/c/a/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}\log(ax-1)}{a^2c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
[Out] -sqrt(-a^2*c)*log(a*x - 1)/(a^2*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.620 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\text{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3(-1+(-1+ax)\text{arctanh}(ax))}{(-2+2ax)\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1 + (-1 + a*x)*ArcTanh[a*x]))/((-2 + 2*a
*x)*(c - a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x-a\ln(ax-1)x-\ln(ax+1)+\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a}$	84

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*x-
1)*x-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)

[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)

[Out] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)

3.621 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (
```

```
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\text{arctanh}(ax)\right)}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,\left(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-3a^2\ln(ax+1)x^2+3a^2\ln(ax-1)x^2-6a^2x^2-3a\ln(ax+1)x+3a\ln(ax-1)x+6ax+3\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+1)*x^3-3*a^3*ln(a*x-1)*x^3-3*a^2*ln(a*x+1)*x^2+3*a^2*ln(a*x-1)*x^2-6*a^2*x^2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(3a^2x^2-3ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.622 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 277

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] 1/24*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(
1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^
2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7
/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a
^2*c*x^2+c)^(7/2)+5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2
+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

$= \{6327, 6328, 46, 213\}$

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\operatorname{arctanh}(ax)}{16\,(c-a^2cx^2)^{7/2}} + \frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\,(c-a^2cx^2)^{7/2}} + \frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\,(c-a^2cx^2)^{7/2}} + \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{24(1-ax)^3\,(c-a^2cx^2)^{7/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (
5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^4(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{8(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{16(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{\left(5a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{-1+a^2x^2}\,dx}{16\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-8-25ax+25a^2x^2+15a^3x^3-15a^4x^4+15(-1+ax)^3(1+ax)^2\operatorname{arctanh}(ax)\right)}{48c^3(-1+ax)^3(1+ax)^2\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
[Out] -1/48*(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15*
a^4*x^4 + 15*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(c^3*(-1 + a*x)^3*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-15\ln(ax+1)x^4a^4+15\ln(ax-1)x^4a^4-30a^4x^4-30a^3\ln(ax+1)x^3+30a^3\ln(ax-}{96\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)^2(}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)
*x^5*a^5-15*ln(a*x-1)*x^5*a^5-15*ln(a*x+1)*x^4*a^4+15*ln(a*x-1)*x^4*a^4-30*
a^4*x^4-30*a^3*ln(a*x+1)*x^3+30*a^3*ln(a*x-1)*x^3+30*a^3*x^3+30*a^2*ln(a*x+
1)*x^2-30*a^2*ln(a*x-1)*x^2+50*a^2*x^2+15*a*ln(a*x+1)*x-15*a*ln(a*x-1)*x-50
*a*x-15*ln(a*x+1)+15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{15\,(a^6x^5-a^5x^4-2\,a^4x^3+2\,a^3x^2+a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(15\,a^4x^4-15\,a^3x^3-25\,a}{96\,(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+a^3c^4x-a^2c^4)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/96*(15*(a^6*x^5 - a^5*x^4 - 2*a^4*x^3 + 2*a^3*x^2 + a^2*x - a)*sqrt(-c)*
log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15*a^4*
x^4 - 15*a^3*x^3 - 25*a^2*x^2 + 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 - a^
6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.623 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 176

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11(c-a^2cx^2)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{77c^{9/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}$$

```
[Out] -77/384*c^3*x*(-a^2*c*x^2+c)^(3/2)-77/480*c^2*x*(-a^2*c*x^2+c)^(5/2)-11/80*
c*x*(-a^2*c*x^2+c)^(7/2)+11/90*(-a^2*c*x^2+c)^(9/2)/a+1/10*(a*x+1)*(-a^2*c*
x^2+c)^(9/2)/a-77/256*c^(9/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-77
/256*c^4*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{77c^{9/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10a}+\frac{11(c-a^2cx^2)^{9/2}}{90a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-77*c^4*x*Sqrt[c - a^2*c*x^2])/256 - (77*c^3*x*(c - a^2*c*x^2)^(3/2))/384
- (77*c^2*x*(c - a^2*c*x^2)^(5/2))/480 - (11*c*x*(c - a^2*c*x^2)^(7/2))/80
+ (11*(c - a^2*c*x^2)^(9/2))/(90*a) + ((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(10
*a) - (77*c^(9/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(256*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$$

$$= -\left(c\int (1+ax)^2\left(c-a^2cx^2\right)^{7/2}\,dx\right)$$

$$= \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{10}(11c)\int (1+ax)\left(c-a^2cx^2\right)^{7/2}\,dx$$

$$= \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{10}(11c)\int \left(c-a^2cx^2\right)^{7/2}\,dx$$

$$= -\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{80}\left(77c^2\right)\int \left(c-a^2cx^2\right)^{5/2}\,dx$$

$$= -\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{96}\left(77c^3\right)\int \left(c-a^2cx^2\right)^{3/2}\,dx$$

$$= -\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{128}\left(77c^4\right)\int \sqrt{c-a^2cx^2}\,dx$$

$$= -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{256}\left(77c^5\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$

$$= -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{256}\left(77c^5\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)$$

$$= -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11(c-a^2cx^2)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{77c^{9/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \frac{c^4\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(2560-10615ax-2185a^2x^2+16390a^3x^3+9210a^4x^4-15048a^5x^5}{11520a\sqrt{1}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(2560 - 10615*a*x - 2185*a^2*x^2 +
16390*a^3*x^3 + 9210*a^4*x^4 - 15048*a^5*x^5 - 10552*a^6*x^6 + 7216*a^7*x^7
 + 5584*a^8*x^8 - 1408*a^9*x^9 - 1152*a^10*x^10) + 6930*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(11520*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(1152a^9x^9+2560a^8x^8-3024a^7x^7-10240a^6x^6+312a^5x^5+15360a^4x^4+6150a^3x^3-10240a^2x^2-8055ax+2560)(a^2x^2-1)c^5}{11520a\sqrt{-c(a^2x^2-1)}} - \frac{77}{}$
default	$\frac{x(-a^2cx^2+c)^{\frac{9}{2}}}{10} + \frac{9c\left(\frac{x(-a^2cx^2+c)^{\frac{7}{2}}}{8} + \frac{7c\left(\frac{x(-a^2cx^2+c)^{\frac{5}{2}}}{6} + \frac{5c\left(\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4} + \frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2} + \frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}$

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/11520*(1152*a^9*x^9+2560*a^8*x^8-3024*a^7*x^7-10240*a^6*x^6+312*a^5*x^5+
15360*a^4*x^4+6150*a^3*x^3-10240*a^2*x^2-8055*a*x+2560)*(a^2*x^2-1)/a/(-c*(
a^2*x^2-1))^(1/2)*c^5-77/256/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x
^2+c)^(1/2))*c^5

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \left[\frac{3465\sqrt{-c}c^4\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(1152\,a^9c^4x^9+2560\,a^8c^4x^8-3\right.}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(3465*sqrt(-c)*c^4*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) + 2*(1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 1
0240*a^6*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 -
 10240*a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a, 1/11
520*(3465*c^(9/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c))
+ (1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 10240*a^6*c^4*x
^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 - 10240*a^2*c^4
*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(162) = 324.

Time = 3.36 (sec) , antiderivative size = 714, normalized size of antiderivative = 4.06

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \begin{cases} \frac{-2c^4\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+6c^4\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{fo}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{ot}\end{cases}\right.}{} \\ -c^{\frac{9}{2}}x \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Piecewise(((-2*c**4*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 6*c**4*Piecewise((sqrt(-a**2*c*x*
*2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x**4/
```

```
4, True)) - 6*c**4*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**6*x**6/7 - a**4*x*
*4/35 - 4*a**2*x**2/105 - 8/105), Ne(c, 0)), (a**6*sqrt(c)*x**6/6, True)) +
 2*c**4*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**8*x**8/9 - a**6*x**6/63 - 2*a
**4*x**4/105 - 8*a**2*x**2/315 - 16/315), Ne(c, 0)), (a**8*sqrt(c)*x**8/8,
True)) + c**4*Piecewise((7*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**
2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True)
)/256 + sqrt(-a**2*c*x**2 + c)*(a**9*x**9/10 - a**7*x**7/80 - 7*a**5*x**5/4
80 - 7*a**3*x**3/384 - 7*a*x/256), Ne(c, 0)), (a**9*sqrt(c)*x**9/9, True))
- 2*c**4*Piecewise((5*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x
**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/128
 + sqrt(-a**2*c*x**2 + c)*(a**7*x**7/8 - a**5*x**5/48 - 5*a**3*x**3/192 - 5
*a*x/128), Ne(c, 0)), (a**7*sqrt(c)*x**7/7, True)) + 2*c**4*Piecewise((c*Pi
ecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c,
0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqr
t(-a**2*c*x**2 + c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c**4*Piecewi
se((a*x*sqrt(-a**2*c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*s
qrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2
), True))/2, Ne(c, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(9/2)*x, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{1}{10}\left(-a^2cx^2+c\right)^{\frac{9}{2}}x-\frac{11}{80}\left(-a^2cx^2+c\right)^{\frac{7}{2}}cx-\frac{77}{480}\left(-a^2cx^2+c\right)^{\frac{5}{2}}c^2x-\frac{77}{384}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^3x-\frac{35}{64}\sqrt{a^2cx^2-4\,acx+3\,c}c^4x+\frac{63}{256}\sqrt{-a^2cx^2+c}c^4x-\frac{35\,c^6\arcsin\left(ax-2\right)}{64\,a\left(-c\right)^{\frac{3}{2}}}+\frac{63\,c^{\frac{9}{2}}\arcsin\left(ax\right)}{256\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{9\,a}+\frac{35\sqrt{a^2cx^2-4\,acx+3\,c}c^4}{32\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/10*(-a^2*c*x^2 + c)^(9/2)*x - 11/80*(-a^2*c*x^2 + c)^(7/2)*c*x - 77/480*(
-a^2*c*x^2 + c)^(5/2)*c^2*x - 77/384*(-a^2*c*x^2 + c)^(3/2)*c^3*x - 35/64*s
qrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4*x + 63/256*sqrt(-a^2*c*x^2 + c)*c^4*x -
35/64*c^6*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 63/256*c^(9/2)*arcsin(a*x)/a + 2
/9*(-a^2*c*x^2 + c)^(9/2)/a + 35/32*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4/a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{77\,c^5\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{256\,\sqrt{-c}|a|}$$

$$+\frac{1}{11520}\sqrt{-a^2cx^2+c}\left(\frac{2560\,c^4}{a}-\left(8055\,c^4+2\left(5120\,ac^4-\left(3075\,a^2c^4+4\left(1920\,a^3c^4+\left(39\,a^4c^4-2\left(640\,a^5c^4\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 77/256*c^5*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/11520*sqrt(-a^2*c*x^2 + c)*(2560*c^4/a - (8055*c^4 + 2*(5120*a*c^4 -
 (3075*a^2*c^4 + 4*(1920*a^3*c^4 + (39*a^4*c^4 - 2*(640*a^5*c^4 + (189*a^6*
c^4 - 8*(9*a^8*c^4*x + 20*a^7*c^4)*x)*x)*x)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{9/2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1), x)
```

3.624 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 153

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{45c^{7/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}$$

```
[Out] -15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)-3/16*c*x*(-a^2*c*x^2+c)^(5/2)+9/56*(-a^2*
c*x^2+c)^(7/2)/a+1/8*(a*x+1)*(-a^2*c*x^2+c)^(7/2)/a-45/128*c^(7/2)*arctan(a
*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{45c^{7/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-45*c^3*x*Sqrt[c - a^2*c*x^2])/128 - (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 -
 (3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1
```

```
+ a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) - (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(128*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx\\
&= -\left(c\int (1+ax)^2\left(c-a^2cx^2\right)^{5/2}\,dx\right)\\
&= \frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int (1+ax)\left(c-a^2cx^2\right)^{5/2}\,dx\\
&= \frac{9(c-a^2cx^2)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int \left(c-a^2cx^2\right)^{5/2}\,dx\\
&= -\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9(c-a^2cx^2)^{7/2}}{56a}\\
&\quad+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{16}\left(15c^2\right)\int \left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9(c-a^2cx^2)^{7/2}}{56a}\\
&\quad+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{64}\left(45c^3\right)\int \sqrt{c-a^2cx^2}\,dx\\
&= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}\\
&\quad+\frac{9(c-a^2cx^2)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{128}\left(45c^4\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9(c-a^2cx^2)^{7/2}}{56a}\\
&\quad+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{128}\left(45c^4\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}\\
&\quad+\frac{9(c-a^2cx^2)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{45c^{7/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(256-837ax-187a^2x^2+978a^3x^3+558a^4x^4-600a^5x^5-424a^6x^6\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]

[Out] (c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978*a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8*x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

method	result
risch	$\frac{(112a^7x^7+256a^6x^6-168a^5x^5-768a^4x^4-210a^3x^3+768a^2x^2+581ax-256)(a^2x^2-1)c^4}{896a\sqrt{-c(a^2x^2-1)}}-\frac{45\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^4}{128\sqrt{a^2c}}$
default	$\frac{x(-a^2cx^2+c)^{\frac{7}{2}}}{8}+\frac{7c\left(\frac{x(-a^2cx^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}\right)}{8}+\frac{2\left(-a^2c\left(x-\frac{1}{a}\right)^2-\right.}{7}$

[In] int((-a^2*c*x^2+c)^(7/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)

```
[Out] 1/896*(112*a^7*x^7+256*a^6*x^6-168*a^5*x^5-768*a^4*x^4-210*a^3*x^3+768*a^2*
x^2+581*a*x-256)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c^4-45/128/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx = \left[\frac{315\sqrt{-c}c^3\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2(112a^7c^3x^7+256a^6c^3x^6-168}{1792a}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, 1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(
c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^
5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256
*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(139) = 278.

Time = 2.97 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.01

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx = \begin{cases} \frac{-2c^3\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+4c^3\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{f}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{o}\end{cases}\right.}{} \\ -c^{\frac{7}{2}}x \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Piecewise(((-2*c**3*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 4*c**3*Piecewise((sqrt(-a**2*c*x*
*2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x**4/
4, True)) - 2*c**3*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**6*x**6/7 - a**4*x*
*4/35 - 4*a**2*x**2/105 - 8/105), Ne(c, 0)), (a**6*sqrt(c)*x**6/6, True)) -
 c**3*Piecewise((5*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2
 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/128 +
sqrt(-a**2*c*x**2 + c)*(a**7*x**7/8 - a**5*x**5/48 - 5*a**3*x**3/192 - 5*a*
x/128), Ne(c, 0)), (a**7*sqrt(c)*x**7/7, True)) + c**3*Piecewise((c*Piecewi
se((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)),
(a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/16 + sqrt(-a**2*c*x**2 + c)*(a**5*
x**5/6 - a**3*x**3/24 - a*x/16), Ne(c, 0)), (a**5*sqrt(c)*x**5/5, True)) +
c**3*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 +
c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/8 + (a**3
*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, T
rue)) - c**3*Piecewise((a*x*sqrt(-a**2*c*x**2 + c)/2 + c*Piecewise((log(-2*
a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*
x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0)), (a*sqrt(c)*x, True)))/a, Ne(a,
0)), (-c**(7/2)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.13

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{1}{8}\left(-a^2cx^2+c\right)^{\frac{7}{2}}x-\frac{3}{16}\left(-a^2cx^2+c\right)^{\frac{5}{2}}cx-\frac{15}{64}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2x-\frac{5}{8}\sqrt{a^2cx^2-4\,acx+3\,c}c^3x+\frac{35}{128}\sqrt{-a^2cx^2+c}c^3x-\frac{5\,c^5\arcsin\left(ax-2\right)}{8\,a\left(-c\right)^{\frac{3}{2}}}+\frac{35\,c^{\frac{7}{2}}\arcsin\left(ax\right)}{128\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{7\,a}+\frac{5\sqrt{a^2cx^2-4\,acx+3\,c}c^3}{4\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-a^2*c*x^2 + c)^(7/2)*x - 3/16*(-a^2*c*x^2 + c)^(5/2)*c*x - 15/64*(-a^
2*c*x^2 + c)^(3/2)*c^2*x - 5/8*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^3*x + 35/1
28*sqrt(-a^2*c*x^2 + c)*c^3*x - 5/8*c^5*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 35
/128*c^(7/2)*arcsin(a*x)/a + 2/7*(-a^2*c*x^2 + c)^(7/2)/a + 5/4*sqrt(a^2*c*
x^2 - 4*a*c*x + 3*c)*c^3/a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}|a|}$$
$$+\frac{1}{896}\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a}-\left(581\,c^3+2\,\left(384\,ac^3-\left(105\,a^2c^3+4\,\left(96\,a^3c^3+\left(21\,a^4c^3-2\,\left(7\,a^6c^3x+16\,a^5c\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] 45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (105
*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)*x)
*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{7/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.625 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7(c-a^2cx^2)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

```
[Out] -7/24*c*x*(-a^2*c*x^2+c)^(3/2)+7/30*(-a^2*c*x^2+c)^(5/2)/a+1/6*(a*x+1)*(-a^
2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-
7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7(c-a^2cx^2)^{5/2}}{30a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(
c - a^2*c*x^2)^(5/2))/(30*a) + ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx\\
&= -\left(c\int (1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)\\
&= \frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int (1+ax)\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= \frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int \left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}\left(7c^2\right)\int \sqrt{c-a^2cx^2}\,dx\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}\\
&\quad+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{16}\left(7c^3\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}\\
&\quad+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{16}\left(7c^3\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}\\
&\quad+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(96-231ax-57a^2x^2+182a^3x^3+106a^4x^4-56a^5x^5-40a^6x^6)+2\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 - 231*a*x - 57*a^2*x^2 + 182*a^
3*x^3 + 106*a^4*x^4 - 56*a^5*x^5 - 40*a^6*x^6) + 210*Sqrt[1 - a*x]*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(40a^5x^5+96a^4x^4-10a^3x^3-192a^2x^2-135ax+96)(a^2x^2-1)c^3}{240a\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^3}{16\sqrt{a^2c}}$
default	$\frac{x(-a^2cx^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}+\frac{\frac{2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}{5}-2ac\left(\right.}{}$

[In] int((-a^2*c*x^2+c)^(5/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)

[Out] -1/240*(40*a^5*x^5+96*a^4*x^4-10*a^3*x^3-192*a^2*x^2-135*a*x+96)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c^3-7/16/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.85

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx=\left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40a^5c^2x^5+96a^4c^2x^4-10a^3c\right.}{480a}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(117) = 234.

Time = 2.53 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.42

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \begin{cases} \frac{-2c^2\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+2c^2\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{for}\ \ldots\\ \frac{a^4\sqrt{c}x^4}{4} & \text{ot}\ldots\end{cases}\right)\ldots}{\ldots} \\ -c^{\frac{5}{2}}x \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Piecewise(((-2*c**2*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 2*c**2*Piecewise((sqrt(-a**2*c*x*
*2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x**4/
4, True)) + c**2*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a*
*2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True
))/16 + sqrt(-a**2*c*x**2 + c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16), Ne(c,
 0)), (a**5*sqrt(c)*x**5/5, True)) - c**2*Piecewise((a*x*sqrt(-a**2*c*x**2
+ c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqr
t(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0)), (a
*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x-\frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx - \frac{3}{4}\sqrt{a^2cx^2-4\,acx+3\,c}c^2x+\frac{5}{16}\sqrt{-a^2cx^2+c}c^2x-\frac{3\,c^4\arcsin\left(ax-2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}} + \frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5\,a}+\frac{3\sqrt{a^2cx^2-4\,acx+3\,c}c^2}{2\,a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(a
^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x - 3/4*c^4
*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a + 2/5*(-a^2*c*
x^2 + c)^(5/2)/a + 3/2*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}|a|}-\frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right)x\right)x-\frac{96\,c^2}{a}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5*a
^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.626 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5(c-a^2cx^2)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

```
[Out] 5/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*(a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2)*
arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5(c-a^2cx^2)^{3/2}}{12a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 + a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\left(c\int(1+ax)^2\sqrt{c-a^2cx^2}\,dx\right)\\
&= \frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int(1+ax)\sqrt{c-a^2cx^2}\,dx\\
&= \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}\\
&\quad-\frac{1}{8}\left(5c^2\right)\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\left(16-25ax-7a^2x^2+10a^3x^3+6a^4x^4\right)+30\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^3
 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqrt
[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+9ax-16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}} - \frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4} + \frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4} + \frac{\frac{2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3} - 2ac\left(-\frac{\left(-2a^2c\left(x-\frac{1}{a}\right)-2ac\right)\sqrt{}}{4}\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/24*(6*a^3*x^3+16*a^2*x^2+9*a*x-16)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c^2-5/8/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.68

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx = \left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(6a^3cx^3+16a^2cx^2+9acx-16\right.}{48a}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/a
]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 2.24 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.34

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx = \begin{cases} \frac{-2c\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases} \frac{c\left(\begin{cases}\frac{\log\left(-2acx+2\sqrt{-c}\sqrt{-a^2cx^2+c}\right)}{\sqrt{-c}} & \text{for } c\neq 0\\ \frac{ax\log(ax)}{\sqrt{-a^2cx^2}} & \text{otherwis}\end{cases}\right)}{8} \\ \frac{a^3\sqrt{c}x^3}{3}\end{cases}\right)}{} \\ -c^{\frac{3}{2}}x \end{cases}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Piecewise(((-2*c*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), Ne(
c, 0)), (a**2*sqrt(c)*x**2/2, True)) - c*Piecewise((c*Piecewise((log(-2*a*c
*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/
sqrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c)
, Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c*Piecewise((a*x*sqrt(-a**2*c*x
**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))
/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0))
, (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(3/2)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx &= \frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x \\ &- \sqrt{a^2cx^2-4\,acx+3\,c}cx + \frac{3}{8}\sqrt{-a^2cx^2+c}cx - \frac{c^3\arcsin(ax-2)}{a(-c)^{\frac{3}{2}}} \\ &+ \frac{3\,c^{\frac{3}{2}}\arcsin(ax)}{8\,a} + \frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a} + \frac{2\sqrt{a^2cx^2-4\,acx+3\,c}c}{a}\end{aligned}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*x + 3/8*sq
rt(-a^2*c*x^2 + c)*c*x - c^3*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)*a
rcsin(a*x)/a + 2/3*(-a^2*c*x^2 + c)^(3/2)/a + 2*sqrt(a^2*c*x^2 - 4*a*c*x +
3*c)*c/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx+8\,ac\right)x+9\,c\right)x-\frac{16\,c}{a}\right) + \frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,\sqrt{-c}|a|}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.627 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^
(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6276, 685, 655, 223, 209}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{(ax+4)\left(a^2x^2-1\right)c}{2a\sqrt{-c\left(a^2x^2-1\right)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}}{a}$	136

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*arc
tan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{}}{2a}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}x - \frac{3\sqrt{c}\arcsin(ax)}{2a} + \frac{2\sqrt{-a^2cx^2+c}}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
+ c)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right) + \frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.628 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*(a*x+1)/a/(-a^2*c*x^2+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6276, 667, 223, 209}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{\sqrt{c-a^2cx^2}}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\right) \\
&= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\operatorname{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right) \\
&= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\left(\sqrt{1+ax}+\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
[Out] (-2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}{a^2c\left(x-\frac{1}{a}\right)}$	79

```
[In] int((a*x+1)/(-a^2*c*x^2+c)^(1/2)/(a*x-1),x,method=_RETURNVERBOSE)
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x-1/a
)*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.59

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \left[-\frac{(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx-ac)}, -\frac{(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx-ac}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] [-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), -((a*x - 1)*sqrt(c)*ar
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{ax+1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral((a*x + 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{-a^2cx^2+c}}{a^2cx-ac} + \frac{\arcsin(ax)}{a\sqrt{c}}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x - a*c) + arcsin(a*x)/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{a\,x+1}{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)), x)
```

3.629 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx = -\frac{2(1+ax)}{3a(c-a^2cx^2)^{3/2}} - \frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] -2/3*(a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6276, 667, 197}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx = -\frac{x}{3c\sqrt{c-a^2cx^2}} - \frac{2(ax+1)}{3a(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] (-2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
```

```
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{5/2}}\,dx\right) \\
&= -\frac{2(1+ax)}{3a\,(c-a^2cx^2)^{3/2}} - \frac{1}{3}\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx \\
&= -\frac{2(1+ax)}{3a\,(c-a^2cx^2)^{3/2}} - \frac{x}{3c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(2-ax)\sqrt{1+ax}\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] -1/3*((2 - a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)^(3/2)*Sqrt[
c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(ax-2)(ax+1)^{2}}{3(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}a}$	31
trager	$\frac{(ax-2)\sqrt{-a^{2}c\,x^{2}+c}}{3c^{2}(ax-1)^{2}a}$	34
default	$\frac{x}{c\sqrt{-a^{2}c\,x^{2}+c}}+\frac{\frac{2}{3ac\left(x-\frac{1}{a}\right)\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}+\frac{2\left(-2a^{2}c\left(x-\frac{1}{a}\right)-2ac\right)}{3a\,c^{2}\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}}{a}$	127

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(a*x-2)*(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(ax-2)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{ax+1}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{x}{3\sqrt{-a^2cx^2+c}c} + \frac{2}{3\left(\sqrt{-a^2cx^2+c}a^2cx - \sqrt{-a^2cx^2+c}ac\right)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x - sqrt(
-a^2*c*x^2 + c)*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(43) = 86.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(ac - 3\sqrt{-a^2c}\sqrt{c}\right)\operatorname{sgn}(x)}{3\left(a^2c^{\frac{5}{2}} - \sqrt{-a^2c}ac^2\right)} - \frac{2\left(2a^2c + 3a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}} - \frac{\sqrt{c}}{x}\right) + 3\left(\sqrt{-a^2c+\frac{c}{x^2}} - \frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c} + \sqrt{-a^2c+\frac{c}{x^2}} - \frac{\sqrt{c}}{x}\right)^3 c\operatorname{sgn}(x)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^2
) - 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqrt
(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sqrt(
c)/x)^3*c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-2)}{3\,a\,c^2\,(a\,x-1)^2}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(a*x - 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x - 2))/(3*a*c^2*(a*x - 1)^2)
```

3.630 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{2(1+ax)}{5a(c-a^2cx^2)^{5/2}} - \frac{x}{5c(c-a^2cx^2)^{3/2}} - \frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

```
[Out] -2/5*(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2/
(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6276, 667, 198, 197}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{2x}{5c^2\sqrt{c-a^2cx^2}} - \frac{x}{5c(c-a^2cx^2)^{3/2}} - \frac{2(ax+1)}{5a(c-a^2cx^2)^{5/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2))
- (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{7/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{5a\,(c-a^2cx^2)^{5/2}} - \frac{3}{5}\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx\\
&= -\frac{2(1+ax)}{5a\,(c-a^2cx^2)^{5/2}} - \frac{x}{5c\,(c-a^2cx^2)^{3/2}} - \frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{5c}\\
&= -\frac{2(1+ax)}{5a\,(c-a^2cx^2)^{5/2}} - \frac{x}{5c\,(c-a^2cx^2)^{3/2}} - \frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=-\frac{2+ax-4a^2x^2+2a^3x^3}{5ac^2(-1+ax)^2\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
[Out] -1/5*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*x
^2])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(ax+1)^2(2a^3x^3-4a^2x^2+ax+2)}{5a(-a^2c\,x^2+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^3x^3-4a^2x^2+ax+2)\sqrt{-a^2c\,x^2+c}}{5c^3(ax-1)^3a(ax+1)}$
default	$\frac{x}{3c(-a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{-a^2c\,x^2+c}}+\frac{\frac{2}{5ac\left(x-\frac{1}{a}\right)\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{8a\left(-\frac{-2a^2c\left(x-\frac{1}{a}\right)-2ac}{6a^2c^2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{\cdots}{3a^2c^3\cdots}\right.}{5}}{a}$

```
[In] int((a*x+1)/(-a^2*c*x^2+c)^(5/2)/(a*x-1),x,method=_RETURNVERBOSE)
[Out] -1/5*(a*x+1)^2*(2*a^3*x^3-4*a^2*x^2+a*x+2)/a/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{\left(2\,a^3x^3-4\,a^2x^2+ax+2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 - 2
*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax-1\right)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{2}{5\left(\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2cx-\left(-a^2cx^2+c\right)^{\frac{3}{2}}ac\right)} - \frac{2\,x}{5\sqrt{-a^2cx^2+c}c^2} - \frac{x}{5\left(-a^2cx^2+c\right)^{\frac{3}{2}}c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] 2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x - (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/(
sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(ax-1\right)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2\right)}{5\,a\,c^3\,{\left(a\,x-1\right)}^3\,\left(a\,x+1\right)}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(a*x - 1)),x)
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2))/(5*a*c^3*(a*x - 1
)^3*(a*x + 1))
```

3.631 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}} - \frac{x}{7c\,(c-a^2cx^2)^{5/2}} - \frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

```
[Out] -2/7*(a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^2
/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6276, 667, 198, 197}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{8x}{21c^3\sqrt{c-a^2cx^2}} - \frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}} - \frac{x}{7c\,(c-a^2cx^2)^{5/2}} - \frac{2(ax+1)}{7a\,(c-a^2cx^2)^{7/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2))
- (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{9/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{5}{7}\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{x}{7c\,(c-a^2cx^2)^{5/2}}-\frac{4\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{7c}\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{x}{7c\,(c-a^2cx^2)^{5/2}}-\frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}}-\frac{8\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{21c^2}
\end{aligned}$$

$$= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\sqrt{1-a^2x^2}\left(6+9ax-24a^2x^2+4a^3x^3+16a^4x^4-8a^5x^5\right)}{21ac^3(1-ax)^{7/2}(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(6 + 9*a*x - 24*a^2*x^2 + 4*a^3*x^3 + 16*a^4*x^4 -
 8*a^5*x^5))/(a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{(ax+1)^2\left(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6\right)}{21a\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$
trager	$\frac{\left(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6\right)\sqrt{-a^2c\,x^2+c}}{21c^4(ax-1)^4(ax+1)^2a}$
default	$\frac{x}{5c\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2c\,x^2+c}}}{c}+\frac{\frac{2}{7ac\left(x-\frac{1}{a}\right)\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}-\frac{12a\left(-\frac{-2a^2c\left(x-\frac{1}{a}\right)-}{10a^2c^2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\right.}\right.}{}}{}$

```
[In] int((a*x+1)/(-a^2*c*x^2+c)^(7/2)/(a*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*(a*x+1)^2*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)/a/(-a^2*
c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a
^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx-(-a^2cx^2+c)^{\frac{5}{2}}ac\right)} - \frac{8\,x}{21\,\sqrt{-a^2cx^2+c}c^3} - \frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2} - \frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x - (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x/
(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/((
-a^2*c*x^2 + c)^(5/2)*c)
```

Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{7}{2}}(ax-1)}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x - 1)), x)

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x-1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x-1)^4} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}+\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,(a\,x-1)\,(a\,x+1)}$$

[In] int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(a*x - 1)),x)

[Out] (c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x - 1)^3) - (c - a^2*c*x^2)^(1/2)/(28*a*c^4*(a*x - 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) + 5/(28*a*c^4)))/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*(a*x + 1))

3.632 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{2(1+ax)}{9a(c-a^2cx^2)^{9/2}} - \frac{x}{9c(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2(c-a^2cx^2)^{5/2}} - \frac{8x}{45c^3(c-a^2cx^2)^{3/2}} - \frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

```
[Out] -2/9*(a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^2
/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6276, 667, 198, 197}

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{16x}{45c^4\sqrt{c-a^2cx^2}} - \frac{8x}{45c^3(c-a^2cx^2)^{3/2}} - \frac{2x}{15c^2(c-a^2cx^2)^{5/2}} - \frac{x}{9c(c-a^2cx^2)^{7/2}} - \frac{2(ax+1)}{9a(c-a^2cx^2)^{9/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-2*(1 + a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2))
- (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2
)) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{11/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{7}{9}\int \frac{1}{(c-a^2cx^2)^{9/2}}\,dx\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx}{3c}\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{15c^2}
\end{aligned}
$$

$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{45c^3}$$

$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = \frac{\sqrt{1-a^2x^2}(-10-25ax+60a^2x^2+10a^3x^3-80a^4x^4+24a^5x^5+32a^6x^6-16a^7x^7)}{45ac^4(1-ax)^{9/2}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 - 25*a*x + 60*a^2*x^2 + 10*a^3*x^3 - 80*a^4*x^4 + 2
4*a^5*x^5 + 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)^(
5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

method	result
gosper	$-\frac{(ax+1)^2\left(16a^7x^7-32a^6x^6-24a^5x^5+80a^4x^4-10a^3x^3-60a^2x^2+25ax+10\right)}{45a\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}$
trager	$\frac{\left(16a^7x^7-32a^6x^6-24a^5x^5+80a^4x^4-10a^3x^3-60a^2x^2+25ax+10\right)\sqrt{-a^2c\,x^2+c}}{45c^5(ax-1)^5(ax+1)^3a}$
default	$\frac{x}{7c\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2c\,x^2+c}}\right)}{7c}}{c}+\frac{\frac{2}{9ac\left(x-\frac{1}{a}\right)\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{7}{2}}}-\cdots}{}$ 16a

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/45*(a*x+1)^2*(16*a^7*x^7-32*a^6*x^6-24*a^5*x^5+80*a^4*x^4-10*a^3*x^3-60*
a^2*x^2+25*a*x+10)/a/(-a^2*c*x^2+c)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{(16\,a^7x^7 - 32\,a^6x^6 - 24\,a^5x^5 + 80\,a^4x^4 - 10\,a^3x^3 - 60\,a^2x^2 + 25\,ax + 10)\sqrt{-a^2cx^2 +}}{45\,(a^9c^5x^8 - 2\,a^8c^5x^7 - 2\,a^7c^5x^6 + 6\,a^6c^5x^5 - 6\,a^4c^5x^3 + 2\,a^3c^5x^2 + 2\,a^2c^5x - ac^5)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
[Out] 1/45*(16*a^7*x^7 - 32*a^6*x^6 - 24*a^5*x^5 + 80*a^4*x^4 - 10*a^3*x^3 - 60*a
^2*x^2 + 25*a*x + 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*x
- a*c^5)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{9}{2}}\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx - (-a^2cx^2+c)^{\frac{7}{2}}ac\right)} - \frac{16\,x}{45\,\sqrt{-a^2cx^2+c}c^4} - \frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3} - \frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2} - \frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
[Out] 2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x - (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*x
/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*x/
((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{9}{2}}(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x-1)^5} - \frac{5\,\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x-1)^4} + \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}+\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}-\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,(a\,x-1)\,(a\,x+1)}$$

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(9/2)*(a*x - 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(72*a*c^5*(a*x - 1)^5) - (5*(c - a^2*c*x^2)^(1/2))/(1
44*a*c^5*(a*x - 1)^4) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) + 5/(24*a*
c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5) -
 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2))/(
45*c^5*(a*x - 1)*(a*x + 1))
```

3.633 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] -8/7*(a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(a*x+1
)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(a*x+1)^9*(-a^2*c
*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(
9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-8*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
```

```
9) - (2*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{(c-a^2cx^2)^{9/2}\int(-1+ax)^3(1+ax)^6\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{(c-a^2cx^2)^{9/2}\int\left(-8(1+ax)^6+12(1+ax)^7-6(1+ax)^8+(1+ax)^9\right)dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= -\frac{8(1+ax)^7(c-a^2cx^2)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8(c-a^2cx^2)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&\quad-\frac{2(1+ax)^9(c-a^2cx^2)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}(c-a^2cx^2)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(1+ax)^7\sqrt{c-a^2cx^2}(-44+98ax-77a^2x^2+21a^3x^3)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{x\left(21a^9x^9+70a^8x^8-240a^6x^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{210(ax+1)^6(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$\frac{\left(21a^9x^9+70a^8x^8-240a^6x^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210\right)x\,c^4\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{210(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3*
x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x+1)^6/(a*x-1)^3/((a*x-1)/(a*x+1))
^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(21\,a^9c^4x^{10}+70\,a^8c^4x^9-240\,a^6c^4x^7-210\,a^5c^4x^6+252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2-}{210\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(21\,a^{11}\sqrt{-c}c^4x^{11}+49\,a^{10}\sqrt{-c}c^4x^{10}-70\,a^9\sqrt{-c}c^4x^9-240\,a^8\sqrt{-c}c^4x^8+30\,a^7\sqrt{-c}c^4x^7+\dots\right.}{210\dots}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 + 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 - 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 + 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 - 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 + 105*a^2*sqrt(-c)*c^4*x^2 + 210*sqrt(-c)*c^4)*(a*x
+ 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.634 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] 2/3*(a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(a*x+1)^
7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^
2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^2(1+ax)^5\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(1+ax)^5-4(1+ax)^6+(1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.45

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(1+ax)^6\left(37-54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
[Out] -1/168*(c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a^
2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(21a^{7}x^{7}+72a^{6}x^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{168\left(ax-1\right)^{2}\left(ax+1\right)^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$-\frac{\left(21a^{7}x^{7}+72a^{6}x^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{168\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x^
2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^5/((a*x-1)/(a*x+1))^(
3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int e^{3\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{7/2}\,dx = -\frac{\left(21\,a^{7}c^{3}x^{8}+72\,a^{6}c^{3}x^{7}+28\,a^{5}c^{3}x^{6}-168\,a^{4}c^{3}x^{5}-210\,a^{3}c^{3}x^{4}+56\,a^{2}c^{3}x^{3}+252\,ac^{3}x^{2}+168\,c^{3}x\right)\sqrt{-a^{2}c}}{168\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2*c
)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^9\sqrt{-c}c^3x^9+51\,a^8\sqrt{-c}c^3x^8-44\,a^7\sqrt{-c}c^3x^7-196\,a^6\sqrt{-c}c^3x^6-42\,a^5\sqrt{-c}c^3x^5+266\,a^4\sqrt{-c}c^3x^4+1\right.}{168\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 + 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 - 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 + 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 - 84*a^2*sqrt(-c)*c^3*x^2 - 168*sq
rt(-c)*c^3)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \int \frac{(-a^2cx^2+c)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.635 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] -2/5*(a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)
^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(c-a^2cx^2)^{5/2}\int(-1+ax)(1+ax)^4\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(c-a^2cx^2)^{5/2}\int\left(-2(1+ax)^4+(1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{2(1+ax)^5(c-a^2cx^2)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6(c-a^2cx^2)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(1+ax)^5(-7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
[Out] (c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{x\left(5a^{5}x^{5}+18a^{4}x^{4}+15a^{3}x^{3}-20a^{2}x^{2}-45ax-30\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}{30\left(ax+1\right)^{4}\left(ax-1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	84
default	$\frac{\left(5a^{5}x^{5}+18a^{4}x^{4}+15a^{3}x^{3}-20a^{2}x^{2}-45ax-30\right)x\,c^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{30\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	86

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+c
)^(5/2)/(a*x+1)^4/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^5c^2x^6+18\,a^4c^2x^5+15\,a^3c^2x^4-20\,a^2c^2x^3-45\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \frac{\left(5\,a^7\sqrt{-c}c^2x^7+13\,a^6\sqrt{-c}c^2x^6-3\,a^5\sqrt{-c}c^2x^5-35\,a^4\sqrt{-c}c^2x^4-25\,a^3\sqrt{-c}c^2x^3+15\,a^2\sqrt{\cdot}\right.}{30\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 + 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 - 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 + 15*a^2*sqrt(-c)*
c^2*x^2 + 30*sqrt(-c)*c^2)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.636 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\frac{(1+ax)^4(c-a^2cx^2)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] 1/4*(a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 32}

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\frac{(ax+1)^4(c-a^2cx^2)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c-a^2cx^2)^{3/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(c-a^2cx^2)^{3/2}\int(1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(1+ax)^4(c-a^2cx^2)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{c\sqrt{c-a^2cx^2}(4+6ax+4a^2x^2+a^3x^3)}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
[Out] -1/4*(c*Sqrt[c - a^2*c*x^2]*(4 + 6*a*x + 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1 -
1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{(ax-1)(ax+1)^2\sqrt{-c(a^2x^2-1)}\,c}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	48
gosper	$\frac{x(a^3x^3+4a^2x^2+6ax+4)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{4(ax+1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	60

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*c/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^3cx^4+4\,a^2cx^3+6\,acx^2+4\,cx\right)\sqrt{-a^2c}}{4\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
[Out] -1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(40) = 80.

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^5\sqrt{-c}cx^5+3\,a^4\sqrt{-c}cx^4+2\,a^3\sqrt{-c}cx^3-2\,a^2\sqrt{-c}cx^2-4\,\sqrt{-c}c\right)(ax+1)^2}{4\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 + 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 - 2*
a^2*sqrt(-c)*c*x^2 - 4*sqrt(-c)*c)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*
x - 1))
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.637 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(3+ax+\frac{4}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$
\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/(Sqrt[1 - 1/
(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x +8 \ln \left(a x -1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x -1\right)}{2 a \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	67

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{-a^2c}}{2\,a^2}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)

[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.638 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] 2*x*(1-1/a^2/x^2)^(1/2)/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(-a*x+1)*(1-1/a^2
/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{1+ax}{(-1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62

$$
\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(\frac{2}{1-ax}+\log(1-ax)\right)}{\sqrt{c-a^2cx^2}}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2/(1 - a*x) + Log[1 - a*x]))/Sqrt[c - a^2*c*x^2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-c(a^{2}x^{2}-1)}\,(a\ln(ax-1)x-\ln(ax-1)-2)}{ac(ax+1)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x-1)*x-ln(a*x-1)-2)/a/c/(a*x+1)^2/((a*x-1)/
(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{\sqrt{-a^2c}((ax-1)\log{(ax-1)}-2)}{a^3cx-a^2c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
[Out] -sqrt(-a^2*c)*((a*x - 1)*log(a*x - 1) - 2)/(a^3*c*x - a^2*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.639 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\,(c-a^2cx^2)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 32}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}} \\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}} \\
&= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)^3(1+ax)}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)^3*(1 + a
*x))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{ax-1}{2a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	39
default	$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)\left(a^{2}x^{2}-1\right)c^{2}a}$	56

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*x-1)/a/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{3/2}}\,dx=-\frac{\sqrt{-a^{2}c}}{2\left(a^{4}c^{2}x^{2}-2\,a^{3}c^{2}x+a^{2}c^{2}\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\frac{1}{2a^3c}+\frac{x}{2a^2c}\right)\sqrt{\frac{ax-1}{ax+1}}}{\frac{\sqrt{c-a^2cx^2}}{a^2}+x^2\sqrt{c-a^2cx^2}-\frac{2x\sqrt{c-a^2cx^2}}{a}}$$

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] ((1/(2*a^3*c) + x/(2*a^2*c))*((a*x - 1)/(a*x + 1))^(1/2))/((c - a^2*c*x^2)^
(1/2)/a^2 + x^2*(c - a^2*c*x^2)^(1/2) - (2*x*(c - a^2*c*x^2)^(1/2))/a)
```

3.640 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\,(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}}$$

```
[Out] 1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-
1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^
(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arc
tanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}} + \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}} + \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(1-ax)^3\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 - a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
```

```
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^4(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}\\
&\quad+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}$$

$$
= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5 \operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10+9ax-3a^2x^2+3(-1+ax)^3\operatorname{arctanh}(ax)\right)}{24c^2(-1+ax)^3\sqrt{c-a^2cx^2}}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[
a*x]))/(24*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\left(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-9a^2\ln(ax+1)x^2+9a^2\ln(ax-1)x^2-6a^2x^2+9a\ln(ax+1)x-9a\ln(ax-1)x+18a\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)\left(a^2x^2-1\right)c^3a}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3
*ln(a*x+1)*x^3-3*a^3*ln(a*x-1)*x^3-9*a^2*ln(a*x+1)*x^2+9*a^2*ln(a*x-1)*x^2-
6*a^2*x^2+9*a*ln(a*x+1)*x-9*a*ln(a*x-1)*x+18*a*x-3*ln(a*x+1)+3*ln(a*x-1)-20
)/(a^2*x^2-1)/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3-3a^3x^2+3a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(3a^2x^2-9ax+10\right)\sqrt{-a^2c}}{48\left(a^5c^3x^3-3a^4c^3x^2+3a^3c^3x-a^2c^3\right)}
$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/48*(3*(a^4*x^3 - 3*a^3*x^2 + 3*a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 - 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{1}{\left(c-a^2\,c\,x^2\right)^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.641 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx &= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\,(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\,(c-a^2cx^2)^{7/2}}\\ &\quad-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\,(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\,(c-a^2cx^2)^{7/2}}\\ &\quad+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)\,(c-a^2cx^2)^{7/2}}-\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{32\,(c-a^2cx^2)^{7/2}}\end{aligned}$$

```
[Out] -1/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x
^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x
^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(
-a^2*c*x^2+c)^(7/2)-5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x
^2+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{6327, 6328, 46, 213\}$

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\operatorname{arctanh}(ax)}{32\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\,(c-a^2cx^2)^{7/2}} + \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)\,(c-a^2cx^2)^{7/2}} - \frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(1-ax)^3\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)^4\,(c-a^2cx^2)^{7/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/16*(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/((1 - a*x)^4*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2))
 - (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2
)) - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
+ (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) -
(5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^5(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{4(-1+ax)^5}-\frac{1}{4(-1+ax)^4}+\frac{3}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{32(1+ax)^2}+\frac{5}{32(-1+a^2x^2)}\right)\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{\left(5a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{-1+a^2x^2}\,dx}{32\left(c-a^2cx^2\right)^{7/2}}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(32-15ax-35a^2x^2+45a^3x^3-15a^4x^4+15(-1+ax)^4(1+ax)\operatorname{arctanh}(ax)\right)}{96c^3(-1+ax)^4(1+ax)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)^4*(1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-45\ln(ax+1)x^4a^4+45\ln(ax-1)x^4a^4-30a^4x^4+30a^3\ln(ax+1)x^3-30a^3\ln(a}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5-45*ln(a*x+1)*x^4*a^4+45*ln(a*x-1)*x^4*a^4-30*a^4*x^4+30*a^3*ln(a*x+1)*x^3-30*a^3*ln(a*x-1)*x^3+90*a^3*x^3+30*a^2*ln(a*x+1)*x^2-30*a^2*ln(a*x-1)*x^2-70*a^2*x^2-45*a*ln(a*x+1)*x+45*a*ln(a*x-1)*x-30*a*x+15*ln(a*x+1)-15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{15\,(a^6x^5-3\,a^5x^4+2\,a^4x^3+2\,a^3x^2-3\,a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(15\,a^4x^4-45\,a^3x^3+3}{192\,(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4x+a^2c^4)}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")

[Out] -1/192*(15*(a^6*x^5 - 3*a^5*x^4 + 2*a^4*x^3 + 2*a^3*x^2 - 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^4*x^4 - 45*a^3*x^3 + 35*a^2*x^2 + 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")

[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

[In] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)

[Out] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

3.642 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] 8/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(-a*x
+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(-a*x+1)^8*(-a^2*
c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(-a*x+1)^9*(-a^2*c*x^2+c)^(
9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^1
0/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (8*(1 - a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^5(1+ax)^4\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(-1+ax)^5+32(-1+ax)^6+24(-1+ax)^7+8(-1+ax)^8+(-1+ax)^9\right)\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(-1+ax)^6\sqrt{c-a^2cx^2}(193+528ax+588a^2x^2+308a^3x^3+63a^4x^4)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
[Out] (c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a^
3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{(63a^9x^9-70a^8x^8-315a^7x^7+360a^6x^6+630a^5x^5-756a^4x^4-630a^3x^3+840a^2x^2+315ax-630)x\,c^4\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{630ax-630}$	113
gosper	$\frac{x(63a^9x^9-70a^8x^8-315a^7x^7+360a^6x^6+630a^5x^5-756a^4x^4-630a^3x^3+840a^2x^2+315ax-630)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{630(ax+1)^4(ax-1)^5}$	116

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/630*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*x^
4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)*((a*x-1
)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(63\,a^9c^4x^{10}-70\,a^8c^4x^9-315\,a^7c^4x^8+360\,a^6c^4x^7+630\,a^5c^4x^6-756\,a^4c^4x^5-630\,a^3c^4x^4}{630\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3 +
315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int{\left(c-a^2\,c\,x^2\right)}^{9/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.643 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] -8/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(-a*x+1)
^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(-a*x+1)^7*(-a^2*c*
x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c*x^2+c)^(7/2
)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-8*(1 - a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
```

```
(6*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{(c-a^2cx^2)^{7/2}\int(-1+ax)^4(1+ax)^3\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{(c-a^2cx^2)^{7/2}\int\left(8(-1+ax)^4+12(-1+ax)^5+6(-1+ax)^6+(-1+ax)^7\right)\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{8(1-ax)^5(c-a^2cx^2)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^6(c-a^2cx^2)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&\quad-\frac{6(1-ax)^7(c-a^2cx^2)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8(c-a^2cx^2)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(-1+ax)^5\sqrt{c-a^2cx^2}(93+185ax+135a^2x^2+35a^3x^3)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/280*(c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 +
35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{\left(35a^7x^7-40a^6x^6-140a^5x^5+168a^4x^4+210a^3x^3-280a^2x^2-140ax+280\right)x\,c^3\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{280(ax-1)}$	97
gosper	$\frac{x\left(35a^7x^7-40a^6x^6-140a^5x^5+168a^4x^4+210a^3x^3-280a^2x^2-140ax+280\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{280(ax-1)^4(ax+1)^3}$	100

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*x
^2-140*a*x+280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=$$
$$-\frac{(35\,a^7c^3x^8-40\,a^6c^3x^7-140\,a^5c^3x^6+168\,a^4c^3x^5+210\,a^3c^3x^4-280\,a^2c^3x^3-140\,ac^3x^2+280\,c^3x)\sqrt{-a^2}}{280\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.644 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] (-a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(-a*x+1)^5*
(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+1)^6*(-a^2*c*x^2
+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^3(1+ax)^2\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(-1+ax)^3+4(-1+ax)^4+(-1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.45

$$
\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(-1+ax)^4\left(11+14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}
$$

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
[Out] (c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sq
rt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30)x\,c^2\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{30ax-30}$	81
gosper	$\frac{x(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{30(ax-1)^3(ax+1)^2}$	84

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*x*c^2*(-c*(a^2*x
^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(5\,a^5c^2x^6-6\,a^4c^2x^5-15\,a^3c^2x^4+20\,a^2c^2x^3+15\,ac^2x^2-30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] 1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.645 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -2/3*(-a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(-a*x+
1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*(1 - a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c-a^2cx^2)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(c-a^2cx^2)^{3/2}\int(-1+ax)^2(1+ax)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(c-a^2cx^2)^{3/2}\int\left(2(-1+ax)^2+(-1+ax)^3\right)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= -\frac{2(1-ax)^3\,(c-a^2cx^2)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\,(c-a^2cx^2)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c(-1+ax)^3(5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
[Out] -1/12*(c*(-1 + a*x)^3*(5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{(3a^3x^3-4a^2x^2-6ax+12)xc\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{12(ax-1)}$	63
gosper	$\frac{x(3a^3x^3-4a^2x^2-6ax+12)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{12(ax+1)(ax-1)^2}$	68

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*x*c*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a
*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx)\sqrt{-a^2c}}{12\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] -1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.646 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6327, 6328}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2-2\,x)}{2\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.647 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] x*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 31}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{1}{1+ax}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\ln(ax+1)\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{c(ax-1)a}$	51

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/c/(a*x-1)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}\log(ax+1)}{a^2c}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
[Out] -sqrt(-a^2*c)*log(a*x + 1)/(a^2*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.648 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a^2*(1-1/a
^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)*(c - a^2*c*x^2)^(3/2)) - (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}} \\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}} \\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(-\frac{1}{2(1+ax)^2}+\frac{1}{2(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}} \\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}} \\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(-1+(1+ax)\text{arctanh}(ax))}{2(c+acx)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (1 + a*x)*ArcTanh[a*x]))/(2*(c + a*c*x)*Sqrt
[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x-a\ln(ax-1)x+\ln(ax+1)-\ln(ax-1)-2)}{4(a^2x^2-1)c^2a}$	84

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*x
-1)*x+ln(a*x+1)-ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\sqrt{-a^2c}}{4\,(a^3c^2x+a^2c^2)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*((a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) - 2*sqrt(-a^2*c))/(a^3*c^2*x + a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

3.649 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\,(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\,(c-a^2cx^2)^{5/2}}+\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}}$$

```
[Out] 1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/
a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1-1/a^2/x^2)^(5/
2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arctanh
(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\,(c-a^2cx^2)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(ax+1)\,(c-a^2cx^2)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) - (a^4
```

```
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (3*a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^2(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{8(-1+ax)^2}+\frac{1}{4(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{3}{8(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(2-3ax-3a^2x^2+3(-1+ax)(1+ax)^2\text{arctanh}(ax))}{8(-1+ax)(c+acx)^2\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2*
ArcTanh[a*x]))/(8*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3+3a^2\ln(ax+1)x^2-3a^2\ln(ax-1)x^2-6a^2x^2-3a\ln(ax+1)x+3a\ln(ax-1}{16(ax+1)(a^2x^2-1)c^3a(ax-1)}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+
1)*x^3-3*a^3*ln(a*x-1)*x^3+3*a^2*ln(a*x+1)*x^2-3*a^2*ln(a*x-1)*x^2-6*a^2*x^
2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x-6*a*x-3*ln(a*x+1)+3*ln(a*x-1)+4)/(a^2*x^2
-1)/c^3/a/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3+a^3x^2-a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(3a^2x^2+3ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/16*(3*(a^4*x^3 + a^3*x^2 - a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 + 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.650 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}} dx = -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}} + \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}} + \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}} + \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] -1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(
1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/24*a^6*(1-1/a^2/x^2)
^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/
(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a
^2*c*x^2+c)^(7/2)-5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2
+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{6327, 6328, 46, 213\}$

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\operatorname{arctanh}(ax)}{16\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\,(c-a^2cx^2)^{7/2}} + \frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\,(c-a^2cx^2)^{7/2}} + \frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\,(c-a^2cx^2)^{7/2}} + \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{24(ax+1)^3\,(c-a^2cx^2)^{7/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/32*(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/((1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) +
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)*(c - a^2*c*x^2)^(7/2))
- (5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2
))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{(c-a^2cx^2)^{7/2}} \\
&= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{(-1+ax)^3(1+ax)^4}\,dx}{(c-a^2cx^2)^{7/2}} \\
&= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int\left(\frac{1}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{8(1+ax)^4}-\frac{3}{16(1+ax)^3}-\frac{3}{16(1+ax)^2}+\frac{5}{16(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{7/2}} \\
&= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)(c-a^2cx^2)^{7/2}} \\
&\quad+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3(c-a^2cx^2)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2(c-a^2cx^2)^{7/2}} \\
&\quad+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)(c-a^2cx^2)^{7/2}}+\frac{\left(5a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{-1+a^2x^2}\,dx}{16(c-a^2cx^2)^{7/2}} \\
&= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)(c-a^2cx^2)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3(c-a^2cx^2)^{7/2}} \\
&\quad+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2(c-a^2cx^2)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)(c-a^2cx^2)^{7/2}}-\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16(c-a^2cx^2)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$
\int\frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-8+25ax+25a^2x^2-15a^3x^3-15a^4x^4+15(-1+ax)^2(1+ax)^3\text{arctanl}}{48(-1+ax)^2(c+acx)^3\sqrt{c-a^2cx^2}}
$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*(-1 + a*x)^2*(c + a*c*x)
^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+15\ln(ax+1)x^4a^4-15\ln(ax-1)x^4a^4-30a^4x^4-30a^3\ln(ax+1)x^3+3}{96(ax+1}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1
)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+15*ln(a*x+1)*x^4*a^4-15*ln(a*x-1)*x^4*a^4-30
*a^4*x^4-30*a^3*ln(a*x+1)*x^3+30*a^3*ln(a*x-1)*x^3-30*a^3*x^3-30*a^2*ln(a*x
+1)*x^2+30*a^2*ln(a*x-1)*x^2+50*a^2*x^2+15*a*ln(a*x+1)*x-15*a*ln(a*x-1)*x+5
0*a*x+15*ln(a*x+1)-15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{15\,(a^6x^5+a^5x^4-2\,a^4x^3-2\,a^3x^2+a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(15\,a^4x^4+15\,a^3x^3-25\,a}{96\,(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3c^4x+a^2c^4)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/96*(15*(a^6*x^5 + a^5*x^4 - 2*a^4*x^3 - 2*a^3*x^2 + a^2*x + a)*sqrt(-c)*
log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^4*
x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 + a^
6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

[In] integrate((((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")

[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

[In] integrate((((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")

[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2),x)

[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2), x)

3.651 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7(c-a^2cx^2)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

```
[Out] -7/24*c*x*(-a^2*c*x^2+c)^(3/2)-7/30*(-a^2*c*x^2+c)^(5/2)/a-1/6*(-a*x+1)*(-a
^2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a
-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6277, 685, 655, 201, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7(c-a^2cx^2)^{5/2}}{30a}$$

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(
c - a^2*c*x^2)^(5/2))/(30*a) - ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx\\
&= -\left(c\int (1-ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)\\
&= -\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int (1-ax)\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int \left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}\left(7c^2\right)\int \sqrt{c-a^2cx^2}\,dx\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}\\
&\quad-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{16}\left(7c^3\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}\\
&\quad-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{16}\left(7c^3\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}\\
&\quad-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}\left(96+39ax-327a^2x^2+202a^3x^3+86a^4x^4-136a^5x^5+40a^6x^6\right)\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202*
a^3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6)) + 210*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(40a^5x^5-96a^4x^4-10a^3x^3+192a^2x^2-135ax-96)(a^2x^2-1)c^3}{240a\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^3}{16\sqrt{a^2c}}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}-\frac{2\left(\frac{\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}{5}+ac\left(\right.\right.}{}$

[In] int((a*x-1)*(-a^2*c*x^2+c)^(5/2)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] -1/240*(40*a^5*x^5-96*a^4*x^4-10*a^3*x^3+192*a^2*x^2-135*a*x-96)*(a^2*x^2-1
)/a/(-c*(a^2*x^2-1))^(1/2)*c^3-7/16/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-
a^2*c*x^2+c)^(1/2))*c^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.84

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx=\left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40a^5c^2x^5-96a^4c^2x^4-10a^3c^2\right.}{480a}\right.$$

[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^
2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 -
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(119) = 238$.

Time = 2.74 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.40

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \begin{cases} \frac{2c^2\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-2c^2\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{fo}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{ot}\end{cases}\right.}{} \\ -c^{\frac{5}{2}}x \end{cases}$$

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise(((2*c**2*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), N
e(c, 0)), (a**2*sqrt(c)*x**2/2, True)) - 2*c**2*Piecewise((sqrt(-a**2*c*x**
2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x**4/4
, True)) + c**2*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**
2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True)
)/16 + sqrt(-a**2*c*x**2 + c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16), Ne(c,
0)), (a**5*sqrt(c)*x**5/5, True)) - c**2*Piecewise((a*x*sqrt(-a**2*c*x**2 +
 c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt
(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0)), (a*
sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= \frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x-\frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx \\ &\quad -\frac{3}{4}\sqrt{a^2cx^2+4\,acx+3\,c}c^2x+\frac{5}{16}\sqrt{-a^2cx^2+c}c^2x+\frac{3\,c^4\arcsin\left(ax+2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}} \\ &\quad +\frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a}-\frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5\,a}-\frac{3\sqrt{a^2cx^2+4\,acx+3\,c}c^2}{2\,a}\end{aligned}$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(a
^2*c*x^2 + 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x + 3/4*c^4
*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a - 2/5*(-a^2*c*
x^2 + c)^(5/2)/a - 3/2*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^2/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}|a|}$$
$$-\frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(135\,c^2-2\left(96\,ac^2-\left(5\,a^2c^2-4\left(5\,a^4c^2x-12\,a^3c^2\right)x\right)x\right)x\right)x+\frac{96\,c^2}{a}\right)$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 - 2*(96*a*c^2 - (5*a^2*c^2 - 4*(5*a
^4*c^2*x - 12*a^3*c^2)*x)*x)*x)*x + 96*c^2/a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.652 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

```
[Out] -5/12*(-a^2*c*x^2+c)^(3/2)/a-1/4*(-a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2
)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6277, 685, 655, 201, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 - a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$= -\left(c\int (1-ax)^2\sqrt{c-a^2cx^2}\,dx\right)$$

$$= -\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} - \frac{1}{4}(5c)\int (1-ax)\sqrt{c-a^2cx^2}\,dx$$

$$= -\frac{5(c-a^2cx^2)^{3/2}}{12a} - \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} - \frac{1}{4}(5c)\int \sqrt{c-a^2cx^2}\,dx$$

$$= -\frac{5}{8}cx\sqrt{c-a^2cx^2} - \frac{5(c-a^2cx^2)^{3/2}}{12a} - \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} - \frac{1}{8}\left(5c^2\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$

$$= -\frac{5}{8}cx\sqrt{c-a^2cx^2} - \frac{5(c-a^2cx^2)^{3/2}}{12a} - \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} - \frac{1}{8}\left(5c^2\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)$$

$$= -\frac{5}{8}cx\sqrt{c-a^2cx^2} - \frac{5(c-a^2cx^2)^{3/2}}{12a} - \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} - \frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(-16+7ax+25a^2x^2-22a^3x^3+6a^4x^4)+30\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqr
t[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

method	result
risch	$\frac{(6a^3x^3-16a^2x^2+9ax+16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}}-\frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}-\frac{2\left(\frac{\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3}+ac\left(-\frac{\left(-2a^2c\left(x+\frac{1}{a}\right)+2ac\right)\sqrt{-}}{4a}\right.\right.}{}$

[In] int((a*x-1)*(-a^2*c*x^2+c)^(3/2)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/24*(6*a^3*x^3-16*a^2*x^2+9*a*x+16)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c^2-5/8/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^2

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.67

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(6a^3cx^3-16a^2cx^2+9acx+16c\right.}{48a}\right.$$

[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/a]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(97) = 194$.

Time = 2.41 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.30

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx = \begin{cases} \frac{2c\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(-2acx+2\sqrt{-c}\sqrt{-a^2cx^2+c}\right)}{\sqrt{-c}} & \text{for } c\neq 0\\ \frac{ax\log(ax)}{\sqrt{-a^2cx^2}} & \text{otherwis}\end{cases}\right)}{8} \\ \frac{a^3\sqrt{c}x^3}{3}\end{cases}\right)}{} \\ -c^{\frac{3}{2}}x \end{cases}$$

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Piecewise(((2*c*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), Ne(c
, 0)), (a**2*sqrt(c)*x**2/2, True)) - c*Piecewise((c*Piecewise((log(-2*a*c*
x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/s
qrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c*Piecewise((a*x*sqrt(-a**2*c*x*
*2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/
sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0)),
 (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(3/2)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx &= \frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x \\ &- \sqrt{a^2cx^2+4\,acx+3\,c}cx + \frac{3}{8}\sqrt{-a^2cx^2+c}cx + \frac{c^3\arcsin(ax+2)}{a(-c)^{\frac{3}{2}}} \\ &+ \frac{3\,c^{\frac{3}{2}}\arcsin(ax)}{8\,a} - \frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a} - \frac{2\sqrt{a^2cx^2+4\,acx+3\,c}c}{a}\end{aligned}$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c*x + 3/8*sq
rt(-a^2*c*x^2 + c)*c*x + c^3*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)*a
rcsin(a*x)/a - 2/3*(-a^2*c*x^2 + c)^(3/2)/a - 2*sqrt(a^2*c*x^2 + 4*a*c*x +
3*c)*c/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx-8\,ac\right)x+9\,c\right)x+\frac{16\,c}{a}\right) + \frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x - 8*a*c)*x + 9*c)*x + 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.653 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^
(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6277, 685, 655, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right) \\
&= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx
\end{aligned}$$

$$= -\frac{3\sqrt{c-a^2cx^2}}{2a} - \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} - \frac{1}{2}(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)$$

$$= -\frac{3\sqrt{c-a^2cx^2}}{2a} - \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} - \frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(4-5ax+a^2x^2)+6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a
*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)}{a}$	127

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*x-4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*arc
tan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$= \left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a}, \frac{\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{}}{2a}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}x - \frac{3\sqrt{c}\arcsin(ax)}{2a} - \frac{2\sqrt{-a^2cx^2+c}}{a}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
+ c)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right) + \frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.654 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*(-a*x+1)/a/(-a^2*c*x^2
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6277, 667, 223, 209}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{3/2}}\,dx\right)\\
&= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\left((-1+ax)\sqrt{1+ax}+\sqrt{1-ax}(1+ax)\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}(1+ax)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*A
rcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*x^
2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}{a^2c\left(x+\frac{1}{a}\right)}$	73

```
[In] int((a*x-1)/(-a^2*c*x^2+c)^(1/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x+1/a
)*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.52

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \left[-\frac{(ax+1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx+ac)}, -\frac{(ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx+ac}\right]$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), -((a*x + 1)*sqrt(c)*ar
```

```
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x + a*c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{ax-1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral((a*x - 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{-a^2cx^2+c}}{a^2cx+ac} + \frac{\arcsin(ax)}{a\sqrt{c}}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x + a*c) + arcsin(a*x)/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{a\,x-1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}\,dx$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)), x)
```

3.655 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{2(1-ax)}{3a\,(c-a^2cx^2)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*(-a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6277, 667, 197}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{2(1-ax)}{3a\,(c-a^2cx^2)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
[Out] (2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
```

```
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\right)\\
&= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{\sqrt{1-ax}(2+ax)\sqrt{1-a^2x^2}}{3ac(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
[Out] (Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c -
 a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{(ax-1)^{2}(ax+2)}{3a(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}}$	31
trager	$\frac{(ax+2)\sqrt{-a^{2}c\,x^{2}+c}}{3c^{2}(ax+1)^{2}a}$	34
default	$\frac{x}{c\sqrt{-a^{2}c\,x^{2}+c}}-\frac{2\left(-\frac{1}{3ac\left(x+\frac{1}{a}\right)\sqrt{-a^{2}c\left(x+\frac{1}{a}\right)^{2}+2\left(x+\frac{1}{a}\right)ac}}-\frac{-2a^{2}c\left(x+\frac{1}{a}\right)+2ac}{3a\,c^{2}\sqrt{-a^{2}c\left(x+\frac{1}{a}\right)^{2}+2\left(x+\frac{1}{a}\right)ac}}\right)}{a}$	115

```
[In] int((a*x-1)/(-a^2*c*x^2+c)^(3/2)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] 1/3*(a*x-1)^2*(a*x+2)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(ax+2)}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{ax-1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{x}{3\sqrt{-a^2cx^2+c}c}+\frac{2}{3\left(\sqrt{-a^2cx^2+c}a^2cx+\sqrt{-a^2cx^2+c}ac\right)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x + sqrt(
-a^2*c*x^2 + c)*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(43) = 86.

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(ac+3\sqrt{-a^2c}\sqrt{c}\right)\operatorname{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}+\sqrt{-a^2c}ac^2\right)} + \frac{2\left(2a^2c-3a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}-\sqrt{-a^2c+\frac{c}{x^2}}+\frac{\sqrt{c}}{x}\right)^3 c\operatorname{sgn}(x)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] -1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) + sqrt(-a^2*c)*a*c^
2) + 2/3*(2*a^2*c - 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqr
t(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) - sqrt(-a^2*c + c/x^2) + sqrt
(c)/x)^3*c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+2)}{3\,a\,c^2\,(a\,x+1)^2}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x + 2))/(3*a*c^2*(a*x + 1)^2)
```

3.656 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

```
[Out] 2/5*(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2/
(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6277, 667, 198, 197}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}+\frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2)) -
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{7/2}}\,dx\right)\\
&= \frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{3}{5}\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx\\
&= \frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{5c}\\
&= \frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{1-a^2x^2}(-2+ax+4a^2x^2+2a^3x^3)}{5ac^2\sqrt{1-ax}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
[Out] -1/5*(Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(a*c^2*Sqrt[1 -
 a*x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{(ax-1)^2(2a^3x^3+4a^2x^2+ax-2)}{5a(-a^2c\,x^2+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^3x^3+4a^2x^2+ax-2)\sqrt{-a^2c\,x^2+c}}{5c^3(ax+1)^3a(ax-1)}$
default	$\frac{x}{3c(-a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{-a^2c\,x^2+c}}-\frac{2\left(-\frac{1}{5ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}+\frac{4a\left(-\frac{-2a^2c\left(x+\frac{1}{a}\right)+2ac}{6a^2c^2\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\cdots\right.}{5}\right.}{a}$

```
[In] int((a*x-1)/(-a^2*c*x^2+c)^(5/2)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/a/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3+4\,a^2x^2+ax-2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2
*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax-1}{(-c(ax-1)(ax+1))^{\frac{5}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx+(-a^2cx^2+c)^{\frac{3}{2}}ac\right)} - \frac{2x}{5\sqrt{-a^2cx^2+c}c^2} - \frac{x}{5(-a^2cx^2+c)^{\frac{3}{2}}c}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] 2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x + (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/(
sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{5}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(2\,a^3\,x^3+4\,a^2\,x^2+a\,x-2)}{5\,a\,c^3\,\left(a\,x-1\right)\,\left(a\,x+1\right)^3}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)),x)
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x + 4*a^2*x^2 + 2*a^3*x^3 - 2))/(5*a*c^3*(a*x - 1
)*(a*x + 1)^3)
```

3.657 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2(1-ax)}{7a(c-a^2cx^2)^{7/2}} - \frac{x}{7c(c-a^2cx^2)^{5/2}} - \frac{4x}{21c^2(c-a^2cx^2)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

```
[Out] 2/7*(-a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^2
/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6277, 667, 198, 197}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{8x}{21c^3\sqrt{c-a^2cx^2}} - \frac{4x}{21c^2(c-a^2cx^2)^{3/2}} - \frac{x}{7c(c-a^2cx^2)^{5/2}} + \frac{2(1-ax)}{7a(c-a^2cx^2)^{7/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2)) -
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{9/2}}\,dx\right)\\
&= \frac{2(1-ax)}{7a\,(c-a^2cx^2)^{7/2}} - \frac{5}{7}\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx\\
&= \frac{2(1-ax)}{7a\,(c-a^2cx^2)^{7/2}} - \frac{x}{7c\,(c-a^2cx^2)^{5/2}} - \frac{4\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{7c}\\
&= \frac{2(1-ax)}{7a\,(c-a^2cx^2)^{7/2}} - \frac{x}{7c\,(c-a^2cx^2)^{5/2}} - \frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}} - \frac{8\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{21c^2}
\end{aligned}$$

$$= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\sqrt{1-a^2x^2}(-6+9ax+24a^2x^2+4a^3x^3-16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{3/2}(1+ax)^{7/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(-6 + 9*a*x + 24*a^2*x^2 + 4*a^3*x^3 - 16*a^4*x^4
- 8*a^5*x^5))/(a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result
gosper	$\frac{(ax-1)^2(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)}{21(-a^2c\,x^2+c)^{\frac{7}{2}}a}$
trager	$\frac{(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)\sqrt{-a^2c\,x^2+c}}{21c^4(ax+1)^4(ax-1)^2a}$
default	$\frac{x}{5c(-a^2c\,x^2+c)^{\frac{5}{2}}} + \frac{\frac{4x}{15c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2c\,x^2+c}}}{c} - \frac{2\left(-\frac{1}{7ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}+\frac{6a\left(-\frac{-2a^2c\left(x+\right.}{10a^2c^2\left(-a^2c\left(x+\frac{1}{a}\right)\right.}\right.}{}\right.}{}$

```
[In] int((a*x-1)/(-a^2*c*x^2+c)^(7/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/(-a^2*c*
x^2+c)^(7/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{(8\,a^5x^5+16\,a^4x^4-4\,a^3x^3-24\,a^2x^2-9\,ax+6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a
^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax-1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}\,(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx+(-a^2cx^2+c)^{\frac{5}{2}}ac\right)} - \frac{8\,x}{21\,\sqrt{-a^2cx^2+c}c^3} - \frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2} - \frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x + (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x/
(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/((
-a^2*c*x^2 + c)^(5/2)*c)
```

Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{ax-1}{(-a^2cx^2+c)^{\frac{7}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x+1)^3}+\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x+1)^4}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}-\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,(a\,x-1)\,(a\,x+1)}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)),x)
[Out] (c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x + 1)^3) + (c - a^2*c*x^2)^(1/2)/(28*a*
c^4*(a*x + 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) - 5/(28*a*c^4)))
/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

3.658 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx=\frac{2(1-ax)}{9a(c-a^2cx^2)^{9/2}}-\frac{x}{9c(c-a^2cx^2)^{7/2}}-\frac{2x}{15c^2(c-a^2cx^2)^{5/2}}-\frac{8x}{45c^3(c-a^2cx^2)^{3/2}}-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

```
[Out] 2/9*(-a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^2
/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6277, 667, 198, 197}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx=-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3(c-a^2cx^2)^{3/2}}-\frac{2x}{15c^2(c-a^2cx^2)^{5/2}}-\frac{x}{9c(c-a^2cx^2)^{7/2}}+\frac{2(1-ax)}{9a(c-a^2cx^2)^{9/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (2*(1 - a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2)) -
 (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2)
) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{11/2}}\,dx\right) \\
&= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{7}{9}\int \frac{1}{(c-a^2cx^2)^{9/2}}\,dx \\
&= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx}{3c} \\
&= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{15c^2}
\end{aligned}
$$

$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{45c^3}$$

$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = \frac{\sqrt{1-a^2x^2}\left(10-25ax-60a^2x^2+10a^3x^3+80a^4x^4+24a^5x^5-32a^6x^6-16a^7x^7\right)}{45ac^4(1-ax)^{5/2}(1+ax)^{9/2}\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(10 - 25*a*x - 60*a^2*x^2 + 10*a^3*x^3 + 80*a^4*x^4 + 24
*a^5*x^5 - 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^(9
/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

method	result
gosper	$-\frac{(ax-1)^2(16a^7x^7+32a^6x^6-24a^5x^5-80a^4x^4-10a^3x^3+60a^2x^2+25ax-10)}{45a(-a^2c\,x^2+c)^{\frac{9}{2}}}$
trager	$\frac{(16a^7x^7+32a^6x^6-24a^5x^5-80a^4x^4-10a^3x^3+60a^2x^2+25ax-10)\sqrt{-a^2c\,x^2+c}}{45c^5(ax+1)^5(ax-1)^3a}$
default	$\frac{x}{7c(-a^2c\,x^2+c)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2c\,x^2+c}}\right)}{7c}}{c}-\frac{2\left(-\frac{1}{9ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{7}{2}}}+\cdots\right.}{\cdots}$

[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/45*(a*x-1)^2*(16*a^7*x^7+32*a^6*x^6-24*a^5*x^5-80*a^4*x^4-10*a^3*x^3+60*a^2*x^2+25*a*x-10)/a/(-a^2*c*x^2+c)^(9/2)

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{(16\,a^7x^7+32\,a^6x^6-24\,a^5x^5-80\,a^4x^4-10\,a^3x^3+60\,a^2x^2+25\,ax-10)\sqrt{-a^2cx^2+}}{45\,(a^9c^5x^8+2\,a^8c^5x^7-2\,a^7c^5x^6-6\,a^6c^5x^5+6\,a^4c^5x^3+2\,a^3c^5x^2-2\,a^2c^5x-ac^5)}$$

[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")

[Out] 1/45*(16*a^7*x^7 + 32*a^6*x^6 - 24*a^5*x^5 - 80*a^4*x^4 - 10*a^3*x^3 + 60*a^2*x^2 + 25*a*x - 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 + 2*a^8*c^5*x^7 - 2*a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 - 2*a^2*c^5*x - a*c^5)

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{9}{2}}\,(ax+1)}\,dx$$

[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)

[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx+(-a^2cx^2+c)^{\frac{7}{2}}ac\right)} - \frac{16\,x}{45\sqrt{-a^2cx^2+c}c^4} - \frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3} - \frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2} - \frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")

[Out] 2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x + (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*x /(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*x/ ((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)

Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{9}{2}}(ax+1)}\,dx$$

[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")

[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)), x)

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{5\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x+1)^4}+\frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x+1)^5}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}-\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}+\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,(a\,x-1)\,(a\,x+1)}$$

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(9/2)*(a*x + 1)),x)
```

```
[Out] (5*(c - a^2*c*x^2)^(1/2))/(144*a*c^5*(a*x + 1)^4) + (c - a^2*c*x^2)^(1/2)/(
72*a*c^5*(a*x + 1)^5) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) - 5/(24*a*
c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5) +
 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2))/(
45*c^5*(a*x - 1)*(a*x + 1))
```

3.659 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] -8/7*(-a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(-a*x
+1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(-a*x+1)^9*(-a^
2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+
c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-8*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
```

```
9) - (2*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9} \\
&= \frac{(c-a^2cx^2)^{9/2}\int(-1+ax)^6(1+ax)^3\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9} \\
&= \frac{(c-a^2cx^2)^{9/2}\int\left(8(-1+ax)^6+12(-1+ax)^7+6(-1+ax)^8+(-1+ax)^9\right)\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9} \\
&= -\frac{8(1-ax)^7\,(c-a^2cx^2)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\,(c-a^2cx^2)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9} \\
&\quad -\frac{2(1-ax)^9\,(c-a^2cx^2)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\,(c-a^2cx^2)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(-1+ax)^7\sqrt{c-a^2cx^2}(44+98ax+77a^2x^2+21a^3x^3)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]

[Out] (c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x^3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x(21a^9x^9-70a^8x^8+240a^6x^6-210a^5x^5-252a^4x^4+420a^3x^3-315ax+210)(-a^2cx^2+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210(ax+1)^3(ax-1)^6}$	100
default	$\frac{(21a^9x^9-70a^8x^8+240a^6x^6-210a^5x^5-252a^4x^4+420a^3x^3-315ax+210)x\,c^4\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210(ax-1)^2}$	102

[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3*x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^3/(a*x-1)^6

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(21\,a^9c^4x^{10}-70\,a^8c^4x^9+240\,a^6c^4x^7-210\,a^5c^4x^6-252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2}{210\,a}$$

[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a^2*c)/a

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(21\,a^{11}\sqrt{-c}c^4x^{11}-49\,a^{10}\sqrt{-c}c^4x^{10}-70\,a^9\sqrt{-c}c^4x^9+240\,a^8\sqrt{-c}c^4x^8+30\,a^7\sqrt{-c}c^4x^7-\cdots\right.}{210\,(\cdots}$$

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 - 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 + 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 - 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 + 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 - 105*a^2*sqrt(-c)*c^4*x^2 - 210*sqrt(-c)*c^4)*(a*x
- 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{9/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.660 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] 2/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(-a*x+1
)^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c
*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-a^2cx^2)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{(c-a^2cx^2)^{7/2}\int(-1+ax)^5(1+ax)^2\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{(c-a^2cx^2)^{7/2}\int\left(4(-1+ax)^5+4(-1+ax)^6+(-1+ax)^7\right)\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{2(1-ax)^6(c-a^2cx^2)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7(c-a^2cx^2)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8(c-a^2cx^2)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(-1+ax)^6\left(37+54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
[Out] -1/168*(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x\left(21a^{7}x^{7}-72a^{6}x^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax+1\right)^{2}\left(ax-1\right)^{5}}$	100
default	$-\frac{\left(21a^{7}x^{7}-72a^{6}x^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax-1\right)^{2}}$	102

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x^
2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-
1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^7c^3x^8-72\,a^6c^3x^7+28\,a^5c^3x^6+168\,a^4c^3x^5-210\,a^3c^3x^4-56\,a^2c^3x^3+252\,ac^3x^2-168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2*c
)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.21

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^9\sqrt{-c}c^3x^9-51\,a^8\sqrt{-c}c^3x^8-44\,a^7\sqrt{-c}c^3x^7+196\,a^6\sqrt{-c}c^3x^6-42\,a^5\sqrt{-c}c^3x^5-266\,a^4\sqrt{-c}c^3x^4+\right.}{168\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 - 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 + 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 - 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 + 84*a^2*sqrt(-c)*c^3*x^2 + 168*sq
rt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \int \left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \int \left(c-a^2\,c\,x^2\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.661 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] -2/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+
1)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^4(1+ax)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(2(-1+ax)^4+(-1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$
\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(-1+ax)^5(7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}
$$

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
[Out] (c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{x\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30\left(ax-1\right)^{4}\left(ax+1\right)}$	84
default	$\frac{\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)x\,c^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30\left(ax-1\right)^{2}}$	86

[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^4/(a*x+1)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{5/2}\,dx=\frac{\left(5\,a^{5}c^{2}x^{6}-18\,a^{4}c^{2}x^{5}+15\,a^{3}c^{2}x^{4}+20\,a^{2}c^{2}x^{3}-45\,ac^{2}x^{2}+30\,c^{2}x\right)\sqrt{-a^{2}c}}{30\,a}$$

[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 45*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{5/2}\,dx=\text{Timed out}$$

[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^7\sqrt{-c}c^2x^7-13\,a^6\sqrt{-c}c^2x^6-3\,a^5\sqrt{-c}c^2x^5+35\,a^4\sqrt{-c}c^2x^4-25\,a^3\sqrt{-c}c^2x^3-15\,a^2\sqrt{-c}\right.}{30\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 - 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 + 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 - 15*a^2*sqrt(-c)*
c^2*x^2 - 30*sqrt(-c)*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.662 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] 1/4*(-a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 32}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c-a^2cx^2)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(c-a^2cx^2)^{3/2}\int(-1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(1-ax)^4(c-a^2cx^2)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c\sqrt{c-a^2cx^2}(-4+6ax-4a^2x^2+a^3x^3)}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
[Out] -1/4*(c*Sqrt[c - a^2*c*x^2]*(-4 + 6*a*x - 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1 -
 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(ax-1)^2\sqrt{-c(a^2x^2-1)}\,c}{4a}$	48
gosper	$\frac{x(a^3x^3-4a^2x^2+6ax-4)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4(ax-1)^3}$	60

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*c/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^3cx^4-4\,a^2cx^3+6\,acx^2-4\,cx\right)\sqrt{-a^2c}}{4\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
[Out] -1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(40) = 80.

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^5\sqrt{-c}cx^5-3\,a^4\sqrt{-c}cx^4+2\,a^3\sqrt{-c}cx^3+2\,a^2\sqrt{-c}cx^2+4\,\sqrt{-c}c\right)(ax-1)^2}{4\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 - 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 + 2*
a^2*sqrt(-c)*c*x^2 + 4*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1))
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.663 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3+ax+\frac{4}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x+8 \ln \left(a x+1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{2 a \left(a x-1\right)^{2}}$	67

[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^2

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2-6\,ax+8\,\log{(ax+1)})\sqrt{-a^2c}}{2\,a^2}$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.664 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] 2*x*(1-1/a^2/x^2)^(1/2)/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(a*x+1)*(1-1/a^2/x
^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(ax+1)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{-1+ax}{(1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \left(-\frac{2}{(1+ax)^2}+\frac{1}{1+ax}\right)\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(\frac{2}{1+ax}+\log(1+ax)\right)}{\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2/(1 + a*x) + Log[1 + a*x]))/Sqrt[c - a^2*c*x^2]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(a \ln \left(a x +1\right) x +\ln \left(a x +1\right)+2\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{a c \left(a x -1\right)^{2}}$	62

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x+ln(a*x+1)+2)*((a*x-1)/(a*x+1))^(3/2)
/a/c/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}((ax+1)\log{(ax+1)}+2)}{a^3cx+a^2c}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
[Out] -sqrt(-a^2*c)*((a*x + 1)*log(a*x + 1) + 2)/(a^3*c*x + a^2*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.665 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 32}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)(1+ax)^3}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)*(1 + a*x
)^3)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}}$	39
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c(a^{2}x^{2}-1)}}{2(ax-1)(a^{2}x^{2}-1)a\,c^{2}}$	56

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}}{2\,(a^4c^2x^2+2\,a^3c^2x+a^2c^2)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,c\,\left(x\,\sqrt{c-a^2\,c\,x^2}+\frac{\sqrt{c-a^2\,c\,x^2}}{a}\right)}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(2*a^2*c*(x*(c - a^2*c*x^2)^(1/2) + (c - a^2*c*
x^2)^(1/2)/a))
```

3.666 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1
/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5
/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arctan
h(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(ax+1)^3\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 + a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
```

```
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)(1+ax)^4}\,dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{2(1+ax)^4}-\frac{1}{4(1+ax)^3}-\frac{1}{8(1+ax)^2}+\frac{1}{8(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}} \\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} \\
&\quad+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

$$
= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

$$
\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10-9ax-3a^2x^2+3(1+ax)^3\operatorname{arctanh}(ax)\right)}{24c^2(1+ax)^3\sqrt{c-a^2cx^2}}
$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
[Out] -1/24*(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*Arc
Tanh[a*x]))/(c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3+9a^2\ln(ax+1)x^2-9a^2\ln(ax-1)x^2-6a^2x^2+9a\ln(ax+1)x-9a\ln(ax-1)x}{48(ax+1)(ax-1)(a^2x^2-1)c^3a}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*
ln(a*x+1)*x^3-3*a^3*ln(a*x-1)*x^3+9*a^2*ln(a*x+1)*x^2-9*a^2*ln(a*x-1)*x^2-6
*a^2*x^2+9*a*ln(a*x+1)*x-9*a*ln(a*x-1)*x-18*a*x+3*ln(a*x+1)-3*ln(a*x-1)-20)
/(a^2*x^2-1)/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$
\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3+3a^3x^2+3a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(3a^2x^2+9ax+10\right)\sqrt{-a^2c}}{48\left(a^5c^3x^3+3a^4c^3x^2+3a^3c^3x+a^2c^3\right)}
$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/48*(3*(a^4*x^3 + 3*a^3*x^2 + 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 + 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.667 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 275

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}} + \frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] 1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/16*a^6*(1-
1/a^2/x^2)^(7/2)*x^7/(a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*(1-1/a^2/x^2)^
(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(
a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a^2
*c*x^2+c)^(7/2)+5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2+c
)^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{6327, 6328, 46, 213\}$

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\operatorname{arctanh}(ax)}{32\,(c-a^2cx^2)^{7/2}} + \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\,(c-a^2cx^2)^{7/2}} - \frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(ax+1)^3\,(c-a^2cx^2)^{7/2}} - \frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)^4\,(c-a^2cx^2)^{7/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)^4*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) - (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) -
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (5*
a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^2(1+ax)^5}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{32(-1+ax)^2}+\frac{1}{4(1+ax)^5}+\frac{1}{4(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{32(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{\left(5a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{-1+a^2x^2}\,dx}{32\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}} + \frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\text{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(32+15ax-35a^2x^2-45a^3x^3-15a^4x^4+15(-1+ax)(1+ax)^4\text{arctanh}(ax)\right)}{96c^3(-1+ax)(1+ax)^4\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
[Out] -1/96*(Sqrt[1 - 1/(a^2*x^2)]*x*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15*
a^4*x^4 + 15*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(c^3*(-1 + a*x)*(1 + a*x
)^4*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+45\ln(ax+1)x^4a^4-45\ln(ax-1)x^4a^4-30a^4x^4+30a^3\ln(ax+1)x^3-30a^3}{192(ax+1)^2(}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(1
5*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+45*ln(a*x+1)*x^4*a^4-45*ln(a*x-1)*
x^4*a^4-30*a^4*x^4+30*a^3*ln(a*x+1)*x^3-30*a^3*ln(a*x-1)*x^3-90*a^3*x^3-30*
a^2*ln(a*x+1)*x^2+30*a^2*ln(a*x-1)*x^2-70*a^2*x^2-45*a*ln(a*x+1)*x+45*a*ln(
a*x-1)*x+30*a*x-15*ln(a*x+1)+15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{15\left(a^6x^5+3\,a^5x^4+2\,a^4x^3-2\,a^3x^2-3\,a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(15\,a^4x^4+45\,a^3x^3+35\right.}{192\left(a^7c^4x^5+3\,a^6c^4x^4+2\,a^5c^4x^3-2\,a^4c^4x^2-3\,a^3c^4x-a^2c^4\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/192*(15*(a^6*x^5 + 3*a^5*x^4 + 2*a^4*x^3 - 2*a^3*x^2 - 3*a^2*x - a)*sqrt
(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15
*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^
5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2), x)
```

3.668 $\int e^{\coth^{-1}(ax)} x^2 \sqrt{c - a^2 c x^2}\, dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int e^{\coth^{-1}(ax)} x^2 \sqrt{c - a^2 c x^2}\, dx = \frac{x^2\sqrt{c - a^2 c x^2}}{3a\sqrt{1 - \frac{1}{a^2x^2}}} + \frac{x^3\sqrt{c - a^2 c x^2}}{4\sqrt{1 - \frac{1}{a^2x^2}}}$$

```
[Out] 1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^(
1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)} x^2 \sqrt{c - a^2 c x^2}\, dx = \frac{x^2\sqrt{c - a^2 c x^2}}{3a\sqrt{1 - \frac{1}{a^2x^2}}} + \frac{x^3\sqrt{c - a^2 c x^2}}{4\sqrt{1 - \frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c
*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x^2(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$
\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2(4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}
$$

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
[Out] (x^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x^{3}(3ax+4)\sqrt{-a^{2}c\,x^{2}+c}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{(3ax+4)x^{3}\sqrt{-c(a^{2}x^{2}-1)}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	48

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{(3\,ax^4+4\,x^3)\sqrt{-a^2c}}{12\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/12*(3*a*x^4 + 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.669 $\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/
2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x
^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x(3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
[Out] (x*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x^{2}(2ax+3)\sqrt{-a^{2}c\,x^{2}+c}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{(2ax+3)x^{2}\sqrt{-c(a^{2}x^{2}-1)}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	48

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,ax^3+3\,x^2)\sqrt{-a^2c}}{6\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+cx}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)

Giac [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+cx}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{x\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

[In] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)

3.670 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6327, 6328}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c(a^2x^2-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2+2\,x)}{2\,a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.671 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 45}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
[Out] (Sqrt[c - a^2*c*x^2]*(x + Log[x]/a))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{(ax+\ln(x))\sqrt{-c(a^2x^2-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

```
[Out] (a*x+ln(x))*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.26

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax+\log(x))}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + log(x))/a
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.672 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 45}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) + (Sqrt[c - a^2*c*x^2]
*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(1/(a*x)) + Log[x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{-c(a^2x^2-1)}}{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}}$	48

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

```
[Out] (a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{-a^2c}(ax\log{(x)}-1)}{ax}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) - 1)/(a*x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.673 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] -3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x
^2+c)^(1/2)/a^2+1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2)
+3/20*(5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6286, 1823, 847, 794, 223, 209}

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}+\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) - (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}x^3\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x^3(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x^3(-9a^2c-10a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}\\
&= \frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x^2(30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x(-72a^4c^3-90a^5c^3x)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}\\
&\quad+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{4a^3}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}\\
&\quad+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{4a^3}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}\\
&\quad+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}(24+15ax+12a^2x^2+10a^3x^3+4a^4x^4)+15\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{20a^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)(a^2x^2-1)c}{20a^4\sqrt{-c(a^2x^2-1)}}-\frac{3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{4a^3\sqrt{a^2c}}$
default	$-\frac{x^2(-a^2c\,x^2+c)^{\frac{3}{2}}}{5a^2c}-\frac{4(-a^2c\,x^2+c)^{\frac{3}{2}}}{5c\,a^4}+\frac{x\sqrt{-a^2c\,x^2+c}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}}{a^3}+\frac{-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{2a^2c}+\frac{\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan}{}}{2a^2}}{a}$

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(4*a^4*x^4+10*a^3*x^3+12*a^2*x^2+15*a*x+24)*(a^2*x^2-1)/a^4/(-c*(a^2*
x^2-1))^(1/2)*c-3/4/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2))*c
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\left(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24\right)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}\right.}{40\,a^4}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c))/a^4, 1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-
a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x^2}{5\,a^2c}+\frac{5\,\sqrt{-a^2cx^2+c}x}{4\,a^3}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{2\,a^3c}-\frac{3\,\sqrt{c}\arcsin{(ax)}}{4\,a^4}+\frac{2\,\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\,(-a^2cx^2+c)^{\frac{3}{2}}}{5\,a^4c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) + 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 -
1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) - 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sqrt
(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.674 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 112

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] -7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+2/3*x^2*(-a^2*c*x
^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)+1/24*(21*a*x+32)*(-a^2*c*x^2+c)^
(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6286, 1823, 847, 794, 223, 209}

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}+\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 + 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}x^2\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x^2(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{\int \frac{x^2(-7a^2c-8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{\int \frac{x(16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{8a^2}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}\\
&\quad -\frac{(7c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{8a^2}\\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}(32+21ax+16a^2x^2+6a^3x^3)+21\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{24a^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{(6a^3x^3+16a^2x^2+21ax+32)(a^2x^2-1)c}{24a^3\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{8a^2\sqrt{a^2c}}$
default	$-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4a^2c}+\frac{\frac{9x\sqrt{-a^2c\,x^2+c}}{8}+\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{8\sqrt{a^2c}}}{a^2}-\frac{2(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^3c}+\frac{2\sqrt{-a^2c(x-\frac{1}{a})^2-2(x-\frac{1}{a})ac}-\frac{2ac\arctan(}{}}{a^3}$

[In] int((-a^2*c*x^2+c)^(1/2)*(a*x+1)*x^2/(a*x-1),x,method=_RETURNVERBOSE)

[Out] -1/24*(6*a^3*x^3+16*a^2*x^2+21*a*x+32)*(a^2*x^2-1)/a^3/(-c*(a^2*x^2-1))^(1/2)*c-7/8/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \left[\frac{2\,(6\,a^3x^3+16\,a^2x^2+21\,ax+32)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},\ \frac{(6}{}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)

[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{9\sqrt{-a^2cx^2+c}\,x}{8\,a^2} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4\,a^2c} - \frac{7\sqrt{c}\arcsin(ax)}{8\,a^3} + \frac{2\sqrt{-a^2cx^2+c}}{a^3} - \frac{2\,(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^3c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/8
*sqrt(c)*arcsin(a*x)/a^3 + 2*sqrt(-a^2*c*x^2 + c)/a^3 - 2/3*(-a^2*c*x^2 + c
)^(3/2)/(a^3*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right) + \frac{7\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,a^2\sqrt{-c}|a|}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.675 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] -arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c
)^(1/2)+1/3*(3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6302, 6286, 1823, 794, 223, 209}

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=-\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}+\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) - (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)} x\sqrt{c-a^2cx^2}\,dx \\
&= -\left(c\int \frac{x(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int \frac{x(-5a^2c-6a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2} \\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}
\end{aligned}$$

$$= \frac{1}{3}x^2\sqrt{c-a^2cx^2} + \frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2} - \frac{c\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}$$

$$= \frac{1}{3}x^2\sqrt{c-a^2cx^2} + \frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2} - \frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{(5+3ax+a^2x^2)\sqrt{c-a^2cx^2} + 3\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{3a^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(a^2x^2+3ax+5)(a^2x^2-1)c}{3a^2\sqrt{-c(a^2x^2-1)}} - \frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{a\sqrt{a^2c}}$
default	$-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^2c} + \frac{x\sqrt{-a^2c\,x^2+c}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}}{a} + \frac{2\sqrt{-a^2c(x-\frac{1}{a})^2-2(x-\frac{1}{a})ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c(x-\frac{1}{a})^2-2(x-\frac{1}{a}\cdots}}\right.}{\sqrt{a^2c}}}{a^2}$

```
[In] int(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a^2*x^2+3*a*x+5)*(a^2*x^2-1)/a^2/(-c*(a^2*x^2-1))^(1/2)*c-1/a/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \left[\frac{2\sqrt{-a^2cx^2+c}(a^2x^2+3\,ax+5)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2}, \frac{\sqrt{-a^2cx^2+c}(a^2x^2\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c *x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^2]

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a**2*c*x**2+c)**(1/2),x)

[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2cx^2+c}x}{a} - \frac{\sqrt{c}\arcsin(ax)}{a^2} + \frac{2\sqrt{-a^2cx^2+c}}{a^2} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] sqrt(-a^2*c*x^2 + c)*x/a - sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c) /a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right) + \frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) + c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.676 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^
(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6276, 685, 655, 223, 209}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\sqrt{c-a^2cx^2}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a} - \frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{3\sqrt{c-a^2cx^2}}{2a} + \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a} - \frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{3\sqrt{c-a^2cx^2}}{2a} + \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a} - \frac{1}{2}(3c)\operatorname{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right) \\
&= \frac{3\sqrt{c-a^2cx^2}}{2a} + \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a} - \frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{(ax+4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}}{a}$	136

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*x+4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*arc
tan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{}}{2a}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}x - \frac{3\sqrt{c}\arcsin(ax)}{2\,a} + \frac{2\sqrt{-a^2cx^2+c}}{a}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
+ c)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right) + \frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.677 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2} - 2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + \sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^
(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6286, 1823, 858, 223, 209, 272, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + \sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right) + \sqrt{c-a^2cx^2}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \sqrt{c-a^2cx^2}+\frac{\int \frac{-a^2c-2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2} \\
&= \sqrt{c-a^2cx^2}-c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx-(2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= \sqrt{c-a^2cx^2}-\frac{1}{2}c\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&\quad -(2ac)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right) \\
&= \sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2} \\
&= \sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(-1+a^2x^2\right)}\right) \\
&\quad -\sqrt{c}\log(x)+\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)
\end{aligned}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(61) = 122$.

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.72

method	result
default	$-\sqrt{-a^2 c\, x^2+c}+\sqrt{c}\, \ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2 c\, x^2+c}}{x}\right)+2\sqrt{-a^2 c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right) ac}-\frac{2ac\arctan\left(\frac{\cdots}{\sqrt{-a^2c(\cdots}}\right)}{\sqrt{\cdots}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \left[2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right) + \frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) + \sqrt{-a^2cx^2+c}, \sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right) + \sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right) + \sqrt{-a^2cx^2+c}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*s
qrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt(
-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 -
 c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
+ sqrt(-a^2*c*x^2 + c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=-a^2\left(\frac{\sqrt{c}\arcsin{(ax)}}{a^2}-\frac{\sqrt{-a^2cx^2+c}}{a^2}\right)-a\left(\frac{\sqrt{c}\arcsin{(ax)}}{a}-\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
[Out] -a^2*(sqrt(c)*arcsin(a*x)/a^2 - sqrt(-a^2*c*x^2 + c)/a^2) - a*(sqrt(c)*arcs
in(a*x)/a - sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x))
/a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=-\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) - 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.678 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + 2a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+2*a*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6286, 1821, 858, 223, 209, 272, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + 2a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right) + \frac{\sqrt{c-a^2cx^2}}{x}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{x}+\int \frac{-2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-(2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx-\left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-(ac)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&\quad -\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a}\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)\\
&\quad -2a\sqrt{c}\log(x)+2a\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)
\end{aligned}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}}-\left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}-\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}+2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)-2a\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right.\right.$

[In] int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/x^2/(a*x-1),x,method=_RETURNVERBOSE)

[Out] -(a^2*x^2-1)/x/(-c*(a^2*x^2-1))^(1/2)*c-(a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(
1/2))/x))*c

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.55

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$=\left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\frac{4a\sqrt{-c}x\arctan\left(\right.}{}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")

[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqrt
(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^2\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^2}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} - \frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
[Out] -4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
 a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a
^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^2\,\left(a\,x-1\right)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

3.679 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(1
/2)/x^2+2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6286, 1821, 821, 272, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{3}{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$

$$= -\left(c\int \frac{(1+ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right)$$

$$
\begin{aligned}
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{1}{2}\int \frac{-4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{1}{2}\left(\frac{(1+4ax)\sqrt{c-a^2cx^2}}{x^2}-3a^2\sqrt{c}\log(x)\right.
\left.+3a^2\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)\right)
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
[Out] (((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\left(4a^3x^3+a^2x^2-4ax-1\right)c}{2x^2\sqrt{-c\left(a^2x^2-1\right)}}+\frac{3a^2\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}$
default	$\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{2c\,x^2}-\frac{3a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}-2a\left(-\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\cdots\right.\right.$

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

[Out] -1/2*(4*a^3*x^3+a^2*x^2-4*a*x-1)/x^2/(-c*(a^2*x^2-1))^(1/2)*c+3/2*a^2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}(4\,ax+1)}{4\,x^2}, \frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+}{2\,x^2}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")

[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2]

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^3\,(ax-1)}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)

[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**3*(a*x - 1)), x)

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^3}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^3), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.56

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^3\,(a\,x-1)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

3.680 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)/
x^3+a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6286, 1821, 849, 821, 272, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{1}{3}\int \frac{-6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\int \frac{10a^2c^2+6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c} \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{(1+3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3}-a^3\sqrt{c}\log(x)+a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
[Out] ((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
+ a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{(5a^4x^4+3a^3x^3-4a^2x^2-3ax-1)c}{3x^3\sqrt{-c(a^2x^2-1)}}+a^3\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{3c\,x^3}-2a^3\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)-2a\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2}\right.}{}\right.$

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/x^4/(a*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(5*a^4*x^4+3*a^3*x^3-4*a^2*x^2-3*a*x-1)/x^3/(-c*(a^2*x^2-1))^(1/2)*c+a
^3*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.66

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\sqrt{-a^2cx^2+c}(5\,a^2x^2+3\,ax+1)}{6\,x^3},\ \frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2c}}{a^2c}\right.}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, 1/3*(3*a^3
*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(
-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^4\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^4}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.53

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c-3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|+12\left(\sqrt{-a^2c}x-\sqrt{-a^2c\cdots}\right.\right.}{3\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\cdots\right.}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
[Out] -2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*c)
*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-
a^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))^2 - c)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^4\,(a\,x-1)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

3.681 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx=\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(1
/2)/x^4+2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2+4/3
*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6286, 1821, 849, 821, 272, 65, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx=\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{1}{4}\int \frac{-8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{\int \frac{21a^2c^2+16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c} \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{\int \frac{-32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2} \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} \\
&\quad + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}\left(7a^4c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} \\
&\quad + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{16}\left(7a^4c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} \\
&\quad + \frac{1}{8}\left(7a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} \\
&\quad + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{7}{8}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}(6+16ax+21a^2x^2+32a^3x^3)}{24x^4} - \frac{7}{8}a^4\sqrt{c}\log(x) + \frac{7}{8}a^4\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(32a^5x^5+21a^4x^4-16a^3x^3-15a^2x^2-16ax-6)c}{24x^4\sqrt{-c(a^2x^2-1)}}+\frac{7a^4\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)}{8}$
default	$\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{4cx^4}-\frac{9a^2\left(-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{2cx^2}-\frac{a^2\left(\sqrt{-a^2cx^2+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)\right)}{2}\right)}{4}-2a^4\left(\sqrt{-a^2cx^2+c}-\sqrt{c}\ \mathrm{l}\right.$

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/x^5/(a*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(32*a^5*x^5+21*a^4*x^4-16*a^3*x^3-15*a^2*x^2-16*a*x-6)/x^4/(-c*(a^2*x
^2-1))^(1/2)*c+7/8*a^4*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{-a^2cx^2+c}}{48\,x^4}, \frac{21\,a^4\sqrt{-c}x^4\ a}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^5\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^5}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(106) = 212.

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c\right.}}{}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
```

```
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 + 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^5\,\left(a\,x-1\right)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

3.682 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 228

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^3
/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3
/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2/x^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) + (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*
x^2]*Log[1 - a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}+\frac{4x}{a^2}+\frac{4x^2}{a}+3x^3+ax^4+\frac{4}{a^3(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}\\
&\quad+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^4}+\frac{2x^2}{a^3}+\frac{4x^3}{3a^2}+\frac{3x^4}{4a}+\frac{x^5}{5}+\frac{4\log(1-ax)}{a^5}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^4 + (2*x^2)/a^3 + (4*x^3)/(3*a^2) + (3*x^4)/(
4*a) + x^5/5 + (4*Log[1 - a*x])/a^5))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(12a^5x^5+45a^4x^4+80a^3x^3+120a^2x^2+240ax+240\ln(ax-1)\right)\sqrt{-c(a^2x^2-1)}\,(ax-1)}{60a^4(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	92

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/60*(12*a^5*x^5+45*a^4*x^4+80*a^3*x^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\left(12\,a^5x^5+45\,a^4x^4+80\,a^3x^3+120\,a^2x^2+240\,ax+240\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{60\,a^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/60*(12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x + 240*lo
g(a*x - 1))*sqrt(-a^2*c)/a^5
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.683 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^2
/(1-1/a^2/x^2)^(1/2)+x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3
*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)
/a^4/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/
```

```
(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt
[c - a^2*c*x^2]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1-ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 + (2*x^2)/a^2 + x^3/a + x^4/4 + (4*Log[1 - a*x])/a^4))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(a^4x^4+4a^3x^3+8a^2x^2+16ax+16\ln(ax-1)\right)\sqrt{-c(a^2x^2-1)}\,(ax-1)}{4a^3(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	83

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\left(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{4\,a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(-a^2*c)/a^4
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.684 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x*(-a^2*c*x^2+c)^(1/2)/a
/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(
-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 78}

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
```

```
(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(ax(24+9ax+2a^2x^2)+24\log(1-ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{(2a^3x^3+9a^2x^2+24ax+24\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{6a^2(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	76

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
[Out] 1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1
)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log(ax-1))\sqrt{-a^2c}}{6\,a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(-a^2*c)/a^3
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.685 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(3+ax+\frac{4}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/(Sqrt[1 - 1/
(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x-1\right)}{2 a \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	67

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\left(a^{2} x^{2} + 6\,a x + 8\,\log\left(a x - 1\right)\right)\sqrt{-a^{2} c}}{2\,a^{2}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.686 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 84}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
[Out] (Sqrt[c - a^2*c*x^2]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/(Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-a x+\ln\left(x\right)-4 \ln\left(a x-1\right)\right) \left(a x-1\right)}{\left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	59

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x)-4*ln(a*x-1))*(a*x-1)/(a*x+1)^2/((a*x-1)
/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.25

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax+4\log(ax-1)-\log(x))}{a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.687 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2} - \frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2)
/x/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*Sqrt[c - a^2*c*x^2]*
Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(S
qrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{ax}-3\log(x)+4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(a*x) - 3*Log[x] + 4*Log[1 - a*x]))/(Sqrt[1 - 1/(a^
2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(3a\ln(x)x-4a\ln(ax-1)x-1)(ax-1)}{(ax+1)^2x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*a*ln(a*x-1)*x-1)*(a*x-1)/(a*x+1)^2/x
/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(4\,ax\log{(ax-1)}-3\,ax\log{(x)}+1)}{ax}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.688 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} - \frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/x
^2/(1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
+4*a*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^2
])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 -
```

```
1/(a^2*x^2)]*x) + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int\frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int\frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int\left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{2ax^2}+\frac{3}{x}-4a\log(x)+4a\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(2*a*x^2) + 3/x - 4*a*Log[x] + 4*a*Log[1 - a*x]))/(
Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\left(8a^{2}\ln(x)x^{2}-8a^{2}\ln(ax-1)x^{2}-6ax-1\right)\sqrt{-c(a^{2}x^{2}-1)}\,(ax-1)}{2(ax+1)^{2}x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	77

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*a^2*ln(a*x-1)*x^2-6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{8\,a^{3}\sqrt{-c}x^{2}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx+\sqrt{-a^{2}c}(2\,ax-1)\sqrt{-c}+ac}{ax^{2}-x}\right)+\sqrt{-a^{2}c}(6\,ax+1)}{2\,ax^{2}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
fricas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
- 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + sqrt(-a^2*c)*(6*a*x + 1))/(a*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.689 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2)
/x^3/(1-1/a^2/x^2)^(1/2)+4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4
*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(-a^
2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x
) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}\\
&\quad -\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{3ax^3}+\frac{3}{2x^2}+\frac{4a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(3*a*x^3) + 3/(2*x^2) + (4*a)/x - 4*a^2*Log[x] + 4*
a^2*Log[1 - a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\left(24a^3\ln(x)x^3-24a^3\ln(ax-1)x^3-24a^2x^2-9ax-2\right)\sqrt{-c(a^2x^2-1)}\,(ax-1)}{6x^3(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	85

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERB
OSE)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*a^3*ln(a*x-1)*x^3-24*a^2*x^2-9*a*x-2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x-1)/x^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{24\,a^4\sqrt{-c}x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+(24\,a^2x^2+9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.690 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 228

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^4
/(1-1/a^2/x^2)^(1/2)+2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a^2
*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(-a^2*c*x^2+c)^(1
/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{6327, 6328, 90\}$

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/(
Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2
)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a^3*
Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a^3*Sqrt[c - a^2
*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{4ax^4}+\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
[Out] (Sqrt[c - a^2*c*x^2]*(1/(4*a*x^4) + x^(-3) + (2*a)/x^2 + (4*a^2)/x - 4*a^3*
Log[x] + 4*a^3*Log[1 - a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(16\ln(x)x^4a^4-16\ln(ax-1)x^4a^4-16a^3x^3-8a^2x^2-4ax-1\right)\sqrt{-c(a^2x^2-1)}\,(ax-1)}{4x^4(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	93

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x,method=_RETURNVERB
OSE)
[Out] -1/4*(16*ln(x)*x^4*a^4-16*ln(a*x-1)*x^4*a^4-16*a^3*x^3-8*a^2*x^2-4*a*x-1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/x^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{16\,a^5\sqrt{-c}x^4\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+(16\,a^3x^3+8\,a^2x^2+4\,ax+1)\sqrt{-a^2c}}{4\,ax^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
fricas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*s
qrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.691 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\,(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\,(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a^2\,(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a^2\,(c-a^2cx^2)^{3/2}}$$

```
[Out] (1-1/a^2/x^2)^(3/2)*x^4/a/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^5/
(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a^2/(-a*x+1)/(-a^2*c*x^2+c
)^(3/2)+7/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)+1/4
*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\,(c-a^2cx^2)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{a\,(c-a^2cx^2)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a^2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{7x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a^2\,(c-a^2cx^2)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a^2\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(a*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2)
)^(3/2)*x^5)/(2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*a
```

```
^2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (7*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
- a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
+ a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{\coth^{-1}(ax)}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{x^4}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int\left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}\\
&\quad+\frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(2\left(\frac{2x}{a}+x^2+\frac{1}{a^2-a^3x}\right)+\frac{7\log(1-ax)}{a^2}+\frac{\log(1+ax)}{a^2}\right)}{4\,(c-a^2cx^2)^{3/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(2*((2*x)/a + x^2 + (a^2 - a^3*x)^(-1)) + (7*L
og[1 - a*x])/a^2 + Log[1 + a*x]/a^2))/(4*(c - a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(2a^3x^3+2a^2x^2+a\ln(ax+1)x+7a\ln(ax-1)x-4ax-\ln(ax+1)-7\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^5}$	106

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3+2*a^2*x^2+a*l
n(a*x+1)*x+7*a*ln(a*x-1)*x-4*a*x-ln(a*x+1)-7*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{(2\,a^3x^3+2\,a^2x^2-4\,ax+(ax-1)\log(ax+1)+7\,(ax-1)\log(ax-1)-2)\sqrt{-a^2c}}{4\,(a^7c^2x-a^6c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*l
og(a*x - 1) - 2)*sqrt(-a^2*c)/(a^7*c^2*x - a^6*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**4/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.692 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\,(c-a^2cx^2)^{3/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a\,(c-a^2cx^2)^{3/2}}$$

```
[Out] (1-1/a^2/x^2)^(3/2)*x^4/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a/
(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+5/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a/(-a^
2*c*x^2+c)^(3/2)-1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a/(-a^2*c*x^2+c)^(3/
2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{(c-a^2cx^2)^{3/2}} + \frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a\,(c-a^2cx^2)^{3/2}} - \frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(c - a^2*c*x^2)^(3/2) + ((1 - 1/(a^2*x^2))^(3
/2)*x^3)/(2*a*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (5*(1 - 1/(a^2*x^2))^(3/2)
```

```
*x^3*Log[1 - a*x])/(4*a*(c - a^2*c*x^2)^(3/2)) - ((1 - 1/(a^2*x^2))^(3/2)*x
^3*Log[1 + a*x])/(4*a*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}\\
&\quad+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(x+\frac{1}{2a-2a^2x}+\frac{5\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}\right)}{(c-a^2cx^2)^{3/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(x + (2*a - 2*a^2*x)^(-1) + (5*Log[1 - a*x])/(
4*a) - Log[1 + a*x]/(4*a)))/(c - a^2*c*x^2)^(3/2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(-4a^2x^2+a\ln(ax+1)x-5a\ln(ax-1)x+4ax-\ln(ax+1)+5\ln(ax-1)+2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^4}$	98

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*a^2*x^2+a*ln(a*x+1)
*x-5*a*ln(a*x-1)*x+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(4\,a^2x^2-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{-a^2c}}{4\,(a^6c^2x-a^5c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)*sqrt(-a^2*c)/(a^6*c^2*x - a^5*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**3/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.693 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\,(c-a^2cx^2)^{3/2}}$$

```
[Out] 1/2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+3/4*(1-1/a^2/x^2)
^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a
*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\,(c-a^2cx^2)^{3/2}} + \frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (3*(1 -
 1/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(
a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^2}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\,(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\,(c-a^2cx^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{2}{1-ax}+3\log(1-ax)+\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(2/(1 - a*x) + 3*Log[1 - a*x] + Log[1 + a*x]))
/(4*(c - a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x+3a\ln(ax-1)x-\ln(ax+1)-3\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^3}$	86

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x+3*a*ln(a*
x-1)*x-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2c}((ax-1)\log(ax+1)+3(ax-1)\log(ax-1)-2)}{4(a^5c^2x-a^4c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(a
^5*c^2*x - a^4*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**2/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.694 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}} - \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

```
[Out] 1/2*a*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a*(1-1/a^2/
x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6327, 6328, 78, 213}

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\,(c-a^2cx^2)^{3/2}} - \frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\text{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) - (a*(1
 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} - \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{1-ax}-\operatorname{arctanh}(ax)\right)}{2\,(c-a^2cx^2)^{3/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a*(1 - 1/(a^2*x^2))^(3/2)*x^3*((1 - a*x)^(-1) - ArcTanh[a*x]))/(2*(c - a^2
*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x-a\ln(ax-1)x-\ln(ax+1)+\ln(ax-1)+2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^2}$	84

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*x
-1)*x-ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\,(a^4c^2x-a^3c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^4*c^2*x - a^3*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.695 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3(-1+(-1+ax)\text{arctanh}(ax))}{(-2+2ax)\,(c-a^2cx^2)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1 + (-1 + a*x)*ArcTanh[a*x]))/((-2 + 2*a
*x)*(c - a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x-a\ln(ax-1)x-\ln(ax+1)+\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a}$	84

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*x-
1)*x-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\,(a^3c^2x-a^2c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.696 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{(c-a^2cx^2)^{3/2}} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4(c-a^2cx^2)^{3/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4(c-a^2cx^2)^{3/2}}$$

```
[Out] 1/2*a^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/
x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-3/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*l
n(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-
a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 84}

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)(c-a^2cx^2)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{(c-a^2cx^2)^{3/2}} - \frac{3a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4(c-a^2cx^2)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
3*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (3*a^3*(1 - 1
```

```
/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}\\
&\quad -\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{2-2ax}+\log(x)-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(1+ax)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*
x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x-4a\ln(x)x+3a\ln(ax-1)x-\ln(ax+1)+4\ln(x)-3\ln(ax-1)+2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2}$	93

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-4*a*ln(x
)*x+3*a*ln(a*x-1)*x-ln(a*x+1)+4*ln(x)-3*ln(a*x-1)+2)/(a^2*x^2-1)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\sqrt{-a^2c}\left((ax-1)\log(ax+1)+3(ax-1)\log(ax-1)-4(ax-1)\log(x)+2\right)}{4\left(a^2c^2x-ac^2\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(a
*x - 1)*log(x) + 2)/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.697 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{(c-a^2cx^2)^{3/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{(c-a^2cx^2)^{3/2}} - \frac{5a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4(c-a^2cx^2)^{3/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4(c-a^2cx^2)^{3/2}}$$

```
[Out] -a^3*(1-1/a^2/x^2)^(3/2)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^4*(1-1/a^2/x^2)^(3/
2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^
2*c*x^2+c)^(3/2)-5/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^
(3/2)+1/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx = \frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)(c-a^2cx^2)^{3/2}} + \frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{(c-a^2cx^2)^{3/2}} - \frac{5a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4(c-a^2cx^2)^{3/2}} + \frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4(c-a^2cx^2)^{3/2}} - \frac{a^3x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*
x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (5*a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^2(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}\\
&\quad -\frac{5a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3\left(-\frac{4}{x}+\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(1+ax)\right)}{4\,(c-a^2cx^2)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*a
*Log[1 - a*x] + a*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(a^2\ln(ax+1)x^2+4a^2\ln(x)x^2-5a^2\ln(ax-1)x^2-a\ln(ax+1)x-4a\ln(x)x+5a\ln(ax-1)x-6ax+4)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2x}$	118

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^2*ln(a*x+1)*x^2+4*a^2
*ln(x)*x^2-5*a^2*ln(a*x-1)*x^2-a*ln(a*x+1)*x-4*a*ln(x)*x+5*a*ln(a*x-1)*x-6*
a*x+4)/(a^2*x^2-1)/c^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}\left(6\,ax-(a^2x^2-ax)\log(ax+1)+5\,(a^2x^2-ax)\log(ax-1)-4\,(a^2x^2-ax)\log(x)-4\right)}{4\,(a^2c^2x^2-ac^2x)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*sqrt(-a^2*c)*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x)
*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a^2*c^2*x^2 - a*c^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.698 $\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^3*(1-1/a^2/x^2)^(3/2)*x/(-a^2*c*x^2+c)^(3/2)-a^4*(1-1/a^2/x^2)^(3/2)
*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^5*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*
x^2+c)^(3/2)+2*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-7/4*a
^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^5*(1-1/a^2
/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{6327, 6328, 90\}$

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{2a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}} - \frac{7a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}} - \frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}} - \frac{a^4x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}} - \frac{a^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/2*(a^3*(1 - 1/(a^2*x^2))^(3/2)*x)/(c - a^2*c*x^2)^(3/2) - (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2) + (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^
3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (2*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3
*Log[x])/(c - a^2*c*x^2)^(3/2) - (7*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 -
 a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 +
 a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^3(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}} + \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}} - \frac{7a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{2}{x^2}-\frac{4a}{x}+\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(1+a\right.}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*
a^2*Log[x] - 7*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/
2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

method	result
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\left(a^3\ln(ax+1)x^3-8a^3\ln(x)x^3+7a^3\ln(ax-1)x^3-a^2\ln(ax+1)x^2+8a^2\ln(x)x^2-7a^2\ln(ax-1)x^2+6a^2x^2-2ax-2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\left(a^2x^2-1\right)c^2x^2}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^3*ln(a*x+1)*x^3-8*a^
3*ln(x)*x^3+7*a^3*ln(a*x-1)*x^3-a^2*ln(a*x+1)*x^2+8*a^2*ln(x)*x^2-7*a^2*ln(
a*x-1)*x^2+6*a^2*x^2-2*a*x-2)/(a^2*x^2-1)/c^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(6\,a^2x^2-2\,ax+(a^3x^3-a^2x^2)\log(ax+1)+7\,(a^3x^3-a^2x^2)\log(ax-1)-8\,(a^3x^3-a^2x^2)\log(x)-2\right)\sqrt{-a^2c}}{4\,(a^2c^2x^3-ac^2x^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 - a
^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)*sqrt(-a^2*c)/(a^2*
c^2*x^3 - a*c^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c - a^2cx^2)^{3/2}} \, dx = \int \frac{1}{(-a^2cx^2 + c)^{\frac{3}{2}} x^3 \sqrt{\frac{ax-1}{ax+1}}} \, dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c - a^2cx^2)^{3/2}} \, dx = \int \frac{1}{x^3 (c - a^2 c x^2)^{3/2} \sqrt{\frac{a x-1}{a x+1}}} \, dx$$

```
[In] int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.699 $\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2\,(c-a^2cx^2)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)\,(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1+ax)\,(c-a^2cx^2)^{5/2}} + \frac{23\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16a\,(c-a^2cx^2)^{5/2}} - \frac{7\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16a\,(c-a^2cx^2)^{5/2}}$$

```
[Out] (1-1/a^2/x^2)^(5/2)*x^6/(-a^2*c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/
(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+(1-1/a^2/x^2)^(5/2)*x^5/a/(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+23/
16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-7/16*(1-1/a^2/
x^2)^(5/2)*x^5*ln(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{6327, 6328, 90\}$

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{x^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{(c-a^2cx^2)^{5/2}} + \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{a(1-ax)\,(c-a^2cx^2)^{5/2}} - \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(ax+1)\,(c-a^2cx^2)^{5/2}} - \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1-ax)^2\,(c-a^2cx^2)^{5/2}} + \frac{23x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16a\,(c-a^2cx^2)^{5/2}} - \frac{7x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16a\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^6)/(c - a^2*c*x^2)^(5/2) - ((1 - 1/(a^2*x^2))^(5
/2)*x^5)/(8*a*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(a^2*x^2))^(5/2)
*x^5)/(a*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - ((1 - 1/(a^2*x^2))^(5/2)*x^5)/(
8*a*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (23*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[
1 - a*x])/(16*a*(c - a^2*c*x^2)^(5/2)) - (7*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log
[1 + a*x])/(16*a*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+ax)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2\,(c-a^2cx^2)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)\,(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1+ax)\,(c-a^2cx^2)^{5/2}} + \frac{23\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16a\,(c-a^2cx^2)^{5/2}} - \frac{7\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16a\,(c-a^2cx^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(x-\frac{1}{8a(-1+ax)^2}+\frac{1}{a-a^2x}-\frac{1}{8a+8a^2x}+\frac{23\log(1-ax)}{16a}-\frac{7\log(1+ax)}{16a}\right)}{(c-a^2cx^2)^{5/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*(x - 1/(8*a*(-1 + a*x)^2) + (a - a^2*x)^(-1) -
 (8*a + 8*a^2*x)^(-1) + (23*Log[1 - a*x])/(16*a) - (7*Log[1 + a*x])/(16*a))
)/(c - a^2*c*x^2)^(5/2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(-16a^4x^4+7a^3\ln(ax+1)x^3-23a^3\ln(ax-1)x^3+16a^3x^3-7a^2\ln(ax+1)x^2+23a^2\ln(ax-1)x^2+34a^2x^2-7a\ln(ax+1)x+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^6(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(-16*a^4*x^4+7*
a^3*ln(a*x+1)*x^3-23*a^3*ln(a*x-1)*x^3+16*a^3*x^3-7*a^2*ln(a*x+1)*x^2+23*a^
2*ln(a*x-1)*x^2+34*a^2*x^2-7*a*ln(a*x+1)*x+23*a*ln(a*x-1)*x-18*a*x+7*ln(a*x
+1)-23*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(16\,a^4x^4 - 16\,a^3x^3 - 34\,a^2x^2 + 18\,ax - 7\,(a^3x^3 - a^2x^2 - ax + 1)\log\left(ax+1\right) + 23\,(a^3x^3 - a^2x^2 - ax + 1}{16\,(a^{10}c^3x^3 - a^9c^3x^2 - a^8c^3x + a^7c^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
12)*sqrt(-a^2*c)/(a^10*c^3*x^3 - a^9*c^3*x^2 - a^8*c^3*x + a^7*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^5}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^5}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.700 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx &= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} \\ &+ \frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} \\ &+ \frac{11\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}} + \frac{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

```
[Out] -1/8*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+3/4*(1-1/a^2/x
^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*(1-1/a^2/x^2)^(5/2)*x^5/(a*
x+1)/(-a^2*c*x^2+c)^(5/2)+11/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)+5/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 90}

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} \\ &+ \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}} - \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} \\ &+ \frac{11x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}} + \frac{5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*((1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (3
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + ((1 - 1
/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (11*(1 - 1/(a^
2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*(1 - 1/(a^2
*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^4}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^4(-1+ax)^3}+\frac{3}{4a^4(-1+ax)^2}+\frac{11}{16a^4(-1+ax)}-\frac{1}{8a^4(1+ax)^2}+\frac{5}{16a^4(1+ax)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}} + \frac{11\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16(c-a^2cx^2)^{5/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16(c-a^2cx^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2(-6+3ax+5a^2x^2)}{(-1+ax)^2(1+ax)}+11\log(1-ax)+5\log(1+ax)\right)}{16\,(c-a^2cx^2)^{5/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*((-2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(
1 + a*x)) + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(12+11a^3\ln(ax-1)x^3+5a^3\ln(ax+1)x^3-11a^2\ln(ax-1)x^2-5a^2\ln(ax+1)x^2-10a^2x^2-11a\ln(ax-1)x-5a\ln(ax+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^5(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(12+11*a^3*ln(
a*x-1)*x^3+5*a^3*ln(a*x+1)*x^3-11*a^2*ln(a*x-1)*x^2-5*a^2*ln(a*x+1)*x^2-10*
a^2*x^2-11*a*ln(a*x-1)*x-5*a*ln(a*x+1)*x-6*a*x+11*ln(a*x-1)+5*ln(a*x+1))/(a
^2*x^2-1)/c^3/a^5/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(10\,a^2x^2+6\,ax-5\,(a^3x^3-a^2x^2-ax+1)\log(ax+1)-11\,(a^3x^3-a^2x^2-ax+1)}{16\,(a^9c^3x^3-a^8c^3x^2-a^7c^3x+a^6c^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)*sqrt(-a^2*c)/(a^9*c^3*x^
3 - a^8*c^3*x^2 - a^7*c^3*x + a^6*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.701 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/2*a*(1-1/a
^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a*(1-1/a^2/x^2)^(5/2)*x
^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a*(1-1/a^2/x^2)^(5/2)*x^5*arctanh(a*x)/
(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6327, 6328, 90, 213}

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}+\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} -\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) +
(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a*(1
```

```
 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a*(1 -
1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^3}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^3(-1+ax)^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{1}{8a^3(1+ax)^2}+\frac{3}{8a^3(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}\\
&\quad-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} + \frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{2+ax-5a^2x^2}{(-1+ax)^2(1+ax)}-3\text{arctanh}(ax)\right)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (a*(1 - 1/(a^2*x^2))^(5/2)*x^5*((2 + a*x - 5*a^2*x^2)/((-1 + a*x)^2*(1 + a*
x)) - 3*ArcTanh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-3a^2\ln(ax+1)x^2+3a^2\ln(ax-1)x^2+10a^2x^2-3a\ln(ax+1)x+3a\ln(ax-1)x-2ax+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^4(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+1
)*x^3-3*a^3*ln(a*x-1)*x^3-3*a^2*ln(a*x+1)*x^2+3*a^2*ln(a*x-1)*x^2+10*a^2*x^
2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x-2*a*x+3*ln(a*x+1)-3*ln(a*x-1)-4)/(a^2*x^2
-1)/c^3/a^4/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(5\,a^2x^2-ax-2)\sqrt{-a^2c}}{16\,(a^8c^3x^3-a^7c^3x^2-a^6c^3x+a^5c^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(5*a^2*x^2 - a*x - 2)*sqrt(-a^2*c
))/(a^8*c^3*x^3 - a^7*c^3*x^2 - a^6*c^3*x + a^5*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.702 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8(c-a^2cx^2)^{5/2}}$$

```
[Out] -1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/4*a^2*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6327, 6328, 90, 213}

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{arctanh}(ax)}{8(c-a^2cx^2)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)(c-a^2cx^2)^{5/2}}-\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(c-a^2cx^2)^{5/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
+ (a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (
```

```
a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^2
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^2}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^2(-1+ax)^3}+\frac{1}{4a^2(-1+ax)^2}-\frac{1}{8a^2(1+ax)^2}-\frac{1}{8a^2(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}\\
&\quad+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}$$

$$
= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$
\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2-3ax-a^2x^2+(-1+ax)^2(1+ax)\text{arctanh}(ax))}{8a^2c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}
$$

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - a^2*x^2 + (-1 + a*x)^2*(1 + a*x)*ArcT
anh[a*x]))/(8*a^2*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(a^3\ln(ax+1)x^3-a^3\ln(ax-1)x^3-a^2\ln(ax+1)x^2+a^2\ln(ax-1)x^2-2a^2x^2-a\ln(ax+1)x+a\ln(ax-1)x-6ax+\ln(ax+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^3(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a^3*ln(a*x+1)
*x^3-a^3*ln(a*x-1)*x^3-a^2*ln(a*x+1)*x^2+a^2*ln(a*x-1)*x^2-2*a^2*x^2-a*ln(a
*x+1)*x+a*ln(a*x-1)*x-6*a*x+ln(a*x+1)-ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a^3/(a*x
+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(a^2x^2+3\,ax-2)\sqrt{-a^2c}}{16\,(a^7c^3x^3-a^6c^3x^2-a^5c^3x+a^4c^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 + 3*a*x - 2)*sqrt(-a^2*c))
/(a^7*c^3*x^3 - a^6*c^3*x^2 - a^5*c^3*x + a^4*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

[In] int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)

[Out] int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)

3.703 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^3*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/8*a^3*(1
-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^3*(1-1/a^2/x^2)^(5
/2)*x^5*arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6327, 6328, 78, 213}

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}} - \frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (
a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a(-1+ax)^3}+\frac{1}{8a(1+ax)^2}-\frac{1}{8a(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx=\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{1}{(-1+ax)^2}-\frac{1}{1+ax}+\operatorname{arctanh}(ax)\right)}{8\,(c-a^2cx^2)^{5/2}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-(-1 + a*x)^(-2) - (1 + a*x)^(-1) + ArcTa
nh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(a^3\ln(ax+1)x^3-a^3\ln(ax-1)x^3-a^2\ln(ax+1)x^2+a^2\ln(ax-1)x^2-2a^2x^2-a\ln(ax+1)x+a\ln(ax-1)x+2ax+\ln(ax}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^2(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a^3*ln(a*x+1)
*x^3-a^3*ln(a*x-1)*x^3-a^2*ln(a*x+1)*x^2+a^2*ln(a*x-1)*x^2-2*a^2*x^2-a*ln(a
*x+1)*x+a*ln(a*x-1)*x+2*a*x+ln(a*x+1)-ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^2/(a*x
+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(a^2x^2-ax+2)\sqrt{-a^2c}}{16\,(a^6c^3x^3-a^5c^3x^2-a^4c^3x+a^3c^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 - a*x + 2)*sqrt(-a^2*c))/(
a^6*c^3*x^3 - a^5*c^3*x^2 - a^4*c^3*x + a^3*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.704 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}} - \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}}$$

```
[Out] -1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6327, 6328, 46, 213}

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\operatorname{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}} - \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\,(c-a^2cx^2)^{5/2}} - \frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (
```

```
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8(-1+a^2x^2)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}\\
&\quad+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\operatorname{arctanh}(ax))}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-3a^2\ln(ax+1)x^2+3a^2\ln(ax-1)x^2-6a^2x^2-3a\ln(ax+1)x+3a\ln(ax-1)x+6ax+3}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+1
)*x^3-3*a^3*ln(a*x-1)*x^3-3*a^2*ln(a*x+1)*x^2+3*a^2*ln(a*x-1)*x^2-6*a^2*x^2
-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-
1)/c^3/a/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(3a^2x^2-3ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.705 $\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{\left(c-a^2cx^2\right)^{5/2}} + \frac{11a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}} + \frac{5a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^5*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/2*a^5*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^5*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a
^2*c*x^2+c)^(5/2)+11/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+
c)^(5/2)+5/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{6327, 6328, 90\}$

$$\int \frac{e^{\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\,(c-a^2cx^2)^{5/2}} - \frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\,(c-a^2cx^2)^{5/2}} - \frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}} - \frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{(c-a^2cx^2)^{5/2}} + \frac{11a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\,(c-a^2cx^2)^{5/2}} + \frac{5a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/8*(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (
a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^5
*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (11*a^5*(1 - 1
/(a^2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*a^5*(1
- 1/(a^2*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x}+\frac{a}{4(-1+ax)^3}-\frac{a}{2(-1+ax)^2}+\frac{11a}{16(-1+ax)}+\frac{a}{8(1+ax)^2}+\frac{5a}{16(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{\left(c-a^2cx^2\right)^{5/2}} + \frac{11a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2}{(-1+ax)^2}+\frac{8}{-1+ax}-\frac{2}{1+ax}-16\log(x)+11\log(1-ax)+5\log(1\right.}{16\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-2/(-1 + a*x)^2 + 8/(-1 + a*x) - 2/(1 + a
*x) - 16*Log[x] + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5
/2))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(5a^3\ln(ax+1)x^3-16a^3\ln(x)x^3+11a^3\ln(ax-1)x^3-5a^2\ln(ax+1)x^2+16a^2\ln(x)x^2-11a^2\ln(ax-1)x^2+6a^2x^2-5a\ln}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3(ax+1)}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*a^3*ln(a*x+
1)*x^3-16*a^3*ln(x)*x^3+11*a^3*ln(a*x-1)*x^3-5*a^2*ln(a*x+1)*x^2+16*a^2*ln(
x)*x^2-11*a^2*ln(a*x-1)*x^2+6*a^2*x^2-5*a*ln(a*x+1)*x+16*a*ln(x)*x-11*a*ln(
a*x-1)*x+2*a*x+5*ln(a*x+1)-16*ln(x)+11*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/(a*x+1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{\left(6\,a^2x^2+2\,ax+5\,(a^3x^3-a^2x^2-ax+1)\log(ax+1)+11\,(a^3x^3-a^2x^2-ax+1)\log(ax-1)-16\,(a^3x\right.}{16\,(a^4c^3x^3-a^3c^3x^2-a^2c^3x+ac^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*x
+ 1)*log(x) - 12)*sqrt(-a^2*c)/(a^4*c^3*x^3 - a^3*c^3*x^2 - a^2*c^3*x + a*c
^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{1}{x\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

[In] int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)

[Out] int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)

3.706 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 307

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx &= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{(c-a^2cx^2)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}\\
&\quad-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{(c-a^2cx^2)^{5/2}}\\
&\quad+\frac{23a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16(c-a^2cx^2)^{5/2}}-\frac{7a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16(c-a^2cx^2)^{5/2}}\end{aligned}$$

```
[Out] a^5*(1-1/a^2/x^2)^(5/2)*x^4/(-a^2*c*x^2+c)^(5/2)-1/8*a^6*(1-1/a^2/x^2)^(5/2
)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-3/4*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x
+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^
2+c)^(5/2)-a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*c*x^2+c)^(5/2)+23/16*a^6
*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-7/16*a^6*(1-1/a^2/
x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{6327, 6328, 90\}$

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{5/2}}\,dx = -\frac{3a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\,(c-a^2cx^2)^{5/2}} - \frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}} - \frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{(c-a^2cx^2)^{5/2}} + \frac{23a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\,(c-a^2cx^2)^{5/2}} - \frac{7a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16\,(c-a^2cx^2)^{5/2}} + \frac{a^5x^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{(c-a^2cx^2)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^4)/(c - a^2*c*x^2)^(5/2) - (a^6*(1 - 1/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) - (3*a^6*(1 - 1/(a^2
*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^6*(1 - 1/(a^2*x^
2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^6*(1 - 1/(a^2*x^2))
^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (23*a^6*(1 - 1/(a^2*x^2))^(5/2)*
x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) - (7*a^6*(1 - 1/(a^2*x^2))^(5/
2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^7}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{1}{x^2(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int\left(-\frac{1}{x^2}-\frac{a}{x}+\frac{a^2}{4(-1+ax)^3}-\frac{3a^2}{4(-1+ax)^2}+\frac{23a^2}{16(-1+ax)}-\frac{a^2}{8(1+ax)^2}-\frac{7a^2}{16(1+ax)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{(c-a^2cx^2)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\,(c-a^2cx^2)^{5/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{(c-a^2cx^2)^{5/2}}+\frac{23a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\,(c-a^2cx^2)^{5/2}}-\frac{7a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\,(c-a^2cx^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.32

$$\int\frac{e^{\coth^{-1}(ax)}}{x^2\,(c-a^2cx^2)^{5/2}}\,dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{16}{x}-\frac{2a}{(-1+ax)^2}+\frac{12a}{-1+ax}+\frac{2a}{1+ax}-16a\log(x)+23a\log(1-ax)-\right.}{16\,(c-a^2cx^2)^{5/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(16/x - (2*a)/(-1 + a*x)^2 + (12*a)/(-1 +
a*x) + (2*a)/(1 + a*x) - 16*a*Log[x] + 23*a*Log[1 - a*x] - 7*a*Log[1 + a*x]
))/(16*(c - a^2*c*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(7\ln(ax+1)x^4a^4+16\ln(x)x^4a^4-23\ln(ax-1)x^4a^4-7a^3\ln(ax+1)x^3-16a^3\ln(x)x^3+23a^3\ln(ax-1)x^3-30a^3x^3-7a^2}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(7*ln(a*x+1)*x^
4*a^4+16*ln(x)*x^4*a^4-23*ln(a*x-1)*x^4*a^4-7*a^3*ln(a*x+1)*x^3-16*a^3*ln(x
)*x^3+23*a^3*ln(a*x-1)*x^3-30*a^3*x^3-7*a^2*ln(a*x+1)*x^2-16*a^2*ln(x)*x^2+
23*a^2*ln(a*x-1)*x^2+22*a^2*x^2+7*a*ln(a*x+1)*x+16*a*ln(x)*x-23*a*ln(a*x-1)
*x+28*a*x-16)/(a^2*x^2-1)/c^3/x/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx =$$
$$-\frac{\left(30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax+1\right)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(\right.\right.}{16\left(a^4c^3x^4-a^3c^3x^3-a^2c^3x^2+ac^3x\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) - 1
6*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)*sqrt(-a^2*c)/(a^4*c^3*x^
4 - a^3*c^3*x^3 - a^2*c^3*x^2 + a*c^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c - a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2 + c)^{\frac{5}{2}} x^2 \sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c - a^2cx^2)^{5/2}}\,dx = \int \frac{1}{x^2 (c - a^2\,c\,x^2)^{5/2} \sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.707 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^
(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -1/3*(x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^
2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int x^2(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int\left(-x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{x^2(-4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (x^2*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x^{3}(3ax-4)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$	47
default	$\frac{(3ax-4)x^{3}\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$	48

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{(3\,ax^4-4\,x^3)\sqrt{-a^2c}}{12\,a}$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*a*x^4 - 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+cx^2}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+cx^2}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int x^2\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.708 $\int e^{-\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx=-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*
c*x^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(-x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{x(-3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (x*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x^{2}(2ax-3)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	47
default	$\frac{(2ax-3)x^{2}\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	48

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,ax^3-3\,x^2)\sqrt{-a^2c}}{6\,a}$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fric
as")
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+cx}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxi
ma")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)

Giac [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+cx}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac
")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

[In] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)

3.709 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6327, 6328}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2-2\,x)}{2\,a}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.710 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
[Out] (Sqrt[c - a^2*c*x^2]*(x - Log[x]/a))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(-ax+\ln(x))\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	46

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x))*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax-\log(x))}{a}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fric
as")
[Out] sqrt(-a^2*c)*(a*x - log(x))/a
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.711 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (Sqrt[c - a^2*c*x^2]*Lo
g[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
[Out] (Sqrt[c - a^2*c*x^2]*(1/(a*x) + Log[x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)x}$	48

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOS
E)
```

```
[Out] (a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(ax\log(x)+1)}{ax}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) + 1)/(a*x)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

3.712 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] 3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x^
2+c)^(1/2)/a^2-1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2)+
3/20*(-5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6287, 1823, 847, 794, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}$$

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) + (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}x^3\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x^3(-9a^2c+10a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}\\
&= -\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x^2(-30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x(-72a^4c^3+90a^5c^3x)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}\\
&\quad+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{4a^3}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}\\
&\quad+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{4a^3}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}\\
&\quad+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c-a^2cx^2}(24-15ax+12a^2x^2-10a^3x^3+4a^4x^4)-15\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{20a^4}$$

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

[Out] (Sqrt[c - a^2*c*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 10*a^3*x^3 + 4*a^4*x^4) - 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*a^4)

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(4a^4x^4-10a^3x^3+12a^2x^2-15ax+24)(a^2x^2-1)c}{20a^4\sqrt{-c(a^2x^2-1)}}+\frac{3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{4a^3\sqrt{a^2c}}$
default	$-\frac{x^2(-a^2c\,x^2+c)^{\frac{3}{2}}}{5a^2c}-\frac{4(-a^2c\,x^2+c)^{\frac{3}{2}}}{5c\,a^4}-\frac{2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{a^3}-\frac{2\left(-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4a^2c}+\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\right.}{a}$

[In] int(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] -1/20*(4*a^4*x^4-10*a^3*x^3+12*a^2*x^2-15*a*x+24)*(a^2*x^2-1)/a^4/(-c*(a^2*x^2-1))^(1/2)*c+3/4/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\,(4\,a^4x^4-10\,a^3x^3+12\,a^2x^2-15\,ax+24)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}\right.}{40\,a^4}\right.$$

[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x^2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^4, 1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^4]

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}{5\,a^2c}-\frac{5\,\sqrt{-a^2cx^2+c}x}{4\,a^3}+\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{2\,a^3c}+\frac{3\,\sqrt{c}\arcsin\left(ax\right)}{4\,a^4}+\frac{2\,\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{5\,a^4c}$$

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) - 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 +
1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) + 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sqrt
(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.713 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 112

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] -7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3-2/3*x^2*(-a^2*c*x
^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)-1/24*(-21*a*x+32)*(-a^2*c*x^2+c)
^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6287, 1823, 847, 794, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}$$

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 - 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}x^2\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{\int \frac{x^2(-7a^2c+8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{\int \frac{x(-16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{8a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}\\
&\quad -\frac{(7c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{8a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}(-32+21ax-16a^2x^2+6a^3x^3)+21\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{24a^3}$$

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{(6a^3x^3-16a^2x^2+21ax-32)(a^2x^2-1)c}{24a^3\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{8a^2\sqrt{a^2c}}$
default	$-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4a^2c}+\frac{\frac{9x\sqrt{-a^2c\,x^2+c}}{8}+\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{8\sqrt{a^2c}}}{a^2}+\frac{2(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^3c}-\frac{2\left(\sqrt{-a^2c(x+\frac{1}{a})^2+2(x+\frac{1}{a})ac}+\frac{ac\arctan\left(\right.}{}\right.}{a^3}$

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*a^3*x^3-16*a^2*x^2+21*a*x-32)*(a^2*x^2-1)/a^3/(-c*(a^2*x^2-1))^(1/
2)*c-7/8/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\,(6\,a^3x^3-16\,a^2x^2+21\,ax-32)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},\frac{(6}{}\right.$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{9\sqrt{-a^2cx^2+c}x}{8\,a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4\,a^2c}-\frac{7\sqrt{c}\arcsin(ax)}{8\,a^3}-\frac{2\sqrt{-a^2cx^2+c}}{a^3}+\frac{2\,(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^3c}$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/8
*sqrt(c)*arcsin(a*x)/a^3 - 2*sqrt(-a^2*c*x^2 + c)/a^3 + 2/3*(-a^2*c*x^2 + c
)^(3/2)/(a^3*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,x-\frac{8}{a}\right)x+\frac{21}{a^2}\right)x-\frac{32}{a^3}\right)+\frac{7\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,a^2\sqrt{-c}|a|}$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x - 8/a)*x + 21/a^2)*x - 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.714 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c)
^(1/2)+1/3*(-3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6302, 6287, 1823, 794, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}+\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}$$

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) + (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\operatorname{arctanh}(ax)} x\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2} + \frac{\int \frac{x(-5a^2c+6a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}
\end{aligned}$$

$$= \frac{1}{3}x^2\sqrt{c-a^2cx^2} + \frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2} + \frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}dx}{a}$$

$$= \frac{1}{3}x^2\sqrt{c-a^2cx^2} + \frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2} + \frac{c\text{Subst}\left(\int \frac{1}{1+a^2cx^2}dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}$$

$$= \frac{1}{3}x^2\sqrt{c-a^2cx^2} + \frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2} + \frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{(5-3ax+a^2x^2)\sqrt{c-a^2cx^2} - 3\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{3a^2}$$

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
[Out] ((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] - 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(a^2x^2-3ax+5)(a^2x^2-1)c}{3a^2\sqrt{-c(a^2x^2-1)}} + \frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{a\sqrt{a^2c}}$
default	$-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^2c} - \frac{2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2} + \frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{a} + \frac{2\sqrt{-a^2c(x+\frac{1}{a})^2+2(x+\frac{1}{a})ac} + \frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c(x+\frac{1}{a})^2+2(x+\right.}}{\sqrt{a^2c}}}{a^2}$

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -1/3*(a^2*x^2-3*a*x+5)*(a^2*x^2-1)/a^2/(-c*(a^2*x^2-1))^(1/2)*c+1/a/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \left[\frac{2\sqrt{-a^2cx^2+c}(a^2x^2-3\,ax+5)+3\sqrt{-c}\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2}, \frac{\sqrt{-a^2cx^2+c}(a^2x\right.$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = -\frac{\sqrt{-a^2cx^2+c}x}{a} + \frac{\sqrt{c}\arcsin(ax)}{a^2} + \frac{2\sqrt{-a^2cx^2+c}}{a^2} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*x/a + sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c
)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x-\frac{3}{a}\right)x+\frac{5}{a^2}\right) - \frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x - 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.715 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^
(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6277, 685, 655, 223, 209}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}\,dx$$

$$= -\left(c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$

$$= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} - \frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx$$

$$= -\frac{3\sqrt{c-a^2cx^2}}{2a} - \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a} - \frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$

$$=-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)$$
$$=-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(4-5ax+a^2x^2)+6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a
*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)}{a}$	127

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*x-4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*arc
tan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4\,a},\frac{\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{}}{2\,a}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin(ax)}{2\,a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
+ c)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right) + \frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.716 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6287, 1823, 858, 223, 209, 272, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = 2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\sqrt{c-a^2cx^2}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \sqrt{c-a^2cx^2}+\frac{\int \frac{-a^2c+2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2} \\
&= \sqrt{c-a^2cx^2}-c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx+(2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= \sqrt{c-a^2cx^2}-\frac{1}{2}c\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&\quad +(2ac)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right) \\
&= \sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2} \\
&= \sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) - \sqrt{c}\log(x)+\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

method	result
default	$-\sqrt{-a^{2} c \,x^{2}+c}+\sqrt{c}\, \ln \left(\frac{2 c+2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)+2 \sqrt{-a^{2} c \left(x+\frac{1}{a}\right)^{2}+2 \left(x+\frac{1}{a}\right) a c}+\frac{2 a c \arctan \left(\cdots\right)}{\cdots}$

```
[In] int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \left[-2\,\sqrt{c}\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+c}a\sqrt{c}x}{a^{2}cx^{2}-c}\right) + \frac{1}{2}\,\sqrt{c}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\sqrt{c}-2\,c}{x^{2}}\right) + \sqrt{-a^{2}cx^{2}+c},\ \sqrt{-c}\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+c}\sqrt{-c}}{a^{2}cx^{2}-c}\right) + \sqrt{-c}\log\left(2\,a^{2}cx^{2}+2\,\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right) + \sqrt{-a^{2}cx^{2}+c}\right]$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt
(-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
- c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
 + sqrt(-a^2*c*x^2 + c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = a^2\left(\frac{\sqrt{c}\arcsin(ax)}{a^2}+\frac{\sqrt{-a^2cx^2+c}}{a^2}\right) + a\left(\frac{\sqrt{c}\arcsin(ax)}{a}+\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] a^2*(sqrt(c)*arcsin(a*x)/a^2 + sqrt(-a^2*c*x^2 + c)/a^2) + a*(sqrt(c)*arcsi
n(a*x)/a + sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x))/
a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} + \sqrt{-a^2cx^2+c}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.717 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-2*a*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6287, 1821, 858, 223, 209, 272, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right) + \frac{\sqrt{c-a^2cx^2}}{x}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{x} + \int \frac{2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x} + (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x} + (ac)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)\\
&\quad - \left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a}\\
&= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) + 2a\sqrt{c}\log(x) - 2a\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}}-\left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}+2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)+2a\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2c}}{x}\right.\right.$

```
[In] int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(a^2*x^2-1)/x/(-c*(a^2*x^2-1))^(1/2)*c-(a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(
1/2))/x))*c
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$=\left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\right.$$
$$-\frac{4a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-a\sqrt{-c}x\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\sqrt{-a^2cx^2+c}}{2x}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqr
t(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
 c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^2\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^2}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} - \frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] 4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a^
2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^2\,\left(a\,x+1\right)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.718 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(1
/2)/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6287, 1821, 821, 272, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{3}{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$

$$= -\left(c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{1}{2}\int \frac{4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{1}{2}\left(\frac{(1-4ax)\sqrt{c-a^2cx^2}}{x^2} - 3a^2\sqrt{c}\log(x) + 3a^2\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)\right)$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (((1 - 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
risch	$\frac{(4a^3x^3-a^2x^2-4ax+1)c}{2x^2\sqrt{-c(a^2x^2-1)}} + \frac{3a^2\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2} - \frac{3a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2} + 2a\Bigg(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx} - 2a^2\Bigg(\frac{x\sqrt{-a^2c\,x^2+c}}{2} + c\,\mathrm{arc}$

```
[In] int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(4*a^3*x^3-a^2*x^2-4*a*x+1)/x^2/(-c*(a^2*x^2-1))^(1/2)*c+3/2*a^2*c^(1/2
)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}(4\,ax-1)}{4\,x^2},\ \frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)}{2\,x^2}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 +
 c)*(4*a*x - 1))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^3\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^3}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c+4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)a}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c + 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 - 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.719 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 101

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)
/x^3-a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6287, 1821, 849, 821, 272, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{\sqrt{c-a^2cx^2}}{3x^3} + a^3\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{1}{3}\int \frac{6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\int \frac{10a^2c^2-6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a^3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{(1-3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\log(x) - a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)
$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
[Out] ((1 - 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x]
- a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{(5a^4x^4-3a^3x^3-4a^2x^2+3ax-1)c}{3x^3\sqrt{-c(a^2x^2-1)}}-a^3\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{3c\,x^3}+2a^3\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)+2a\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2+\right.}{}\right.$

[In] int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*(5*a^4*x^4-3*a^3*x^3-4*a^2*x^2+3*a*x-1)/x^3/(-c*(a^2*x^2-1))^(1/2)*c-a
^3*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}(5\,a^2x^2-3\,ax+1)}{6\,x^3},\ -\frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2cx^2+c}(5\,a^2x^2-3\,ax+1)}{3\,x^3}\right]$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")

[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, -1/3*(3*a^
3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt
(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^4\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^4}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.48

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c+3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|-12\left(\sqrt{-a^2c}x-\sqrt{-a^2\right.}{3\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-\right.}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
[Out] 2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c + 3*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) - 12*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))*a^3*c^3 + 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^2 - c)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^4\,\left(a\,x+1\right)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.720 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(1
/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2-4/3
*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6287, 1821, 849, 821, 272, 65, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{1}{4}\int \frac{8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\int \frac{21a^2c^2-16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\int \frac{32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}\\
&\quad -\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{1}{8}\left(7a^4c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}\\
&\quad -\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{1}{16}\left(7a^4c\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}\\
&\quad +\frac{1}{8}\left(7a^2\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}\\
&\quad -\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}(6-16ax+21a^2x^2-32a^3x^3)}{24x^4} - \frac{7}{8}a^4\sqrt{c}\log(x) + \frac{7}{8}a^4\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 - 16*a*x + 21*a^2*x^2 - 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)c}{24x^4\sqrt{-c(a^2x^2-1)}}+\frac{7a^4\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{8}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{4c\,x^4}-\frac{9a^2\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}\right)}{4}-2a^4\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\text{l}\right.$

```
[In] int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(32*a^5*x^5-21*a^4*x^4-16*a^3*x^3+15*a^2*x^2-16*a*x+6)/x^4/(-c*(a^2*x^
2-1))^(1/2)*c+7/8*a^4*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{-a^2cx^2+c}}{48\,x^4}, \frac{21\,a^4\sqrt{-c}x^4\,a}{}\right.$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^5\,(ax+1)}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^5}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(106) = 212.

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}} + \frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2-96\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c\right.}}{}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 - 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
```

```
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 + 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 - 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.721 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)-2*x*(-a^2*c*x^2+c)^(1/2)/a^3
/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3
/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2/x^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{6327, 6328, 90\}$

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) - (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) - (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - a^2*c*
x^2]*Log[1 + a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}-\frac{4x}{a^2}+\frac{4x^2}{a}-3x^3+ax^4-\frac{4}{a^3(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^4}-\frac{2x^2}{a^3}+\frac{4x^3}{3a^2}-\frac{3x^4}{4a}+\frac{x^5}{5}-\frac{4\log(1+ax)}{a^5}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^4 - (2*x^2)/a^3 + (4*x^3)/(3*a^2) - (3*x^4)/(
4*a) + x^5/5 - (4*Log[1 + a*x])/a^5))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{(-12a^5x^5+45a^4x^4-80a^3x^3+120a^2x^2-240ax+240\ln(ax+1))\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{60(ax-1)^2a^4}$	92

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
[Out] -1/60*(-12*a^5*x^5+45*a^4*x^4-80*a^3*x^3+120*a^2*x^2-240*a*x+240*ln(a*x+1))
*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{(12\,a^5x^5-45\,a^4x^4+80\,a^3x^3-120\,a^2x^2+240\,ax-240\,\log(ax+1))\sqrt{-a^2c}}{60\,a^5}$$

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x - 240*lo
g(a*x + 1))*sqrt(-a^2*c)/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.722 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
2/(1-1/a^2/x^2)^(1/2)-x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^
3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)
/a^4/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1
```

```
/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqr
t[c - a^2*c*x^2]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{4x}{a^3}+\frac{2x^2}{a^2}-\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1+ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a^3 + (2*x^2)/a^2 - x^3/a + x^4/4 + (4*Log[1 +
 a*x])/a^4))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(a^4x^4-4a^3x^3+8a^2x^2-16ax+16\ln(ax+1)\right)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4a^3(ax-1)^2}$	83

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\left(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{4\,a^4}$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(-a^2*
c)/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

```
[In] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.723 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x*(-a^2*c*x^2+c)^(1/2)/a
/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln(
a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 78}

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
```

```
(a^2*x^2)]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.43

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\left(-2a^3x^3+9a^2x^2-24ax+24\ln(ax+1)\right)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6a^2(ax-1)^2}$	76

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x
+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log{(ax+1)})\sqrt{-a^2c}}{6\,a^3}$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fric
as")
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(-a^2*c)/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\,\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

```
[In] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.724 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3+ax+\frac{4}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x +8 \ln \left(a x +1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{2} a}$	67

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/(a*x-1)^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2 - 6\,ax + 8\,\log{(ax+1)})\sqrt{-a^2c}}{2\,a^2}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)

[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)

3.725 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 84}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}x}
$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(-ax+4\ln(ax+1)-\ln(x))(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	61

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+4*ln(a*x+1)-ln(x))*(a*x+1)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.23

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax-4\log(ax+1)+\log(x))}{a}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fric
as")
[Out] sqrt(-a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxi ma")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac ")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)

[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)

3.726 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2
)/x/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) - (3*Sqrt[c - a^2*c*x^
2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{ax}-3\log(x)+4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]

[Out] (Sqrt[c - a^2*c*x^2]*(-(1/(a*x)) - 3*Log[x] + 4*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{(4a\ln(ax+1)x-3a\ln(x)x-1)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2x}$	64

[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] (4*a*ln(a*x+1)*x-3*a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{-a^2c}\,(4\,ax\log(ax+1)-3\,ax\log(x)-1)}{ax}$$

[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")

[Out] sqrt(-a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.727 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 152

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2
)-4*a*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/2*Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*
x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[
```

```
1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{2ax^2}+\frac{3}{x}+4a\log(x)-4a\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/2*1/(a*x^2) + 3/x + 4*a*Log[x] - 4*a*Log[1 + a*x])
)/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\left(8a^2\ln(ax+1)x^2-8a^2\ln(x)x^2-6ax+1\right)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2(ax-1)^2x^2}$	77

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(8*a^2*ln(a*x+1)*x^2-8*a^2*ln(x)*x^2-6*a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{8\,a^3\sqrt{-c}x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+\sqrt{-a^2c}(6\,ax-1)}{2\,ax^2}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
+ 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + sqrt(-a^2*c)*(6*a*x - 1))/(a*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.728 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2
)/x^3/(1-1/a^2/x^2)^(1/2)-4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-
4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(-a^
2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] -1/3*Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*
x^2])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1
/(a^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)
]*x) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} \\
&\quad -\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{3ax^3}+\frac{3}{2x^2}-\frac{4a}{x}-4a^2\log(x)+4a^2\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/3*1/(a*x^3) + 3/(2*x^2) - (4*a)/x - 4*a^2*Log[x] +
 4*a^2*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(24a^3\ln(ax+1)x^3-24a^3\ln(x)x^3-24a^2x^2+9ax-2\right)\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6x^3(ax-1)^2}$	85

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*(24*a^3*ln(a*x+1)*x^3-24*a^3*ln(x)*x^3-24*a^2*x^2+9*a*x-2)*(-c*(a^2*x^2
-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{24\,a^4\sqrt{-c}x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.729 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^4} -\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} +\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^3\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[Out] -1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^
4/(1-1/a^2/x^2)^(1/2)-2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a^
2*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(-a^2*c*x^2+c)^(
1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

= {6327, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a^3\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/4*Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2
]/(Sqrt[1 - 1/(a^2*x^2)]*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*
x^2)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a
^3*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^3*Sqrt[c -
a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^3\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.35

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{4ax^4}+\frac{1}{x^3}-\frac{2a}{x^2}+\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-1/4*1/(a*x^4) + x^(-3) - (2*a)/x^2 + (4*a^2)/x + 4*a
^3*Log[x] - 4*a^3*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(16\ln(ax+1)x^4a^4-16\ln(x)x^4a^4-16a^3x^3+8a^2x^2-4ax+1\right)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4x^4(ax-1)^2}$	93

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOS
E)
[Out] -1/4*(16*ln(a*x+1)*x^4*a^4-16*ln(x)*x^4*a^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{16\,a^5\sqrt{-c}x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+(16\,a^3x^3-8\,a^2x^2+4\,ax-1)\sqrt{-a^2c}}{4\,ax^4}$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*s
qrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.730 $\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 136

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+
c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)-4*x^m*hypergeom([1, 1+m],[2+m],a*x)*(-a^
2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6327, 6328, 90, 66}

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) - (4*x^m*Sqrt[c - a^2*
c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(a*(1 + m)*Sqrt[1 - 1/(a^2*
x^2)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(3x^m+ax^{1+m}+\frac{4x^m}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

$$= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^m\sqrt{c-a^2cx^2}(6+ax+m(3+ax)-4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, a*x]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [F]

$$\int \frac{x^m\sqrt{-a^2c\,x^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\,x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{x^m\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.731 $\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}-\frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}$$

```
[Out] -c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^
2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)-2*a*c*x^(2+m)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)+x^
(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6286, 1823, 822, 372, 371}

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^m(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\int \frac{x^m(-a^2c(3+2m)-2a^3c(2+m)x)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - (2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - \frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx}{(2+m)\sqrt{c-a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} - \frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\left(\frac{2\sqrt{1-ax}\sqrt{-c(1+ax)}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,ax,-ax\right)}{\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/2
, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[c - a^2*c*x^2
]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]
))/(1 + m)
```

Maple [F]

$$\int \frac{(ax+1)\,x^m\sqrt{-a^2c\,x^2+c}}{ax-1}dx$$

```
[In] int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
[Out] int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.732 $\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c)
^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 45}

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*Sq
rt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x^m(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(x^m+ax^{1+m}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{x^m(2+m+ax+amx)\sqrt{c-a^2cx^2}}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
[Out] (x^m*(2 + m + a*x + a*m*x)*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*(2 + m)*Sqrt[1 -
 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{1+m}\sqrt{-a^{2}c\,x^{2}+c}\,(amx+ax+m+2)}{(1+m)(2+m)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	62
risch	$-\frac{\sqrt{-\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax-1)c(amx+ax+m+2)x\,x^{m}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{-c}\,(2+m)(1+m)}$	95

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] x^(1+m)/(1+m)/(2+m)/(a*x+1)*(-a^2*c*x^2+c)^(1/2)*(a*m*x+a*x+m+2)/((a*x-1)/(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=-\frac{\sqrt{-a^2cx^2+c}((am+a)x^2+(m+2)x)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-(am^2+3\,am+2\,a)x+3\,m+2}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 + (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)

[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\left(a\sqrt{-c}(m+1)x^2+\sqrt{-c}(m+2)x\right)(ax+1)x^m}{(m^2+3m+2)ax+m^2+3m+2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 + sqrt(-c)*(m + 2)*x)*(a*x + 1)*x^m/((m^2 + 3*m + 2
)*a*x + m^2 + 3*m + 2)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,(m+1)}{m^2+3\,m+2}+\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,(m+2)}{a\,(m^2+3\,m+2)}\right)}{x-\frac{1}{a}}$$

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
+ m^2 + 2) + (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/(x
 - 1/a)
```

3.733 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 83

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = -\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c
)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -((x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)])) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x^m(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-x^m+ax^{1+m}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= -\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{x^{1+m}}{a(1+m)}+\frac{x^{2+m}}{2+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(x^(1 + m)/(a*(1 + m))) + x^(2 + m)/(2 + m)))/(Sqrt[
1 - 1/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{x^{1+m}\sqrt{-a^{2}c\,x^{2}+c}\,(amx+ax-m-2)\sqrt{\frac{ax-1}{ax+1}}}{(1+m)(2+m)(ax-1)}$	64
risch	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax+1)c(amx+ax-m-2)x\,x^{m}}{\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{-c}\,(2+m)(1+m)}$	97

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] x^(1+m)/(1+m)/(2+m)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)*(a*m*x+a*x-m-2)*((a*x-1)/(
a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}x^{m}\sqrt{c-a^{2}cx^{2}}\,dx=-\frac{\sqrt{-a^{2}cx^{2}+c}((am+a)x^{2}-(m+2)x)x^{m}\sqrt{\frac{ax-1}{ax+1}}}{m^{2}-(am^{2}+3\,am+2\,a)x+3\,m+2}$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 - (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^{m}\sqrt{c-a^{2}cx^{2}}\,dx=\text{Timed out}$$

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\left(a\sqrt{-c}(m+1)x^2-\sqrt{-c}(m+2)x\right)(ax-1)x^m}{(m^2+3\,m+2)ax-m^2-3\,m-2}$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 - sqrt(-c)*(m + 2)*x)*(a*x - 1)*x^m/((m^2 + 3*m + 2
)*a*x - m^2 - 3*m - 2)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,(m+1)}{m^2+3\,m+2}-\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,(m+2)}{a\,(m^2+3\,m+2)}\right)}{x-\frac{1}{a}}$$

```
[In] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
+ m^2 + 2) - (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/(x
 - 1/a)
```

3.734 $\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} + \frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}$$

```
[Out] -c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^
2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)+2*a*c*x^(2+m)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)+x^
(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6287, 1823, 822, 372, 371}

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = -\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}} + \frac{2ac\sqrt{1-a^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}} + \frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\int \frac{x^m(-a^2c(3+2m)+2a^3c(2+m)x)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + (2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - \frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx}{(2+m)\sqrt{c-a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} + \frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\left(-\frac{2\sqrt{c-acx}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-ax,ax\right)}{\sqrt{1-ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
[Out] (x^(1 + m)*((-2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x), a
*x])/Sqrt[1 - a*x] + (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2
, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

Maple [F]

$$\int \frac{x^m \sqrt{-a^2 c\, x^2+c}\,(a x-1)}{a x+1}dx$$

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}\, dx$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx = \int \frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\, dx$$

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}\, dx$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c - a^2cx^2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c - a^2cx^2}\, dx = \int \frac{x^m \sqrt{c - a^2\,c\,x^2}\,(a\,x - 1)}{a\,x + 1}\, dx$$

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.735 $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} +\frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)+4*x^m*hypergeom([1, 1+m],[2+m],-a*x)*(-
a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6327, 6328, 90, 66}

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}} +\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^m*Sqrt[c - a^2
*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(a*(1 + m)*Sqrt[1 - 1/(
a^2*x^2)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m+ax^{1+m}+\frac{4x^m}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}
$$

$$= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^m\sqrt{c-a^2cx^2}(-6+ax+m(-3+ax)+4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hypergeometri
c2F1[1, 1 + m, 2 + m, -(a*x)]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [F]

$$\int x^m\sqrt{-a^2c\,x^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.736 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(8,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] -256*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([8, 4-1/2*n],[5
-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6326, 6330, 133}

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\text{Hypergeometric2F1}\left(8,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-256*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[8, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(a^6c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)\\ &= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{3+\frac{n}{2}}}{x^8}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(8,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(81) = 162.

Time = 2.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.30

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{c^3e^{n\coth^{-1}(ax)}\left(-912n+58n^3-n^5-5040ax+912an^2x-58an^4x+an^6x+1368a^2nx^2-64a^2n^3x^2+a\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/5040*(c^3*E^(n*ArcCoth[a*x])*(-912*n + 58*n^3 - n^5 - 5040*a*x + 912*a*n
^2*x - 58*a*n^4*x + a*n^6*x + 1368*a^2*n*x^2 - 64*a^2*n^3*x^2 + a^2*n^5*x^2
 + 5040*a^3*x^3 - 152*a^3*n^2*x^3 + 2*a^3*n^4*x^3 - 576*a^4*n*x^4 + 6*a^4*n
^3*x^4 - 3024*a^5*x^5 + 24*a^5*n^2*x^5 + 120*a^6*n*x^6 + 720*a^7*x^7 + E^(2
*ArcCoth[a*x])*n*(-1152 + 576*n + 104*n^2 - 52*n^3 - 2*n^4 + n^5)*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (-2304 + 784*n^2 - 56*n
^4 + n^6)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/a
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)} \left(-a^2 c\,x^2+c\right)^3 dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)} \left(c-a^2cx^2\right)^3 \, dx = \int -\left(a^2cx^2-c\right)^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x + 1)/(a
*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)} \left(c-a^2cx^2\right)^3 \, dx = -c^3 \left(\int 3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int \left(-3a^4x^4e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx + \int a^6x^6e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int \left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-3*a**4*x**4*e
xp(n*acoth(a*x)), x) + Integral(a**6*x**6*exp(n*acoth(a*x)), x) + Integral(
-exp(n*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int -\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -integrate((a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int -\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-(a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^3\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3,x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3, x)
```

3.737 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx = \frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(6,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] 64*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([6, 3-1/2*n],[4-1
/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6326, 6330, 133}

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx = \frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\text{Hypergeometric2F1}\left(6,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (64*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[6, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\left(a^4c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx\\&=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{2+\frac{n}{2}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)\\&=\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(6,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(81) = 162.

Time = 1.40 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$

$$=\frac{c^2e^{n\coth^{-1}(ax)}\left(22n-n^3+120ax-22an^2x+an^4x-28a^2nx^2+a^2n^3x^2-80a^3x^3+2a^3n^2x^3+6a^4nx^4+2\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(22*n - n^3 + 120*a*x - 22*a*n^2*x + a*n^4*x - 28*a
^2*n*x^2 + a^2*n^3*x^2 - 80*a^3*x^3 + 2*a^3*n^2*x^3 + 6*a^4*n*x^4 + 24*a^5*
x^5 + E^(2*ArcCoth[a*x])*n*(32 - 16*n - 2*n^2 + n^3)*Hypergeometric2F1[1, 1
 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (64 - 20*n^2 + n^4)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/(120*a)
```

Maple [F]

$$\int e^{n \,\operatorname{arccoth}(a x)} \left(-a^{2} c \,x^{2}+c \right)^{2}d x$$

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(a x)} \left(c - a^{2} c x^{2}\right)^{2} \, dx = \int { {\left(a^{2} c x^{2} - c\right)}^{2} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n} \,d x }$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n),
 x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(a x)} \left(c - a^{2} c x^{2}\right)^{2} \, dx = c^{2} \left(\int \left(- 2 a^{2} x^{2} e^{n \operatorname{acoth}{\left(a x \right)}}\right)\, dx + \int a^{4} x^{4} e^{n \operatorname{acoth}{\left(a x \right)}}\, dx + \int e^{n \operatorname{acoth}{\left(a x \right)}}\, dx\right)$$

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(-2*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(a**4*x**4*exp(
n*acoth(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int\left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int\left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\left(c-a^2\,c\,x^2\right)^2\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2,x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2, x)
```

3.738 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(4,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] -16*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([4, 2-1/2*n],[3-1/
2*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6326, 6330, 133}

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(4,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-16*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[4, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(\left(a^2c\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^4}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(4,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx =$$

$$-\frac{ce^{n\coth^{-1}(ax)}\left(-n-6ax+an^2x+a^2nx^2+2a^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},\right.\right.}{6a}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -1/6*(c*E^(n*ArcCoth[a*x])*(-n - 6*a*x + a*n^2*x + a^2*n*x^2 + 2*a^3*x^3 +
E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*A
rcCoth[a*x])] + (-4 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[
a*x])]))/a
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)} \left(-a^{2} c\,x^{2}+c\right) dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(\int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c),x)
[Out] -c*(Integral(a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)),
 x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2),x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2), x)
```

3.739 $\int e^{n\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 78

$$\int e^{n\coth^{-1}(ax)}\,dx=\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] 4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(a-1/x)/(a+1/x))/a/(2-n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6305, 133}

$$\int e^{n\coth^{-1}(ax)}\,dx=\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
```

```
|| !SumSimplerQ[p, 1]) && !ILtQ[m, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}\,dx$$
$$= \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1, 1+\frac{n}{2}, 2+\frac{n}{2}, e^{2\coth^{-1}(ax)}\right)+(2+n)\left(ax+\text{Hypergeometr}\right.\right.}{a(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}dx$$

```
[In] int(exp(n*arccoth(a*x)),x)
[Out] int(exp(n*arccoth(a*x)),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int e^{n\operatorname{acoth}(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x)),x)
[Out] Integral(exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

```
[In] int(exp(n*acoth(a*x)),x)
[Out] int(exp(n*acoth(a*x)), x)
```

3.740 $\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{n\coth^{-1}(ax)}}{acn}$$

```
[Out] exp(n*arccoth(a*x))/a/c/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6318}

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{n\coth^{-1}(ax)}}{acn}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\text{integral}=\frac{e^{n\coth^{-1}(ax)}}{acn}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{n\coth^{-1}(ax)}}{acn}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{n\,\operatorname{arccoth}(ax)}}{acn}$	18
parallelrisch	$\frac{e^{n\,\operatorname{arccoth}(ax)}}{acn}$	18
risch	$\frac{(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}}{can}$	29

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] exp(n*arccoth(a*x))/a/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acn}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] ((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \begin{cases} \frac{x}{c} & \text{for } a=0\wedge n=0\\ \frac{xe^{\frac{i\pi n}{2}}}{c} & \text{for } a=0\\ -\frac{\log\left(x-\frac{1}{a}\right)}{2ac}+\frac{\log\left(x+\frac{1}{a}\right)}{2ac} & \text{for } n=0\\ \frac{e^{n\operatorname{acoth}(ax)}}{acn} & \text{otherwise}\end{cases}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise((x/c, Eq(a, 0) & Eq(n, 0)), (x*exp(I*pi*n/2)/c, Eq(a, 0)), (-log(
x - 1/a)/(2*a*c) + log(x + 1/a)/(2*a*c), Eq(n, 0)), (exp(n*acoth(a*x))/(a*c
*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{\left(\frac{1}{2}n\log(ax+1)-\frac{1}{2}n\log(ax-1)\right)}}{acn}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] e^(1/2*n*log(a*x + 1) - 1/2*n*log(a*x - 1))/(a*c*n)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{\left(\frac{1}{a\,x}+1\right)^{n/2}}{a\,c\,n\,\left(1-\frac{1}{a\,x}\right)^{n/2}}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2),x)
[Out] (1/(a*x) + 1)^(n/2)/(a*c*n*(1 - 1/(a*x))^(n/2))
```

3.741 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{2e^{n\coth^{-1}(ax)}}{ac^2n\,(4-n^2)}-\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2\,(4-n^2)\,(1-a^2x^2)}$$

```
[Out] 2*exp(n*arccoth(a*x))/a/c^2/n/(-n^2+4)-exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^2
/(-n^2+4)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{2e^{n\coth^{-1}(ax)}}{ac^2n\,(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2\,(4-n^2)\,(1-a^2x^2)}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^(n*ArcCoth[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)} + \frac{2\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c(4-n^2)}\\ &= \frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)} - \frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{e^{n\coth^{-1}(ax)}(-2+n^2-2anx+2a^2x^2)}{ac^2n(-4+n^2)(-1+a^2x^2)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] -((E^(n*ArcCoth[a*x])*(-2 + n^2 - 2*a*n*x + 2*a^2*x^2))/(a*c^2*n*(-4 + n^2)
*(-1 + a^2*x^2)))
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{e^{n\ \mathrm{arccoth}(ax)}(2a^2x^2-2anx+n^2-2)}{(a^2x^2-1)c^2an(n^2-4)}$	55
risch	$-\frac{(2a^2x^2-2anx+n^2-2)(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}}{(a^2x^2-1)c^2an(n^2-4)}$	66
parallelrisch	$\frac{-2x^2e^{n\ \mathrm{arccoth}(ax)}a^2+2x\,e^{n\ \mathrm{arccoth}(ax)}an-e^{n\ \mathrm{arccoth}(ax)}n^2+2\,e^{n\ \mathrm{arccoth}(ax)}}{c^2(a^2x^2-1)an(n^2-4)}$	78

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -exp(n*arccoth(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{(2\,a^2x^2 - 2\,anx + n^2 - 2)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^2n^3 - 4\,ac^2n - (a^3c^2n^3 - 4\,a^3c^2n)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*x^2 - 2*a*n*x + n^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^3 -
4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$$

$$= \begin{cases} \frac{xe^{\frac{i\pi n}{2}}}{c^2} \\ -\frac{a^2x^2\operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)}-4ac^2e^{2\operatorname{acoth}(ax)}} - \frac{2ax\operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)}-4ac^2e^{2\operatorname{acoth}(ax)}} + \frac{ax}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)}-4ac^2e^{2\operatorname{acoth}(ax)}} - \cdots \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} + \frac{a^2x^2\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{2ax}{4a^3c^2x^2-4ac^2} + \frac{\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} \\ \frac{\int \frac{e^{2\operatorname{acoth}(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2} \\ -\frac{2a^2x^2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \frac{2anxe^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} - \frac{n^2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \cdots \end{cases}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((x*exp(I*pi*n/2)/c**2, Eq(a, 0)), (-a**2*x**2*acoth(a*x)/(4*a**3*
c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - 2*a*x*acoth(a*x
)/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + a*x/(
4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - acoth(a*
x)/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + 2/(4
*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))), Eq(n, -2))
, (-a**2*x**2*log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) + a**2*x**2*log(x
+ 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - 2*a*x/(4*a**3*c**2*x**2 - 4*a*c**2)
+ log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - log(x + 1/a)/(4*a**3*c**2*x*
*2 - 4*a*c**2), Eq(n, 0)), (Integral(exp(2*acoth(a*x))/(a**4*x**4 - 2*a**2*
x**2 + 1), x)/c**2, Eq(n, 2)), (-2*a**2*x**2*exp(n*acoth(a*x))/(a**3*c**2*n
**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) + 2*a*n*x*exp(n*a
coth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c*
*2*n) - n**2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 -
a*c**2*n**3 + 4*a*c**2*n) + 2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a*
*3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n), True))
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c - a^2cx^2)^2}\, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(a^2cx^2 - c)^2}\, dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c - a^2cx^2)^2}\, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(a^2cx^2 - c)^2}\, dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c - a^2cx^2)^2}\, dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,x^2}{a\,c^2\,n\,(n^2-4)} - \frac{2\,x}{a^2\,c^2\,(n^2-4)} + \frac{n^2-2}{a^3\,c^2\,n\,(n^2-4)}\right)}{\left(\frac{1}{a^2} - x^2\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^2,x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((2*x^2)/(a*c^2*n*(n^2 - 4)) - (2*x)/(a^2*c^2*(n^2
 - 4)) + (n^2 - 2)/(a^3*c^2*n*(n^2 - 4))))/((1/a^2 - x^2)*((a*x - 1)/(a*x))
^(n/2))
```

3.742 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{24e^{n\coth^{-1}(ax)}}{ac^3n(64-20n^2+n^4)} - \frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}$$

```
[Out] 24*exp(n*arccoth(a*x))/a/c^3/n/(n^4-20*n^2+64)-exp(n*arccoth(a*x))*(-4*a*x+
n)/a/c^3/(-n^2+16)/(-a^2*x^2+1)^2-12*exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^3/(
n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12(n-2ax)e^{n\coth^{-1}(ax)}}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)} + \frac{24e^{n\coth^{-1}(ax)}}{ac^3n(n^4-20n^2+64)}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (24*E^(n*ArcCoth[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcCoth[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcCoth[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2}+\frac{12\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{c(16-n^2)}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}+\frac{24\int\frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^2(64-20n^2+n^4)}\\ &= \frac{24e^{n\coth^{-1}(ax)}}{ac^3n(64-20n^2+n^4)}-\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(n^4-4an^3x+24(-1+a^2x^2)^2-8anx(-5+3a^2x^2)+4n^2(-4+3a^2x^2)\right)}{ac^3n(-16+n^2)(-4+n^2)(-1+a^2x^2)^2}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*x^2)^2 - 8*a*n*x*(-5 +
3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*n*(-16 + n^2)*(-4 + n^2)*(-1 +
 a^2*x^2)^2)
```

Maple [A] (verified)

Time = 17.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

method	result
gosper	$\frac{(24a^4x^4-24a^3x^3n+12a^2n^2x^2-4a\,n^3x-48a^2x^2+n^4+40anx-16n^2+24)\,\mathrm{e}^{n\,\operatorname{arccoth}(ax)}}{(a^2x^2-1)^2c^3a(n^2-16)(n^2-4)n}$
risch	$\frac{(24a^4x^4-24a^3x^3n+12a^2n^2x^2-4a\,n^3x-48a^2x^2+n^4+40anx-16n^2+24)\,(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}}{(a^2x^2-1)^2c^3a(n^2-16)(n^2-4)n}$
parallelrisch	$\frac{24x^4\mathrm{e}^{n\,\operatorname{arccoth}(ax)}a^4+40x\,\mathrm{e}^{n\,\operatorname{arccoth}(ax)}an-48x^2\mathrm{e}^{n\,\operatorname{arccoth}(ax)}a^2-24a^3x^3\mathrm{e}^{n\,\operatorname{arccoth}(ax)}n+12x^2\mathrm{e}^{n\,\operatorname{arccoth}(ax)}a^2n^2-4x\,\mathrm{e}^{n\,\operatorname{arc}}}{c^3(a^2x^2-1)^2a(n^2-16)(n^2-4)n}$

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] (24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-1
6*n^2+24)*exp(n*arccoth(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.37

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{(24\,a^4x^4-24\,a^3nx^3+n^4+12\,(a^2n^2-4\,a^2)x^2-16\,n^2-4\,(an^3-10\,an)x+24)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n)x^4-2\,(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] (24*a^4*x^4 - 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 - 4*(a
*n^3 - 10*a*n)*x + 24)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^3*
n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2*(a
^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^3}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^3}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.51

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{24\,x^4}{a\,c^3\,n\,(n^4-20\,n^2+64)}-\frac{4\,x\,(n^2-10)}{a^4\,c^3\,(n^4-20\,n^2+64)}-\frac{24\,x^3}{a^2\,c^3\,(n^4-20\,n^2+64)}+\frac{n^4-16\,n^2+24}{a^5\,c^3\,n\,(n^4-20\,n^2+64)}+\frac{x^2\,(12\,n^2-48)}{a^3\,c^3\,n\,(n^4-20\,n^2+64)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{1}{a^4}+x^4-\frac{2\,x^2}{a^2}\right)}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^3,x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((24*x^4)/(a*c^3*n*(n^4 - 20*n^2 + 64)) - (4*x*(n^
2 - 10))/(a^4*c^3*(n^4 - 20*n^2 + 64)) - (24*x^3)/(a^2*c^3*(n^4 - 20*n^2 +
64)) + (n^4 - 16*n^2 + 24)/(a^5*c^3*n*(n^4 - 20*n^2 + 64)) + (x^2*(12*n^2 -
 48))/(a^3*c^3*n*(n^4 - 20*n^2 + 64))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^4 + x
^4 - (2*x^2)/a^2))
```

3.743 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^4} dx = \frac{720e^{n \coth^{-1}(ax)}}{ac^4n(36-n^2)(64-20n^2+n^4)} - \frac{e^{n \coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n \coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2} - \frac{360e^{n \coth^{-1}(ax)}(n-2ax)}{ac^4(4-n^2)(16-n^2)(36-n^2)(1-a^2x^2)}$$

```
[Out] 720*exp(n*arccoth(a*x))/a/c^4/n/(-n^2+36)/(n^4-20*n^2+64)-exp(n*arccoth(a*x
))*(-6*a*x+n)/a/c^4/(-n^2+36)/(-a^2*x^2+1)^3-30*exp(n*arccoth(a*x))*(-4*a*x
+n)/a/c^4/(n^4-52*n^2+576)/(-a^2*x^2+1)^2-360*exp(n*arccoth(a*x))*(-2*a*x+n
)/a/c^4/(-n^2+36)/(n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^4} dx = -\frac{(n-6ax)e^{n \coth^{-1}(ax)}}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{360(n-2ax)e^{n \coth^{-1}(ax)}}{ac^4(4-n^2)(16-n^2)(36-n^2)(1-a^2x^2)} - \frac{30(n-4ax)e^{n \coth^{-1}(ax)}}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2} + \frac{720e^{n \coth^{-1}(ax)}}{ac^4n(36-n^2)(n^4-20n^2+64)}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcCoth[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcCoth[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Coth[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcCoth[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3}+\frac{30\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{c(36-n^2)}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3}-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2}+\frac{360\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{c^2(576-52n^2+n^4)}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3}-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2}\\
&\quad-\frac{360e^{n\coth^{-1}(ax)}(n-2ax)}{ac^4(4-n^2)(576-52n^2+n^4)(1-a^2x^2)}+\frac{720\int\frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^3(4-n^2)(576-52n^2+n^4)}\\
&= \frac{720e^{n\coth^{-1}(ax)}}{ac^4n(4-n^2)(576-52n^2+n^4)}-\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3}\\
&\quad-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2}-\frac{360e^{n\coth^{-1}(ax)}(n-2ax)}{ac^4(4-n^2)(576-52n^2+n^4)(1-a^2x^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.77

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}\left(n^6-6an^5x-120an^3x(-2+a^2x^2)+720(-1+a^2x^2)^3+10n^4(-5+3a^2x^2)-48anx(33-\right.}{ac^4n\,(-36+n^2)\,(-16+n^2)\,(-4+n^2)\,(-1+a^2x^2)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n^6 - 6*a*n^5*x - 120*a*n^3*x*(-2 + a^2*x^2) + 720*(
-1 + a^2*x^2)^3 + 10*n^4*(-5 + 3*a^2*x^2) - 48*a*n*x*(33 - 40*a^2*x^2 + 15*
a^4*x^4) + 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a*c^4*n*(-36 + n^2)*(-1
6 + n^2)*(-4 + n^2)*(-1 + a^2*x^2)^3))
```

Maple [A] (verified)

Time = 59.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{(720a^6x^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160a^4x^4+30a^2n^4x^2+1920a^3x^3n-6a\,n^5x-840a^2n^2x^2+n^6+240a\,n^3x+2}{(a^2x^2-1)^3c^4an(n^6-56n^4+784n^2-2304)}$
risch	$-\frac{(720a^6x^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160a^4x^4+30a^2n^4x^2+1920a^3x^3n-6a\,n^5x-840a^2n^2x^2+n^6+240a\,n^3x+2}{c^4(n^2-36)(n^2-16)(n^2-4)an(a^2x^2-1)^3}$
parallelrisch	$\frac{-720x^6e^{n\ \mathrm{arccoth}(ax)}a^6+2160x^4e^{n\ \mathrm{arccoth}(ax)}a^4+720a^5e^{n\ \mathrm{arccoth}(ax)}x^5n-360x^4e^{n\ \mathrm{arccoth}(ax)}a^4n^2+120x^3e^{n\ \mathrm{arccoth}(ax)}a^3n}{}$

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*
a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arccoth(a*x))/(a^2*x^2-1)^3/c^
4/a/n/(n^6-56*n^4+784*n^2-2304)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.57

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{(720\,a^6x^6-720\,a^5nx^5+n^6+360\,(a^4n^2-6\,a^4)x^4-50\,n^4-120\,(a^3n^3-16\,a^3n)x^3+30\,(a}{ac^4n^7-56\,ac^4n^5+784\,ac^4n^3-(a^7c^4n^7-56\,a^7c^4n^5+784\,a^7c^4n^3-2304\,a^7c^4n)x^6-2304\,ac^4n+3\,(a^5c^4n^7}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 - 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 - 1
20*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 - 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x + 1)/(a*x - 1))^(1/2*n)
/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*n^5
+ 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^7 - 5
6*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^7 - 56
*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^4}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^4}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.59

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^6-50\,n^4+544\,n^2-720}{a^7\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{720\,x^5}{a^2\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{x^3\,\left(120\,n^2-1920\right)}{a^4\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}+\frac{720}{a\,c^4\,n\,\left(n^6-56\,n^4\right.}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^6}-x^6+\right.}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^4,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((544*n^2 - 50*n^4 + n^6 - 720)/(a^7*c^4*n*(784*n^
2 - 56*n^4 + n^6 - 2304)) - (720*x^5)/(a^2*c^4*(784*n^2 - 56*n^4 + n^6 - 23
04)) - (x^3*(120*n^2 - 1920))/(a^4*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (
720*x^6)/(a*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) - (6*x*(n^4 - 40*n^2 + 2
64))/(a^6*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^2*(30*n^4 - 840*n^2 + 2
160))/(a^5*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^4*(360*n^2 - 2160))/
(a^3*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304))))/(((a*x - 1)/(a*x))^(n/2)*(1/a
^6 - x^6 + (3*x^4)/a^2 - (3*x^2)/a^4))
```

3.744 $\int e^{n \coth^{-1}(ax)}\left(c-a^2 c x^2\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{n \coth^{-1}(ax)}\left(c-a^2 c x^2\right)^{3/2} dx=\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}\left(c-a^2cx^2\right)^{3/2}\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] 32*(1-1/a/x)^(5/2-1/2*n)*(1+1/a/x)^(-5/2+1/2*n)*(-a^2*c*x^2+c)^(3/2)*hyperg
eom([5, 5/2-1/2*n],[7/2-1/2*n],(a-1/x)/(a+1/x))/a^4/(5-n)/(1-1/a^2/x^2)^(3/
2)/x^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6330, 133}

$$\int e^{n \coth^{-1}(ax)}\left(c-a^2 c x^2\right)^{3/2} dx=\frac{32\left(c-a^2cx^2\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}}\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(3
/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule steps

$$\text{integral} = \frac{\left(c-a^2cx^2\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+\frac{n}{2}}}{x^5}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}\left(c-a^2cx^2\right)^{3/2}\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(116) = 232.

Time = 2.93 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.41

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx = \frac{c^2\left(96a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x(n+ax)+2e^{(1+n)\coth^{-1}(ax)}(-1+n)\,\mathrm{Hyp}\right.\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c^2*(96*a^3*c*(1 - 1/(a^2*x^2))^(3/2)*x^3*(a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1
/(a^2*x^2)]*x*(n + a*x) + 2*E^((1 + n)*ArcCoth[a*x])*(-1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]) - c*(-1 + a^2*x^2)*(2*E^
(n*ArcCoth[a*x])*(-1 + a^2*x^2)^2*(-(a*(-21 + n^2)*x) + 2*n*(1 - n^2 + (3 +
 n^2)*Cosh[2*ArcCoth[a*x]]) + a*(3 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*Ar
cCoth[a*x]]) + 16*a*E^((1 + n)*ArcCoth[a*x])*(-3 + 3*n - n^2 + n^3)*Sqrt[1
- 1/(a^2*x^2)]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*
x])])))/(192*a*(c - a^2*c*x^2)^(3/2))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx = \int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n
), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}e^{n\operatorname{acoth}(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int e^{n\operatorname{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2), x)
```

3.745 $\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 8*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*hypergeom([3, 3/2-1/2*n],[5/
2-1/2*n],(a-1/x)/(a+1/x))*(-a^2*c*x^2+c)^(1/2)/a^2/(3-n)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6330, 133}

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a^
2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-a^2cx^2}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= -\frac{\sqrt{c-a^2cx^2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^3}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{ce^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\left(a\sqrt{1-\frac{1}{a^2x^2}}x(n+ax)+2e^{\coth^{-1}(ax)}(-1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\right.\right.}{2\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] -1/2*(c*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*(a*Sqrt[1 - 1/(a^2*x^2)]
*x*(n + a*x) + 2*E^ArcCoth[a*x]*(-1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, E^(2*ArcCoth[a*x])]))/Sqrt[c - a^2*c*x^2]
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}\sqrt{-a^2c\,x^2+c}dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-c\,(ax-1)\,(ax+1)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

3.746 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] 2*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[
3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6330, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/
2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])
/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int\frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}\\&=-\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{\sqrt{c-a^2cx^2}}\\&=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2e^{(1+n)\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,(a^2cx+a^2cnx)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
[Out] (-2*E^((1 + n)*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(Sqrt[1 - 1/(a^2*x^2)]*(a^2*c*x + a^2
*c*n*x))
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{-a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c
), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2), x)
```

3.747 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6319}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx = -\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\,(-1+n^2)\,\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2cx^2+c)^{\frac{3}{2}}}$	49

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(\frac{x}{c\,(n^2-1)}-\frac{n}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
[Out] -((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2*
c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.748 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} - \frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6320, 6319}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}} - \frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}}+\frac{6\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c\,(9-n^2)}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2\,(1-n^2)\,(9-n^2)\,\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n\coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-\right.}{4ac^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(ax-1)(ax+1)(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{(6\,a^3x^3-6\,a^2nx^2-n^3+3\,(an^2-3\,a)x+7\,n)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^4+9\,ac^3-2\,(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
[Out] -((((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/(
a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9))
- (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*
(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.749 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{120e^{n\coth^{-1}(ax)}(n-ax)}{ac^3\,(1-n^2)\,(9-n^2)\,(25-n^2)\,\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-5*a*x+n)/a/c/(-n^2+25)/(-a^2*c*x^2+c)^(5/2)-20*exp(n
*arccoth(a*x))*(-3*a*x+n)/a/c^2/(n^4-34*n^2+225)/(-a^2*c*x^2+c)^(3/2)-120*e
xp(n*arccoth(a*x))*(-a*x+n)/a/c^3/(-n^2+25)/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6320, 6319}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=-\frac{120(n-ax)e^{n\coth^{-1}(ax)}}{ac^3\,(1-n^2)\,(9-n^2)\,(25-n^2)\,\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\coth^{-1}(ax)}}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
```

```
c*x^2)^(3/2)) - (120*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}+\frac{20\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx}{c\,(25-n^2)}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}}+\frac{120\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c^2\,(9-n^2)\,(25-n^2)}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}}\\
&\quad-\frac{120e^{n\coth^{-1}(ax)}(n-ax)}{ac^3\,(1-n^2)\,(9-n^2)\,(25-n^2)\,\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.80

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{a^2e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{10(225-34n^2+n^4)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2250n}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{340n^3}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{10n^5}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+15(25-26n^2+n^4\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/16*(a^2*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))^(3/2)*x^3*((-10*(225 - 34*n
^2 + n^4))/Sqrt[1 - 1/(a^2*x^2)] + (2250*n)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (
340*n^3)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (10*n^5)/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
 + 15*(25 - 26*n^2 + n^4)*Cosh[3*ArcCoth[a*x]] - 45*Cosh[5*ArcCoth[a*x]] +
50*n^2*Cosh[5*ArcCoth[a*x]] - 5*n^4*Cosh[5*ArcCoth[a*x]] - 125*n*Sinh[3*Arc
Coth[a*x]] + 130*n^3*Sinh[3*ArcCoth[a*x]] - 5*n^5*Sinh[3*ArcCoth[a*x]] + 9*
n*Sinh[5*ArcCoth[a*x]] - 10*n^3*Sinh[5*ArcCoth[a*x]] + n^5*Sinh[5*ArcCoth[a
*x]]))/(c^2*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*(c - a^2*c*x
^2)^(3/2))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{(ax-1)(ax+1)(120a^5x^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300a^3x^3+5a\,n^4x+260n\,x^2a^2-n^5-110n^2xa+30n^3+225ax-149n)e^{n\,a}}{a(n^6-35n^4+259n^2-225)(-a^2c\,x^2+c)^{\frac{7}{2}}}$

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-30
0*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*exp
(n*arccoth(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.75

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{(120\,a^5x^5-120\,a^4nx^4-n^5+60\,(a^3n^2-5\,a^3)x^3+30\,n^3-20\,(a^2n^3-13\,a^2n)x}{ac^4n^6-35\,ac^4n^4+259\,ac^4n^2-(a^7c^4n^6-35\,a^7c^4n^4+259\,a^7c^4n^2-225\,a^7c^4)x^6-225\,ac^4+3\,(a^5c^4n^6-3}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -(120*a^5*x^5 - 120*a^4*n*x^4 - n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 + 30*n^3 - 2
0*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x - 149*n)*sqrt(-a
^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 + 259
*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*c^4)
*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 - 225*
a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 225*a^3*
c^4)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.74

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{120\,x^5}{c^3\,(n^6-35\,n^4+259\,n^2-225)}-\frac{120\,n\,x^4}{a\,c^3\,(n^6-35\,n^4+259\,n^2-225)}+\frac{x^3\,(60\,n^2-300)}{a^2\,c^3\,(n^6-35\,n^4+259\,n^2-225)}-\frac{n\,(n^4-30\,n^2+149)}{a^5\,c^3\,(n^6-35\,n^4+259\,n^2-22}\right.}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/}}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*((120*x^5)/(c^3*(259*n^2 - 35*n^4 + n^6 - 225)) -
 (120*n*x^4)/(a*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (x^3*(60*n^2 - 300))/
(a^2*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) - (n*(n^4 - 30*n^2 + 149))/(a^5*c^
3*(259*n^2 - 35*n^4 + n^6 - 225)) + (5*x*(n^4 - 22*n^2 + 45))/(a^4*c^3*(259
*n^2 - 35*n^4 + n^6 - 225)) - (20*n*x^2*(n^2 - 13))/(a^3*c^3*(259*n^2 - 35*
n^4 + n^6 - 225))))/(((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2*c*x^2)^(1/2)
 - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2)*((a*x - 1)/(a*x))^(n/2))
```

3.750 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 239

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{e^{n \coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}} - \frac{42e^{n \coth^{-1}(ax)}(n-5ax)}{ac^2\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{840e^{n \coth^{-1}(ax)}(n-3ax)}{ac^3\,(9-n^2)\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{5040e^{n \coth^{-1}(ax)}(n-ax)}{ac^4\,(1-n^2)\,(9-n^2)\,(25-n^2)\,(49-n^2)\,\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-7*a*x+n)/a/c/(-n^2+49)/(-a^2*c*x^2+c)^(7/2)-42*exp(n
*arccoth(a*x))*(-5*a*x+n)/a/c^2/(n^4-74*n^2+1225)/(-a^2*c*x^2+c)^(5/2)-840*
exp(n*arccoth(a*x))*(-3*a*x+n)/a/c^3/(-n^2+49)/(n^4-34*n^2+225)/(-a^2*c*x^2
+c)^(3/2)-5040*exp(n*arccoth(a*x))*(-a*x+n)/a/c^4/(n^4-74*n^2+1225)/(n^4-10
*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used

$= \{6320, 6319\}$

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{5040(n-ax)e^{n\coth^{-1}(ax)}}{ac^4(1-n^2)(9-n^2)(25-n^2)(49-n^2)\sqrt{c-a^2cx^2}} - \frac{840(n-3ax)e^{n\coth^{-1}(ax)}}{ac^3(9-n^2)(25-n^2)(49-n^2)(c-a^2cx^2)^{3/2}} - \frac{42(n-5ax)e^{n\coth^{-1}(ax)}}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} - \frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac(49-n^2)(c-a^2cx^2)^{7/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
- (42*E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c^2*(25 - n^2)*(49 - n^2)*(c - a^2
*c*x^2)^(5/2)) - (840*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^3*(9 - n^2)*(25
- n^2)*(49 - n^2)*(c - a^2*c*x^2)^(3/2)) - (5040*E^(n*ArcCoth[a*x])*(n - a*
x))/(a*c^4*(1 - n^2)*(9 - n^2)*(25 - n^2)*(49 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\text{integral} = -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} + \frac{42\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx}{c(49-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} + \frac{840\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx}{c^2(25-n^2)(49-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} - \frac{840e^{n\coth^{-1}(ax)}(n-3ax)}{ac^3(9-n^2)(25-n^2)(49-n^2)(c-a^2cx^2)^{3/2}} + \frac{5040\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c^3(9-n^2)(25-n^2)(49-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{840e^{n\coth^{-1}(ax)}(n-3ax)}{ac^3\,(9-n^2)\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{5040e^{n\coth^{-1}(ax)}(n-ax)}{ac^4\,(1-n^2)\,(9-n^2)\,(25-n^2)\,(49-n^2)\,\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{ae^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\left(-\frac{35n}{-1+n^2}+\frac{35ax}{-1+n^2}-\frac{63a\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)}{-9+n^2}+\frac{35a\sqrt{1-\frac{1}{a^2x^2}}}{}\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))*x^2*((-35*n)/(-1 + n^2) + (35*a*x)/
(-1 + n^2) - (63*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]])/(-9 + n^2)
 + (35*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[5*ArcCoth[a*x]])/(-25 + n^2) - (7*a*S
qrt[1 - 1/(a^2*x^2)]*x*Cosh[7*ArcCoth[a*x]])/(-49 + n^2) + (21*a*n*Sqrt[1 -
 1/(a^2*x^2)]*x*Sinh[3*ArcCoth[a*x]])/(-9 + n^2) - (7*a*n*Sqrt[1 - 1/(a^2*x
^2)]*x*Sinh[5*ArcCoth[a*x]])/(-25 + n^2) + (a*n*Sqrt[1 - 1/(a^2*x^2)]*x*Sin
h[7*ArcCoth[a*x]])/(-49 + n^2)))/(64*c^3*(c - a^2*c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.91

method	result
gosper	$\frac{(ax-1)(ax+1)(5040a^7x^7-5040n\,a^6x^6+2520a^5n^2x^5-840a^4n^3x^4-17640a^5x^5+210a^3n^4x^3+15960n\,a^4x^4-42a^2n^5x^2-7140a^3n^2x^3+7}{a(n^8-84n^6+1974n^4-12916n^2+}$

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*(a*x+1)*(5040*a^7*x^7-5040*a^6*n*x^6+2520*a^5*n^2*x^5-840*a^4*n^3*x
^4-17640*a^5*x^5+210*a^3*n^4*x^3+15960*a^4*n*x^4-42*a^2*n^5*x^2-7140*a^3*n^
2*x^3+7*a*n^6*x+2100*a^2*n^3*x^2-n^7+22050*a^3*x^3-455*a*n^4*x-17178*a^2*n*
x^2+77*n^5+6433*a*n^2*x-1519*n^3-11025*a*x+6483*n)*exp(n*arccoth(a*x))/a/(n
^8-84*n^6+1974*n^4-12916*n^2+11025)/(-a^2*c*x^2+c)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.90

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx =$$

$$-\frac{(5040\,a^7x^7 - 5040\,a^6nx^6 - n^7 + 2520\,(a^5n^2 - 7\,a^5)x^5 +}{ac^5n^8 - 84\,ac^5n^6 + 1974\,ac^5n^4 + (a^9c^5n^8 - 84\,a^9c^5n^6 + 1974\,a^9c^5n^4 - 12916\,a^9c^5n^2 + 11025\,a^9c^5)x^8 - 12}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -(5040*a^7*x^7 - 5040*a^6*n*x^6 - n^7 + 2520*(a^5*n^2 - 7*a^5)*x^5 + 77*n^5
 - 840*(a^4*n^3 - 19*a^4*n)*x^4 + 210*(a^3*n^4 - 34*a^3*n^2 + 105*a^3)*x^3
- 1519*n^3 - 42*(a^2*n^5 - 50*a^2*n^3 + 409*a^2*n)*x^2 + 7*(a*n^6 - 65*a*n^
4 + 919*a*n^2 - 1575*a)*x + 6483*n)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x -
1))^(1/2*n)/(a*c^5*n^8 - 84*a*c^5*n^6 + 1974*a*c^5*n^4 + (a^9*c^5*n^8 - 84*
a^9*c^5*n^6 + 1974*a^9*c^5*n^4 - 12916*a^9*c^5*n^2 + 11025*a^9*c^5)*x^8 - 1
2916*a*c^5*n^2 - 4*(a^7*c^5*n^8 - 84*a^7*c^5*n^6 + 1974*a^7*c^5*n^4 - 12916
*a^7*c^5*n^2 + 11025*a^7*c^5)*x^6 + 11025*a*c^5 + 6*(a^5*c^5*n^8 - 84*a^5*c
^5*n^6 + 1974*a^5*c^5*n^4 - 12916*a^5*c^5*n^2 + 11025*a^5*c^5)*x^4 - 4*(a^3
*c^5*n^8 - 84*a^3*c^5*n^6 + 1974*a^3*c^5*n^4 - 12916*a^3*c^5*n^2 + 11025*a^
3*c^5)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{9}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.85

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{5040\,x^7}{c^4\,(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025)}+\frac{-n^7+77\,n^5-1519\,n^3+6483\,n}{a^7\,c^4\,(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025)}-\frac{5040\,n\,x^6}{a\,c^4\,(n^8-84\,n^6+1974\,n^4-12916\,n^2}\right.}{}$$

[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(9/2),x)

[Out] -(((a*x + 1)/(a*x))^(n/2)*((5040*x^7)/(c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (6483*n - 1519*n^3 + 77*n^5 - n^7)/(a^7*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) - (5040*n*x^6)/(a*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (x^5*(2520*n^2 - 17640))/(a^2*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (x^3*(210*n^4 - 7140*n^2 + 22050))/(a^4*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (7*x*(919*n^2 - 65*n^4 + n^6 - 1575))/(a^6*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) - (840*n*x^4*(n^2 - 19))/(a^3*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) - (42*n*x^2*(n^4 - 50*n^2 + 409))/(a^5*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025))))/(((a*x - 1)/(a*x))^(n/2)*((c - a^2*c*x^2)^(1/2)/a^6 - x^6*(c - a^2*c*x^2)^(1/2) + (3*x^4*(c - a^2*c*x^2)^(1/2))/a^2 - (3*x^2*(c - a^2*c*x^2)^(1/2))/a^4))

3.751 $\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 359

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx &= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} \\ &+ \frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}} \\ &+ \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} \\ &- \frac{2n\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}}\end{aligned}$$

```
[Out] -(2+n)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^
3/a/(1+n)/(-a^2*c*x^2+c)^(3/2)+(n^2+2*n+2)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1
/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/a/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)+(1-1/a
^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^4/(-a^2*c*x^2
+c)^(3/2)-2*n*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2
*n)*x^3*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/a/(1-n)/(-a^
2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6327, 6329, 105, 160, 12, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{2nx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(n^2+2n+2\right)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{a(1-n)(n+1)\left(c-a^2cx^2\right)^{3/2}} + \frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{\left(c-a^2cx^2\right)^{3/2}} - \frac{(n+2)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{a(n+1)\left(c-a^2cx^2\right)^{3/2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(((2 + n)*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3)/(a*(1 + n)*(c - a^2*c*x^2)^(3/2))) + ((2 + 2*n + n^2)*(1
 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*
x^3)/(a*(1 - n)*(1 + n)*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*(
1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^4)/(c - a^2*c*x^2)^(
3/2) - (2*n*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (a + x^(-1))/
(a - x^(-1))])/(a*(1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\frac{\left(-\frac{n}{a}-\frac{2x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\left(\frac{n(1+n)}{a^2}+\frac{(2+n)x}{a^3}\right)}{x}\,dx,x,\frac{1}{x}\right)}{(1+n)\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\frac{n\left(1-n^2\right)\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{a^3x}\,dx,x,\frac{1}{x}\right)}{(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(n\left(1-n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{a(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}}$$
$$+\frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}}$$
$$+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}}$$
$$-\frac{2n\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.37

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\frac{ce^{n\coth^{-1}(ax)}(-1+anx)}{-1+n^2}-\frac{c(-1+a^2x^2)\left(e^{n\coth^{-1}(ax)}(1+n)+\frac{2e^{(1+n)\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax)}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\right)}{1+n}}{a^4c^2\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((c*E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(-1 + n^2) - (c*(-1 + a^2*x^2)*(E^(n*A
rcCoth[a*x])*(1 + n) + (2*E^((1 + n)*ArcCoth[a*x])*n*Hypergeometric2F1[1, (
1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(a*Sqrt[1 - 1/(a^2*x^2)]*x)))/(1
+ n))/(a^4*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}x^3}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}}\, dx = \int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\, dx$$

[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")

[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}}\, dx = \int \frac{x^3 e^{n\,\text{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\, dx$$

[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)

[Out] Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}}\, dx = \int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\, dx$$

[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")

[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

```
[In] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
[Out] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

3.752 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c(1-n^2)\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a^3/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)-2*(1-1/a/
x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[3/2-1/2*n
],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a^2/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6324, 6327, 6330, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}} - \frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) - (
2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2
)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6324

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\,(1-n^2)\,\sqrt{c-a^2cx^2}}-\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\,(1-n^2)\,\sqrt{c-a^2cx^2}}-\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{a^2c\sqrt{c-a^2cx^2}}
\end{aligned}
$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\,(1-n^2)\,\sqrt{c-a^2cx^2}} + \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{a^2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\,(1-n^2)\,\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx =$$

$$-\frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-n+ax) + 2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right)\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e\right.\right.}{a^4c(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-n + a*x) + 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2
*ArcCoth[a*x])]))/(a^4*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c -
a^2*c*x^2]))
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}x^2}{(-a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)

[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

```
[In] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
[Out] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

3.753 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c\,(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] exp(n*arccoth(a*x))*(-a*n*x+1)/a^2/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6321}

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c\,(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
[Out] (E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6321

```
Int[(E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcCoth[a*x])/(a^2*c*(n^2 - 1)*Sqrt[c + d*
x^2])), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rubi steps

$$\text{integral}=\frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c\,(1-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(-1+anx)}{a^2c\,(-1+n^2)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
[Out] (E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(a^2*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{(ax-1)(ax+1)(anx-1)e^{n\ \mathrm{arccoth}(ax)}}{a^2(n^2-1)(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arccoth(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^(
3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(anx-1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-(a^4c^2n^2-a^4c^2)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] sqrt(-a^2*c*x^2 + c)*(a*n*x - 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^2
 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(\frac{1}{a^2\,c\,(n^2-1)}-\frac{n\,x}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
[Out] -((1/(a^2*c*(n^2 - 1)) - (n*x)/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((
c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.754 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6319}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\,(-1+n^2)\,\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2cx^2+c)^{\frac{3}{2}}}$	49

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(\frac{x}{c\,(n^2-1)}-\frac{n}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
[Out] -((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2*
c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.755 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1-n^2)\left(c-a^2cx^2\right)^{3/2}} - \frac{2^{\frac{1+n}{2}}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/
(1+n)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1
+1/a/x)^(-1/2+1/2*n)*x^3/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)-2^(1/2+1/2*n)*a^3*(1
-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*x^3*hypergeom([1/2-1/2*n, 1/2-1/2*n
],[3/2-1/2*n],1/2*(a-1/x)/a)/(1-n)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{6327, 6330, 91, 80, 71\}$

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^3 2^{\frac{n+1}{2}} x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} \text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)(c-a^2cx^2)^{3/2}} + \frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(1-n^2)(c-a^2cx^2)^{3/2}} - \frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n+1)(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-
1 + n)/2)*x^3)/((1 + n)*(c - a^2*c*x^2)^(3/2))) + (a^3*(1 - 1/(a^2*x^2))^(3
/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^3)/((1 - n^2)*(c
 - a^2*c*x^2)^(3/2)) - (2^((1 + n)/2)*a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a
*x))^((1 - n)/2)*x^3*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a
- x^(-1))/(2*a)])/((1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{(c-a^2cx^2)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(c-a^2cx^2)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\,(c-a^2cx^2)^{3/2}} + \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\left(\frac{n}{a}+\frac{(1+n)x}{a^2}\right)dx,x,\frac{1}{x}\right)}{(1+n)\,(c-a^2cx^2)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\,(c-a^2cx^2)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}(-1+n)}dx,x,\frac{1}{x}\right)}{(c-a^2cx^2)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1-n^2)\left(c-a^2cx^2\right)^{3/2}} - \frac{2^{\frac{1+n}{2}}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.46

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-1+anx)-2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right)\text{Hypergeon}\right.}{ac(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*n*x) - 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(
2*ArcCoth[a*x])]))/(a*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a
^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 -
2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)), x)
```

3.756 $\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 463

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} + \frac{(15+6n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(18+7n-2n^2-n^3)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}} - \frac{2\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n
)/(-a^2*c*x^2+c)^(5/2)-(6+n)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+(n^2+6*n+15)*(1-1/
a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2
+n+3)/(-a^2*c*x^2+c)^(5/2)-(-n^3-2*n^2+7*n+18)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x
)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2
)-2*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5*hy
pergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/(1-n)/(-a^2*c*x^2+c)^(
5/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6327, 6330, 136, 160, 12, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}} + \frac{(n^2+6n+15)\,x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(1-n)(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}} - \frac{(-n^3-2n^2+7n+18)\,x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{(n^4-10n^2+9)\left(c-a^2cx^2\right)^{5/2}} - \frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}} - \frac{(n+6)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3 +
n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - ((6 + n)*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) + ((15 + 6*n + n^2)*(1 - 1/(a^2*x^2))^(5/2)*(1
- 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 - n)*(1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) - ((18 + 7*n - 2*n^2 - n^3)*(1 - 1/(a^2*x^2))^(
5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2
 + n^4)*(c - a^2*c*x^2)^(5/2)) - (2*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^(
(1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5*Hypergeometric2F1[1, (-1 + n)/2,
(1 + n)/2, (a + x^(-1))/(a - x^(-1))])/((1 - n)*(c - a^2*c*x^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
```

```
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 136

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimp
lerQ[p, 1]))
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral}=\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$=-\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{(c-a^2cx^2)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}}+\frac{\left(a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(-\frac{3+n}{a}-\frac{3x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{(3+n)\,(c-a^2cx^2)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\,(c-a^2cx^2)^{5/2}}-\frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\left(\frac{(1+n)(3+n)}{a^2}+\frac{2(6+n)x}{a^3}\right)}{x}\,dx,x,\frac{1}{x}\right)}{(1+n)(3+n)\,(c-a^2cx^2)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\,(c-a^2cx^2)^{5/2}}+\frac{(15+6n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1-n)(1+n)(3+n)\,(c-a^2cx^2)^{5/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\left(\frac{(1-n)(1+n)(3+n)}{a^3}-\frac{(15+6n+n^2)x}{a^4}\right)}{x}\,dx,x,\frac{1}{x}\right)}{(1-n)(1+n)(3+n)\,(c-a^2cx^2)^{5/2}}$$

$$
\begin{aligned}
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad +\frac{(15+6n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{(18+7n-2n^2-n^3)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{(9-10n^2+n^4)\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{a^4x}\,dx,x,\frac{1}{x}\right)}{(1-n)(3-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad +\frac{(15+6n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{(18+7n-2n^2-n^3)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{\left((9-10n^2+n^4)\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{(1-n)(3-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad +\frac{(15+6n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{(18+7n-2n^2-n^3)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}} \\
&\quad -\frac{2\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.43

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(-1+a^2x^2)\left(\frac{8e^{n\coth^{-1}(ax)}(n-ax)}{-1+n^2}+\frac{e^{n\coth^{-1}(ax)}\left(26n-2n^3-27ax+3an^2x+2n(-1+n^2)\cosh(2\coth^{-1}(\cdots\right.}{9-10n^2+n^4}\cdots\right.}{4\cdots}$$

[In] Integrate[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]

[Out] ((-1 + a^2*x^2)*((8*E^(n*ArcCoth[a*x])*(n - a*x))/(-1 + n^2) + (E^(n*ArcCot h[a*x])*(26*n - 2*n^3 - 27*a*x + 3*a*n^2*x + 2*n*(-1 + n^2)*Cosh[2*ArcCoth[a*x]] - 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))/(9 - 10*n^2 + n^4) - (8*a*E^((1 + n)*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*Hyper geometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(1 + n)))/(4*a^5 *c*(c - a^2*c*x^2)^(3/2))

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}x^4}{(-a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

[In] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)

[Out] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas ")

[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x ^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*x**4/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**4*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

```
[In] int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
[Out] int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2), x)
```

3.757 $\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 330

$$\begin{aligned}\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{5/2}} \, dx = &-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} \\ &-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\,(c-a^2cx^2)^{5/2}} \\ &+\frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(1-n^2)\,(c-a^2cx^2)^{5/2}} \\ &-\frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\,(c-a^2cx^2)^{5/2}}\end{aligned}$$

```
[Out] -a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3
+n)/(-a^2*c*x^2+c)^(5/2)-3*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a*(1-1/a^2/x^2)^
(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a
^2*c*x^2+c)^(5/2)-6*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(
-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6327, 6330, 47, 37}

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{5/2}} \, dx = -\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\,(c-a^2cx^2)^{5/2}} + \frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)\,(1-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{(n^4-10n^2+9)\,(c-a^2cx^2)^{5/2}} - \frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\,(c-a^2cx^2)^{5/2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a*(1 - 1/(a^2*x^2))^(5/2
)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n + n^
2)*(c - a^2*c*x^2)^(5/2)) + (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1
- n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^
(5/2)) - (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{\left(3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(3+n)\,(c-a^2cx^2)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\,(c-a^2cx^2)^{5/2}} - \frac{\left(6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(1+n)(3+n)\,(c-a^2cx^2)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}} + \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n-3n^2-n^3)\left(c-a^2cx^2\right)^{5/2}} + \frac{\left(6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1-n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}dx, x, \frac{1}{x}\right)}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}} + \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n-3n^2-n^3)\left(c-a^2cx^2\right)^{5/2}} - \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.33

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}\left(3(10-2n^2-9anx+an^3x)-6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+an(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}x\cosh\right.}{4a^4c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/4*(E^(n*ArcCoth[a*x])*(3*(10 - 2*n^2 - 9*a*n*x + a*n^3*x) - 6*(-1 + n^2)
*Cosh[2*ArcCoth[a*x]] + a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCo
th[a*x]]))/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

method	result	size
gosper	$-\frac{(ax-1)(ax+1)\left(a^{3} n^{3} x^{3}-7 a^{3} x^{3} n-3 a^{2} n^{2} x^{2}+9 a^{2} x^{2}+6 a n x-6\right) {\mathrm e}^{n \,\mathrm{arccoth}(a x)}}{a^{4} \left(n^{4}-10 n^{2}+9\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}$	93

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-6
)*exp(n*arccoth(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{\left(c-a^2 c x^2\right)^{5/2}}\, dx = \frac{\sqrt{-a^2cx^2+c}\left((a^3n^3-7\,a^3n)x^3+6\,anx-3\,(a^2n^2-3\,a^2)x^2-6\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}}}{a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3+(a^8c^3n^4-10\,a^8c^3n^2+9\,a^8c^3)x^4-2\,(a^6c^3n^4-10\,a^6c^3n}$$

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
[Out] sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x - 3*(a^2*n^2 - 3*a^
2)*x^2 - 6)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 + 9
*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n^4
- 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{\left(c-a^2 c x^2\right)^{5/2}}\, dx = \int \frac{x^{3} e^{n \operatorname{acoth}{\left(a x \right)}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6}{a^6\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x}{a^5\,c^2\,(n^4-10\,n^2+9)}+\frac{x^2\,(3\,n^2-9)}{a^4\,c^2\,(n^4-10\,n^2+9)}-\frac{n\,x^3\,(n^2-7)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

```
[In] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*(6/(a^6*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x)/(a^5*c^
2*(n^4 - 10*n^2 + 9)) + (x^2*(3*n^2 - 9))/(a^4*c^2*(n^4 - 10*n^2 + 9)) - (n
*x^3*(n^2 - 7))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2
- x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.758 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}+\frac{e^{n\coth^{-1}(ax)}(3-n^2)(n-ax)}{a^3c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a^3/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+exp(n*a
rccoth(a*x))*(-n^2+3)*(-a*x+n)/a^3/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6324, 6319}

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(3-n^2)(n-ax)e^{n\coth^{-1}(ax)}}{a^3c^2(n^4-10n^2+9)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rule 6324

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{(3-n^2)\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{a^2c\,(9-n^2)}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{e^{n\coth^{-1}(ax)}(3-n^2)\,(n-ax)}{a^3c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int\frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\Big(10n-2n^3-9ax+an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1+}{4a^3c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(10*n - 2*n^3 - 9*a*x + a*n^2*x - 2*n*(-1 + n^2)*Cosh[2
*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]
]))/(4*a^3*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{(ax-1)(ax+1)(a^3n^2x^3-a^2n^3x^2-3a^3x^3+3n\,x^2a^2+2n^2xa-2n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^4-10n^2+9)a^3(-a^2c\,x^2+c)^{\frac{5}{2}}}$	96

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2*
n)*exp(n*arccoth(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}\left(2\,an^2x+(a^3n^2-3\,a^3)x^3-(a^2n^3-3\,a^2n)x^2-2\,n\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3)x^4-2\,(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 - (a^2*n^3 - 3*a^2
*n)*x^2 - 2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^2
+ 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2e^{n\operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,n}{a^5\,c^2\,(n^4-10\,n^2+9)}-\frac{x^3\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{2\,n^2\,x}{a^4\,c^2\,(n^4-10\,n^2+9)}+\frac{n\,x^2\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

[In] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)

[Out] (((a*x + 1)/(a*x))^(n/2)*((2*n)/(a^5*c^2*(n^4 - 10*n^2 + 9)) - (x^3*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n^2*x)/(a^4*c^2*(n^4 - 10*n^2 + 9)) + (n*x^2*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))

3.759 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}+\frac{2e^{n\coth^{-1}(ax)}n(n-ax)}{a^2c^2\,(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

```
[Out] exp(n*arccoth(a*x))*(-a*n*x+3)/a^2/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+2*exp(n*
arccoth(a*x))*n*(-a*x+n)/a^2/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6322, 6319}

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx=\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2\,(n^4-10n^2+9)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}$$

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
(2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2
*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rule 6322

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(2*(p + 1) + a*n*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^2*
c*(n^2 - 4*(p + 1)^2))), x] - Dist[n*((2*p + 3)/(a*c*(n^2 - 4*(p + 1)^2))),
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{(2n)\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{ac\,(9-n^2)}\\ &= \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{2e^{n\coth^{-1}(ax)}n(n-ax)}{a^2c^2\,(9-10n^2+n^4)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(6+2n^2-9anx+an^3x+6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)-an(-1+n^2)\right.}{4a^2c^2\,(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(6 + 2*n^2 - 9*a*n*x + a*n^3*x + 6*(-1 + n^2)*Cosh[2*Ar
cCoth[a*x]] - a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))
/(4*a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{(ax-1)(ax+1)(2a^3x^3n-2a^2n^2x^2+a\,n^3x-3anx-n^2+3)\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a^2(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	86

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*ar
ccoth(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3nx^3-2\,a^2n^2x^2-n^2+(an^3-3\,an)x+3)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3)x^4-2\,(a^4c^3n^4-10\,a^4c^3n}$$

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] (2*a^3*n*x^3 - 2*a^2*n^2*x^2 - n^2 + (a*n^3 - 3*a*n)*x + 3)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2*c
^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 10*a
^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.81

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^2-3}{a^4\,c^2\,(n^4-10\,n^2+9)}+\frac{2\,n^2\,x^2}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{2\,n\,x^3}{a\,c^2\,(n^4-10\,n^2+9)}-\frac{n\,x\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

[In] int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)

[Out] -(((a*x + 1)/(a*x))^(n/2)*((n^2 - 3)/(a^4*c^2*(n^4 - 10*n^2 + 9)) + (2*n^2*x^2)/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n*x^3)/(a*c^2*(n^4 - 10*n^2 + 9)) - (n*x*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))

3.760 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} - \frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6320, 6319}

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}} - \frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}}+\frac{6\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c(9-n^2)}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int\frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n\coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1+n^2)\right.}{4ac^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(ax-1)(ax+1)(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{(6\,a^3x^3-6\,a^2nx^2-n^3+3\,(an^2-3\,a)x+7\,n)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^4+9\,ac^3-2\,(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3)x^2}$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\,\operatorname{acoth}(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)

[Out] -(((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/(a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))

3.761 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 944

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)(c-a^2cx^2)^{5/2}}$$
$$-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)(c-a^2cx^2)^{5/2}}$$
$$+\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)(1-n^2)(c-a^2cx^2)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)(c-a^2cx^2)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)(c-a^2cx^2)^{5/2}}$$
$$+\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+4n+n^2)(c-a^2cx^2)^{5/2}}$$
$$-\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)(1-n^2)(c-a^2cx^2)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+n)(c-a^2cx^2)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+4n+n^2)(c-a^2cx^2)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}x^5}{(3+n)(c-a^2cx^2)^{5/2}}$$
$$-\frac{2^{\frac{5+n}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-n),\frac{1}{2}(-3-n),\frac{1}{2}(-1-n),\frac{a-\frac{1}{x}}{2a}\right)}{(3+n)(c-a^2cx^2)^{5/2}}$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/
(3+n)/(-a^2*c*x^2+c)^(5/2)-3*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)
*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a^5*(1-1/a^2
/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+
3)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)+4*a^5*(1-1/a^2/
x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(3+n)/(-a^2*c*
x^2+c)^(5/2)+8*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1
/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)-8*a^5*(1-1/a^2/x^2)^(5/2)*(1
-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a^2*c*x^2
```

```
+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(1/2+1
/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-
1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+4*a^5
*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(3/2+1/2*n)*x^5/(3+n)
/(-a^2*c*x^2+c)^(5/2)-2^(5/2+1/2*n)*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2
-1/2*n)*x^5*hypergeom([-3/2-1/2*n, -3/2-1/2*n],[-1/2-1/2*n],1/2*(a-1/x)/a)/
(3+n)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 944, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6327, 6330, 128, 47, 37, 71}

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx =$$

$$-\frac{2^{\frac{n+5}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3),\frac{1}{2}(-n-1),\frac{a-\frac{1}{x}}{2a}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n+3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}-\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{(n^4-10n^2+9)\left(c-a^2cx^2\right)^{5/2}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -((a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-
3 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a^5*(1 - 1/(a^2*x^2))^
(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n
+ n^2)*(c - a^2*c*x^2)^(5/2)) + (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x)
)^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c
*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1
+ 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2)) + (
4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-1
 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) + (8*a^5*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + 4*n +
n^2)*(c - a^2*c*x^2)^(5/2)) - (8*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^
((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x
^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 +
 1/(a*x))^((1 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/
(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*x^5)/
((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) + (4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1
 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2)*x^5)/((3 + n)*(c - a^2*c
*x^2)^(5/2)) - (2^((5 + n)/2)*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-
3 - n)/2)*x^5*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/2, (-1 - n)/2, (a - x^
(-1))/(2*a)])/((3 + n)*(c - a^2*c*x^2)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int x^4\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}-4a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\right.\right.}{}$$

$$= -\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{\left(4a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{\left(4a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{\left(6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{2^{\frac{5+n}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-n),\frac{1}{2}(-3-n),\frac{1}{2}(-1-n),\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{\left(3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{\left(6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{\left(8a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{2^{\frac{5+n}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-n),\frac{1}{2}(-3-n),\frac{1}{2}(-1-n)\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{\left(6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}dx,x,\frac{1}{x}\right)}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{\left(8a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}dx,x,\frac{1}{x}\right)}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n-3n^2-n^3)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n-3n^2-n^3)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$-\frac{2^{\frac{5+n}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-n),\frac{1}{2}(-3-n),\frac{1}{2}(-1-n),\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$
$$+\frac{\left(6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1-n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}dx,x,\frac{1}{x}\right)}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}} + \frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n-3n^2-n^3)\left(c-a^2cx^2\right)^{5/2}} - \frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}} + \frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} + \frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}} - \frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n-3n^2-n^3)\left(c-a^2cx^2\right)^{5/2}} - \frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}} + \frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{2^{\frac{5+n}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-n),\frac{1}{2}(-3-n),\frac{1}{2}(-1-n)\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(42-2n^2-45anx+5an^3x)+6a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}x\right.}{}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(42 - 2*n^2 - 45*a*n*x + 5*a
*n^3*x) + 6*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[2*ArcCoth[a*x]] - n*(
-1 + n^2)*(-1 + a^2*x^2)*Cosh[3*ArcCoth[a*x]]) - 8*E^((1 + n)*ArcCoth[a*x])
*(9 - 9*n - n^2 + n^3)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 +
```

```
n)/2, -E^(2*ArcCoth[a*x])])/(4*a*c^2*(-1 + n)*(1 + n)*(-9 + n^2)*Sqrt[1 - 1
/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^7 -
 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)),x)
[Out] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)), x)
```

3.762 $\int e^{n \coth^{-1}(ax)}(c-a^2cx^2)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{n \coth^{-1}(ax)}(c-a^2cx^2)^p \, dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x(c-a^2cx^2)^p \text{Hypergeometric2F1}\left(-1-2p,\frac{1}{2}(n-\right.}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2*n-p)*(1-1/a/x)^(-1/2*n+p)*(1+1/a/x)^(1+1/2*n+p)*x*(-
a^2*c*x^2+c)^p*hypergeom([-1-2*p, 1/2*n-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-
1/a^2/x^2)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6331, 134}

$$\int e^{n \coth^{-1}(ax)}(c-a^2cx^2)^p \, dx = \frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}(c-a^2cx^2)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}(n-\right.}{2p+1}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-1/2*n + p)*(1 +
1/(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n -
 2*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+p}dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-1-\right.}{1+2p}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{e^{(-2+n)\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)(-1+a^2x^2)\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(1,-\frac{n}{2}-p,2-\frac{n}{2}+p\right.}{a(n-2(1+p))}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((E^((-2 + n)*ArcCoth[a*x])*(-1 + E^(2*ArcCoth[a*x]))*(-1 + a^2*x^2)*(c -
a^2*c*x^2)^p*Hypergeometric2F1[1, -1/2*n - p, 2 - n/2 + p, E^(-2*ArcCoth[a*
x])])/(a*(n - 2*(1 + p))))
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^p dx$$

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \left(-c(ax-1)(ax+1)\right)^p e^{n\operatorname{acoth}(ax)}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")

[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")

[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p,x)

[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p, x)

3.763 $\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

```
[Out] (1+1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6327, 6331, 37}

$$\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}(c-a^2cx^2)^p}{2p+1}$$

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1+\frac{x}{a}\right)^{2p}dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{e^{2p\coth^{-1}(ax)}(1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] (E^(2*p*ArcCoth[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{(ax+1)\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(1+2p)}$
parallelrisch	$-\frac{-\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}x\left(-a^{2}c\,x^{2}+c\right)^{p}a-\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(1+2p)}$
risch	$(ax+1)(ax+1)^{2p}(ax-1)^{-p}c^{p}(ax-1)^{p}\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(i(ax-1)(ax+1))^{3}-\mathrm{csgn}(i(ax-1))\,\mathrm{csgn}(i(ax-1)(ax+1))^{2}-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{csgn}(\right.}{}}$

[In] int(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x,method=_RETURNVERBOSE)

[Out] (a*x+1)/a/(1+2*p)*exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{p}\,dx=\frac{(ax+1)(-a^{2}cx^{2}+c)^{p}\left(\frac{ax+1}{ax-1}\right)^{p}}{2\,ap+a}$$

[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")

[Out] (a*x + 1)*(-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p/(2*a*p + a)

Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{p}\,dx$$
$$=\begin{cases}-\frac{ix}{\sqrt{c}} & \text{for}\ a=0\wedge p=-\frac{1}{2}\\ c^{p}xe^{i\pi p} & \text{for}\ a=0\\ \int\frac{e^{-\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx & \text{for}\ p=-\frac{1}{2}\\ \frac{ax(-a^{2}cx^{2}+c)^{p}e^{2p\operatorname{acoth}(ax)}}{2ap+a}+\frac{(-a^{2}cx^{2}+c)^{p}e^{2p\operatorname{acoth}(ax)}}{2ap+a} & \text{otherwise}\end{cases}$$

[In] integrate(exp(2*p*acoth(a*x))*(-a**2*c*x**2+c)**p,x)

[Out] Piecewise((-I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(I*pi*p), Eq(a , 0)), (Integral(exp(-acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, - 1/2)), (a*x*(-a**2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a) + (-a**2* c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a), True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(a(-c)^p\,x+(-c)^p)(ax+1)^{2\,p}}{a(2\,p+1)}$$

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x-1}{a\,x}\right)^p}$$

```
[In] int(exp(2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
[Out] ((c - a^2*c*x^2)^p*(a*x + 1)*((a*x + 1)/(a*x))^p)/(a*(2*p + 1)*((a*x - 1)/(
a*x))^p)
```

3.764 $\int e^{-2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

```
[Out] (1-1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6327, 6331, 37}

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}(c-a^2cx^2)^p}{2p+1}$$

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Int
egersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx\\
&= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{2p}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{e^{-2p\coth^{-1}(ax)}(-1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
[Out] ((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcCoth[a*x])*(a + 2*a*p))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(ax-1)\left(-a^{2}c\,x^{2}+c\right)^{p} \mathrm{e}^{-2p \,\operatorname{arccoth}(ax)}}{a(1+2p)}$
parallelrisch	$\frac{\left(x\left(-a^{2}c\,x^{2}+c\right)^{p} a-\left(-a^{2}c\,x^{2}+c\right)^{p}\right) \mathrm{e}^{-2p \,\operatorname{arccoth}(ax)}}{a(1+2p)}$
risch	$(ax-1)\left((ax-1)^{p}\right)^{2} c^{p} \mathrm{e}^{-\frac{i p\pi\left(\operatorname{csgn}(i(ax-1)(ax+1))^{3}-\operatorname{csgn}(i(ax-1))\operatorname{csgn}(i(ax-1)(ax+1))^{2}-\operatorname{csgn}(i(ax-1)(ax+1))^{2}\operatorname{csgn}(i(ax+1))+\operatorname{csgn}(i(a\right.}{}}$

[In] int((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x,method=_RETURNVERBOSE)

[Out] (a*x-1)/a/(1+2*p)*(-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(ax-1)(-a^2cx^2+c)^p}{(2\,ap+a)\left(\frac{ax+1}{ax-1}\right)^p}$$

[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")

[Out] (a*x - 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x + 1)/(a*x - 1))^p)

Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\begin{cases}\frac{ix}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2}\\ c^pxe^{-i\pi p} & \text{for } a=0\\ \int\frac{e^{\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{ax(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}}-\frac{(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}} & \text{otherwise}\end{cases}$$

[In] integrate((-a**2*c*x**2+c)**p/exp(2*p*acoth(a*x)),x)

[Out] Piecewise((I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(-I*pi*p), Eq(a, 0)), (Integral(exp(acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -1/2)), (a*x*(-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*acoth(a*x))) - (-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*acoth(a*x))), True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(a(-c)^p\,x-(-c)^p)(ax-1)^{2\,p}}{a(2\,p+1)}$$

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
[Out] (a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(c-a^2\,c\,x^2)^p\,(a\,x-1)\,\left(\frac{a\,x-1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}$$

```
[In] int(exp(-2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
[Out] ((c - a^2*c*x^2)^p*(a*x - 1)*((a*x - 1)/(a*x))^p)/(a*(2*p + 1)*((a*x + 1)/(
a*x))^p)
```

3.765 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[Out] 2^(2+p)*c*(a*x+1)^(1-p)*(-a^2*c*x^2+c)^(-1+p)*hypergeom([-1+p, -2-p],[p],-1
/2*a*x+1/2)/a/(1-p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6276, 692, 71}

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx \\
&= c^2\int(1+ax)^4\left(c-a^2cx^2\right)^{-2+p}\,dx \\
&= \left(c^2(1+ax)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\right)\int(1+ax)^{2+p}(c-acx)^{-2+p}\,dx \\
&= \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{2^{2+p}(1-ax)^{-1+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(-1+p)}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] -((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p,
 -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

Maple [F]

$$\int \frac{(ax+1)^2\left(-a^2c\,x^2+c\right)^p}{(ax-1)^2}dx$$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
[Out] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \frac{(-c(ax-1)(ax+1))^p(ax+1)^2}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Maxima [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Giac [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2, x)
```

3.766 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{3}{2}-p,-2p\right.}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(3/2-p)*(1-1/a/x)^(-3/2+p)*(1+1/a/x)^(5/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, 3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)
^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6331, 134}

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-2p-1,\frac{3}{2}-p,-2p,\right.}{2p+1}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*x
))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+p}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{3}{2}\right.}{1+2p}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = -\frac{4^{1+p}e^{5\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric2F}}{5a+2ap}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(5*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/
2 + p, 2 + 2*p, 7/2 + p, E^(2*ArcCoth[a*x])])/((5*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \int \frac{(-a^2cx^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))
/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-c(ax-1)\,(ax+1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.767 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}$$

```
[Out] 2^(p+1)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[p+1],-1/2*a*x+1/2)/a/p/((a*x+
1)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6276, 692, 71}

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)/
2])/(a*p*(1 + a*x)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx\\
&= -\left(c\int (1+ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)\\
&= -\left(\left(c(1+ax)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1+ax)^{1+p}(c-acx)^{-1+p}\,dx\right)\\
&= \frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{1+p}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p
, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p)
```

Maple [F]

$$\int \frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{ax-1}dx$$

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.07 (sec) , antiderivative size = 648, normalized size of antiderivative = 12.00

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**(
```

```
2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2
 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) + c**p*x**2*gamma(p)
*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*
gamma(-p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(1/(a**
2*x**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*
x))/a**2 - a**(2*p - 1)*c**p*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p -
 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)
*gamma(p + 1)) + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2
), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)), True)) + Piecew
ise((0**p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(
p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(
2*gamma(-p)*gamma(p + 1)) - a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*ga
mma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2
*gamma(3/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 +
1)/(2*a) - 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2,
1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1))
- a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hype
r((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*gamma(3/2 - p)*gamma(p +
1)), True))
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")

[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)

Giac [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")

[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1), x)
```

3.768 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{1}{2}-p,-2\right.}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2-p)*(1-1/a/x)^(-1/2+p)*(1+1/a/x)^(3/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, 1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)
^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6327, 6331, 134}

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}-p,-2p\right.}{2p+1}$$

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
[Out] (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x
))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+p}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{1}{2}-\right.}{1+2p}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{4^{1+p}e^{3\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2}}{3a+2ap}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(3*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/
2 + p, 2 + 2*p, 5/2 + p, E^(2*ArcCoth[a*x])])/((3*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

Maple [F]

$$\int \frac{\left(-a^2c\,x^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1),
x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int\frac{(-c(ax-1)(ax+1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.769 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,-\frac{1}{2}-p,-\right.}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(-1/2-p)*(1-1/a/x)^(1/2+p)*(1+1/a/x)^(1/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, -1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2
)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6331, 134}

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{1}{2},-\right.}{2p+1}$$

```
[In] Int[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*x
))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2*p
, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+p}dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-1-2p,-\frac{1}{2}\right.}{1+2p}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{4^{1+p}e^{\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric2I}}{a+2ap}$$

```
[In] Integrate[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
[Out] -((4^(1 + p)*E^ArcCoth[a*x]*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/
(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2 +
p, 2 + 2*p, 3/2 + p, E^(2*ArcCoth[a*x])])/((a + 2*a*p)*(a*Sqrt[1 - 1/(a^2*x
^2)]*x)^(2*p)))
```

Maple [F]

$$\int \left(-a^2c\,x^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}dx$$

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \sqrt{\frac{ax-1}{ax+1}}\left(-c(ax-1)\,(ax+1)\right)^p\,dx$$

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p, x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(c-a^2\,c\,x^2\right)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.770 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1+ax)\right)}{ap}$$

```
[Out] -2^(p+1)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[p+1],1/2*a*x+1/2)/a/p/((-a*x
+1)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6277, 692, 71}

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(ax+1)\right)}{ap}$$

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x
)/2])/(a*p*(1 - a*x)^p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx\\ &= -\left(c\int(1-ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)\\ &= -\left(\left(c(1-ax)^{-p}(c+acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{1+p}(c+acx)^{-1+p}\,dx\right)\\ &= -\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1+ax)\right)}{ap}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{2^{-1+p}(1-ax)^{2+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(1-p,2+p,3+p,\frac{1}{2}(1-ax)\right)}{a(2+p)}$$

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
[Out] (2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p, 2
+ p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p)
```

Maple [F]

$$\int \frac{\left(-a^2c\,x^2+c\right)^p(ax-1)}{ax+1}dx$$

```
[In] int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
[Out] int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.31 (sec) , antiderivative size = 648, normalized size of antiderivative = 11.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate((-a**2*c*x**2+c)**p*(a*x-1)/(a*x+1),x)
[Out] a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**(
```

```
2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2
 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) - c**p*x**2*gamma(p)
*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*
gamma(-p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**
2*x**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*
x))/a**2 - a**(2*p - 1)*c**p*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p -
 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)
*gamma(p + 1)) - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2
), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)), True)) - Piecew
ise((0**p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(
p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(
2*gamma(-p)*gamma(p + 1)) + a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*ga
mma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2
*gamma(3/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 +
1)/(2*a) + 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2,
1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1))
+ a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hype
r((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*gamma(3/2 - p)*gamma(p +
1)), True))
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

```
[In] int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1), x)
```

3.771 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-1-2p,-\frac{3}{2}-p,-\right.}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2+p)*(1+1/a/x)^(-1/2+p)*x*(-a^2*c*x
^2+c)^p*hypergeom([-1-2*p, -3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^
2)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6327, 6331, 134}

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{3}{2},-\right.}{2p+1}$$

```
[In] Int[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*x
))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+p}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-2p,\right.}{1+2p}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{4^{1+p}e^{-\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}}{a-2ap}$$

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4^(1 + p)*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/(-1 + E^(2*ArcCot
h[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2 + p, 2 + 2*p, 1/2
+ p, E^(2*ArcCoth[a*x])])/(E^ArcCoth[a*x]*(a - 2*a*p)*(a*Sqrt[1 - 1/(a^2*x^
2)]*x)^(2*p))
```

Maple [F]

$$\int \left(-a^2c\,x^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \int \left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1),
x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \text{Timed out}$$

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \int (-a^2cx^2+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \int (-a^2cx^2+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \int (c-a^2\,c\,x^2)^p\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.772 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 342

$$\begin{aligned}
&\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx\\
&=-\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a}-\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a}\\
&\quad-\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}\\
&\quad+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a}\\
&\quad+c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{35c^4\csc^{-1}(ax)}{16a}+\frac{c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}$$

```
[Out] 47/42*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)/a+8/7*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(9/2)/a+c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x+35/16*c^4*arccsc(a*x)/a+c^
4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-67/48*c^4*(1+1/a/x)^(3/2)*(1-1
/a/x)^(1/2)/a-91/120*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-131/280*c^4*(1+1
/a/x)^(7/2)*(1-1/a/x)^(1/2)/a+61/70*c^4*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-5
1/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$$
$$=\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{7a}+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}{42a}$$
$$+c^4x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{280a}-\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{120a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-51*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (67*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(48*a) - (91*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(120*a) - (131*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) +
 (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (47*c^4*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2))/(42*a) + (8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(
a*x))^(9/2))/(7*a) + c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x + (35*c^
4*ArcCsc[a*x])/(16*a) + (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/
a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^4\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^4\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int\frac{\left(\frac{7}{a^2}-\frac{47x}{a^3}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{42}\left(a^2c^4\right)\text{Subst}\left(\int\frac{\left(\frac{42}{a^3}-\frac{183x}{a^4}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{210}\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(\frac{210}{a^4}-\frac{393x}{a^5}\right)\left(1+\frac{x}{a}\right)^{7/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{1}{840}\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{840}{a^5}+\frac{1911x}{a^6}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{\left(a^5c^4\right)\text{Subst}\left(\int\frac{\left(\frac{2520}{a^6}-\frac{7035x}{a^7}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2520}$$

$$= -\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{\left(a^6c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{5040}{a^7}+\frac{16065x}{a^8}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5040}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{(a^7c^4)\,\text{Subst}\left(\int \frac{\frac{5040}{a^8}-\frac{11025x}{a^9}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5040}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{(35c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^2} - \frac{c^4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx\right.}{a}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{c^4\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2} + \frac{(35c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.}{16a^2}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{35c^4\csc^{-1}(ax)}{16a} + \frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.35

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(240+280ax-1056a^2x^2-1330a^3x^3+1952a^4x^4+3045a^5x^5-2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\arcsin\left(\frac{1}{ax}\right)+1680a^6\log\right.}{1680a^7}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(240 + 280*a*x - 1056*a^2*x^2 - 1330*a^3*x^3 +
 1952*a^4*x^4 + 3045*a^5*x^5 - 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^6
*ArcSin[1/(a*x)] + 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{(ax-1)(2816a^6x^6-3045a^5x^5-1952a^4x^4+1330a^3x^3+1056a^2x^2-280ax-240)c^4}{1680x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{35a^7\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{16}\right.}{a^8\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3675a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}+3675a^7x^7\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+16\right.}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(a*x-1)*(2816*a^6*x^6-3045*a^5*x^5-1952*a^4*x^4+1330*a^3*x^3+1056*a
^2*x^2-280*a*x-240)/x^7*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)+(a^8*ln(a^2*x/(a^2)
^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+35/16*a^7*arctan(1/(a^2*x^2-1)^(1/2))
+a^7*((a*x-1)*(a*x+1))^(1/2))*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(1680\,a^8}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 - 1136*a^7*c^4*x^7 + 229*a^6*c^4*x^6
+ 4997*a^5*c^4*x^5 + 622*a^4*c^4*x^4 - 2386*a^3*c^4*x^3 - 776*a^2*c^4*x^2 +
 520*a*c^4*x + 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{c^4\left(\int\frac{a^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^2}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^6}{x^2\sqrt{\frac{ax}{ax+1}-}}\right.\right.}{a^8}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] c**4*(Integral(a**8/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a**2/(x**6*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**4/(x**4*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x) + Integral(-4*a**6/(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)), x))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{5355\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (5355*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 31465*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 72051*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 71801*c^4*((a*x - 1)/(
a*x + 1))^(9/2) + 4569*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 17619*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 10185*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 1995*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.35

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$= -\frac{35\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\mathrm{sgn}\,(ax+1)} - \frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\,(ax+1)}$$
$$-\frac{3045\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|+6720\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4+6860\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a|+20}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^4*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 -
1)*c^4/(a*sgn(a*x + 1)) - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4
*abs(a) + 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 + 6860*(x*abs(a) - s
qrt(a^2*x^2 - 1))^11*c^4*abs(a) + 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a
*c^4 + 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) + 49280*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^8*a*c^4 + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4
- 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) + 38976*(x*abs(a) - sqrt
(a^2*x^2 - 1))^4*a*c^4 - 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a) +
 12992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 3045*(x*abs(a) - sqrt(a^2*x
^2 - 1))*c^4*abs(a) + 2816*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^7
*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{19\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{97\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{839\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{1523\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{71801\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{840}+\frac{3431\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}+\frac{899\,c^4\,(\ldots}{\ldots}}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{14\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}-\frac{14\,a\,{(a\,x-1)}^5}{{(a\,x+1)}^5}-\frac{14\,a\,{(a\,x-1)}^6}{{(a\,x+1)}^6}-\frac{6\,a\,{(a\,x-1)}^7}{{(a\,x+1)}^7}-\frac{a\,(a\,x\ldots}{(a\,x+\ldots}}$$
$$-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((19*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (97*c^4*((a*x - 1)/(a*x + 1))^(3/
2))/8 + (839*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (1523*c^4*((a*x - 1)/(a*
x + 1))^(7/2))/280 + (71801*c^4*((a*x - 1)/(a*x + 1))^(9/2))/840 + (3431*c^
4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (899*c^4*((a*x - 1)/(a*x + 1))^(13/2))
/24 + (51*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x +
1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (14*
a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x - 1
)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/(8*a) + (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.773 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 268

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{15c^3\csc^{-1}(ax)}{8a}+\frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] 6/5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+c^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(7/
2)*x+15/8*c^3*arccsc(a*x)/a+c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-
31/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-43/60*c^3*(1+1/a/x)^(5/2)*(1-1/
a/x)^(1/2)/a+23/20*c^3*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-23/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{5a}+c^3x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{60a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{24a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-23*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (31*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(24*a) - (43*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2))/(60*a) + (23*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*
c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(5*a) + c^3*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(7/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) + (c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - c^3\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \frac{\left(\frac{5}{a^2}-\frac{23x}{a^3}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{20}\left(a^2c^3\right)\text{Subst}\left(\int \frac{\left(\frac{20}{a^3}-\frac{43x}{a^4}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x + \frac{1}{60}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(-\frac{60}{a^4}+\frac{155x}{a^5}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$=-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{1}{120}\left(a^4c^3\right)\text{Subst}\left(\int\frac{\left(\frac{120}{a^5}-\frac{345x}{a^6}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$=-\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{1}{120}\left(a^5c^3\right)\text{Subst}\left(\int\frac{-\frac{120}{a^6}+\frac{225x}{a^7}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$=-\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{\left(15c^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{8a^2}-\frac{c^3\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx\right.}{a}$$

$$=-\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{c^3\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}+\frac{\left(15c^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-}}\right.}{8a^2}$$

$$=-\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{15c^3\csc^{-1}(ax)}{8a}+\frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$=\frac{c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(-24-30ax+88a^2x^2+135a^3x^3-184a^4x^4+120a^5x^5)}{x^4}+225a^4\arcsin\left(\frac{1}{ax}\right)+120a^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a^5}$$

[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]

[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*(-24 - 30*a*x + 88*a^2*x^2 + 135*a^3*x^3 - 184*a^4*x^4 + 120*a^5*x^5))/x^4 + 225*a^4*ArcSin[1/(a*x)] + 120*a^4*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{(ax-1)(184a^4x^4-135a^3x^3-88a^2x^2+30ax+24)c^3}{120x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+a^5\sqrt{(ax-1)(ax+1)}\right)}{a^6\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$\frac{(ax-1)c^3\left(-120\sqrt{a^2x^2-1}\sqrt{a^2}a^6x^6+120(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^4x^4+225a^5x^5\sqrt{a^2}\sqrt{a^2x^2-1}+225a^5x^5\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+120\ln\left(\right.\right.}{120\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(}}$

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)

[Out] -1/120*(a*x-1)*(184*a^4*x^4-135*a^3*x^3-88*a^2*x^2+30*a*x+24)/x^5*c^3/a^6/((a*x-1)/(a*x+1))^(1/2)+(a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+15/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3/a^6/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3x^6}{120\,a^6x^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 - 64*a^5*c^3*x^5 - 49*a^4*c^3*x^4 + 223*a^
3*c^3*x^3 + 58*a^2*c^3*x^2 - 54*a*c^3*x - 24*c^3)*sqrt((a*x - 1)/(a*x + 1))
)/(a^6*x^5)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(\int\frac{a^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a^2}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^6}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**3,x)
[Out] c**3*(Integral(a**6/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*
*6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a**2/(x**4*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**4/(x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))), x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+134}{4(}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
 (345*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(9/
2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 86*c^3*((a*x - 1)/(a*x + 1))^(5
/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 105*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.32

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$= -\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)} - \frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)}$$
$$- \frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a| + 360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3 + 150\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a| + 720\left(x|a\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^3*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 -
1)*c^3/(a*sgn(a*x + 1)) - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*ab
s(a) + 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 + 150*(x*abs(a) - sqrt(a^
2*x^2 - 1))^7*c^3*abs(a) + 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3 + 112
0*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 - 150*(x*abs(a) - sqrt(a^2*x^2 - 1
))^3*c^3*abs(a) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 135*(x*abs(a
) - sqrt(a^2*x^2 - 1))*c^3*abs(a) + 184*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 -
 1))^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{\frac{7c^3\sqrt{\frac{ax-1}{ax+1}}}{4}+\frac{61c^3\left(\frac{ax-1}{ax+1}\right)^{3/2}}{12}+\frac{43c^3\left(\frac{ax-1}{ax+1}\right)^{5/2}}{30}+\frac{827c^3\left(\frac{ax-1}{ax+1}\right)^{7/2}}{30}+\frac{269c^3\left(\frac{ax-1}{ax+1}\right)^{9/2}}{12}+\frac{23c^3\left(\frac{ax-1}{ax+1}\right)^{11/2}}{4}}{a+\frac{4a(ax-1)}{ax+1}+\frac{5a(ax-1)^2}{(ax+1)^2}-\frac{5a(ax-1)^4}{(ax+1)^4}-\frac{4a(ax-1)^5}{(ax+1)^5}-\frac{a(ax-1)^6}{(ax+1)^6}}$$
$$-\frac{15c^3\,\mathrm{atan}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{4a}+\frac{2c^3\,\mathrm{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((7*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (61*c^3*((a*x - 1)/(a*x + 1))^(3/2
))/12 + (43*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (827*c^3*((a*x - 1)/(a*x
+ 1))^(7/2))/30 + (269*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (23*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/(4*a) + (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.774 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx &= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}\\&\quad+c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x+\frac{3c^2\csc^{-1}(ax)}{2a}+\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

```
[Out] c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x+3/2*c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+4/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx &= \frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+c^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}\\&\quad+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}\\&\quad-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}\end{aligned}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (7*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(6*a) + (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(3*a) + c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) + (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^2\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{4x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}\\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(\frac{3}{a^2}-\frac{7x}{a^3}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}\\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(-\frac{6}{a^3}+\frac{15x}{a^4}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}\\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{6}\left(a^3c^2\right)\text{Subst}\left(\int \frac{\frac{6}{a^4}-\frac{9x}{a^5}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2} - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,\right.}{a}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\ldots}\right.}{2a^2}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{3c^2\csc^{-1}(ax)}{2a} + \frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(2+3ax-8a^2x^2+6a^3x^3)+9a^2x^2\arcsin\left(\frac{1}{ax}\right)+6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^2
*ArcSin[1/(a*x)] + 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x^
2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{(ax-1)(8a^2x^2-3ax-2)c^2}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{(ax-1)(ax+1)}}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^2\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+9a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\frac{a^2x+\sqrt{a^2x}}{\sqrt{a}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)*(8*a^2*x^2-3*a*x-2)/x^3*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)+(a^4*l
n(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+3/2*a^3*arctan(1/(a^2*x^
2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)*((
a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^2x^4-2\,a^3}{6\,a^4x^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 5*a^2*c^2*x^2 + 5*a*c^2*x + 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a**2/(x**2*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{1}{3}a\left(\frac{9c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+29c^2\left(\frac{ax-}{ax+}\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-}{(ax-}}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (1
5*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2) + c^
2*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.28

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{3c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\operatorname{sgn}(ax+1)}-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)}+\frac{\sqrt{a^2x^2-1}c^2}{a\operatorname{sgn}(ax+1)} - \frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2+12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-3\left(x|a|-\sqrt{a^2x^2}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\operatorname{sgn}(ax+1)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^2/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a) +
 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 + 12*(x*abs(a) - sqrt(a^2*x^2 -
1))^2*a*c^2 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) + 8*a*c^2)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+5\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c^2*((a*x - 1)/(a*x + 1))^(1/2) + (c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 + (2
9*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.775 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\csc^{-1}(ax)}{a}+\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a+c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/x)^(3
/2)*x*(1-1/a/x)^(1/2)-2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6329, 99, 159, 21, 132, 41, 222, 94, 214}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a}+\frac{c\csc^{-1}(ax)}{a}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(3/2)*x + (c*ArcCsc[a*x])/a + (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - c\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x\\
&\quad + (ac)\text{Subst}\left(\int \frac{-\frac{1}{a^2}+\frac{x}{a^3}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x\\
&\quad + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}
\end{aligned}$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{c\csc^{-1}(ax)}{a} + \frac{c\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)+\arcsin\left(\frac{1}{ax}\right)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{(ax-1)c}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{(ax-1)(ax+1)}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] -(a*x-1)/x*c/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))*c/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) + a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 - c)*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)}-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c}{a\text{sgn}\left(ax+1\right)}-\frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\text{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c*log(abs(-x*
abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/(a
*sgn(a*x + 1)) - 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn(a*x
 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

```
[In] int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

3.776 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-2*(1+1/a/x)^(1/2)/a/c/(1-1/a/x
)^(1/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6329, 105, 21, 96, 94, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}+\frac{x\sqrt{\frac{1}{ax}+1}}{c\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{ac\sqrt{1-\frac{1}{ax}}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)])/(a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*S
qrt[1 - 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}x}{c\sqrt{1-\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\sqrt{1-\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-2+ax)}{-1+ax} + \frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(-1 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]/a)/c
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,ax+4\ln\Big(}{2a\sqrt{a^2}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)

[Out] 1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")

[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*c)

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)

[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*c*
sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) + l
og(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

```
[In] int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 4)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.777 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx = -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

```
[Out] arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-4/3/a/c^2/(1-1/a/x)^(3/2)/(1
+1/a/x)^(1/2)+x/c^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)-11/3/a/c^2/(1-1/a/x)^(1
/2)/(1+1/a/x)^(1/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}} - \frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -4/(3*a*c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) - 11/(3*a*c^2*Sqrt[1 - 1
/(a*x)]*Sqrt[1 + 1/(a*x)]) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*x
)]) + x/(c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a
*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\text{Subst}\left(\int \frac{\frac{3}{a^2}+\frac{8x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{3}{a^3}-\frac{11x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^3\text{Subst}\left(\int -\frac{3}{a^4x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(8-5ax-7a^2x^2+3a^3x^3)}{3(-1+ax)^2(1+ax)} + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 + a
*x)^2*(1 + a*x)) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.21

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}} + \frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}} + \frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{4a^6\left(x+\frac{1}{a}\right)} - \frac{19\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{12a^6\left(x-\frac{1}{a}\right)} - \frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{6a^7\left(x-\frac{1}{a}\right)^2}\right)a^4\sqrt{(ax-1)(ax}}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+21((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)+1/4/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1
```

```
9/12/a^6/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/6/a^7/(x-1/a)^2*((x-1/
a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+
1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-7\,a^2x^2-5\,ax+}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))),
x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^2) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

```
[In] int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^2) - ((17*(a*x - 1))/(3*(a*x + 1)) - (14
*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 4*a
*c^2*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^2)
```

3.778 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 254

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

```
[Out] -6/5/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-29/15/a/c^3/(1-1/a/x)^(3/2)/(1+1
/a/x)^(3/2)+x/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)+arctanh((1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2))/a/c^3-34/5/a/c^3/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+21/5*(1-1/
a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{6329, 105, 157, 12, 94, 214\}$

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}} + \frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}} - \frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -6/(5*a*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) - 29/(15*a*c^3*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) - 34/(5*a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(3/2)) + (21*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sq
rt[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(5/2)*(
1 + 1/(a*x))^(3/2)) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}\\
&\quad - \frac{a\text{Subst}\left(\int \frac{\frac{5}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^3}\\
&= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}\\
&\quad + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{15}{a^3}-\frac{87x}{a^4}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{15c^3}
\end{aligned}$$

$$=-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{a^3\text{Subst}\left(\int\frac{\frac{15}{a^4}+\frac{204x}{a^5}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}dx,x,\frac{1}{x}\right)}{15c^3}$$

$$=-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{a^4\text{Subst}\left(\int\frac{\frac{45}{a^5}+\frac{189x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{45c^3}$$

$$=-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{a^5\text{Subst}\left(\int\frac{45}{a^6x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{45c^3}$$

$$=-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{ac^3}$$

$$=-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-48+33ax+87a^2x^2-52a^3x^3-38a^4x^4+15a^5x^5)}{15(-1+ax)^3(1+ax)^2} + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a^
4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^3*(1 + a*x)^2) + Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{20a^{10}\left(x-\frac{1}{a}\right)^3}-\frac{23\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{60a^9\left(x-\frac{1}{a}\right)^2}-\frac{493\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{240a^8\left(x-\frac{1}{a}\right)}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right.}}{24a^9}\right.}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7-240\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^8x^7+285((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+525\sqrt{(ax-1)(ax+1)}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(1/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/20/a^10/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-23/60/a^9/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-493/240/a^8/(x-1/a
)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/24/a^9/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(
```

```
x+1/a))^(1/2)+25/48/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^6/c^3/
((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^5x}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x - 48)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{a^6\int \frac{x^6}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x**
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{1}{240}\,a\left(\frac{\frac{37\,(ax-1)}{ax+1}+\frac{410\,(ax-1)^2}{(ax+1)^2}-\frac{930\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+24\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240\ \mathrm{l}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*((37*(a*x - 1)/(a*x + 1) + 410*(a*x - 1)^2/(a*x + 1)^2 - 930*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
 - 1)/(a*x + 1))^(5/2)) + 5*(((a*x - 1)/(a*x + 1))^(3/2) + 24*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}-\frac{\frac{82\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{62\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{37\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^3}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^3}$$

```
[In] int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(2*a*c^3) - ((82*(a*x - 1)^2)/(3*(a*x + 1)^2) -
 (62*(a*x - 1)^3)/(a*x + 1)^3 + (37*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(16*a*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + (
(a*x - 1)/(a*x + 1))^(3/2)/(48*a*c^3) - (atan(((a*x - 1)/(a*x + 1))^(1/2)*1
i)*2i)/(a*c^3)
```

3.779 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 328

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx = &-\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}\\&-\frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}\\&+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}}\\&+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

```
[Out] -8/7/a/c^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(5/2)-11/7/a/c^4/(1-1/a/x)^(5/2)/(1+1/
a/x)^(5/2)-62/21/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^4/(1-1/a/x)^(7/2
)/(1+1/a/x)^(5/2)+arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-269/21/a/c
^4/(1+1/a/x)^(5/2)/(1-1/a/x)^(1/2)+262/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(
5/2)+163/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2)/a/
c^4/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -8/(7*a*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) - 11/(7*a*c^4*(1 - 1/(
a*x))^(5/2)*(1 + 1/(a*x))^(5/2)) - 62/(21*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/
(a*x))^(5/2)) - 269/(21*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (262
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (163*Sqrt[1 - 1/(a*x)]
)/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) + ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\text{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{48x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{35}{a^3}-\frac{275x}{a^4}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{35c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a^3\text{Subst}\left(\int \frac{\frac{105}{a^4}+\frac{1240x}{a^5}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{105c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^4\text{Subst}\left(\int \frac{-\frac{105}{a^5}-\frac{4035x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{105c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^5\text{Subst}\left(\int \frac{-\frac{525}{a^6}-\frac{7860x}{a^7}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{525c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^6\text{Subst}\left(\int \frac{-\frac{1575}{a^7}-\frac{7335x}{a^8}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{1575c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^7\text{Subst}\left(\int -\frac{1575}{a^8x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{1575c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^4}$$

$$
\begin{aligned}
&= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} \\
&\quad - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} \\
&\quad + \frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} \\
&\quad + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.35

$$
\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(384-279ax-1065a^2x^2+715a^3x^3+965a^4x^4-559a^5x^5-281a^6x^6+105a^7x^7\right)}{105(-1+ax)^4(1+ax)^3} + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}
$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 + 9
65*a^4*x^4 - 559*a^5*x^5 - 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^4*(1
 + a*x)^3) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.12

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}} + \frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}} - \frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{56a^{13}\left(x-\frac{1}{a}\right)^4} - \frac{17\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{112a^{12}\left(x-\frac{1}{a}\right)^3} - \frac{211\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{336a^{11}\left(x-\frac{1}{a}\right)^2} - \frac{1657\sqrt{(x-}}{672}\right.}{c^4\sqrt{\frac{ax-}{ax-}}}$
default	Expression too large to display

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(1/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/56/a^13/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-17/112/a^12/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-211/336/a^11/(x-
```

```
1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1657/672/a^10/(x-1/a)*((x-1/a)^2*a
^2+2*(x-1/a)*a)^(1/2)-7/60/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)
+379/480/a^10/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+1/80/a^12/(x+1/a)^3
*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4/((a*x-1)/(a*x+1))^(1/2)*((a*x-1
)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a\right.}{105\left(a^7c^4x^6-2\,a^6c^4\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1
)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 +
 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (105
*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 - 1065*a^2
*x^2 - 279*a*x + 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 2*a^6*c^4*x
^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{x^8}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x*
*6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{6720}\,a\left(\frac{5\left(\frac{39\,(ax-1)}{ax+1}+\frac{287\,(ax-1)^2}{(ax+1)^2}+\frac{2611\,(ax-1)^3}{(ax+1)^3}-\frac{5628\,(ax-1)^4}{(ax+1)^4}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{7\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+50\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+705\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(5*(39*(a*x - 1)/(a*x + 1) + 287*(a*x - 1)^2/(a*x + 1)^2 + 2611*(a
*x - 1)^3/(a*x + 1)^3 - 5628*(a*x - 1)^4/(a*x + 1)^4 + 3)/(a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 7*(3*((a*x - 1
)/(a*x + 1))^(5/2) + 50*((a*x - 1)/(a*x + 1))^(3/2) + 705*sqrt((a*x - 1)/(a
*x + 1)))/(a^2*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 6
720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{47\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{41\,(a\,x-1)^2}{3\,(a\,x+1)^2}+\frac{373\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{268\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{13\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}+\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^4}$$

```
[In] int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (47*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((41*(a*x - 1)^2)/(3*(a*x + 1
)^2) + (373*(a*x - 1)^3)/(3*(a*x + 1)^3) - (268*(a*x - 1)^4)/(a*x + 1)^4 +
(13*(a*x - 1))/(7*(a*x + 1)) + 1/7)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2) -
 64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2)) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(9
6*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*2i)/(a*c^4)
```

3.780 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

```
[Out] -1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/a
^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/a^3/x^2+3*c^5/a^2/x+c^5*x+2*c^5*
ln(x)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^
7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4
*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx\\
&= \frac{c^5\int\frac{e^{2\operatorname{arctanh}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}\,dx}{a^{10}}\\
&= \frac{c^5\int\frac{(1-ax)^4(1+ax)^6}{x^{10}}\,dx}{a^{10}}\\
&= \frac{c^5\int\left(a^{10}+\frac{1}{x^{10}}+\frac{2a}{x^9}-\frac{3a^2}{x^8}-\frac{8a^3}{x^7}+\frac{2a^4}{x^6}+\frac{12a^5}{x^5}+\frac{2a^6}{x^4}-\frac{8a^7}{x^3}-\frac{3a^8}{x^2}+\frac{2a^9}{x}\right)dx}{a^{10}}\\
&= -\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x\\
&\quad+\frac{2c^5\log(x)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = -\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^
7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4
*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^5\left(a^{10}x-\frac{1}{9x^9}+2a^9\ln(x)-\frac{3a^5}{x^4}-\frac{2a^6}{3x^3}+\frac{4a^3}{3x^6}+\frac{4a^7}{x^2}+\frac{3a^8}{x}+\frac{3a^2}{7x^7}-\frac{a}{4x^8}-\frac{2a^4}{5x^5}\right)}{a^{10}}$
risch	$c^5x+\frac{3a^8c^5x^8+4a^7c^5x^7-\frac{2}{3}a^6c^5x^6-3a^5c^5x^5-\frac{2}{5}a^4c^5x^4+\frac{4}{3}a^3c^5x^3+\frac{3}{7}a^2c^5x^2-\frac{1}{4}a\,c^5x-\frac{1}{9}c^5}{a^{10}x^9}+\frac{2c^5\ln(x)}{a}$
norman	$\frac{a^9c^5x^{10}-\frac{c^5}{9a}-\frac{c^5x}{4}+\frac{3a\,c^5x^2}{7}-\frac{2a^3c^5x^4}{5}-3a^4c^5x^5-\frac{2a^5c^5x^6}{3}+4a^6c^5x^7+3a^7c^5x^8+\frac{4c^5a^2x^3}{3}}{a^9x^9}+\frac{2c^5\ln(x)}{a}$
parallelrisch	$\frac{1260a^{10}c^5x^{10}+2520c^5\ln(x)a^9x^9+3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-}{1260a^{10}x^9}$
meijerg	$-\frac{c^5(-ax-\ln(-ax+1))}{a}+\frac{5c^5(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{10c^5\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{10c^5}{}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5/a^10*(a^10*x-1/9/x^9+2*a^9*ln(x)-3*a^5/x^4-2/3*a^6/x^3+4/3*a^3/x^6+4*a^
7/x^2+3*a^8/x+3/7*a^2/x^7-1/4*a/x^8-2/5*a^4/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{1260\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)+3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4}{1260\,a^{10}x^9}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 + 5
040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 16
80*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{a^{10}c^5x+2a^9c^5\log(x)+\frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 2*a**9*c**5*log(x) + (3780*a**8*c**5*x**8 + 5040*a**7*c**5*
x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 1680
*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x**9)
)/a**10
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = c^5x+\frac{2c^5\log(x)}{a} + \frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-315\,ac}{1260\,a^{10}x^9}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 2*c^5*log(x)/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840*
a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540*a
^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=c^5x+\frac{2\,c^5\log\left(|x|\right)}{a}+\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-315\,a}{1260\,a^{10}x^9}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] c^5*x + 2*c^5*log(abs(x))/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 -
 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 +
540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5\left(\frac{3\,a^2\,x^2}{7}-\frac{a\,x}{4}+\frac{4\,a^3\,x^3}{3}-\frac{2\,a^4\,x^4}{5}-3\,a^5\,x^5-\frac{2\,a^6\,x^6}{3}+4\,a^7\,x^7+3\,a^8\,x^8+a^{10}\,x^{10}+2\,a^9\,x^9\,\ln\left(x\right)-\frac{1}{9}\right)}{a^{10}\,x^9}$$

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^5*((3*a^2*x^2)/7 - (a*x)/4 + (4*a^3*x^3)/3 - (2*a^4*x^4)/5 - 3*a^5*x^5 -
 (2*a^6*x^6)/3 + 4*a^7*x^7 + 3*a^8*x^8 + a^10*x^10 + 2*a^9*x^9*log(x) - 1/9
))/(a^10*x^9)
```

3.781 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

```
[Out] 1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/a^3/x
^2+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx\\
&= -\frac{c^4\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8}\\
&= -\frac{c^4\int \frac{(1-ax)^3(1+ax)^5}{x^8}\,dx}{a^8}\\
&= -\frac{c^4\int \left(-a^8+\frac{1}{x^8}+\frac{2a}{x^7}-\frac{2a^2}{x^6}-\frac{6a^3}{x^5}+\frac{6a^5}{x^3}+\frac{2a^6}{x^2}-\frac{2a^7}{x}\right)dx}{a^8}\\
&= \frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(a^8x+2a^7\ln(x)-\frac{3a^3}{2x^4}+\frac{a}{3x^6}+\frac{3a^5}{x^2}+\frac{2a^6}{x}+\frac{1}{7x^7}-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6+3a^5c^4x^5-\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2+\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}+\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}+\frac{c^4x}{3}-\frac{2a\,c^4x^2}{5}-\frac{3a^2c^4x^3}{2}+3a^4c^4x^5+2a^5c^4x^6}{a^7x^7}+\frac{2c^4\ln(x)}{a}$
parallelrisch	$\frac{210a^8c^4x^8+420c^4\ln(x)a^7x^7+420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70a\,c^4x+30c^4}{210a^8x^7}$
meijerg	$-\frac{c^4(-ax-\ln(-ax+1))}{a}+\frac{4c^4(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{6c^4\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{4c^4\left(-\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)

[Out] c^4/a^8*(a^8*x+2*a^7*ln(x)-3/2*a^3/x^4+1/3*a/x^6+3*a^5/x^2+2*a^6/x+1/7/x^7-2/5*a^2/x^5)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")

[Out] 1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{a^8c^4x+2a^7c^4\log(x)+\frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**4,x)

```
[Out] (a**8*c**4*x + 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 + 630*a**5*c**4*x**
5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = c^4x + \frac{2\,c^4\log(x)}{a} + \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
[Out] c^4*x + 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = c^4x + \frac{2\,c^4\log(|x|)}{a} + \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
[Out] c^4*x + 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\frac{a\,x}{3}-\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5+2\,a^6\,x^6+a^8\,x^8+2\,a^7\,x^7\,\ln(x)+\frac{1}{7}\right)}{a^8\,x^7}$$

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1))/(a*x - 1),x)
[Out] (c^4*((a*x)/3 - (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 + 2*a^6*x^6 + a^8
*x^8 + 2*a^7*x^7*log(x) + 1/7))/(a^8*x^7)
```

3.782 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx=-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

```
[Out] -1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/a^3/x^2+c^3/a^2/x+c^
3*x+2*c^3*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx=-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] -1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx\\
&= \frac{c^3\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}\\
&= \frac{c^3\int \frac{(1-ax)^2(1+ax)^4}{x^6}\,dx}{a^6}\\
&= \frac{c^3\int \left(a^6+\frac{1}{x^6}+\frac{2a}{x^5}-\frac{a^2}{x^4}-\frac{4a^3}{x^3}-\frac{a^4}{x^2}+\frac{2a^5}{x}\right) dx}{a^6}\\
&= -\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
[Out] -1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result
default	$\frac{c^3\left(a^6x+2a^5\ln(x)-\frac{a}{2x^4}+\frac{a^2}{3x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}-\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{a^4c^3x^4+2a^3c^3x^3+\frac{1}{3}a^2c^3x^2-\frac{1}{2}a\,c^3x-\frac{1}{5}c^3}{a^6x^5}+\frac{2c^3\ln(x)}{a}$
norman	$\frac{a^3c^3x^4+a^5c^3x^6-\frac{c^3}{5a}-\frac{c^3x}{2}+\frac{a\,c^3x^2}{3}+2a^2c^3x^3}{a^5x^5}+\frac{2c^3\ln(x)}{a}$
parallelrisch	$\frac{30a^6c^3x^6+60c^3\ln(x)a^5x^5+30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15a\,c^3x-6c^3}{30a^6x^5}$
meijerg	$-\frac{c^3(-ax-\ln(-ax+1))}{a}+\frac{3c^3(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{3c^3\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{c^3\left(-\ln\right.}{}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^6*(a^6*x+2*a^5*ln(x)-1/2*a/x^4+1/3*a^2/x^3+2*a^3/x^2+a^4/x-1/5/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx =\frac{30\,a^6c^3x^6+60\,a^5c^3x^5\log\left(x\right)+30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{a^6c^3x+2a^5c^3\log\left(x\right)+\frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = c^3x + \frac{2c^3\log(x)}{a} + \frac{30a^4c^3x^4 + 60a^3c^3x^3 + 10a^2c^3x^2 - 15ac^3x - 6c^3}{30a^6x^5}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = c^3x + \frac{2c^3\log(|x|)}{a} + \frac{30a^4c^3x^4 + 60a^3c^3x^3 + 10a^2c^3x^2 - 15ac^3x - 6c^3}{30a^6x^5}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] c^3*x + 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\frac{a^2x^2}{3} - \frac{ax}{2} + 2a^3x^3 + a^4x^4 + a^6x^6 + 2a^5x^5\ln(x) - \frac{1}{5}\right)}{a^6x^5}$$

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^3*((a^2*x^2)/3 - (a*x)/2 + 2*a^3*x^3 + a^4*x^4 + a^6*x^6 + 2*a^5*x^5*log
(x) - 1/5))/(a^6*x^5)
```

3.783 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

```
[Out] 1/3*c^2/a^4/x^3+c^2/a^3/x^2+c^2*x+2*c^2*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 76}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx \\
&= -\frac{c^2\int \frac{e^{2\text{arctanh}(ax)}(1-a^2x^2)^2}{x^4}\,dx}{a^4} \\
&= -\frac{c^2\int \frac{(1-ax)(1+ax)^3}{x^4}\,dx}{a^4} \\
&= -\frac{c^2\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx}{a^4} \\
&= \frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^{2}\left(a^{4}x+2a^{3}\ln\left(x\right)+\frac{1}{3x^{3}}+\frac{a}{x^{2}}\right)}{a^{4}}$
risch	$c^{2}x+\frac{a\,c^{2}x+\frac{1}{3}c^{2}}{a^{4}x^{3}}+\frac{2c^{2}\ln\left(x\right)}{a}$
norman	$\frac{c^{2}x+a^{3}c^{2}x^{4}+\frac{c^{2}}{3a}}{a^{3}x^{3}}+\frac{2c^{2}\ln\left(x\right)}{a}$
parallelrisch	$\frac{3a^{4}c^{2}x^{4}+6c^{2}\ln\left(x\right)a^{3}x^{3}+3a\,c^{2}x+c^{2}}{3a^{4}x^{3}}$
meijerg	$-\frac{c^{2}\left(-ax-\ln\left(-ax+1\right)\right)}{a}+\frac{2c^{2}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)\right)}{a}-\frac{c^{2}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)-\frac{1}{2a^{2}x^{2}}-\frac{1}{ax}\right)}{a}+\frac{c^{2}\ln(-a}{a}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)

[Out] c^2/a^4*(a^4*x+2*a^3*ln(x)+1/3/x^3+a/x^2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log\left(x\right)+3\,ac^2x+c^2}{3\,a^4x^3}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")

[Out] 1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x+2a^3c^2\log\left(x\right)+\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**2,x)

[Out] (a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x + c**2)/(3*x**3))/a**4

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = c^2x+\frac{2\,c^2\log\left(x\right)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
[Out] c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = c^2x+\frac{2\,c^2\log\left(|x|\right)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
[Out] c^2*x + 2*c^2*log(abs(x))/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\,\left(a\,x+a^4\,x^4+2\,a^3\,x^3\,\ln\left(x\right)+\frac{1}{3}\right)}{a^4\,x^3}$$

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1))/(a*x - 1),x)
[Out] (c^2*(a*x + a^4*x^4 + 2*a^3*x^3*log(x) + 1/3))/(a^4*x^3)
```

3.784 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

```
[Out] -c/a^2/x+c*x+2*c*ln(x)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6292, 6285, 45}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx\\
&= \frac{c\int \frac{e^{2\text{arctanh}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}\\
&= \frac{c\int \frac{(1+ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)\,dx}{a^2}\\
&= -\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c\left(a^{2}x+2a\ln\left(x\right)-\frac{1}{x}\right)}{a^{2}}$	22
risch	$-\frac{c}{a^{2}x}+cx+\frac{2c\ln\left(x\right)}{a}$	22
parallelrisch	$\frac{a^{2}c\,x^{2}+2c\ln\left(x\right)ax-c}{a^{2}x}$	27
norman	$\frac{ac\,x^{2}-\frac{c}{a}}{ax}+\frac{2c\ln\left(x\right)}{a}$	30
meijerg	$-\frac{c\left(-ax-\ln\left(-ax+1\right)\right)}{a}+\frac{c\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)\right)}{a}+\frac{c\ln\left(-ax+1\right)}{a}-\frac{c\left(\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)+\frac{1}{ax}\right)}{a}$	86

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] c/a^2*(a^2*x+2*a*ln(x)-1/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2+2\,acx\log\left(x\right)-c}{a^2x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx+2ac\log\left(x\right)-\frac{c}{x}}{a^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2),x)
[Out] (a**2*c*x + 2*a*c*log(x) - c/x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{2\,c\log{(x)}}{a}-\frac{c}{a^2x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] c*x + 2*c*log(x)/a - c/(a^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{2\,c\log{(|x|)}}{a}-\frac{c}{a^2x}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="giac")
[Out] c*x + 2*c*log(abs(x))/a - c/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\,\left(a^2\,x^2+2\,a\,x\,\ln{(x)}-1\right)}{a^2\,x}$$

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1))/(a*x - 1),x)
[Out] (c*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

3.785 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}$$

```
[Out] x/c+1/a/c/(-a*x+1)+2*ln(-a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}+\frac{x}{c}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c + 1/(a*c*(1 - a*x)) + (2*Log[1 - a*x])/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= \frac{a^2\int \frac{e^{2\text{arctanh}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= \frac{a^2\int \frac{x^2}{(1-ax)^2}\,dx}{c} \\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x+\frac{1}{a-a^2x}+\frac{2\log(1-ax)}{a}}{c}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] (x + (a - a^2*x)^(-1) + (2*Log[1 - a*x])/a)/c
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac(ax-1)}+\frac{2\ln(ax-1)}{ac}$	36
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{1}{a^{3}(ax-1)}+\frac{2\ln(ax-1)}{a^{3}}\right)}{c}$	37
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{2x}{c}}{ax-1}+\frac{2\ln(ax-1)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}+2a\ln(ax-1)x-2ax-2\ln(ax-1)}{c(ax-1)a}$	45

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] x/c-1/a/c/(a*x-1)+2/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2x^2-ax+2\,(ax-1)\log{(ax-1)}-1}{a^2cx-ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=a^2\left(-\frac{1}{a^4cx-a^3c}+\frac{x}{a^2c}+\frac{2\log{(ax-1)}}{a^3c}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2),x)
[Out] a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c} - \frac{1}{a^2cx-ac} + \frac{2\log(ax-1)}{ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] x/c - 1/(a^2*c*x - a*c) + 2*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c} + \frac{2\log(|ax-1|)}{ac} - \frac{1}{(ax-1)ac}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] x/c + 2*log(abs(a*x - 1))/(a*c) - 1/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c} + \frac{1}{a\,(c-a\,c\,x)} + \frac{2\ln(a\,x-1)}{a\,c}$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))*(a*x - 1)),x)
[Out] x/c + 1/(a*(c - a*c*x)) + (2*log(a*x - 1))/(a*c)
```

3.786 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

```
[Out] x/c^2-1/4/a/c^2/(-a*x+1)^2+7/4/a/c^2/(-a*x+1)+17/8*ln(-a*x+1)/a/c^2-1/8*ln(
a*x+1)/a/c^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx \\
&= -\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2} \\
&= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2} \\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2} \\
&= \frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{a^{4}\left(-\frac{\ln(ax+1)}{8a^{5}}+\frac{x}{a^{4}}+\frac{17\ln(ax-1)}{8a^{5}}-\frac{1}{4a^{5}(ax-1)^{2}}-\frac{7}{4a^{5}(ax-1)}\right)}{c^{2}}$	60
risch	$\frac{x}{c^{2}}+\frac{-\frac{7c^{2}x}{4}+\frac{3c^{2}}{2a}}{c^{4}(ax-1)^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}+\frac{17\ln(-ax+1)}{8a\,c^{2}}$	62
norman	$\frac{\frac{a^{3}x^{4}}{c}+\frac{9x}{4c}-\frac{5a\,x^{2}}{4c}-\frac{5a^{2}x^{3}}{2c}}{(ax+1)c(ax-1)^{2}}+\frac{17\ln(ax-1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	85
parallelrisch	$\frac{8a^{3}x^{3}+17a^{2}\ln(ax-1)x^{2}-a^{2}\ln(ax+1)x^{2}-28a^{2}x^{2}-34a\ln(ax-1)x+2a\ln(ax+1)x+18ax+17\ln(ax-1)-\ln(ax+1)}{8c^{2}(ax-1)^{2}a}$	101

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^2*(-1/8*ln(a*x+1)/a^5+x/a^4+17/8/a^5*ln(a*x-1)-1/4/a^5/(a*x-1)^2-7/4/
a^5/(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{8\,a^3x^3-16\,a^2x^2-6\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+17\,(a^2x^2-2\,ax+1)\log(ax-1)+12}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*
c^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{7\,ax-6}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{17\,\log\left(ax-1\right)}{8\,ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 - 1/8*log(a*x
+ 1)/(a*c^2) + 17/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{17\,\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 1/8*log(abs(a*x + 1))/(a*c^2) + 17/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^2}-\frac{\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^2*(a*x - 1)),x)
```

```
[Out] x/c^2 - ((7*x)/4 - 3/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (17*log(a*x -
 1))/(8*a*c^2) - log(a*x + 1)/(8*a*c^2)
```

3.787 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

```
[Out] x/c^3+1/12/a/c^3/(-a*x+1)^3-5/8/a/c^3/(-a*x+1)^2+39/16/a/c^3/(-a*x+1)-1/16/
a/c^3/(a*x+1)+9/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}-\frac{5}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 + 1/(12*a*c^3*(1 - a*x)^3) - 5/(8*a*c^3*(1 - a*x)^2) + 39/(16*a*c^3*(
1 - a*x)) - 1/(16*a*c^3*(1 + a*x)) + (9*Log[1 - a*x])/(4*a*c^3) - Log[1 + a
*x]/(4*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx\\
&= \frac{a^6\int \frac{e^{2\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}\\
&= \frac{a^6\int \frac{x^6}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{4a^6(-1+ax)^4}+\frac{5}{4a^6(-1+ax)^3}+\frac{39}{16a^6(-1+ax)^2}+\frac{9}{4a^6(-1+ax)}+\frac{1}{16a^6(1+ax)^2}-\frac{1}{4a^6(1+ax)}\right)dx}{c^3}\\
&= \frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}\\
&\quad -\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\frac{2(-11+7ax+24a^2x^2-15a^3x^3-12a^4x^4+6a^5x^5)}{(-1+ax)^3(1+ax)}+27\log(1-ax)-3\log(1+ax)}{12ac^3}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((2*(-11 + 7*a*x + 24*a^2*x^2 - 15*a^3*x^3 - 12*a^4*x^4 + 6*a^5*x^5))/((-1
+ a*x)^3*(1 + a*x)) + 27*Log[1 - a*x] - 3*Log[1 + a*x])/(12*a*c^3)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

method	result
default	$\frac{a^6\left(-\frac{\ln(ax+1)}{4a^7}-\frac{1}{16a^7(ax+1)}+\frac{x}{a^6}-\frac{39}{16a^7(ax-1)}+\frac{9\ln(ax-1)}{4a^7}-\frac{1}{12a^7(ax-1)^3}-\frac{5}{8a^7(ax-1)^2}\right)}{c^3}$
risch	$\frac{x}{c^3}+\frac{-\frac{5a^2c^3x^3}{2}+2a\,c^3x^2+\frac{13c^3x}{6}-\frac{11c^3}{6a}}{c^6(ax-1)^2(a^2x^2-1)}+\frac{9\ln(-ax+1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$
norman	$\frac{\frac{a^5x^6}{c}-\frac{5x}{2c}+\frac{3a\,x^2}{2c}+\frac{31a^2x^3}{6c}-\frac{8a^3x^4}{3c}-\frac{17a^4x^5}{6c}}{(ax-1)^3(ax+1)^2c^2}+\frac{9\ln(ax-1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$
parallelrisch	$\frac{12a^5x^5+27\ln(ax-1)x^4a^4-3\ln(ax+1)x^4a^4-46a^4x^4-54a^3\ln(ax-1)x^3+6a^3\ln(ax+1)x^3+14a^3x^3+48a^2x^2+54a\ln(ax-1)x-6a}{12c^3(ax-1)^2(a^2x^2-1)a}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^6/c^3*(-1/4*ln(a*x+1)/a^7-1/16/a^7/(a*x+1)+x/a^6-39/16/a^7/(a*x-1)+9/4/a^
7*ln(a*x-1)-1/12/a^7/(a*x-1)^3-5/8/a^7/(a*x-1)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{12\,a^5x^5-24\,a^4x^4-30\,a^3x^3+48\,a^2x^2+14\,ax-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)+27\left(a^4x^4-2\,a\right.}{12\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*x
^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-15a^3x^3+12a^2x^2+13ax-11}{6a^{11}c^3x^4-12a^{10}c^3x^3+12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-15*a**3*x**3 + 12*a**2*x**2 + 13*a*x - 11)/(6*a**11*c**3*x**4 - 12*
a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(x
- 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{\log\left(ax+1\right)}{4\,ac^3}+\frac{9\,\log\left(ax-1\right)}{4\,ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
 2*a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 9/4*log(a*x - 1)
/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{\log\left(|ax+1|\right)}{4\,ac^3}+\frac{9\,\log\left(|ax-1|\right)}{4\,ac^3}-\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(ax+1)(ax-1)^3ac^3}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 9/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{\frac{13\,x}{6}+2\,a\,x^2-\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}+\frac{9\,\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^3*(a*x - 1)),x)
```

```
[Out] x/c^3 - ((13*x)/6 + 2*a*x^2 - 11/(6*a) - (5*a^2*x^3)/2)/(c^3 + 2*a^3*c^3*x^
3 - a^4*c^3*x^4 - 2*a*c^3*x) + (9*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)/(4
*a*c^3)
```

3.788 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}-\frac{11}{64ac^4(1+ax)}+\frac{303\log(1-ax)}{128ac^4}-\frac{47\log(1+ax)}{128ac^4}$$

```
[Out] x/c^4-1/32/a/c^4/(-a*x+1)^4+13/48/a/c^4/(-a*x+1)^3-35/32/a/c^4/(-a*x+1)^2+9
9/32/a/c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a*x+1)+303/128*ln(-a*
x+1)/a/c^4-47/128*ln(a*x+1)/a/c^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{99}{32ac^4(1-ax)}-\frac{11}{64ac^4(ax+1)}-\frac{35}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}+\frac{13}{48ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}+\frac{303\log(1-ax)}{128ac^4}-\frac{47\log(ax+1)}{128ac^4}+\frac{x}{c^4}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 - 1/(32*a*c^4*(1 - a*x)^4) + 13/(48*a*c^4*(1 - a*x)^3) - 35/(32*a*c^4
*(1 - a*x)^2) + 99/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) - 11/(64
*a*c^4*(1 + a*x)) + (303*Log[1 - a*x])/(128*a*c^4) - (47*Log[1 + a*x])/(128
*a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= -\frac{a^8\int \frac{e^{2\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= -\frac{a^8\int \frac{x^8}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{8a^8(-1+ax)^5}-\frac{13}{16a^8(-1+ax)^4}-\frac{35}{16a^8(-1+ax)^3}-\frac{99}{32a^8(-1+ax)^2}-\frac{303}{128a^8(-1+ax)}+\frac{1}{32a^8(1+ax)^3}-\right.}{c^4}$$

$$= \frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)} + \frac{1}{64ac^4(1+ax)^2}-\frac{11}{64ac^4(1+ax)}+\frac{303\log(1-ax)}{128ac^4}-\frac{47\log(1+ax)}{128ac^4}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{2(400-275ax-1258a^2x^2+866a^3x^3+1254a^4x^4-819a^5x^5-384a^6x^6+192a^7x^7)}{(-1+ax)^4(1+ax)^2}+909\log(1-ax)-141\log(1+ax)}{384ac^4}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(400 - 275*a*x - 1258*a^2*x^2 + 866*a^3*x^3 + 1254*a^4*x^4 - 819*a^5*x^
5 - 384*a^6*x^6 + 192*a^7*x^7))/((-1 + a*x)^4*(1 + a*x)^2) + 909*Log[1 - a*
x] - 141*Log[1 + a*x])/(384*a*c^4)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^8\left(\frac{1}{64a^9(ax+1)^2}-\frac{11}{64a^9(ax+1)}-\frac{47\ln(ax+1)}{128a^9}+\frac{x}{a^8}-\frac{1}{32a^9(ax-1)^4}-\frac{13}{48a^9(ax-1)^3}-\frac{35}{32a^9(ax-1)^2}-\frac{99}{32a^9(ax-1)}+\frac{303\ln(ax-1)}{128a^9}\right)}{c^4}$
risch	$\frac{x}{c^4}+\frac{-\frac{209a^4c^4x^5}{64}+\frac{81a^3c^4x^4}{32}+\frac{529a^2c^4x^3}{96}-\frac{437a\,c^4x^2}{96}-\frac{467c^4x}{192}+\frac{25c^4}{12a}}{c^8(ax-1)^2(a^2x^2-1)^2}+\frac{303\ln(-ax+1)}{128a\,c^4}-\frac{47\ln(ax+1)}{128a\,c^4}$
norman	$\frac{\frac{a^7x^8}{c}+\frac{175x}{64c}-\frac{111a\,x^2}{64c}-\frac{199a^2x^3}{24c}+\frac{115a^3x^4}{24c}+\frac{545a^4x^5}{64c}-\frac{803a^5x^6}{192c}-\frac{37a^6x^7}{12c}}{c^3(ax-1)^4(ax+1)^3}+\frac{303\ln(ax-1)}{128a\,c^4}-\frac{47\ln(ax+1)}{128a\,c^4}$
parallelrisch	$282a\ln(ax+1)x+141a^2\ln(ax+1)x^2-38a^5x^5-1468a^3x^3+282\ln(ax+1)x^5a^5-141\ln(ax+1)x^6a^6+141\ln(ax+1)x^4a^4+909\ln($

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^8/c^4*(1/64/a^9/(a*x+1)^2-11/64/a^9/(a*x+1)-47/128/a^9*ln(a*x+1)+1/a^8*x-
1/32/a^9/(a*x-1)^4-13/48/a^9/(a*x-1)^3-35/32/a^9/(a*x-1)^2-99/32/a^9/(a*x-1
)+303/128/a^9*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{384\,a^7x^7 - 768\,a^6x^6 - 1638\,a^5x^5 + 2508\,a^4x^4 + 1732\,a^3x^3 - 2516\,a^2x^2 - 550\,ax - 141\,(a^6x^6 - 2\,a^5x^5 - a^4x}{384\,(a^7c^4x^6 - 2\,a^6c^4x^5 - a^5c}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*x
^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^
3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
+ 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2*a^6
*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = a^8\left(\frac{-627a^5x^5 + 486a^4x^4 + 1058a^3x^3 - 874a^2x^2 - 467ax + 400}{192a^{15}c^4x^6 - 384a^{14}c^4x^5 - 192a^{13}c^4x^4 + 768a^{12}c^4x^3 - 192a^{11}c^4x^2 - 384a^{10}c^4x + 192a^9c^4} + \frac{x}{a^8c^4} + \frac{\frac{303\log\left(x-\frac{1}{a}\right)}{128} - \frac{47\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-627*a**5*x**5 + 486*a**4*x**4 + 1058*a**3*x**3 - 874*a**2*x**2 - 46
7*a*x + 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/(a**
9*c**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)} + \frac{x}{c^4}-\frac{47\,\log{(ax+1)}}{128\,ac^4}+\frac{303\,\log{(ax-1)}}{128\,ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x -
400)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x
^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 47/128*log(a*x + 1)/(a*c^4) + 303/128*l
og(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}-\frac{47\,\log{(|ax+1|)}}{128\,ac^4}+\frac{303\,\log{(|ax-1|)}}{128\,ac^4} - \frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 47/128*log(abs(a*x + 1))/(a*c^4) + 303/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x
 - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}+\frac{\frac{467\,x}{192}+\frac{437\,a\,x^2}{96}-\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}-\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4} + \frac{303\,\ln\left(a\,x-1\right)}{128\,a\,c^4}-\frac{47\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^4*(a*x - 1)),x)
```

```
[Out] x/c^4 + ((467*x)/192 + (437*a*x^2)/96 - 25/(12*a) - (529*a^2*x^3)/96 - (81*
a^3*x^4)/32 + (209*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^
4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) + (303*log(a*x - 1))/(128*
a*c^4) - (47*log(a*x + 1))/(128*a*c^4)
```

3.789 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 343

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}\\ &\quad-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}\\ &\quad+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}\\ &\quad+c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x+\frac{15c^4\csc^{-1}(ax)}{16a}+\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

```
[Out] 8/7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)/a+c^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(1
1/2)*x+15/16*c^4*arccsc(a*x)/a+3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2
))/a-37/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-61/40*c^4*(1+1/a/x)^(5/2)*
(1-1/a/x)^(1/2)/a-303/280*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-57/70*c^4*(
1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a+15/14*c^4*(1+1/a/x)^(11/2)*(1-1/a/x)^(1/2)
/a-63/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{7a} +c^4x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{14a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{280a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-63*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (37*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(40*a) - (303*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) -
(57*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (15*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(11/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(11/2))/(7*a) + c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x + (15*c^
4*ArcCsc[a*x])/(16*a) + (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^4\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x-c^4\text{Subst}\left(\int \frac{\left(\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}\\
&\quad +c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x-\frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int \frac{\left(\frac{21}{a^2}-\frac{45x}{a^3}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}{x}\,dx,x,\frac{1}{x}\right)
\end{aligned}
$$

$$=\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x-\frac{1}{42}\left(a^2c^4\right)\text{Subst}\left(\int\frac{\left(\frac{126}{a^3}-\frac{171x}{a^4}\right)\left(1+\frac{x}{a}\right)^{9/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$=-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x+\frac{1}{210}\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{630}{a^4}+\frac{909x}{a^5}\right)\left(1+\frac{x}{a}\right)^{7/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$=-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x-\frac{1}{840}\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(\frac{2520}{a^5}-\frac{3843x}{a^6}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$=-\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x+\frac{\left(a^5c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{7560}{a^6}+\frac{11655x}{a^7}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{2520}$$

$$=-\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x-\frac{\left(a^6c^4\right)\text{Subst}\left(\int\frac{\left(\frac{15120}{a^7}-\frac{19845x}{a^8}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{5040}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x + \frac{(a^7c^4)\,\text{Subst}\left(\int \frac{-\frac{15120}{a^8}+\frac{4725x}{a^9}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5040}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x + \frac{(15c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^2} - \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{\cdots}}\right.}{a}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x + \frac{(15c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{16a^2} + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{\cdots}\right.}{a^2}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x + \frac{15c^4\csc^{-1}(ax)}{16a} + \frac{3c^4\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.37

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}(-80-280ax-96a^2x^2+770a^3x^3+992a^4x^4-525a^5x^5-2496a^6x^6+560a^7x^7)+525a^6x^6\right.}{560a^7x^6}$$

[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]

[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(-80 - 280*a*x - 96*a^2*x^2 + 770*a^3*x^3 + 992*a^4*x^4 - 525*a^5*x^5 - 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1/(a*x)] + 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(ax-1)(560a^7x^7-2496a^6x^6-525a^5x^5+992a^4x^4+770a^3x^3-96a^2x^2-280ax-80)c^4}{560x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^7\arctan\left(\frac{1}{\sqrt{a^2x}}\right.}{16}\right.}{a^8(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+525a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}+525a^7x^7\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+1680\right.}{}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)

[Out] 1/560*(a*x-1)*(560*a^7*x^7-2496*a^6*x^6-525*a^5*x^5+992*a^4*x^4+770*a^3*x^3-96*a^2*x^2-280*a*x-80)/x^7*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)+(3*a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+15/16*a^7*arctan(1/(a^2*x^2-1)^(1/2)))*c^4/a^8/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(560\,a^8c\right.}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 - 1936*a^7*c^4*x^7 - 3021*a^6*c^4*x^6 +
 467*a^5*c^4*x^5 + 1762*a^4*c^4*x^4 + 674*a^3*c^4*x^3 - 376*a^2*c^4*x^2 - 3
60*a*c^4*x - 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$= \frac{c^4\left(\int\left(-\frac{4a^2}{\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^6}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{}}{}}\right.\right.}{a^8}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4/(
a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6/(a*x**3*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x) + Integral(a**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**9*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**8*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$
$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2205\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}}{}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
```

```
2 - (2205*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 13615*c^4*((a*x - 1)/(a*x + 1)
)^(13/2) + 33621*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 39071*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 12799*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 20811*c^4*((a*x -
1)/(a*x + 1))^(5/2) - 7665*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 1155*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.34

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= -\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\mathrm{sgn}\left(ax+1\right)} - \frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\left(ax+1\right)} + \frac{525\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a| - 4480\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4 - 980\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a| - 20160}{}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^4*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
- 1)*c^4/(a*sgn(a*x + 1)) + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^
4*abs(a) - 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 - 980*(x*abs(a) - s
qrt(a^2*x^2 - 1))^11*c^4*abs(a) - 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a
*c^4 + 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) - 38080*(x*abs(a) -
sqrt(a^2*x^2 - 1))^8*a*c^4 - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4 -
 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) - 32256*(x*abs(a) - sqrt(a
^2*x^2 - 1))^4*a*c^4 + 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a) - 12
992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 525*(x*abs(a) - sqrt(a^2*x^2 -
 1))*c^4*abs(a) - 2496*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^7*a*a
bs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{12799\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}-\frac{219\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}-\frac{2973\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{33\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{39071\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{4803\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\cdots}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\cdots}$$
$$-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((12799*c^4*((a*x - 1)/(a*x + 1))^(7/2))/280 - (219*c^4*((a*x - 1)/(a*x + 1
))^(3/2))/8 - (2973*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 - (33*c^4*((a*x - 1
)/(a*x + 1))^(1/2))/8 + (39071*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (4803
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (389*c^4*((a*x - 1)/(a*x + 1))^(13/
2))/8 + (63*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x
+ 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (1
4*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x -
 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((a*x - 1)
/(a*x + 1))^(1/2)))/(8*a) + (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.790 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx &= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}\\&\quad-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}\\&\quad+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{3c^3\csc^{-1}(ax)}{8a}+\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

```
[Out] c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x+3/8*c^3*arccsc(a*x)/a+3*c^3*arctanh((
1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-17/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/
a-29/20*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-21/20*c^3*(1+1/a/x)^(7/2)*(1-
1/a/x)^(1/2)/a+6/5*c^3*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-27/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{6329, 99, 159, 163, 41, 222, 94, 214\}$

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^3x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2} + \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{5a} - \frac{21c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{20a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{8a} - \frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{8a} + \frac{3c^3\csc^{-1}(ax)}{8a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-27*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (17*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(8*a) - (29*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(20*a) - (21*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*c
^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(5*a) + c^3*(1 - 1/(a*x))^(3/2)*(
1 + 1/(a*x))^(9/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) + (3*c^3*ArcTanh[Sqrt[1 -
1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^3\text{Subst}\left(\int \frac{\left(\frac{3}{a}-\frac{6x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x$$
$$-\frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int\frac{\left(\frac{15}{a^2}-\frac{21x}{a^3}\right)\left(1+\frac{x}{a}\right)^{7/2}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}$$
$$+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{1}{20}\left(a^2c^3\right)\text{Subst}\left(\int\frac{\left(-\frac{60}{a^3}+\frac{87x}{a^4}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}$$
$$+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x-\frac{1}{60}\left(a^3c^3\right)\text{Subst}\left(\int\frac{\left(\frac{180}{a^4}-\frac{255x}{a^5}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$
$$-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}$$
$$+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{1}{120}\left(a^4c^3\right)\text{Subst}\left(\int\frac{\left(-\frac{360}{a^5}+\frac{405x}{a^6}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$
$$-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}$$
$$+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x-\frac{1}{120}\left(a^5c^3\right)\text{Subst}\left(\int\frac{\frac{360}{a^6}-\frac{45x}{a^7}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$
$$-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}$$
$$+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}-\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+}\right.}{a}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{(3c^3)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2} + \frac{(3c^3)\,\mathrm{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{3c^3\csc^{-1}(ax)}{8a} + \frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(8+30ax+24a^2x^2-55a^3x^3-152a^4x^4+40a^5x^5)+15a^4x^4\arcsin\left(\frac{1}{ax}\right)+120a^4x^4\log\left(\left(1-\right.\right.\right.}{40a^5x^4}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(8 + 30*a*x + 24*a^2*x^2 - 55*a^3*x^3 - 152*a^4
*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] + 120*a^4*x^4*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{(ax-1)(152a^4x^4+55a^3x^3-24a^2x^2-30ax-8)c^3}{40x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+a^5\sqrt{(ax-1)(ax+1)}\right)c^3}{a^6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+15a^5x^5\sqrt{a^2}\,\sqrt{a^2x^2-1}+15a^5x^5\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+120\ln\left(\right.\right.}{40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(a*x-1)*(152*a^4*x^4+55*a^3*x^3-24*a^2*x^2-30*a*x-8)/x^5*c^3/a^6/((a*
x-1)/(a*x+1))^(1/2)+(3*a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1
/2)+3/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3/a^
6/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(40\,a^6c^3x^6}{40\,a^6x^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 - 112*a^5*c^3*x^5 - 207*a^4*c^3*x^4 - 31*a^3*
c^3*x^3 + 54*a^2*c^3*x^2 + 38*a*c^3*x + 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(
a^6*x^5)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(\int\frac{3a^2}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-}}{ax+1}}\right.}{a^6}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] c**3*(Integral(3*a**2/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4/(
a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**6*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{135\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+575\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
(135*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 575*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 + 842*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 298*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 465*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)} + \frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|-200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3-10\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|-720\left(x|a|-\right.}{20}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^3*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 -
 1)*c^3/(a*sgn(a*x + 1)) + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*ab
s(a) - 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 - 10*(x*abs(a) - sqrt(a^2
*x^2 - 1))^7*c^3*abs(a) - 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3 - 800*
(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^
3*c^3*abs(a) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 55*(x*abs(a) -
sqrt(a^2*x^2 - 1))*c^3*abs(a) - 152*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 - 1))
^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{\frac{149\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{10}-\frac{93\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}-\frac{21\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{421\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{10}+\frac{115\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}+\frac{27\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{4}}{a+\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{5\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{a\,(a\,x-1)^6}{(a\,x+1)^6}} - \frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((149*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 - (21*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (421*c^3*((a*x - 1)/(a*x
 + 1))^(7/2))/10 + (115*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 + (27*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2
)))/(4*a) + (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.791 $\int e^{3 \coth^{-1}(ax)} \left(c-\frac{c}{a^2 x^2}\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int e^{3 \coth^{-1}(ax)} \left(c-\frac{c}{a^2 x^2}\right)^2 \, dx = -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2} x - \frac{c^2\csc^{-1}(ax)}{2a} + \frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] -1/2*c^2*arccsc(a*x)/a+3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-11/
6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-4/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(
1/2)/a+c^2*(1+1/a/x)^(7/2)*x*(1-1/a/x)^(1/2)-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a
/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{3 \coth^{-1}(ax)} \left(c-\frac{c}{a^2 x^2}\right)^2 \, dx = \frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^2 x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a} - \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a} - \frac{c^2\csc^{-1}(ax)}{2a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (11*c^2*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/(6*a) - (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(3*a) + c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x - (c^2*ArcCsc[a*x])
/(2*a) + (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^2\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - c^2\text{Subst}\left(\int \frac{\left(\frac{3}{a}-\frac{4x}{a^2}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x + \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(-\frac{9}{a^2}+\frac{11x}{a^3}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(\frac{18}{a^3}-\frac{15x}{a^4}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x + \frac{1}{6}\left(a^3c^2\right)\text{Subst}\left(\int \frac{-\frac{18}{a^4}-\frac{3x}{a^5}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{c^2\csc^{-1}(ax)}{2a} + \frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-2-9ax-16a^2x^2+6a^3x^3)-3a^2x^2\arcsin\left(\frac{1}{ax}\right)+18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 - 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x
^2*ArcSin[1/(a*x)] + 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3
*x^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(ax-1)\left(16a^{2}x^{2}+9ax+2\right)c^{2}}{6x^{3}a^{4}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^{4}\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}-\frac{a^{3}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}+a^{3}\sqrt{(ax-1)(ax+1)}\right)c^{2}\sqrt{(ax-1)(ax+1)}}{a^{4}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^{2}c^{2}\left(-18\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}+18\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{2}x^{2}-3\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}-3a^{3}x^{3}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+18\ln\left(\frac{a^{2}x+\sqrt{\cdots}}{\cdots}\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^{4}x^{3}\sqrt{a^{2}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)*(16*a^2*x^2+9*a*x+2)/x^3*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)+(3*a^
4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/2*a^3*arctan(1/(a^2
*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4/(a*x+1)/((a*x-1)/(a*x+1
))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)^{2}dx =\frac{6\,a^{3}c^{2}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+18\,a^{3}c^{2}x^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-18\,a^{3}c^{2}x^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^{4}c^{2}x^{4}-10\,a^{3}c^{2}\cdots\right.}{6\,a^{4}x^{3}}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 25*a^2*c^2*x^2 - 11*a*c^2*x - 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{c^2\left(\int\left(-\frac{2a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\cdots}}{ax+1}}\cdots}{a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**2,x)
[Out] c**2*(Integral(-2*a**2/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4/(a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.14

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+37c^2\left(\frac{ax-1}{ax+1}\cdots\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax\cdots}{(ax\cdots}}\cdots\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (15
*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 37*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 17*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 21*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.27

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\left(ax+1\right)}$$
$$+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2-36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
^2/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a) -
12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 - 36*(x*abs(a) - sqrt(a^2*x^2 - 1
))^2*a*c^2 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) - 16*a*c^2)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{\frac{17\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+5\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((17*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(1/2)
 + (37*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c^2
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.792 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] -3*c*arccsc(a*x)/a+3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/
x)^(3/2)*x*(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6329, 100, 12, 132, 41, 222, 94, 214}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{3c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x - (3*c*ArcCsc[a*x])/a + (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + c\text{Subst}\left(\int -\frac{3\sqrt{1+\frac{x}{a}}}{ax\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{(3c)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{(3c)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&\quad - \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{(3c)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&\quad + \frac{(3c)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{3c\csc^{-1}(ax)}{a} + \frac{3c\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(1+ax)-3\arcsin\left(\frac{1}{ax}\right)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) - 3*ArcSin[1/(a*x)] + 3*Log[(1 + Sqrt[1
 - 1/(a^2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(ax-1)c}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{(ax-1)(ax+1)}}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+3ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] (a*x-1)/x*c/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(3*a*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)-3*arctan(1/(a^2*x^2-1)^(1
/2)))*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+2\,acx+c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 + 2*
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int \frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2),x)
[Out] c*(Integral(a**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**3*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1)), x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/(
a*sgn(a*x + 1)) + 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn(a*
x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

```
[In] int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

3.793 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-5/3*(1+1/a/x)^(1/2)/a/c/(1-1
/a/x)^(3/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(3/2)-14/3*(1+1/a/x)^(1/2)/a/c/(1
-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 101, 157, 12, 94, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}+\frac{x\sqrt{\frac{1}{ax}+1}}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{\frac{1}{ax}+1}}{3ac\sqrt{1-\frac{1}{ax}}}-\frac{5\sqrt{\frac{1}{ax}+1}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (-5*Sqrt[1 + 1/(a*x)])/(3*a*c*(1 - 1/(a*x))^(3/2)) - (14*Sqrt[1 + 1/(a*x)])
/(3*a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*(1 - 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{3}{a}+\frac{2x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{a\text{Subst}\left(\int \frac{-\frac{9}{a^2}-\frac{5x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(14-19ax+3a^2x^2\right)}{(-1+ax)^2} + \frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 - 19*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + (9*Log[
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^5\left(x-\frac{1}{a}\right)^2}-\frac{13\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-9\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+6\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+27\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-
1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1
3/3/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c/((a*x-1)/(a*x+1))^
(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax+\right.}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int \frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{1}{3}a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a
*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a
^2*c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)}-\frac{6\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

```
[In] int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((11*(a*x - 1))/(3*(a*x + 1)
) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) - a
*c*((a*x - 1)/(a*x + 1))^(5/2))
```

3.794 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-6/5*(1+1/a/x)^(1/2)/a/c^2/
(1-1/a/x)^(5/2)-9/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(3/2)+x*(1+1/a/x)^(1/2)
/c^2/(1-1/a/x)^(5/2)-24/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 105, 21, 101, 157, 12, 94, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2} + \frac{x\sqrt{\frac{1}{ax}+1}}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{24\sqrt{\frac{1}{ax}+1}}{5ac^2\sqrt{1-\frac{1}{ax}}} - \frac{9\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{6\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-6*Sqrt[1 + 1/(a*x)])/(5*a*c^2*(1 - 1/(a*x))^(5/2)) - (9*Sqrt[1 + 1/(a*x)]
)/(5*a*c^2*(1 - 1/(a*x))^(3/2)) - (24*Sqrt[1 + 1/(a*x)])/(5*a*c^2*Sqrt[1 -
1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c^2*(1 - 1/(a*x))^(5/2)) + (3*ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{6\text{Subst}\left(\int \frac{-\frac{5}{2}-\frac{2x}{a}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\text{Subst}\left(\int \frac{\frac{15}{2a}+\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{(2a)\text{Subst}\left(\int -\frac{15}{2a^2x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-24+57ax-39a^2x^2+5a^3x^3)}{5(-1+ax)^3} + 3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(5*(-1
 + a*x)^3) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.24

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^8\left(x-\frac{1}{a}\right)^3}-\frac{6\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^7\left(x-\frac{1}{a}\right)^2}-\frac{24\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^6\left(x-\frac{1}{a}\right)}\right)a^4\sqrt{(ax-}}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4+125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4-480\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3-85((ax-1)(ax+1}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/5/a^8/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)
-6/5/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-24/5/a^6/(x-1/a)*((x-1
/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$=\frac{15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(5\,a^4}{5\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{a^4 \int \frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^2|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^2\mathrm{sgn}\left(ax+1\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.67

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{15\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{25\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{9\,\left(a\,x-1\right)}{5\,\left(a\,x+1\right)}+\frac{1}{5}}{4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

```
[In] int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((15*(a*x - 1)^2)/(a*x + 1
)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(4*
a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

3.795 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-8/7/a/c^3/(1-1/a/x)^(7/2)/
(1+1/a/x)^(1/2)-53/35/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-88/35/a/c^3/(1-
1/a/x)^(3/2)/(1+1/a/x)^(1/2)+x/c^3/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)-281/35/a
/c^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}} +\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}} -\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -8/(7*a*c^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - 53/(35*a*c^3*(1 - 1/(a
*x))^(5/2)*Sqrt[1 + 1/(a*x)]) - 88/(35*a*c^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1
/(a*x)]) - 281/(35*a*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (176*Sqrt[1
 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(7/2)*Sqrt
[1 + 1/(a*x)]) + (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\text{Subst}\left(\int \frac{\frac{21}{a^2}+\frac{32x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{7c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{105}{a^3}-\frac{159x}{a^4}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{35c^3}$$

$$=-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{a^3\text{Subst}\left(\int\frac{\frac{315}{a^4}+\frac{528x}{a^5}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{105c^3}$$

$$=-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{a^4\text{Subst}\left(\int\frac{-\frac{315}{a^5}-\frac{843x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{105c^3}$$

$$=-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{a^5\text{Subst}\left(\int-\frac{315}{a^6x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{105c^3}$$

$$=-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{3\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{ac^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(176-423ax+125a^2x^2+368a^3x^3-286a^4x^4+35a^5x^5)}{35(-1+ax)^4(1+ax)} + 3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(176 - 423*a*x + 125*a^2*x^2 + 368*a^3*x^3 - 28
6*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)^4*(1 + a*x)) + 3*Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x])/(a*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.17

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{6}\sqrt{a^{2}}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{14a^{11}\left(x-\frac{1}{a}\right)^{4}}-\frac{71\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{140a^{10}\left(x-\frac{1}{a}\right)^{3}}-\frac{477\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{280a^{9}\left(x-\frac{1}{a}\right)^{2}}-\frac{2931\sqrt{\left(x-\right.}}{56}\right.}{c^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{7}x^{7}-3360\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{8}x^{7}+2555((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{5}x^{5}+11025\sqrt{(ax-1)(}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(3/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/14/a^11/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-71/140/a^10/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-477/280/a^9/(x-1
/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-2931/560/a^8/(x-1/a)*((x-1/a)^2*a^2
+2*(x-1/a)*a)^(1/2)+1/16/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^6
/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.80

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{105\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\right.}{35\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - 105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 + 125
*a^2*x^2 - 423*a*x + 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c
^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{a^6\int \frac{x^6}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\cdots}}{\cdots}}}{c^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1) - 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1)), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{1}{560}a\left(\frac{\frac{51(ax-1)}{ax+1}+\frac{294(ax-1)^2}{(ax+1)^2}+\frac{2170(ax-1)^3}{(ax+1)^3}-\frac{3640(ax-1)^4}{(ax+1)^4}+5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{1680\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\log\left(\sqrt{\frac{ax-1}{ax+1}}\cdots\right.}{a^2c^3}\cdots\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/560*a*((51*(a*x - 1)/(a*x + 1) + 294*(a*x - 1)^2/(a*x + 1)^2 + 2170*(a*x
- 1)^3/(a*x + 1)^3 - 3640*(a*x - 1)^4/(a*x + 1)^4 + 5)/(a^2*c^3*((a*x - 1)/
(a*x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 1680*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c^3) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
[Out] integrate(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^3}-\frac{\frac{42\,(a\,x-1)^2}{5\,(a\,x+1)^2}+\frac{62\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{104\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{51\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}+\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}$$

```
[In] int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(16*a*c^3) - ((42*(a*x - 1)^2)/(5*(a*x + 1)^2)
+ (62*(a*x - 1)^3)/(a*x + 1)^3 - (104*(a*x - 1)^4)/(a*x + 1)^4 + (51*(a*x -
 1))/(35*(a*x + 1)) + 1/7)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 16*a*c^3
*((a*x - 1)/(a*x + 1))^(9/2)) + (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c
^3)
```

3.796 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 329

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx =& -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}\\ &-\frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}\\ &-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}\\ &+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

```
[Out] -10/9/a/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)-29/21/a/c^4/(1-1/a/x)^(7/2)/(1+
1/a/x)^(3/2)-208/105/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-1147/315/a/c^4/(
1-1/a/x)^(3/2)/(1+1/a/x)^(3/2)+x/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)+3*arct
anh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-1462/105/a/c^4/(1+1/a/x)^(3/2)/(
1-1/a/x)^(1/2)+2609/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1
/a/x)^(1/2)/a/c^4/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -10/(9*a*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)) - 29/(21*a*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)) - 208/(105*a*c^4*(1 - 1/(a*x))^(5/2)*(1
+ 1/(a*x))^(3/2)) - 1147/(315*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)
) - 1462/(105*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (2609*Sqrt[1 -
 1/(a*x)])/(315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*
a*c^4*Sqrt[1 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{\frac{27}{a^2}+\frac{60x}{a^3}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{9c^4}
\end{aligned}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{189}{a^3}-\frac{435x}{a^4}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{63c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^3\text{Subst}\left(\int \frac{\frac{945}{a^4}+\frac{2496x}{a^5}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{315c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{a^4\text{Subst}\left(\int \frac{-\frac{2835}{a^5}-\frac{10323x}{a^6}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{945c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^5\text{Subst}\left(\int \frac{\frac{2835}{a^6}+\frac{26316x}{a^7}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{945c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^6\text{Subst}\left(\int \frac{\frac{8505}{a^7}+\frac{23481x}{a^8}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2835c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^7\text{Subst}\left(\int \frac{8505}{a^8x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2835c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^4}$$

$$
\begin{aligned}
&= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} \\
&\quad - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} \\
&\quad - \frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}} \\
&\quad + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^4} \\
&= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} \\
&\quad - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} \\
&\quad - \frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}} \\
&\quad + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1664+4047ax+339a^2x^2-7399a^3x^3+4029a^4x^4+2967a^5x^5-2669a^6x^6+315a^7x^7)}{315(-1+ax)^5(1+ax)^2}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1664 + 4047*a*x + 339*a^2*x^2 - 7399*a^3*x^3
+ 4029*a^4*x^4 + 2967*a^5*x^5 - 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x
)^5*(1 + a*x)^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.13

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{36a^{14}\left(x-\frac{1}{a}\right)^5}-\frac{59\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{252a^{13}\left(x-\frac{1}{a}\right)^4}-\frac{1507\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{1680a^{12}\left(x-\frac{1}{a}\right)^3}-\frac{691\sqrt{\left(x-\right.}}{315a}\right.}{c^4\sqrt{\frac{ax}{ax}}}$
default	$-\frac{-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^9x^9-120960\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^7x^7+416745\sqrt{(a}}{}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(3/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-1/36/a^14/(x-1/a)^5*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-59/252/a^13/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1507/1680/a^12/(
x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-691/315/a^11/(x-1/a)^2*((x-1/a)^
2*a^2+2*(x-1/a)*a)^(1/2)-113591/20160/a^10/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)
*a)^(1/2)-1/96/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+31/192/a^10
/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4/((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{945\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2\right.}{315\left(a^7c^4x^6-4\,a^6c\right.}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2
*x^2 + 4*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^7*x^7 - 2669*
a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3*x^3 + 339*a^2*x^2 + 4047*a
*x - 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*
c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{20160}a\left(\frac{\frac{415\,(ax-1)}{ax+1}+\frac{2511\,(ax-1)^2}{(ax+1)^2}+\frac{11739\,(ax-1)^3}{(ax+1)^3}+\frac{80745\,(ax-1)^4}{(ax+1)^4}-\frac{135765\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{105\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+30\sqrt{}\right.}{a^2c^4}\right.$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
[Out] 1/20160*a*((415*(a*x - 1)/(a*x + 1) + 2511*(a*x - 1)^2/(a*x + 1)^2 + 11739*
(a*x - 1)^3/(a*x + 1)^3 + 80745*(a*x - 1)^4/(a*x + 1)^4 - 135765*(a*x - 1)^
5/(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x -
 1)/(a*x + 1))^(9/2)) + 105*(((a*x - 1)/(a*x + 1))^(3/2) + 30*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4
) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
[Out] integrate(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{5\sqrt{\frac{a\,x-1}{a\,x+1}}}{32\,a\,c^4} - \frac{\frac{279\,(a\,x-1)^2}{35\,(a\,x+1)^2}+\frac{559\,(a\,x-1)^3}{15\,(a\,x+1)^3}+\frac{769\,(a\,x-1)^4}{3\,(a\,x+1)^4}-\frac{431\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{83\,(a\,x-1)}{63\,(a\,x+1)}+\frac{1}{9}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{192\,a\,c^4} - \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^4}$$

```
[In] int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (5*((a*x - 1)/(a*x + 1))^(1/2))/(32*a*c^4) - ((279*(a*x - 1)^2)/(35*(a*x +
1)^2) + (559*(a*x - 1)^3)/(15*(a*x + 1)^3) + (769*(a*x - 1)^4)/(3*(a*x + 1)
^4) - (431*(a*x - 1)^5)/(a*x + 1)^5 + (83*(a*x - 1))/(63*(a*x + 1)) + 1/9)/
(64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(11/
2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(192*a*c^4) - (atan(((a*x - 1)/(a*x + 1))
^(1/2)*1i)*6i)/(a*c^4)
```

3.797 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/a^3/x^2-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx\\
&= -\frac{c^5\int\frac{e^{4\text{arctanh}(ax)}(1-a^2x^2)^5}{x^{10}}\,dx}{a^{10}}\\
&= -\frac{c^5\int\frac{(1-ax)^3(1+ax)^7}{x^{10}}\,dx}{a^{10}}\\
&= -\frac{c^5\int\left(-a^{10}+\frac{1}{x^{10}}+\frac{4a}{x^9}+\frac{3a^2}{x^8}-\frac{8a^3}{x^7}-\frac{14a^4}{x^6}+\frac{14a^6}{x^4}+\frac{8a^7}{x^3}-\frac{3a^8}{x^2}-\frac{4a^9}{x}\right)dx}{a^{10}}\\
&= \frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^5\left(a^{10}x+\frac{1}{9x^9}+4a^9\ln(x)+\frac{14a^6}{3x^3}-\frac{4a^3}{3x^6}+\frac{4a^7}{x^2}-\frac{3a^8}{x}+\frac{3a^2}{7x^7}+\frac{a}{2x^8}-\frac{14a^4}{5x^5}\right)}{a^{10}}$
risch	$c^5x+\frac{-3a^8c^5x^8+4a^7c^5x^7+\frac{14}{3}a^6c^5x^6-\frac{14}{5}a^4c^5x^4-\frac{4}{3}a^3c^5x^3+\frac{3}{7}a^2c^5x^2+\frac{1}{2}a\,c^5x+\frac{1}{9}c^5}{a^{10}x^9}+\frac{4c^5\ln(x)}{a}$
parallelrisch	$\frac{630a^{10}c^5x^{10}+2520c^5\ln(x)a^9x^9-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315a\,c^5x+70}{630a^{10}x^9}$
norman	$\frac{-4a^9c^5x^{10}+a^{10}c^5x^{11}-\frac{c^5}{9a}-\frac{7c^5x}{18}+\frac{a\,c^5x^2}{14}+\frac{22a^3c^5x^4}{15}-\frac{14a^4c^5x^5}{5}-\frac{14a^5c^5x^6}{3}+\frac{2a^6c^5x^7}{3}+7a^7c^5x^8+\frac{37c^5a^2x^3}{21}}{(ax-1)a^9x^9}+\frac{4c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{4c^5x}{-ax+1}-\frac{5c^5\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{5c^5\left(-\frac{7a}{-7a}\right.}{}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5/a^10*(a^10*x+1/9/x^9+4*a^9*ln(x)+14/3*a^6/x^3-4/3*a^3/x^6+4*a^7/x^2-3*a
^8/x+3/7*a^2/x^7+1/2*a/x^8-14/5*a^4/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{630\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)-1890\,a^8c^5x^8+2520\,a^7c^5x^7+2940\,a^6c^5x^6-1764\,a^4c^5x^4-840\,a^3c^5x^3}{630\,a^{10}x^9}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 252
0*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 270
*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx = \frac{a^{10}c^5x+4a^9c^5\log{(x)}+\frac{-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315ac^5x+70c^5}{630x^9}}{a^{10}}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 4*a**9*c**5*log(x) + (-1890*a**8*c**5*x**8 + 2520*a**7*c**5
*x**7 + 2940*a**6*c**5*x**6 - 1764*a**4*c**5*x**4 - 840*a**3*c**5*x**3 + 27
0*a**2*c**5*x**2 + 315*a*c**5*x + 70*c**5)/(630*x**9))/a**10
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx = c^5x+\frac{4\,c^5\log{(x)}}{a} - \frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x-70\,c^5}{630\,a^{10}x^9}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940*
a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*a*
c^5*x - 70*c^5)/(a^10*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{4\,c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(630\,c^5+\frac{4049\,c^5}{ax-1}+\frac{6201\,c^5}{(ax-1)^2}-\frac{18036\,c^5}{(ax-1)^3}-\frac{89124\,c^5}{(ax-1)^4}-\frac{160146\,c^5}{(ax-1)^5}-\frac{153090\,c^5}{(ax-1)^6}-\frac{80220\,c^5}{(ax-1)^7}-\frac{21420\,c^5}{(ax-1)^8}-\frac{2520\,c^5}{(ax-1)^9}\right)(ax-}{630\,a\left(\frac{1}{ax-1}+1\right)^9}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] -4*c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^5*log(abs(-1/(a*x - 1
) - 1))/a + 1/630*(630*c^5 + 4049*c^5/(a*x - 1) + 6201*c^5/(a*x - 1)^2 - 18
036*c^5/(a*x - 1)^3 - 89124*c^5/(a*x - 1)^4 - 160146*c^5/(a*x - 1)^5 - 1530
90*c^5/(a*x - 1)^6 - 80220*c^5/(a*x - 1)^7 - 21420*c^5/(a*x - 1)^8 - 2520*c
^5/(a*x - 1)^9)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^9)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5\left(\frac{a\,x}{2}+\frac{3\,a^2\,x^2}{7}-\frac{4\,a^3\,x^3}{3}-\frac{14\,a^4\,x^4}{5}+\frac{14\,a^6\,x^6}{3}+4\,a^7\,x^7-3\,a^8\,x^8+a^{10}\,x^{10}+4\,a^9\,x^9\,\ln\left(x\right)+\frac{1}{9}\right)}{a^{10}\,x^9}$$

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^5*((a*x)/2 + (3*a^2*x^2)/7 - (4*a^3*x^3)/3 - (14*a^4*x^4)/5 + (14*a^6*x^
6)/3 + 4*a^7*x^7 - 3*a^8*x^8 + a^10*x^10 + 4*a^9*x^9*log(x) + 1/9))/(a^10*x
^9)
```

3.798 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/a^3/x^2-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x
^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x +
(4*c^4*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$

$$= \frac{c^4\int \frac{e^{4\text{arctanh}(ax)}(1-a^2x^2)^4}{x^8}\,dx}{a^8}$$

$$= \frac{c^4\int \frac{(1-ax)^2(1+ax)^6}{x^8}\,dx}{a^8}$$

$$= \frac{c^4\int \left(a^8+\frac{1}{x^8}+\frac{4a}{x^7}+\frac{4a^2}{x^6}-\frac{4a^3}{x^5}-\frac{10a^4}{x^4}-\frac{4a^5}{x^3}+\frac{4a^6}{x^2}+\frac{4a^7}{x}\right)dx}{a^8}$$

$$= -\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x
^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x +
(4*c^4*Log[x])/a
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(a^8x+4a^7\ln(x)+\frac{a^3}{x^4}+\frac{10a^4}{3x^3}-\frac{2a}{3x^6}+\frac{2a^5}{x^2}-\frac{4a^6}{x}-\frac{1}{7x^7}-\frac{4a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{-4a^6c^4x^6+2a^5c^4x^5+\frac{10}{3}a^4c^4x^4+a^3c^4x^3-\frac{4}{5}a^2c^4x^2-\frac{2}{3}a\,c^4x-\frac{1}{7}c^4}{a^8x^7}+\frac{4c^4\ln(x)}{a}$
parallelrisch	$\frac{105a^8c^4x^8+420c^4\ln(x)a^7x^7-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70a\,c^4x-15c^4}{105a^8x^7}$
norman	$\frac{-5a^7c^4x^8+a^8c^4x^9+\frac{c^4}{7a}+\frac{11c^4x}{21}+\frac{2a\,c^4x^2}{15}-\frac{9a^2c^4x^3}{5}-\frac{7a^3c^4x^4}{3}+\frac{4a^4c^4x^5}{3}+6a^5c^4x^6}{(ax-1)a^7x^7}+\frac{4c^4\ln(x)}{a}$
meijerg	$-\frac{c^4\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{3c^4x}{-ax+1}-\frac{2c^4\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}-\frac{2c^4\left(-\frac{5ax}{-5ax+\cdots}\right.}{}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^8*(a^8*x+4*a^7*ln(x)+a^3/x^4+10/3*a^4/x^3-2/3*a/x^6+2*a^5/x^2-4*a^6/x
-1/7/x^7-4/5*a^2/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{105\,a^8c^4x^8+420\,a^7c^4x^7\log(x)-420\,a^6c^4x^6+210\,a^5c^4x^5+350\,a^4c^4x^4+105\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x\cdots}{105\,a^8x^7}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^5
*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x
- 15*c^4)/(a^8*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{a^8c^4x+4a^7c^4\log{(x)}+\frac{-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70ac^4x-15c^4}{105x^7}}{a^8}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 4*a**7*c**4*log(x) + (-420*a**6*c**4*x**6 + 210*a**5*c**4*x*
*5 + 350*a**4*c**4*x**4 + 105*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**
4*x - 15*c**4)/(105*x**7))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = c^4x+\frac{4\,c^4\log{(x)}}{a} - \frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^4
*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x^7
)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$= -\frac{4\,c^4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^4\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$
$$+\frac{\left(105\,c^4+\frac{659\,c^4}{ax-1}+\frac{1253\,c^4}{(ax-1)^2}-\frac{231\,c^4}{(ax-1)^3}-\frac{3885\,c^4}{(ax-1)^4}-\frac{5250\,c^4}{(ax-1)^5}-\frac{2730\,c^4}{(ax-1)^6}-\frac{420\,c^4}{(ax-1)^7}\right)(ax-1)}{105\,a\left(\frac{1}{ax-1}+1\right)^7}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -4*c^4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^4*log(abs(-1/(a*x - 1
) - 1))/a + 1/105*(105*c^4 + 659*c^4/(a*x - 1) + 1253*c^4/(a*x - 1)^2 - 231
*c^4/(a*x - 1)^3 - 3885*c^4/(a*x - 1)^4 - 5250*c^4/(a*x - 1)^5 - 2730*c^4/(
a*x - 1)^6 - 420*c^4/(a*x - 1)^7)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(a^3\,x^3-\frac{4\,a^2\,x^2}{5}-\frac{2\,a\,x}{3}+\frac{10\,a^4\,x^4}{3}+2\,a^5\,x^5-4\,a^6\,x^6+a^8\,x^8+4\,a^7\,x^7\,\ln\left(x\right)-\frac{1}{7}\right)}{a^8\,x^7}$$

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^4*(a^3*x^3 - (4*a^2*x^2)/5 - (2*a*x)/3 + (10*a^4*x^4)/3 + 2*a^5*x^5 - 4*
a^6*x^6 + a^8*x^8 + 4*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

3.799 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 76}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\
&= -\frac{c^3\int\frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}\\
&= -\frac{c^3\int\frac{(1-ax)(1+ax)^5}{x^6}\,dx}{a^6}\\
&= -\frac{c^3\int\left(-a^6+\frac{1}{x^6}+\frac{4a}{x^5}+\frac{5a^2}{x^4}-\frac{5a^4}{x^2}-\frac{4a^5}{x}\right)dx}{a^6}\\
&= \frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^{3}\left(a^{6} x+4 a^{5} \ln \left(x\right)+\frac{a}{x^{4}}+\frac{5 a^{2}}{3 x^{3}}-\frac{5 a^{4}}{x}+\frac{1}{5 x^{5}}\right)}{a^{6}}$
risch	$c^{3} x+\frac{-5 a^{4} c^{3} x^{4}+\frac{5}{3} a^{2} c^{3} x^{2}+a \,c^{3} x+\frac{1}{5} c^{3}}{a^{6} x^{5}}+\frac{4 c^{3} \ln \left(x\right)}{a}$
parallelrisch	$\frac{15 a^{6} c^{3} x^{6}+60 c^{3} \ln \left(x\right) a^{5} x^{5}-75 a^{4} c^{3} x^{4}+25 a^{2} c^{3} x^{2}+15 a \,c^{3} x+3 c^{3}}{15 a^{6} x^{5}}$
norman	$\frac{-6 a^{5} c^{3} x^{6}+a^{6} c^{3} x^{7}-\frac{c^{3}}{5 a}-\frac{4 c^{3} x}{5}-\frac{2 a \,c^{3} x^{2}}{3}+\frac{5 a^{2} c^{3} x^{3}}{3}+5 a^{3} c^{3} x^{4}}{\left(a x -1\right) a^{5} x^{5}}+\frac{4 c^{3} \ln \left(x\right)}{a}$
meijerg	$-\frac{c^{3}\left(-\frac{a x \left(-3 a x +6\right)}{3 \left(-a x +1\right)}-2 \ln \left(-a x +1\right)\right)}{a}-\frac{2 c^{3} x}{-a x +1}-\frac{2 c^{3}\left(-\frac{5 a x}{-5 a x +5}+4 \ln \left(-a x +1\right)-1-4 \ln \left(x \right)-4 \ln \left(-a \right)+\frac{1}{3 x^{3} a^{3}}+\frac{1}{a^{2} x^{2}}+\frac{3}{a x}\right)}{a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^6*(a^6*x+4*a^5*ln(x)+a/x^4+5/3*a^2/x^3-5*a^4/x+1/5/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{15\,a^{6}c^{3}x^{6}+60\,a^{5}c^{3}x^{5}\log\left(x\right)-75\,a^{4}c^{3}x^{4}+25\,a^{2}c^{3}x^{2}+15\,ac^{3}x+3\,c^{3}}{15\,a^{6}x^{5}}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3*
x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{a^{6}c^{3}x+4a^{5}c^{3}\log\left(x\right)+\frac{-75a^{4}c^{3}x^{4}+25a^{2}c^{3}x^{2}+15ac^{3}x+3c^{3}}{15x^{5}}}{a^{6}}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 4*a**5*c**3*log(x) + (-75*a**4*c**3*x**4 + 25*a**2*c**3*x**2
 + 15*a*c**3*x + 3*c**3)/(15*x**5))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = c^3x+\frac{4\,c^3\log\left(x\right)}{a}-\frac{75\,a^4c^3x^4-25\,a^2c^3x^2-15\,ac^3x-3\,c^3}{15\,a^6x^5}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*x
 - 3*c^3)/(a^6*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\begin{aligned}&\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\&=-\frac{4\,c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}\\&\quad+\frac{\left(15\,c^3+\frac{107\,c^3}{ax-1}+\frac{235\,c^3}{(ax-1)^2}+\frac{170\,c^3}{(ax-1)^3}-\frac{30\,c^3}{(ax-1)^4}-\frac{60\,c^3}{(ax-1)^5}\right)(ax-1)}{15\,a\left(\frac{1}{ax-1}+1\right)^5}\end{aligned}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -4*c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^3*log(abs(-1/(a*x - 1
) - 1))/a + 1/15*(15*c^3 + 107*c^3/(a*x - 1) + 235*c^3/(a*x - 1)^2 + 170*c^
3/(a*x - 1)^3 - 30*c^3/(a*x - 1)^4 - 60*c^3/(a*x - 1)^5)*(a*x - 1)/(a*(1/(a
*x - 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(a\,x+\frac{5\,a^2\,x^2}{3}-5\,a^4\,x^4+a^6\,x^6+4\,a^5\,x^5\,\ln\left(x\right)+\frac{1}{5}\right)}{a^6\,x^5}$$

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^3*(a*x + (5*a^2*x^2)/3 - 5*a^4*x^4 + a^6*x^6 + 4*a^5*x^5*log(x) + 1/5))/
(a^6*x^5)
```

3.800 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/a^3/x^2-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*L
og[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int e^{4\operatorname{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\
&= \frac{c^2\int \frac{e^{4\operatorname{arctanh}(ax)}(1-a^2x^2)^2}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int \frac{(1+ax)^4}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int \left(a^4+\frac{1}{x^4}+\frac{4a}{x^3}+\frac{6a^2}{x^2}+\frac{4a^3}{x}\right)dx}{a^4}\\
&= -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*L
og[x])/a
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^2\left(a^4x+4a^3\ln(x)-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{6a^2}{x}\right)}{a^4}$
risch	$c^2x+\frac{-6a^2c^2x^2-2a\,c^2x-\frac{1}{3}c^2}{a^4x^3}+\frac{4c^2\ln(x)}{a}$
parallelrisch	$\frac{3a^4c^2x^4+12c^2\ln(x)a^3x^3-18a^2c^2x^2-6a\,c^2x-c^2}{3a^4x^3}$
norman	$\frac{-7a^3c^2x^4+a^4c^2x^5+\frac{c^2}{3a}+\frac{5c^2x}{3}+4a\,c^2x^2}{(ax-1)a^3x^3}+\frac{4c^2\ln(x)}{a}$
meijerg	$-\frac{c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{c^2x}{-ax+1}+\frac{c^2\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{2c^2\left(\frac{ax}{-ax+1}\right.}{}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2/a^4*(a^4*x+4*a^3*ln(x)-1/3/x^3-2*a/x^2-6*a^2/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4+12\,a^3c^2x^3\log\left(x\right)-18\,a^2c^2x^2-6\,ac^2x-c^2}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x - c
^2)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x+4a^3c^2\log\left(x\right)+\frac{-18a^2c^2x^2-6ac^2x-c^2}{3x^3}}{a^4}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 4*a**3*c**2*log(x) + (-18*a**2*c**2*x**2 - 6*a*c**2*x - c**2
)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x+\frac{4\,c^2\log\left(x\right)}{a}-\frac{18\,a^2c^2x^2+6\,ac^2x+c^2}{3\,a^4x^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
[Out] c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{4\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(3\,c^2+\frac{34\,c^2}{ax-1}+\frac{66\,c^2}{(ax-1)^2}+\frac{36\,c^2}{(ax-1)^3}\right)(ax-1)}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
[Out] -4*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^2*log(abs(-1/(a*x - 1
) - 1))/a + 1/3*(3*c^2 + 34*c^2/(a*x - 1) + 66*c^2/(a*x - 1)^2 + 36*c^2/(a*
x - 1)^3)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2\left(6\,a\,x+18\,a^2\,x^2-3\,a^4\,x^4-12\,a^3\,x^3\,\ln\left(x\right)+1\right)}{3\,a^4\,x^3}$$

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] -(c^2*(6*a*x + 18*a^2*x^2 - 3*a^4*x^4 - 12*a^3*x^3*log(x) + 1))/(3*a^4*x^3)
```

3.801 $\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx=\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

```
[Out] c/a^2/x+c*x-4*c*ln(x)/a+8*c*ln(-a*x+1)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6292, 6285, 90}

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx=\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{4\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx\\ &= -\frac{c\int\frac{e^{4\text{arctanh}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}\\ &= -\frac{c\int\frac{(1+ax)^3}{x^2(1-ax)}\,dx}{a^2}\\ &= -\frac{c\int\left(-a^2+\frac{1}{x^2}+\frac{4a}{x}-\frac{8a^2}{-1+ax}\right)dx}{a^2}\\ &= \frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result
default	$\frac{c\left(a^{2}x+\frac{1}{x}-4a\ln\left(x\right)+8a\ln\left(ax-1\right)\right)}{a^{2}}$
risch	$\frac{c}{a^{2}x}+cx-\frac{4c\ln\left(x\right)}{a}+\frac{8c\ln\left(-ax+1\right)}{a}$
parallelrisch	$-\frac{-a^{2}c\,x^{2}+4c\ln\left(x\right)ax-8c\ln\left(ax-1\right)ax-c}{a^{2}x}$
norman	$\frac{a^{2}c\,x^{3}-\frac{c}{a}}{x\left(ax-1\right)a}-\frac{4c\ln\left(x\right)}{a}+\frac{8c\ln\left(ax-1\right)}{a}$
meijerg	$-\frac{c\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln\left(-ax+1\right)\right)}{a}-\frac{2c\left(\frac{2ax}{-2ax+2}-\ln\left(-ax+1\right)+1+\ln\left(x\right)+\ln\left(-a\right)\right)}{a}+ \frac{c\left(-\frac{}{-}\right.}{}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] c/a^2*(a^2*x+1/x-4*a*ln(x)+8*a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2+8\,acx\log\left(ax-1\right)-4\,acx\log\left(x\right)+c}{a^2x}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{4c\left(-\log\left(x\right)+2\log\left(x-\frac{1}{a}\right)\right)}{a}+\frac{c}{a^2x}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2),x)
[Out] c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{8\,c\log\left(ax-1\right)}{a}-\frac{4\,c\log\left(x\right)}{a}+\frac{c}{a^2x}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{4\,c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{(ax-1)c}{a\left(\frac{1}{ax-1}+1\right)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
[Out] -4*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4*c*log(abs(-1/(a*x - 1) -
1))/a + (a*x - 1)*c/(a*(1/(a*x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=c\,x+\frac{c}{a^2\,x}-\frac{4\,c\,\ln\left(x\right)}{a}+\frac{8\,c\,\ln\left(a\,x-1\right)}{a}$$

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c*x + c/(a^2*x) - (4*c*log(x))/a + (8*c*log(a*x - 1))/a
```

3.802 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

```
[Out] x/c-1/a/c/(-a*x+1)^2+5/a/c/(-a*x+1)+4*ln(-a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 78}

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c}\\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c}\\
&= \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{c}+\frac{-5cx+\frac{4c}{a}}{c^{2}(ax-1)^{2}}+\frac{4\ln(ax-1)}{ac}$	43
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{1}{a^{3}(ax-1)^{2}}-\frac{5}{a^{3}(ax-1)}+\frac{4\ln(ax-1)}{a^{3}}\right)}{c}$	49
norman	$\frac{\frac{a^{2}x^{3}}{c}-\frac{6a\,x^{2}}{c}+\frac{4x}{c}}{(ax-1)^{2}}+\frac{4\ln(ax-1)}{ac}$	50
parallelrisch	$\frac{a^{3}x^{3}+4a^{2}\ln(ax-1)x^{2}-6a^{2}x^{2}-8a\ln(ax-1)x+4ax+4\ln(ax-1)}{(ax-1)^{2}ca}$	67

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] x/c+(-5*c*x+4*c/a)/c^2/(a*x-1)^2+4/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log{(ax-1)}+4}{a^3cx^2-2\,a^2cx+ac}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{-5ax+4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log{(ax-1)}}{ac}$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2),x)
[Out] (-5*a*x + 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log\left(ax-1\right)}{ac}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] -(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{ax-1}{ac}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{\frac{5\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}}{a^4c^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
[Out] (a*x - 1)/(a*c) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - (5*a^3*c
/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{5\,x-\frac{4}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c}$$

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))*(a*x - 1)^2),x)
[Out] x/c - (5*x - 4/a)/(c + a^2*c*x^2 - 2*a*c*x) + (4*log(a*x - 1))/(a*c)
```

3.803 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

```
[Out] x/c^2+1/3/a/c^2/(-a*x+1)^3-2/a/c^2/(-a*x+1)^2+6/a/c^2/(-a*x+1)+4*ln(-a*x+1)
/a/c^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx \\
&= \frac{a^4\int \frac{e^{4\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2} \\
&= \frac{a^4\int \frac{x^4}{(1-ax)^4}\,dx}{c^2} \\
&= \frac{a^4\int \left(\frac{1}{a^4}+\frac{1}{a^4(-1+ax)^4}+\frac{4}{a^4(-1+ax)^3}+\frac{6}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c^2} \\
&= \frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{-13+27ax-9a^2x^2-9a^3x^3+3a^4x^4+12(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
[Out] (-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1 -
 a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-6 a \,c^{2} x^{2}+10 c^{2} x -\frac{13 c^{2}}{3 a}}{c^{4} \left(a x -1\right)^{3}}+\frac{4 \ln \left(a x -1\right)}{a \,c^{2}}$	56
default	$\frac{a^{4} \left(\frac{x}{a^{4}}-\frac{1}{3 a^{5} \left(a x -1\right)^{3}}-\frac{2}{a^{5} \left(a x -1\right)^{2}}-\frac{6}{a^{5} \left(a x -1\right)}+\frac{4 \ln \left(a x -1\right)}{a^{5}}\right)}{c^{2}}$	61
norman	$\frac{\frac{a^{4} x^{5}}{c}+\frac{6 a \,x^{2}}{c}-\frac{4 x}{c}+\frac{8 a^{2} x^{3}}{3 c}-\frac{19 a^{3} x^{4}}{3 c}}{\left(a x -1\right)^{3} c \left(a x +1\right)}+\frac{4 \ln \left(a x -1\right)}{a \,c^{2}}$	82
parallelrisch	$\frac{3 a^{4} x^{4}+12 a^{3} \ln \left(a x -1\right) x^{3}-22 a^{3} x^{3}-36 a^{2} \ln \left(a x -1\right) x^{2}+30 a^{2} x^{2}+36 a \ln \left(a x -1\right) x -12 a x -12 \ln \left(a x -1\right)}{3 \left(a x -1\right)^{3} c^{2} a}$	91

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2+(-6*a*c^2*x^2+10*c^2*x-13/3*c^2/a)/c^4/(a*x-1)^3+4/a/c^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\,a^4x^4-9\,a^3x^3-9\,a^2x^2+27\,ax+12\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log{(ax-1)}-13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2 +
 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x -
 a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-18a^2x^2+30ax-13}{3a^8c^2x^3-9a^7c^2x^2+9a^6c^2x-3a^5c^2}+\frac{x}{a^4c^2}+\frac{4\log{(ax-1)}}{a^5c^2}\right)$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-18*a**2*x**2 + 30*a*x - 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{18\,a^2x^2-30\,ax+13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{4\,\log\left(ax-1\right)}{ac^2}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{ax-1}{ac^2}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{18\,a^5c^4}{ax-1}+\frac{6\,a^5c^4}{(ax-1)^2}+\frac{a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(18*a^5*c^4/(a*x - 1) + 6*a^5*c^4/(a*x - 1)^2 + a^5*c^4/(a*x - 1)^3)/(a^6*c
^6)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\,a\,x^2-10\,x+\frac{13}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^2*(a*x - 1)^2),x)
```

```
[Out] (6*a*x^2 - 10*x + 13/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x)
 + x/c^2 + (4*log(a*x - 1))/(a*c^2)
```

3.804 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1+ax)}{32ac^3}$$

```
[Out] x/c^3-1/8/a/c^3/(-a*x+1)^4+11/12/a/c^3/(-a*x+1)^3-49/16/a/c^3/(-a*x+1)^2+11
1/16/a/c^3/(-a*x+1)+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e^{4\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx \\
&= -\frac{a^6\int \frac{e^{4\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3} \\
&= -\frac{a^6\int \frac{x^6}{(1-ax)^5(1+ax)}\,dx}{c^3} \\
&= -\frac{a^6\int \left(-\frac{1}{a^6}-\frac{1}{2a^6(-1+ax)^5}-\frac{11}{4a^6(-1+ax)^4}-\frac{49}{8a^6(-1+ax)^3}-\frac{111}{16a^6(-1+ax)^2}-\frac{129}{32a^6(-1+ax)}+\frac{1}{32a^6(1+ax)}\right)dx}{c^3} \\
&= \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2} \\
&\quad +\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1+ax)}{32ac^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{2(224-701ax+660a^2x^2-45a^3x^3-192a^4x^4+48a^5x^5)+387(-1+ax)^4\log(1-ax)-3(-1+ax)^4\log}{96ac^3(-1+ax)^4}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-1
+ a*x)^4)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{c^3}+\frac{-\frac{111a^2c^3x^3}{16}+\frac{71a\,c^3x^2}{4}-\frac{749c^3x}{48}+\frac{14c^3}{3a}}{c^6(ax-1)^4}+\frac{129\ln(-ax+1)}{32a\,c^3}-\frac{\ln(ax+1)}{32a\,c^3}$
default	$\frac{a^6\left(-\frac{\ln(ax+1)}{32a^7}+\frac{x}{a^6}-\frac{1}{8a^7(ax-1)^4}-\frac{11}{12a^7(ax-1)^3}-\frac{49}{16a^7(ax-1)^2}-\frac{111}{16a^7(ax-1)}+\frac{129\ln(ax-1)}{32a^7}\right)}{c^3}$
norman	$\frac{\frac{a^6x^7}{c}+\frac{65x}{16c}-\frac{49a\,x^2}{8c}-\frac{161a^2x^3}{24c}+\frac{301a^3x^4}{24c}+\frac{67a^4x^5}{48c}-\frac{20a^5x^6}{3c}}{(ax+1)^2(ax-1)^4c^2}+\frac{129\ln(ax-1)}{32a\,c^3}-\frac{\ln(ax+1)}{32a\,c^3}$
parallelrisch	$\frac{12a\ln(ax+1)x-18a^2\ln(ax+1)x^2+96a^5x^5+1702a^3x^3-3\ln(ax+1)x^4a^4+387\ln(ax-1)x^4a^4+12a^3\ln(ax+1)x^3+390ax-1548a}{96(ax-1)^4c^3a}$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^3+(-111/16*a^2*c^3*x^3+71/4*a*c^3*x^2-749/48*c^3*x+14/3*c^3/a)/c^6/(a*x
-1)^4+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right)}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac\right.}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*(
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*a^4
*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-333a^3x^3+852a^2x^2-749ax+224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-333*a**3*x**3 + 852*a**2*x**2 - 749*a*x + 224)/(48*a**11*c**3*x**4
- 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c**3
) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\,\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{x}{c^3}-\frac{\log\left(ax+1\right)}{32\,ac^3}+\frac{129\,\log\left(ax-1\right)}{32\,ac^3}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a*c
^3) + 129/32*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{ax-1}{ac^3} - \frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3} - \frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3} - \frac{\frac{333\,a^{11}c^9}{ax-1}+\frac{147\,a^{11}c^9}{(ax-1)^2}+\frac{44\,a^{11}c^9}{(ax-1)^3}+\frac{6\,a^{11}c^9}{(ax-1)^4}}{48\,a^{12}c^{12}}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/32
*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(333*a^11*c^9/(a*x - 1) + 147*a^
11*c^9/(a*x - 1)^2 + 44*a^11*c^9/(a*x - 1)^3 + 6*a^11*c^9/(a*x - 1)^4)/(a^1
2*c^12)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3} - \frac{\frac{749\,x}{48}-\frac{71\,a\,x^2}{4}-\frac{14}{3\,a}+\frac{111\,a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3} + \frac{129\,\ln(a\,x-1)}{32\,a\,c^3} - \frac{\ln(a\,x+1)}{32\,a\,c^3}$$

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^3*(a*x - 1)^2),x)
```

```
[Out] x/c^3 - ((749*x)/48 - (71*a*x^2)/4 - 14/(3*a) + (111*a^2*x^3)/16)/(c^3 + 6*
a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (129*log(a*x - 1))
/(32*a*c^3) - log(a*x + 1)/(32*a*c^3)
```

3.805 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx = \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{261\log(1-ax)}{64ac^4}-\frac{5\log(1+ax)}{64ac^4}$$

```
[Out] x/c^4+1/20/a/c^4/(-a*x+1)^5-7/16/a/c^4/(-a*x+1)^4+83/48/a/c^4/(-a*x+1)^3-67
/16/a/c^4/(-a*x+1)^2+501/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+261/64*ln(-a*
x+1)/a/c^4-5/64*ln(a*x+1)/a/c^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx = \frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-ax)}{64ac^4}-\frac{5\log(ax+1)}{64ac^4}+\frac{x}{c^4}$$

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = \int \frac{e^{4\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= \frac{a^8\int \frac{e^{4\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$

$$= \frac{a^8\int \left(\frac{1}{a^8}+\frac{1}{4a^8(-1+ax)^6}+\frac{7}{4a^8(-1+ax)^5}+\frac{83}{16a^8(-1+ax)^4}+\frac{67}{8a^8(-1+ax)^3}+\frac{501}{64a^8(-1+ax)^2}+\frac{261}{64a^8(-1+ax)}+\cdots\right.}{c^4}$$

$$= \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{261\log(1-ax)}{64ac^4}-\frac{5\log(1+ax)}{64ac^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.67

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{2(-2384+7541ax-4900a^2x^2-6800a^3x^3+9300a^4x^4-1365a^5x^5-1920a^6x^6+480a^7x^7)}{(-1+ax)^5(1+ax)}+3915\log(1-ax)-75\log(1+ax)}{960ac^4}$$

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*a
^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log[1
 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^8\left(-\frac{1}{64a^9(ax+1)}-\frac{5\ln(ax+1)}{64a^9}+\frac{x}{a^8}-\frac{1}{20a^9(ax-1)^5}-\frac{7}{16a^9(ax-1)^4}-\frac{83}{48a^9(ax-1)^3}-\frac{67}{16a^9(ax-1)^2}-\frac{501}{64a^9(ax-1)}+\frac{261\ln(ax-1)}{64a^9}\right)}{c^4}$
risch	$\frac{x}{c^4}+\frac{-\frac{251a^4c^4x^5}{32}+\frac{155a^3c^4x^4}{8}-\frac{55a^2c^4x^3}{6}-\frac{341a\,c^4x^2}{24}+\frac{8021c^4x}{480}-\frac{149c^4}{30a}}{c^8(ax-1)^4(a^2x^2-1)}-\frac{5\ln(ax+1)}{64a\,c^4}+\frac{261\ln(-ax+1)}{64a\,c^4}$
norman	$\frac{\frac{a^8x^9}{c}-\frac{115a^3x^4}{6c}-\frac{133x}{32c}+\frac{101a\,x^2}{16c}+\frac{1049a^2x^3}{96c}-\frac{3869a^4x^5}{480c}+\frac{4709a^5x^6}{240c}+\frac{43a^6x^7}{480c}-\frac{209a^7x^8}{30c}}{(ax+1)^3(ax-1)^5c^3}+\frac{261\ln(ax-1)}{64a\,c^4}-\frac{5\ln(ax+1)}{64a\,c^4}$
parallelrisch	$-300a\ln(ax+1)x+375a^2\ln(ax+1)x^2+16342a^5x^5-13600a^3x^3+300\ln(ax+1)x^5a^5-75\ln(ax+1)x^6a^6-375\ln(ax+1)x^4a^4+3915$

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^8/c^4*(-1/64/a^9/(a*x+1)-5/64/a^9*ln(a*x+1)+1/a^8*x-1/20/a^9/(a*x-1)^5-7/
16/a^9/(a*x-1)^4-83/48/a^9/(a*x-1)^3-67/16/a^9/(a*x-1)^2-501/64/a^9/(a*x-1)
+261/64/a^9*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{960\,a^7x^7 - 3840\,a^6x^6 - 2730\,a^5x^5 + 18600\,a^4x^4 - 13600\,a^3x^3 - 9800\,a^2x^2 + 15082\,ax - 75\,(a^6x^6 - 4\,a^5x^5}{960\,(a^7c^4x^6 - 4\,a^6c^4x^5 +}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a^
3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*
a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 +
 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = a^8\left(\frac{-3765a^5x^5 + 9300a^4x^4 - 4400a^3x^3 - 6820a^2x^2 + 8021ax - 2384}{480a^{15}c^4x^6 - 1920a^{14}c^4x^5 + 2400a^{13}c^4x^4 - 2400a^{11}c^4x^2 + 1920a^{10}c^4x - 480a^9c^4} + \frac{x}{a^8c^4} + \frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64} - \frac{5\log\left(x+\frac{1}{a}\right)}{64}}{a^9c^4}\right)$$

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-3765*a**5*x**5 + 9300*a**4*x**4 - 4400*a**3*x**3 - 6820*a**2*x**2 +
 8021*a*x - 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**13*
c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) + x/(
a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)} + \frac{x}{c^4} - \frac{5\,\log(ax+1)}{64\,ac^4} + \frac{261\,\log(ax-1)}{64\,ac^4}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*
a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x - 1
)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{(ax-1)\left(\frac{257}{ax-1}+128\right)}{128\,ac^4\left(\frac{2}{ax-1}+1\right)} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{5\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4} - \frac{\frac{7515\,a^{19}c^{16}}{ax-1}+\frac{4020\,a^{19}c^{16}}{(ax-1)^2}+\frac{1660\,a^{19}c^{16}}{(ax-1)^3}+\frac{420\,a^{19}c^{16}}{(ax-1)^4}+\frac{48\,a^{19}c^{16}}{(ax-1)^5}}{960\,a^{20}c^{20}}$$

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/128*(a*x - 1)*(257/(a*x - 1) + 128)/(a*c^4*(2/(a*x - 1) + 1)) - 4*log(abs
(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 5/64*log(abs(-2/(a*x - 1) - 1))/(
a*c^4) - 1/960*(7515*a^19*c^16/(a*x - 1) + 4020*a^19*c^16/(a*x - 1)^2 + 166
0*a^19*c^16/(a*x - 1)^3 + 420*a^19*c^16/(a*x - 1)^4 + 48*a^19*c^16/(a*x - 1
)^5)/(a^20*c^20)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{\frac{341\,a\,x^2}{24}-\frac{8021\,x}{480}+\frac{149}{30\,a}+\frac{55\,a^2\,x^3}{6}-\frac{155\,a^3\,x^4}{8}+\frac{251\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{x}{c^4}+\frac{261\,\ln\left(a\,x-1\right)}{64\,a\,c^4}-\frac{5\,\ln\left(a\,x+1\right)}{64\,a\,c^4}$$

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^4*(a*x - 1)^2),x)
```

```
[Out] ((341*a*x^2)/24 - (8021*x)/480 + 149/(30*a) + (55*a^2*x^3)/6 - (155*a^3*x^4
)/8 + (251*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 + 4*a^5*c^4*x^
5 - a^6*c^4*x^6 - 4*a*c^4*x) + x/c^4 + (261*log(a*x - 1))/(64*a*c^4) - (5*l
og(a*x + 1))/(64*a*c^4)
```

3.806 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 343

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}\\&\quad+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}\\&\quad+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}\\&\quad+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{35c^4\csc^{-1}(ax)}{16a}-\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

```
[Out] 29/30*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+7/6*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(7/2)/a+8/7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)/a+c^4*(1-1/a/x)^(9/2)*(1
+1/a/x)^(7/2)*x+35/16*c^4*arccsc(a*x)/a-c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2))/a-1/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+7/40*c^4*(1+1/a/x)^(
5/2)*(1-1/a/x)^(1/2)/a+19/40*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-19/16*c^
4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+c^4x\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{7/2}}{7a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{5/2}}{6a}+\frac{29c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{3/2}}{30a}+\frac{19c^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{1-\frac{1}{ax}}}{40a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{40a}-\frac{c^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{16a}-\frac{19c^4\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{16a}+\frac{35c^4\csc^{-1}(ax)}{16a}$$

```
[In] Int[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-19*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (c^4*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(16*a) + (7*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(40*a) + (19*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(40*a) + (29*c^
4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(30*a) + (7*c^4*(1 - 1/(a*x))^(5
/2)*(1 + 1/(a*x))^(7/2))/(6*a) + (8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
7/2))/(7*a) + c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x + (35*c^4*ArcCs
c[a*x])/(16*a) - (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^4\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$=c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-c^4\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int\frac{\left(-\frac{7}{a^2}-\frac{49x}{a^3}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{1}{42}\left(a^2c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{42}{a^3}-\frac{203x}{a^4}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{1}{210}\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{210}{a^4}-\frac{399x}{a^5}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{1}{840}\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{840}{a^5}+\frac{441x}{a^6}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{\left(a^5c^4\right)\text{Subst}\left(\int\frac{\left(\frac{2520}{a^6}+\frac{315x}{a^7}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2520}$$

$$=-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{\left(a^6c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{5040}{a^7}-\frac{5985x}{a^8}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5040}$$

$$=-\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{(a^7c^4)\,\text{Subst}\left(\int\frac{\frac{5040}{a^8}+\frac{11025x}{a^9}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5040}$$

$$=-\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{(35c^4)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2}+\frac{c^4\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx\right.}{a}$$

$$=-\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x-\frac{c^4\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}+\frac{(35c^4)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-}}\right.}{16a^2}$$

$$=-\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{35c^4\csc^{-1}(ax)}{16a}-\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(-240+280ax+1056a^2x^2-1330a^3x^3-1952a^4x^4+3045a^5x^5+2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\arcsin\left(\frac{1}{ax}\right)-1680a^6 l\right.}{1680a^7}$$

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(-240 + 280*a*x + 1056*a^2*x^2 - 1330*a^3*x^3
- 1952*a^4*x^4 + 3045*a^5*x^5 + 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^
6*ArcSin[1/(a*x)] - 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7
)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(ax+1)(2816a^6x^6+3045a^5x^5-1952a^4x^4-1330a^3x^3+1056a^2x^2+280ax-240)c^4\sqrt{\frac{ax-1}{ax+1}}}{1680x^7a^8}+\frac{\left(-\frac{a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{35a^7\arctan}{}\right.}{}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^6x^6-3675a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}-3675a^7x^7\sqrt{a^2}\arctan\left(\frac{}{\sqrt{a^2}}\right.\right.}{}$

```
[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*(a*x+1)*(2816*a^6*x^6+3045*a^5*x^5-1952*a^4*x^4-1330*a^3*x^3+1056*a^
2*x^2+280*a*x-240)/x^7*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)+(-a^8*ln(a^2*x/(a^2)
^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+35/16*a^7*arctan(1/(a^2*x^2-1)^(1/2))
+a^7*((a*x-1)*(a*x+1))^(1/2))*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(1680\,a^8}{}$$

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 + 4496*a^7*c^4*x^7 + 5861*a^6*c^4*x^6
 + 1093*a^5*c^4*x^5 - 3282*a^4*c^4*x^4 - 274*a^3*c^4*x^3 + 1336*a^2*c^4*x^2
 + 40*a*c^4*x - 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{c^4\left(\int a^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^8}\,dx+\int\left(-\frac{4a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{6a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a^6}{}\right.\right.}{a^8}$$

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(1/2),x)
[Out] c**4*(Integral(a**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**8, x) + Integral(-4*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**6, x) + Integral(6*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/x**4, x) + Integral(-4*a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a*
*8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{1995\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (1995*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10185*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 17619*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 4569*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 71801*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 72051*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 31465*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 5355*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.53

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{35\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{8\,a} + \frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^4\operatorname{sgn}(ax+1)}{a} - \frac{3045\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|\operatorname{sgn}(ax+1) - 6720\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4\operatorname{sgn}(ax+1) + 6860\left(x|a|-\right.}{}$$

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^4*sgn(a*x + 1)/a - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(a
)*sgn(a*x + 1) - 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
+ 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) - 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 9065*(x*abs(a) - sqrt(
a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 -
1))^8*a*c^4*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn
(a*x + 1) - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) -
 38976*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) - 6860*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) - 12992*(x*abs(a) - sqrt(a^
2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 3045*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*
abs(a)*sgn(a*x + 1) - 2816*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 -
 1))^2 + 1)^7*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{51\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{899\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{24}+\frac{3431\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}+\frac{71801\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{840}+\frac{1523\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{839\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\frac{97\,c^4\,(\cdots}{\cdots}}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a\,x\cdots}{(a\,x+\cdots}}$$
$$-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((51*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (899*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/24 + (3431*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (71801*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/840 + (1523*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (839*
c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (97*c^4*((a*x - 1)/(a*x + 1))^(13/2)
)/8 + (19*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x +
1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (14*
a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x - 1
)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/(8*a) - (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.807 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}+\frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}+\frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a}+\frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x+\frac{15c^3\csc^{-1}(ax)}{8a}-\frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] 5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+6/5*c^3*(1-1/a/x)^(5/2)*(1+1/a/x)
^(5/2)/a+c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x+15/8*c^3*arccsc(a*x)/a-c^3*a
rctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x
)^(1/2)/a+11/12*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-7/8*c^3*(1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{6329, 99, 159, 163, 41, 222, 94, 214\}$

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^3x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2} + \frac{6c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{5a} + \frac{5c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a} + \frac{11c^3\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{12a} + \frac{c^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{24a} - \frac{7c^3\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{8a} + \frac{15c^3\csc^{-1}(ax)}{8a}$$

```
[In] Int[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (c^3*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(24*a) + (11*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(12*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(4*a) + (6*c^3*
(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(5*a) + c^3*(1 - 1/(a*x))^(7/2)*(1
 + 1/(a*x))^(5/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) - (c^3*ArcTanh[Sqrt[1 - 1/
(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^3 \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2} \left(1+\frac{x}{a}\right)^{5/2}}{x^2} \, dx, x, \frac{1}{x}\right)\right)$$

$$= c^3 \left(1-\frac{1}{ax}\right)^{7/2} \left(1+\frac{1}{ax}\right)^{5/2} x - c^3 \text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{6x}{a^2}\right) \left(1-\frac{x}{a}\right)^{5/2} \left(1+\frac{x}{a}\right)^{3/2}}{x} \, dx, x, \frac{1}{x}\right)$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \frac{\left(-\frac{5}{a^2}-\frac{25x}{a^3}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{20}\left(a^2c^3\right)\text{Subst}\left(\int \frac{\left(-\frac{20}{a^3}-\frac{55x}{a^4}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{60}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(-\frac{60}{a^4}+\frac{5x}{a^5}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{1}{120}\left(a^4c^3\right)\text{Subst}\left(\int \frac{\left(\frac{120}{a^5}+\frac{105x}{a^6}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{120}\left(a^5c^3\right)\text{Subst}\left(\int \frac{-\frac{120}{a^6}-\frac{225x}{a^7}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{\left(15c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2} + \frac{c^3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx\right.}{a}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{c^3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2} + \frac{(15c^3)\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{8a^2}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{15c^3\csc^{-1}(ax)}{8a} - \frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(24-30ax-88a^2x^2+135a^3x^3+184a^4x^4+120a^5x^5)+225a^4x^4\arcsin\left(\frac{1}{ax}\right)-120a^4x^4\log\right.}{120a^5x^4}$$

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(24 - 30*a*x - 88*a^2*x^2 + 135*a^3*x^3 + 184*a
^4*x^4 + 120*a^5*x^5) + 225*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 +
Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5*x^4)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
risch	$\frac{(ax+1)(184a^4x^4+135a^3x^3-88a^2x^2-30ax+24)c^3\sqrt{\frac{ax-1}{ax+1}}}{120x^5a^6} + \frac{\left(-\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}} + \frac{15a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8} + a^5\sqrt{(ax-1)(a\right.}{a^6(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-225a^5x^5\sqrt{a^2}\,\sqrt{a^2x^2-1}-225a^5x^5\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.\right.}{120\sqrt{(ax-1}}$

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(a*x+1)*(184*a^4*x^4+135*a^3*x^3-88*a^2*x^2-30*a*x+24)/x^5*c^3/a^6*((
a*x-1)/(a*x+1))^(1/2)+(-a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(
1/2)+15/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3/
a^6*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3x^6}{120\,a^6x^5}$$

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 + 304*a^5*c^3*x^5 + 319*a^4*c^3*x^4 + 47*a
^3*c^3*x^3 - 118*a^2*c^3*x^2 - 6*a*c^3*x + 24*c^3)*sqrt((a*x - 1)/(a*x + 1)
))/(a^6*x^5)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(\int a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^6}$$

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**3*(Integral(a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**6, x) + Integral(3*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**4, x) + Integral(-3*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/x**2, x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+3\cdots}{\cdots}\right.$$

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
 (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(9/2
) + 86*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 345*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.46

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{4\,a} + \frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a} - \frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\operatorname{sgn}(ax+1)-360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\operatorname{sgn}(ax+1)+150\left(x|a|-\sqrt{a^2\cdots}\right.}{\cdots}$$

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^3*sgn(a*x + 1)/a - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) - 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) + 150
*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) - 720*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) - 1120*(x*abs(a) - sqrt(a^2*x^2 -
```

```
1))^4*a*c^3*sgn(a*x + 1) - 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*
sgn(a*x + 1) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 13
5*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) - 184*a*c^3*sgn(a*
x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{\frac{23\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{7\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}} - \frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((23*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (269*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/12 + (827*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (43*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/30 + (61*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (7*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/(4*a) - (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.808 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{3c^2\csc^{-1}(ax)}{2a}-\frac{c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] 4/3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/
2)*x+3/2*c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+3
/2*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-1/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/x)^
(1/2)/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^2x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2} + \frac{4c^2\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{3a} + \frac{3c^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{2a} - \frac{c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a} + \frac{3c^2\csc^{-1}(ax)}{2a}$$

```
[In] Int[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + (3*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(2*a) + (4*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2
))/(3*a) + c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) - (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^2 \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x - c^2 \text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}$$
$$+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int\frac{\left(-\frac{3}{a^2}-\frac{9x}{a^3}\right)\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}$$
$$+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(-\frac{6}{a^3}-\frac{3x}{a^4}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}$$
$$+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{1}{6}\left(a^3c^2\right)\text{Subst}\left(\int\frac{\frac{6}{a^4}+\frac{9x}{a^5}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}$$
$$+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{\left(3c^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a^2}+\frac{c^2\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,\right.}{a}$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}$$
$$+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{c^2\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}+\frac{\left(3c^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\right.}}\right.}{2a^2}$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}$$
$$+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{3c^2\csc^{-1}(ax)}{2a}-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-2+3ax+8a^2x^2+6a^3x^3)+9a^2x^2\arcsin\left(\frac{1}{ax}\right)-6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^
2*ArcSin[1/(a*x)] - 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x
^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

method	result
risch	$\frac{(ax+1)(8a^2x^2+3ax-2)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-}}{a^4(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)c^2\left(-6\sqrt{a^2x^2-1}\sqrt{a^2}a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^2x^2-9\sqrt{a^2x^2-1}\sqrt{a^2}a^3x^3-9a^3x^3\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\right.}{6\sqrt{(ax-1)(ax+1)}a^4x^3\sqrt{a^2}}$

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x+1)*(8*a^2*x^2+3*a*x-2)/x^3*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)+(-a^4*l
n(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+3/2*a^3*arctan(1/(a^2*x^
2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)*((
a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^2x^4+14\,a}{6\,a^4x^3}$$

[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^2*x^4 + 14*a^3*c^2*x^3 + 11*a^2*c^2*x^2 + a*c^2*x - 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^4}$$

[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(1/2),x)

[Out] c**2*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-2*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**2, x))/a**4

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{1}{3}a\left(\frac{9c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-}{(ax-}}\right.$$

[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3
c^2((a*x - 1)/(a*x + 1))^(7/2) + c^2*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a} - \frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\operatorname{sgn}(ax+1) - 12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\operatorname{sgn}(ax+1) - 12\left(x|a|-\sqrt{a^2x^2-\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)}$$

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^2*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*
sgn(a*x + 1)/a - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
 + 1) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) - 12*(x*abs(
a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*x^2 -
 1))*c^2*abs(a)*sgn(a*x + 1) - 8*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} - \frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^2*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.809 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x+\frac{c\csc^{-1}(ax)}{a}-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a-c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/x)^(3
/2)*x*(1+1/a/x)^(1/2)+2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6329, 99, 159, 21, 132, 41, 222, 94, 214}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}+\frac{2c\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{a}+\frac{c\csc^{-1}(ax)}{a}$$

```
[In] Int[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*(1 - 1/(a*x))^(3/2)*Sqrt[1
+ 1/(a*x)]*x + (c*ArcCsc[a*x])/a - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - c\text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - (ac)\text{Subst}\left(\int \frac{-\frac{1}{a^2}-\frac{x}{a^3}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{c\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x\\
&\quad + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x \\
&\quad + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2} \\
&= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x \\
&\quad + \frac{c\csc^{-1}(ax)}{a} - \frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$
\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(1+ax)+\arcsin\left(\frac{1}{ax}\right)-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}
$$

```
[In] Integrate[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) + ArcSin[1/(a*x)] - Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x]))/a
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

method	result
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{\cdots}}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*x+1)/x*c/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(-a*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/2)
))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+2\,acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) - a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 + 2*a*c
*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)\,dx\right)}{a^2}$$

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}+\frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a*
x + 1)/a + 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*c*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

3.810 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+2*(1-1/a/x)^(1/2)/a/c/(1+1/a/
x)^(1/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 21, 96, 94, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}+\frac{x\sqrt{1-\frac{1}{ax}}}{c\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{\frac{1}{ax}+1}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)])/(a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*Sq
rt[1 + 1/(a*x)]) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}x(2+ax)}{1+ax} - \frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x))/(1 + a*x) - Log[(1 + Sqrt[1 - 1/(a^2*x
^2)])*x]/a)/c
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}+\frac{\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{a^{4}\left(x+\frac{1}{a}\right)}\right)a^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(-3\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}+2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}-6\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,ax+4\ln(}{2a\sqrt{a^{2}}\,(ax+1)c\sqrt{(ax-1)(ax+}}$

[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)

[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+1/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^2/c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")

[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2\int\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)

[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{c-\frac{c}{a^2x^2}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2)),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + ((a*x -
 1)/(a*x + 1))^(1/2)/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.811 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

```
[Out] -arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-2/a/c^2/(1+1/a/x)^(3/2)/(1-
1/a/x)^(1/2)+x/c^2/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+5/3*(1-1/a/x)^(1/2)/a/c^
2/(1+1/a/x)^(3/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}+\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -2/(a*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (5*Sqrt[1 - 1/(a*x)])/(3
*a*c^2*(1 + 1/(a*x))^(3/2)) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*
x)]) + x/(c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - ArcTanh[Sqrt[1 - 1/(
a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{1}{a^2}+\frac{4x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{-\frac{3}{a^3}+\frac{5x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^3\text{Subst}\left(\int -\frac{3}{a^4x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$
\begin{aligned}
&= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} \\
&\quad+\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} \\
&\quad+\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$
\int\frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-8-5ax+7a^2x^2+3a^3x^3)}{3(-1+ax)(1+ax)^2}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}
$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 5*a*x + 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
a*x)*(1 + a*x)^2) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.19

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{6a^7\left(x+\frac{1}{a}\right)^2}+\frac{19\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{12a^6\left(x+\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{4a^6\left(x-\frac{1}{a}\right)}\right)a^4\sqrt{\frac{ax-1}{ax+1}}}{c^2(ax-1)}$
default	$-\frac{\left(-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5+24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+21((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/6/a^7/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2
```

```
)+19/12/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/4/a^6/(x-1/a)*((x-1
/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{3\,(a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3\,a^3x^3+7\,a^2x^2-5\,ax-8)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-ac^2)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
[Out] -1/3*(3*(a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^2 - 1)*
log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*sq
rt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x*
*2 + 1), x)/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{1}{12}\,a\left(\frac{3\left(\frac{9\,(ax-1)}{ax+1}-1\right)}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+18\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^2c^2}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*a*(3*(9*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - (((a*x - 1)/(a*x + 1))^(3/2) + 18*s
qrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 12*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{9\,(a\,x-1)}{a\,x+1}-1}{4\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}+\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^2}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^2,x)
```

```
[Out] ((9*(a*x - 1))/(a*x + 1) - 1)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(1/2) - 4*a*c^
2*((a*x - 1)/(a*x + 1))^(3/2)) + (3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^2)
+ ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^2) + (atan(((a*x - 1)/(a*x + 1))^(1/2
)*1i)*2i)/(a*c^2)
```

3.812 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

```
[Out] -4/3/a/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^
(5/2)-arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-13/3/a/c^3/(1+1/a/x)^(
5/2)/(1-1/a/x)^(1/2)+14/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+11/5*(1-1/a
/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used

$= \{6329, 105, 157, 12, 94, 214\}$

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}} - \frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -4/(3*a*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) - 13/(3*a*c^3*Sqrt[1 -
 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (14*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*
x))^(5/2)) + (11*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sqr
t[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(3/2)*(1
 + 1/(a*x))^(5/2)) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^3}\\
&= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^3}\\
&= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}\\
&\quad -\frac{a\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{16x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{3c^3}\\
&= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}\\
&\quad +\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{a^2\text{Subst}\left(\int \frac{\frac{3}{a^3}-\frac{39x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{3c^3}
\end{aligned}
$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^3\text{Subst}\left(\int \frac{\frac{15}{a^4}-\frac{84x}{a^5}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{15c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^4\text{Subst}\left(\int \frac{\frac{45}{a^5}-\frac{99x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{45c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^5\text{Subst}\left(\int \frac{45}{a^6x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{45c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(48+33ax-87a^2x^2-52a^3x^3+38a^4x^4+15a^5x^5)}{15(-1+ax)^2(1+ax)^3} - \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 33*a*x - 87*a^2*x^2 - 52*a^3*x^3 + 38*a^4
*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^2*(1 + a*x)^3) - Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x])/(a*c^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3} + \frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}} + \frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{20a^{10}\left(x+\frac{1}{a}\right)^3} - \frac{23\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{60a^9\left(x+\frac{1}{a}\right)^2} + \frac{493\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{240a^8\left(x+\frac{1}{a}\right)} - \cdots\right.}{c^3(ax-1)}$
default	$-\frac{\left(-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7+240\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^8x^7+285((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-525\sqrt{(ax-1)(ax+\cdots}\right.}{\cdots}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-1/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)+1/20/a^10/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1
/2)-23/60/a^9/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+493/240/a^8/(x+1/
a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/24/a^9/(x-1/a)^2*((x-1/a)^2*a^2+2*(x
-1/a)*a)^(1/2)-25/48/a^8/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^6/c^3
*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,(a^4x^4-2\,a^2x^2+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^4x^4-2\,a^2x^2+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(15\,a^5x^5+38\,a^4x^4}{15\,(a^5c^3x^4-2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (15*a^5*x^5
 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x + 48)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{a^6\int \frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x*
*4 + 3*a**2*x**2 - 1), x)/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{1}{240}\,a\left(\frac{5\left(\frac{23\,(ax-1)}{ax+1}-\frac{120\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+450\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}+\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*(5*(23*(a*x - 1)/(a*x + 1) - 120*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (3
*((a*x - 1)/(a*x + 1))^(5/2) + 40*((a*x - 1)/(a*x + 1))^(3/2) + 450*sqrt((a
*x - 1)/(a*x + 1)))/(a^2*c^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2
*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\frac{23\,(a\,x-1)}{3\,(a\,x+1)}-\frac{40\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^3,x)
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(8*a*c^3) - ((23*(a*x - 1))/(3*(a*x + 1))
- (40*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2)
 - 16*a*c^3*((a*x - 1)/(a*x + 1))^(5/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(6*a
*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) + (atan(((a*x - 1)/(a*x + 1)
)^(1/2)*1i)*2i)/(a*c^3)
```

3.813 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 329

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = &-\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}\\&-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}}\\&+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}}\\&+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

```
[Out] -6/5/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-31/15/a/c^4/(1-1/a/x)^(3/2)/(1+1
/a/x)^(7/2)+x/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-arctanh((1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2))/a/c^4-28/3/a/c^4/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+115/21*(1-
1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+122/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(
5/2)+93/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2)/a/c
^4/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -6/(5*a*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - 31/(15*a*c^4*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)) - 28/(3*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(7/2)) + (115*Sqrt[1 - 1/(a*x)])/(21*a*c^4*(1 + 1/(a*x))^(7/2)) + (122
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (93*Sqrt[1 - 1/(a*x)])
/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
+ 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{-\frac{5}{a^2}+\frac{36x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{5c^4}
\end{aligned}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{a^2\text{Subst}\left(\int \frac{\frac{15}{a^3}-\frac{155x}{a^4}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{15c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^3\text{Subst}\left(\int \frac{-\frac{15}{a^4}+\frac{560x}{a^5}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{15c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^4\text{Subst}\left(\int \frac{-\frac{105}{a^5}+\frac{1725x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{105c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^5\text{Subst}\left(\int \frac{-\frac{525}{a^6}+\frac{3660x}{a^7}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{525c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^6\text{Subst}\left(\int \frac{-\frac{1575}{a^7}+\frac{4185x}{a^8}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{1575c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^7\text{Subst}\left(\int -\frac{1575}{a^8x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{1575c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^4}$$

$$
\begin{aligned}
&= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} \\
&\quad - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}} \\
&\quad + \frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}} \\
&\quad + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$
\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-384-279ax+1065a^2x^2+715a^3x^3-965a^4x^4-559a^5x^5+281a^6x^6+105a^7x^7\right)}{105(-1+ax)^3(1+ax)^4} - \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}
$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-384 - 279*a*x + 1065*a^2*x^2 + 715*a^3*x^3 -
965*a^4*x^4 - 559*a^5*x^5 + 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^3*(
1 + a*x)^4) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4} + \frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}} - \frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{56a^{13}\left(x+\frac{1}{a}\right)^4} + \frac{17\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{112a^{12}\left(x+\frac{1}{a}\right)^3} - \frac{211\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{336a^{11}\left(x+\frac{1}{a}\right)^2} + \frac{1657}{}\right.}{}$
default	Expression too large to display

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-1/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/56/a^13/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1
/2)+17/112/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-211/336/a^11/(x
```

```
+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+1657/672/a^10/(x+1/a)*(a^2*(x+1/a
)^2-2*a*(x+1/a))^(1/2)-7/60/a^11/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2
)-379/480/a^10/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/80/a^12/(x-1/a)^
3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^8/c^4*((a*x-1)/(a*x+1))^(1/2)*((a*x-
1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.62

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{105\left(a^7c^4x^6-3\,a^5c^4x^4+\right.}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x +
 1)) + 1) - 105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 + 71
5*a^3*x^3 + 1065*a^2*x^2 - 279*a*x - 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c
^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x**
*6 + 6*a**4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{6720}\,a\left(\frac{7\left(\frac{47\,(ax-1)}{ax+1}+\frac{655\,(ax-1)^2}{(ax+1)^2}-\frac{2625\,(ax-1)^3}{(ax+1)^3}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+329\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2940\,\sqrt{\cdots}\right.}{a^2c^4}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(7*(47*(a*x - 1)/(a*x + 1) + 655*(a*x - 1)^2/(a*x + 1)^2 - 2625*(a
*x - 1)^3/(a*x + 1)^3 + 3)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*(
(a*x - 1)/(a*x + 1))^(5/2)) + 5*(3*((a*x - 1)/(a*x + 1))^(7/2) + 42*((a*x -
 1)/(a*x + 1))^(5/2) + 329*((a*x - 1)/(a*x + 1))^(3/2) + 2940*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4)
 + 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{35\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{\frac{131\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{175\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{47\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}} + \frac{47\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{192\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{448\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^4}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^4,x)
```

```
[Out] (35*((a*x - 1)/(a*x + 1))^(1/2))/(16*a*c^4) - ((131*(a*x - 1)^2)/(3*(a*x +
1)^2) - (175*(a*x - 1)^3)/(a*x + 1)^3 + (47*(a*x - 1))/(15*(a*x + 1)) + 1/5
)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(7
/2)) + (47*((a*x - 1)/(a*x + 1))^(3/2))/(192*a*c^4) + ((a*x - 1)/(a*x + 1))
^(5/2)/(32*a*c^4) + ((a*x - 1)/(a*x + 1))^(7/2)/(448*a*c^4) + (atan(((a*x -
 1)/(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

3.814 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

```
[Out] 1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/a^3/x
^2+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

```
[In] Int[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\operatorname{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx \\
&= -\frac{c^4\int \frac{e^{-2\operatorname{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8} \\
&= -\frac{c^4\int \frac{(1-ax)^5(1+ax)^3}{x^8}\,dx}{a^8} \\
&= -\frac{c^4\int \left(-a^8+\frac{1}{x^8}-\frac{2a}{x^7}-\frac{2a^2}{x^6}+\frac{6a^3}{x^5}-\frac{6a^5}{x^3}+\frac{2a^6}{x^2}+\frac{2a^7}{x}\right) dx}{a^8} \\
&= \frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(a^8x-2a^7\ln(x)+\frac{3a^3}{2x^4}-\frac{a}{3x^6}-\frac{3a^5}{x^2}+\frac{2a^6}{x}+\frac{1}{7x^7}-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6-3a^5c^4x^5+\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2-\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}-\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}-\frac{c^4x}{3}-\frac{2a\,c^4x^2}{5}+\frac{3a^2c^4x^3}{2}-3a^4c^4x^5+2a^5c^4x^6}{a^7x^7}-\frac{2c^4\ln(x)}{a}$
parallelrisch	$-\frac{-210a^8c^4x^8+420c^4\ln(x)a^7x^7-420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3+84a^2c^4x^2+70a\,c^4x-30c^4}{210a^8x^7}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{c^4\ln(ax+1)}{a}-\frac{4c^4(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{4c^4\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{6c^4\left(-\ln(ax\right.}{}$

```
[In] int((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^8*(a^8*x-2*a^7*ln(x)+3/2*a^3/x^4-1/3*a/x^6-3*a^5/x^2+2*a^6/x+1/7/x^7-
2/5*a^2/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^5
*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{a^8c^4x - 2a^7c^4\log{(x)} + \frac{420a^6c^4x^6 - 630a^5c^4x^5 + 315a^3c^4x^3 - 84a^2c^4x^2 - 70ac^4x + 30c^4}{210x^7}}{a^8}$$

```
[In] integrate((c-c/a**2/x**2)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**8*c**4*x - 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 - 630*a**5*c**4*x**
5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*x**
*7))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = c^4x - \frac{2\,c^4\log{(x)}}{a} + \frac{420\,a^6c^4x^6 - 630\,a^5c^4x^5 + 315\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = c^4x - \frac{2\,c^4\log{(|x|)}}{a} + \frac{420\,a^6c^4x^6 - 630\,a^5c^4x^5 + 315\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{c^4\left(\frac{a\,x}{3}+\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5-2\,a^6\,x^6-a^8\,x^8+2\,a^7\,x^7\,\ln\left(x\right)-\frac{1}{7}\right)}{a^8\,x^7}$$

```
[In] int(((c - c/(a^2*x^2))^4*(a*x - 1))/(a*x + 1),x)
[Out] -(c^4*((a*x)/3 + (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 - 2*a^6*x^6 - a^
8*x^8 + 2*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

3.815 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

```
[Out] -1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/a^3/x^2+c^3/a^2/x+c^
3*x-2*c^3*ln(x)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

```
[In] Int[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\
&= \frac{c^3\int \frac{e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}\\
&= \frac{c^3\int \frac{(1-ax)^4(1+ax)^2}{x^6}\,dx}{a^6}\\
&= \frac{c^3\int \left(a^6+\frac{1}{x^6}-\frac{2a}{x^5}-\frac{a^2}{x^4}+\frac{4a^3}{x^3}-\frac{a^4}{x^2}-\frac{2a^5}{x}\right)dx}{a^6}\\
&= -\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
[Out] -1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result
default	$\frac{c^{3}\left(a^{6} x-2 a^{5} \ln \left(x \right)+\frac{a}{2 x^{4}}+\frac{a^{2}}{3 x^{3}}-\frac{2 a^{3}}{x^{2}}+\frac{a^{4}}{x}-\frac{1}{5 x^{5}}\right)}{a^{6}}$
risch	$c^{3} x +\frac{a^{4} c^{3} x^{4}-2 a^{3} c^{3} x^{3}+\frac{1}{3} a^{2} c^{3} x^{2}+\frac{1}{2} a \,c^{3} x -\frac{1}{5} c^{3}}{a^{6} x^{5}}-\frac{2 c^{3} \ln \left(x \right)}{a}$
norman	$\frac{a^{3} c^{3} x^{4}+a^{5} c^{3} x^{6}-\frac{c^{3}}{5 a}+\frac{c^{3} x}{2}+\frac{a \,c^{3} x^{2}}{3}-2 a^{2} c^{3} x^{3}}{a^{5} x^{5}}-\frac{2 c^{3} \ln \left(x \right)}{a}$
parallelrisch	$-\frac{-30 a^{6} c^{3} x^{6}+60 c^{3} \ln \left(x \right) a^{5} x^{5}-30 a^{4} c^{3} x^{4}+60 a^{3} c^{3} x^{3}-10 a^{2} c^{3} x^{2}-15 a \,c^{3} x +6 c^{3}}{30 a^{6} x^{5}}$
meijerg	$\frac{c^{3} \left(a x -\ln \left(a x +1\right)\right)}{a}-\frac{c^{3} \ln \left(a x +1\right)}{a}-\frac{3 c^{3} \left(-\ln \left(a x +1\right)+\ln \left(x \right)+\ln \left(a \right)\right)}{a}+\frac{3 c^{3} \left(\ln \left(a x +1\right)-\ln \left(x \right)-\ln \left(a \right)-\frac{1}{a x}\right)}{a}+\frac{3 c^{3} \left(-\ln (ax+1\right.}{}$

```
[In] int((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^6*(a^6*x-2*a^5*ln(x)+1/2*a/x^4+1/3*a^2/x^3-2*a^3/x^2+a^4/x-1/5/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{30\,a^{6}c^{3}x^{6}-60\,a^{5}c^{3}x^{5}\log\left(x\right)+30\,a^{4}c^{3}x^{4}-60\,a^{3}c^{3}x^{3}+10\,a^{2}c^{3}x^{2}+15\,ac^{3}x-6\,c^{3}}{30\,a^{6}x^{5}}$$

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{a^{6}c^{3}x-2a^{5}c^{3}\log\left(x\right)+\frac{30a^{4}c^{3}x^{4}-60a^{3}c^{3}x^{3}+10a^{2}c^{3}x^{2}+15ac^{3}x-6c^{3}}{30x^{5}}}{a^{6}}$$

```
[In] integrate((c-c/a**2/x**2)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**6*c**3*x - 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = c^3x - \frac{2c^3\log(x)}{a} + \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30a^6x^5}$$

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = c^3x - \frac{2c^3\log(|x|)}{a} + \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30a^6x^5}$$

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\frac{a\,x}{2}+\frac{a^2\,x^2}{3}-2\,a^3\,x^3+a^4\,x^4+a^6\,x^6-2\,a^5\,x^5\,\ln(x)-\frac{1}{5}\right)}{a^6\,x^5}$$

```
[In] int(((c - c/(a^2*x^2))^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (c^3*((a*x)/2 + (a^2*x^2)/3 - 2*a^3*x^3 + a^4*x^4 + a^6*x^6 - 2*a^5*x^5*log
(x) - 1/5))/(a^6*x^5)
```

3.816 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

```
[Out] 1/3*c^2/a^4/x^3-c^2/a^3/x^2+c^2*x-2*c^2*ln(x)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 76}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-\frac{2c^2\log(x)}{a}+c^2x$$

```
[In] Int[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\ &= -\frac{c^2\int\frac{e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\ &= -\frac{c^2\int\frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4}\\ &= -\frac{c^2\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}\\ &= \frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^{2}\left(a^{4} x-2 a^{3} \ln \left(x \right)+\frac{1}{3 x^{3}}-\frac{a}{x^{2}}\right)}{a^{4}}$
risch	$c^{2} x+\frac{-a \,c^{2} x+\frac{1}{3} c^{2}}{a^{4} x^{3}}-\frac{2 c^{2} \ln \left(x \right)}{a}$
norman	$\frac{a^{3} c^{2} x^{4}+\frac{c^{2}}{3 a}-c^{2} x}{a^{3} x^{3}}-\frac{2 c^{2} \ln \left(x \right)}{a}$
parallelrisch	$-\frac{-3 a^{4} c^{2} x^{4}+6 c^{2} \ln \left(x \right) a^{3} x^{3}+3 a \,c^{2} x -c^{2}}{3 a^{4} x^{3}}$
meijerg	$\frac{c^{2} \left(a x -\ln \left(a x +1\right)\right)}{a}-\frac{c^{2} \ln \left(a x +1\right)}{a}-\frac{2 c^{2} \left(-\ln \left(a x +1\right)+\ln \left(x \right)+\ln \left(a \right)\right)}{a}+\frac{2 c^{2} \left(\ln \left(a x +1\right)-\ln \left(x \right)-\ln \left(a \right)-\frac{1}{a x}\right)}{a}+\frac{c^{2} \left(-\ln \left(a x +1\right)\right.}{}$

```
[In] int((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^2/a^4*(a^4*x-2*a^3*ln(x)+1/3/x^3-a/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4-6\,a^3c^2x^3\log\left(x\right)-3\,ac^2x+c^2}{3\,a^4x^3}$$

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x-2a^3c^2\log\left(x\right)+\frac{-3ac^2x+c^2}{3x^3}}{a^4}$$

```
[In] integrate((c-c/a**2/x**2)**2*(a*x-1)/(a*x+1),x)
[Out] (a**4*c**2*x - 2*a**3*c**2*log(x) + (-3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = c^2x - \frac{2\,c^2\log{(x)}}{a} - \frac{3\,ac^2x - c^2}{3\,a^4x^3}$$

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = c^2x - \frac{2\,c^2\log{(|x|)}}{a} - \frac{3\,ac^2x - c^2}{3\,a^4x^3}$$

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{c^2\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln{(x)}-1\right)}{3\,a^4\,x^3}$$

```
[In] int(((c - c/(a^2*x^2))^2*(a*x - 1))/(a*x + 1),x)
[Out] -(c^2*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

3.817 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

```
[Out] -c/a^2/x+c*x-2*c*ln(x)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6292, 6285, 45}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

```
[In] Int[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx\\ &= \frac{c\int \frac{e^{-2\text{arctanh}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}\\ &= \frac{c\int \frac{(1-ax)^2}{x^2}\,dx}{a^2}\\ &= \frac{c\int \left(a^2+\frac{1}{x^2}-\frac{2a}{x}\right)\,dx}{a^2}\\ &= -\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

```
[In] Integrate[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c\left(a^{2} x-2 a \ln \left(x\right)-\frac{1}{x}\right)}{a^{2}}$	22
risch	$-\frac{c}{a^{2} x}+c x-\frac{2 c \ln \left(x\right)}{a}$	22
parallelrisch	$-\frac{-a^{2} c \,x^{2}+2 c \ln \left(x\right) a x+c}{a^{2} x}$	27
norman	$\frac{a c \,x^{2}-\frac{c}{a}}{a x}-\frac{2 c \ln \left(x\right)}{a}$	30
meijerg	$\frac{c \left(a x-\ln \left(a x+1\right)\right)}{a}-\frac{c \ln \left(a x+1\right)}{a}-\frac{c \left(-\ln \left(a x+1\right)+\ln \left(x\right)+\ln \left(a\right)\right)}{a}+\frac{c \left(\ln \left(a x+1\right)-\ln \left(x\right)-\ln \left(a\right)-\frac{1}{a x}\right)}{a}$	78

```
[In] int((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c/a^2*(a^2*x-2*a*ln(x)-1/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2-2\,acx\log\left(x\right)-c}{a^2x}$$

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx-2ac\log\left(x\right)-\frac{c}{x}}{a^2}$$

```
[In] integrate((c-c/a**2/x**2)*(a*x-1)/(a*x+1),x)
[Out] (a**2*c*x - 2*a*c*log(x) - c/x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx-\frac{2\,c\log\left(x\right)}{a}-\frac{c}{a^2x}$$

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c*x - 2*c*log(x)/a - c/(a^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx-\frac{2\,c\log\left(|x|\right)}{a}-\frac{c}{a^2x}$$

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c*x - 2*c*log(abs(x))/a - c/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c\,\left(2\,a\,x\,\ln\left(x\right)-a^2\,x^2+1\right)}{a^2\,x}$$

```
[In] int(((c - c/(a^2*x^2))*(a*x - 1))/(a*x + 1),x)
[Out] -(c*(2*a*x*log(x) - a^2*x^2 + 1))/(a^2*x)
```

3.818 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}$$

```
[Out] x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-\frac{1}{ac(ax+1)}-\frac{2\log(ax+1)}{ac}+\frac{x}{c}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] x/c - 1/(a*c*(1 + a*x)) - (2*Log[1 + a*x])/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{c-\frac{c}{a^2x^2}}\,dx\\
&= \frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= \frac{a^2\int \frac{x^2}{(1+ax)^2}\,dx}{c}\\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(1+ax)^2}-\frac{2}{a^2(1+ax)}\right)dx}{c}\\
&= \frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x-\frac{1}{a+a^2x}-\frac{2\log(1+ax)}{a}}{c}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
[Out] (x - (a + a^2*x)^(-1) - (2*Log[1 + a*x])/a)/c
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac(ax+1)}-\frac{2\ln(ax+1)}{ac}$	36
default	$\frac{a^{2}\left(-\frac{2\ln(ax+1)}{a^{3}}-\frac{1}{a^{3}(ax+1)}+\frac{x}{a^{2}}\right)}{c}$	37
norman	$\frac{\frac{a\,x^{2}}{c}+\frac{2x}{c}}{ax+1}-\frac{2\ln(ax+1)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}-2a\ln(ax+1)x+2ax-2\ln(ax+1)}{c(ax+1)a}$	45

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2x^2+ax-2\,(ax+1)\log{(ax+1)}-1}{a^2cx+ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=a^2\left(-\frac{1}{a^4cx+a^3c}+\frac{x}{a^2c}-\frac{2\log{(ax+1)}}{a^3c}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{a^2cx+ac}-\frac{2\,\log\left(ax+1\right)}{ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] x/c - 1/(a^2*c*x + a*c) - 2*log(a*x + 1)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{2\,\log\left(|ax+1|\right)}{ac}-\frac{1}{(ax+1)ac}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] x/c - 2*log(abs(a*x + 1))/(a*c) - 1/((a*x + 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{a\,\left(c+a\,c\,x\right)}-\frac{2\,\ln\left(a\,x+1\right)}{a\,c}$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))*(a*x + 1)),x)
[Out] x/c - 1/(a*(c + a*c*x)) - (2*log(a*x + 1))/(a*c)
```

3.819 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}$$

```
[Out] x/c^2+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)+1/8*ln(-a*x+1)/a/c^2-17/8*ln(a*
x+1)/a/c^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=-\frac{7}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] x/c^2 + 1/(4*a*c^2*(1 + a*x)^2) - 7/(4*a*c^2*(1 + a*x)) + Log[1 - a*x]/(8*a
*c^2) - (17*Log[1 + a*x])/(8*a*c^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\
&= -\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}\\
&= -\frac{a^4\int \frac{x^4}{(1-ax)(1+ax)^3}\,dx}{c^2}\\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{8a^4(-1+ax)}+\frac{1}{2a^4(1+ax)^3}-\frac{7}{4a^4(1+ax)^2}+\frac{17}{8a^4(1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx
= \frac{2(-6-3ax+8a^2x^2+4a^3x^3)+(1+ax)^2\log(1-ax)-17(1+ax)^2\log(1+ax)}{8a(c+acx)^2}
$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
[Out] (2*(-6 - 3*a*x + 8*a^2*x^2 + 4*a^3*x^3) + (1 + a*x)^2*Log[1 - a*x] - 17*(1
+ a*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{a^{4}\left(-\frac{17\ln\left(ax+1\right)}{8a^{5}}+\frac{1}{4a^{5}\left(ax+1\right)^{2}}-\frac{7}{4a^{5}\left(ax+1\right)}+\frac{x}{a^{4}}+\frac{\ln\left(ax-1\right)}{8a^{5}}\right)}{c^{2}}$	60
risch	$\frac{x}{c^{2}}+\frac{-\frac{7c^{2}x}{4}-\frac{3c^{2}}{2a}}{c^{4}\left(ax+1\right)^{2}}-\frac{17\ln\left(ax+1\right)}{8a\,c^{2}}+\frac{\ln\left(-ax+1\right)}{8a\,c^{2}}$	62
norman	$\frac{\frac{a^{3}x^{4}}{c}-\frac{9x}{4c}-\frac{5a\,x^{2}}{4c}+\frac{5a^{2}x^{3}}{2c}}{\left(ax+1\right)^{2}c\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{8a\,c^{2}}-\frac{17\ln\left(ax+1\right)}{8a\,c^{2}}$	85
parallelrisch	$\frac{8a^{3}x^{3}+a^{2}\ln\left(ax-1\right)x^{2}-17a^{2}\ln\left(ax+1\right)x^{2}+28a^{2}x^{2}+2a\ln\left(ax-1\right)x-34a\ln\left(ax+1\right)x+18ax+\ln\left(ax-1\right)-17\ln\left(ax+1\right)}{8c^{2}\left(ax+1\right)^{2}a}$	98

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^2*(-17/8*ln(a*x+1)/a^5+1/4/a^5/(a*x+1)^2-7/4/a^5/(a*x+1)+x/a^4+1/8/a^
5*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{8\,a^3x^3+16\,a^2x^2-6\,ax-17\,(a^2x^2+2\,ax+1)\log\left(ax+1\right)+(a^2x^2+2\,ax+1)\log\left(ax-1\right)-12}{8\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*
c^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-7ax-6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-7*a*x - 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{7\,ax+6}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{17\,\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + x/c^2 - 17/8*log(a*x
 + 1)/(a*c^2) + 1/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{17\,\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax+6}{4\,(ax+1)^2ac^2}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 17/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x + 6)/((a*x + 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\frac{7\,x}{4}+\frac{3}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}+\frac{\ln\left(a\,x-1\right)}{8\,a\,c^2}-\frac{17\,\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^2*(a*x + 1)),x)
```

```
[Out] x/c^2 - ((7*x)/4 + 3/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) + log(a*x - 1)/
(8*a*c^2) - (17*log(a*x + 1))/(8*a*c^2)
```

3.820 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2}-\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(1+ax)}{4ac^3}$$

```
[Out] x/c^3+1/16/a/c^3/(-a*x+1)-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3/(a*x+1)^2-39/16/a/
c^3/(a*x+1)+1/4*ln(-a*x+1)/a/c^3-9/4*ln(a*x+1)/a/c^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{1}{16ac^3(1-ax)}-\frac{39}{16ac^3(ax+1)}+\frac{5}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] x/c^3 + 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) + 5/(8*a*c^3*(1 +
 a*x)^2) - 39/(16*a*c^3*(1 + a*x)) + Log[1 - a*x]/(4*a*c^3) - (9*Log[1 + a*
x])/(4*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx \\
&= \frac{a^6\int \frac{e^{-2\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3} \\
&= \frac{a^6\int \frac{x^6}{(1-ax)^2(1+ax)^4}\,dx}{c^3} \\
&= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{16a^6(-1+ax)^2}+\frac{1}{4a^6(-1+ax)}+\frac{1}{4a^6(1+ax)^4}-\frac{5}{4a^6(1+ax)^3}+\frac{39}{16a^6(1+ax)^2}-\frac{9}{4a^6(1+ax)}\right)dx}{c^3} \\
&= \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2} \\
&\quad -\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(1+ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{2(11+7ax-24a^2x^2-15a^3x^3+12a^4x^4+6a^5x^5)+3(-1+ax)(1+ax)^3\log(1-ax)-27(-1+ax)(1+ax)^3\log(1+ax)}{12a(-1+ax)(c+acx)^3}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) + 3*(-1
+ a*x)*(1 + a*x)^3*Log[1 - a*x] - 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])/(
12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

method	result
default	$\frac{a^6\left(-\frac{9\ln(ax+1)}{4a^7}-\frac{1}{12a^7(ax+1)^3}+\frac{5}{8a^7(ax+1)^2}-\frac{39}{16a^7(ax+1)}+\frac{x}{a^6}-\frac{1}{16a^7(ax-1)}+\frac{\ln(ax-1)}{4a^7}\right)}{c^3}$
risch	$\frac{x}{c^3}+\frac{-\frac{5a^2c^3x^3}{2}-2a\,c^3x^2+\frac{13c^3x}{6}+\frac{11c^3}{6a}}{c^6(ax+1)^2(a^2x^2-1)}+\frac{\ln(-ax+1)}{4a\,c^3}-\frac{9\ln(ax+1)}{4a\,c^3}$
norman	$\frac{\frac{a^5x^6}{c}+\frac{5x}{2c}+\frac{3a\,x^2}{2c}-\frac{31a^2x^3}{6c}-\frac{8a^3x^4}{3c}+\frac{17a^4x^5}{6c}}{c^2(ax+1)^3(ax-1)^2}+\frac{\ln(ax-1)}{4a\,c^3}-\frac{9\ln(ax+1)}{4a\,c^3}$
parallelrisch	$\frac{12a^5x^5+3\ln(ax-1)x^4a^4-27\ln(ax+1)x^4a^4+46a^4x^4+6a^3\ln(ax-1)x^3-54a^3\ln(ax+1)x^3+14a^3x^3-48a^2x^2-6a\ln(ax-1)x+54a}{12c^3(ax+1)^2(a^2x^2-1)a}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^6/c^3*(-9/4*ln(a*x+1)/a^7-1/12/a^7/(a*x+1)^3+5/8/a^7/(a*x+1)^2-39/16/a^7/
(a*x+1)+x/a^6-1/16/a^7/(a*x-1)+1/4/a^7*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{12\,a^5x^5+24\,a^4x^4-30\,a^3x^3-48\,a^2x^2+14\,ax-27\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log(ax+1)+3\,(a^4x^4+2\,a}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4*
x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x
- 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-15a^3x^3-12a^2x^2+13ax+11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-15*a**3*x**3 - 12*a**2*x**2 + 13*a*x + 11)/(6*a**11*c**3*x**4 + 12*
a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x -
1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{9\,\log\left(ax+1\right)}{4\,ac^3}+\frac{\log\left(ax-1\right)}{4\,ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3) + x/c^3 - 9/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)
/(a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{9\,\log\left(|ax+1|\right)}{4\,ac^3}+\frac{\log\left(|ax-1|\right)}{4\,ac^3}-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(ax+1)^3(ax-1)ac^3}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 9/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{\frac{13\,x}{6}-2\,a\,x^2+\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}+\frac{\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{9\,\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^3*(a*x + 1)),x)
```

```
[Out] x/c^3 - ((13*x)/6 - 2*a*x^2 + 11/(6*a) - (5*a^2*x^3)/2)/(c^3 - 2*a^3*c^3*x^
3 - a^4*c^3*x^4 + 2*a*c^3*x) + log(a*x - 1)/(4*a*c^3) - (9*log(a*x + 1))/(4
*a*c^3)
```

3.821 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)^2}-\frac{99}{32ac^4(1+ax)}+\frac{47\log(1-ax)}{128ac^4}-\frac{303\log(1+ax)}{128ac^4}$$

```
[Out] x/c^4-1/64/a/c^4/(-a*x+1)^2+11/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4-13/48
/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/32/a/c^4/(a*x+1)+47/128*ln(-a*x+1
)/a/c^4-303/128*ln(a*x+1)/a/c^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{11}{64ac^4(1-ax)}-\frac{99}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{35}{32ac^4(ax+1)^2}-\frac{13}{48ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}+\frac{47\log(1-ax)}{128ac^4}-\frac{303\log(ax+1)}{128ac^4}+\frac{x}{c^4}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] x/c^4 - 1/(64*a*c^4*(1 - a*x)^2) + 11/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1
 + a*x)^4) - 13/(48*a*c^4*(1 + a*x)^3) + 35/(32*a*c^4*(1 + a*x)^2) - 99/(32
*a*c^4*(1 + a*x)) + (47*Log[1 - a*x])/(128*a*c^4) - (303*Log[1 + a*x])/(128
*a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= -\frac{a^8\int \frac{e^{-2\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= -\frac{a^8\int \frac{x^8}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{32a^8(-1+ax)^3}-\frac{11}{64a^8(-1+ax)^2}-\frac{47}{128a^8(-1+ax)}+\frac{1}{8a^8(1+ax)^5}-\frac{13}{16a^8(1+ax)^4}+\frac{35}{16a^8(1+ax)^3}-\ldots\right.}{c^4}$$

$$= \frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)^2}-\frac{99}{32ac^4(1+ax)}+\frac{47\log(1-ax)}{128ac^4}-\frac{303\log(1+ax)}{128ac^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{2(-400-275ax+1258a^2x^2+866a^3x^3-1254a^4x^4-819a^5x^5+384a^6x^6+192a^7x^7)+141(-1+ax)^2(1}{384a(-1+ax)^2(c+acx)^4}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (2*(-400 - 275*a*x + 1258*a^2*x^2 + 866*a^3*x^3 - 1254*a^4*x^4 - 819*a^5*x^
5 + 384*a^6*x^6 + 192*a^7*x^7) + 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x]
- 909*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*c*x
)^4)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
default	$\frac{a^8\left(\frac{1}{32a^9(ax+1)^4}-\frac{13}{48a^9(ax+1)^3}+\frac{35}{32a^9(ax+1)^2}-\frac{99}{32a^9(ax+1)}-\frac{303\ln(ax+1)}{128a^9}+\frac{x}{a^8}-\frac{1}{64a^9(ax-1)^2}-\frac{11}{64a^9(ax-1)}+\frac{47\ln(ax-1)}{128a^9}\right)}{c^4}$
risch	$\frac{x}{c^4}+\frac{-\frac{209a^4c^4x^5}{64}-\frac{81a^3c^4x^4}{32}+\frac{529a^2c^4x^3}{96}+\frac{437a\,c^4x^2}{96}-\frac{467c^4x}{192}-\frac{25c^4}{12a}}{c^8(ax+1)^2(a^2x^2-1)^2}+\frac{47\ln(-ax+1)}{128a\,c^4}-\frac{303\ln(ax+1)}{128a\,c^4}$
norman	$\frac{\frac{a^7x^8}{c}-\frac{175x}{64c}-\frac{111a\,x^2}{64c}+\frac{199a^2x^3}{24c}+\frac{115a^3x^4}{24c}-\frac{545a^4x^5}{64c}-\frac{803a^5x^6}{192c}+\frac{37a^6x^7}{12c}}{(ax-1)^3c^3(ax+1)^4}+\frac{47\ln(ax-1)}{128a\,c^4}-\frac{303\ln(ax+1)}{128a\,c^4}$
parallelrisch	$-1818a\ln(ax+1)x+909a^2\ln(ax+1)x^2-38a^5x^5-1468a^3x^3-1818\ln(ax+1)x^5a^5-909\ln(ax+1)x^6a^6+909\ln(ax+1)x^4a^4+141$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^8/c^4*(1/32/a^9/(a*x+1)^4-13/48/a^9/(a*x+1)^3+35/32/a^9/(a*x+1)^2-99/32/a
^9/(a*x+1)-303/128/a^9*ln(a*x+1)+1/a^8*x-1/64/a^9/(a*x-1)^2-11/64/a^9/(a*x-
1)+47/128/a^9*ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\,(a^6x^6+2\,a^5x^5-a^4x}{384\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*x
^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^
3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= a^8\left(\frac{-627a^5x^5-486a^4x^4+1058a^3x^3+874a^2x^2-467ax-400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}-\frac{303\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-627*a**5*x**5 - 486*a**4*x**4 + 1058*a**3*x**3 + 874*a**2*x**2 - 46
7*a*x - 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/(a**
9*c**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)} + \frac{x}{c^4} - \frac{303\,\log\left(ax+1\right)}{128\,ac^4} + \frac{47\,\log\left(ax-1\right)}{128\,ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x +
400)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x
^2 + 2*a^2*c^4*x + a*c^4) + x/c^4 - 303/128*log(a*x + 1)/(a*c^4) + 47/128*l
og(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} - \frac{303\,\log\left(|ax+1|\right)}{128\,ac^4} + \frac{47\,\log\left(|ax-1|\right)}{128\,ac^4} - \frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(ax+1)^4(ax-1)^2ac^4}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 303/128*log(abs(a*x + 1))/(a*c^4) + 47/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x
 + 400)/((a*x + 1)^4*(a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} - \frac{\frac{467\,x}{192}-\frac{437\,a\,x^2}{96}+\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}+\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4} + \frac{47\,\ln\left(a\,x-1\right)}{128\,a\,c^4} - \frac{303\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^4*(a*x + 1)),x)
```

```
[Out] x/c^4 - ((467*x)/192 - (437*a*x^2)/96 + 25/(12*a) - (529*a^2*x^3)/96 + (81*
a^3*x^4)/32 + (209*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^
4*x^4 + 2*a^5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) + (47*log(a*x - 1))/(128*a
*c^4) - (303*log(a*x + 1))/(128*a*c^4)
```

3.822 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 343

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}+\frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}\\ &\quad+\frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}+\frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a}\\ &\quad+\frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}\\ &\quad+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x+\frac{15c^4\csc^{-1}(ax)}{16a}-\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

```
[Out] 5/8*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+11/10*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(5/2)/a+17/14*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)/a+8/7*c^4*(1-1/a/x)^(9
/2)*(1+1/a/x)^(5/2)/a+c^4*(1-1/a/x)^(11/2)*(1+1/a/x)^(5/2)*x+15/16*c^4*arcc
sc(a*x)/a-3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+27/16*c^4*(1+1/a
/x)^(3/2)*(1-1/a/x)^(1/2)/a-3/8*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a+33/16
*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^4x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{11/2} + \frac{8c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{7a} + \frac{17c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}}{14a} + \frac{11c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{10a} + \frac{5c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{8a} - \frac{3c^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{8a} + \frac{27c^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{16a} + \frac{33c^4\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{16a} + \frac{15c^4\csc^{-1}(ax)}{16a}$$

```
[In] Int[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (33*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) + (27*c^4*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(8*a) + (5*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(8*a) + (11*c
^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(10*a) + (17*c^4*(1 - 1/(a*x))^
(7/2)*(1 + 1/(a*x))^(5/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x)
)^(5/2))/(7*a) + c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x + (15*c^4*A
rcCsc[a*x])/(16*a) - (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^4 \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$=c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x-c^4\text{Subst}\left(\int\frac{\left(-\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$=\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}$$
$$+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x-\frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int\frac{\left(-\frac{21}{a^2}-\frac{51x}{a^3}\right)\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$=\frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}$$
$$+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x-\frac{1}{42}\left(a^2c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{126}{a^3}-\frac{231x}{a^4}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$=\frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a}+\frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}$$
$$+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x-\frac{1}{210}\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{630}{a^4}-\frac{525x}{a^5}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$=\frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}+\frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a}$$
$$+\frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}$$
$$+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x-\frac{1}{840}\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{2520}{a^5}+\frac{945x}{a^6}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$=-\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}+\frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}$$
$$+\frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a}$$
$$+\frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}$$
$$+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x-\frac{\left(a^5c^4\right)\text{Subst}\left(\int\frac{\left(-\frac{7560}{a^6}+\frac{8505x}{a^7}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{2520}$$

$$= \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{(a^6c^4)\,\text{Subst}\left(\int \frac{\left(\frac{15120}{a^7}-\frac{10395x}{a^8}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5040}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{(a^7c^4)\,\text{Subst}\left(\int \frac{-\frac{15120}{a^8}-\frac{4725x}{a^9}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5040}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{(15c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^2} + \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1}}\right.}{a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{(15c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{16a^2} - \frac{(3c^4)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{}\right.}{a^2}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{15c^4\csc^{-1}(ax)}{16a} - \frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.37

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}(80-280ax+96a^2x^2+770a^3x^3-992a^4x^4-525a^5x^5+2496a^6x^6+560a^7x^7)+525a^6x^6\,\text{ar}\right.}{560a^7x^6}$$

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(80 - 280*a*x + 96*a^2*x^2 + 770*a^3*x^3 - 992*
a^4*x^4 - 525*a^5*x^5 + 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1/
(a*x)] - 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(ax+1)(560a^7x^7+2496a^6x^6-525a^5x^5-992a^4x^4+770a^3x^3+96a^2x^2-280ax+80)c^4\sqrt{\frac{ax-1}{ax+1}}}{560x^7a^8} + \frac{\left(-\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^7\,\text{arc}}{}\right.}{a}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-525a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}-525a^7x^7\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x}}\right.\right.}{}$

```
[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/560*(a*x+1)*(560*a^7*x^7+2496*a^6*x^6-525*a^5*x^5-992*a^4*x^4+770*a^3*x^3
+96*a^2*x^2-280*a*x+80)/x^7*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)+(-3*a^8*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+15/16*a^7*arctan(1/(a^2*x^2-1)
^(1/2)))*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(560\,a}{}$$

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 + 3056*a^7*c^4*x^7 + 1971*a^6*c^4*x^6 -
 1517*a^5*c^4*x^5 - 222*a^4*c^4*x^4 + 866*a^3*c^4*x^3 - 184*a^2*c^4*x^2 - 2
00*a*c^4*x + 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^9+x^8}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^8+x^7}\,dx+\int\frac{4a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\right.\right.}{}$$

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**9 + x**8), x) + Int
egral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**8 + x**7), x) + Integral(4*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Integral(-4*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(-6*a**4
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(6*a**5*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(4*a**6*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-4*a**7*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**8*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**9*x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1), x))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.10

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{1155\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 + (1155*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 7665*c^4*((a*x - 1)/(a*x + 1))
^(13/2) + 20811*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 12799*c^4*((a*x - 1)/(a*
x + 1))^(9/2) - 39071*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 33621*c^4*((a*x - 1
)/(a*x + 1))^(5/2) - 13615*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 2205*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.53

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{8\,a} + \frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^4\operatorname{sgn}(ax+1)}{a} + \frac{525\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|\operatorname{sgn}(ax+1)+4480\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4\operatorname{sgn}(ax+1)-980\left(x|a|-\sqrt{a}\right.}{}$$

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^4*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^4*sgn(a*x + 1)/a + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(
a)*sgn(a*x + 1) + 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
```

```
 - 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) + 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 945*(x*abs(a) - sqrt(a
^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) + 38080*(x*abs(a) - sqrt(a^2*x^2 - 1
))^8*a*c^4*sgn(a*x + 1) + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn(
a*x + 1) - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) + 3
2256*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 980*(x*abs(a) -
sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) + 12992*(x*abs(a) - sqrt(a^2*x
^2 - 1))^2*a*c^4*sgn(a*x + 1) - 525*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(
a)*sgn(a*x + 1) + 2496*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))
^2 + 1)^7*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{63\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{389\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{4803\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{39071\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{12799\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{280}-\frac{2973\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}-21}{a+\frac{6\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{14\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{14\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{14\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{14\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{6\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}-a(}$$
$$-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((63*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (389*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/8 + (4803*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (39071*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/280 + (12799*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 - (2973
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 - (219*c^4*((a*x - 1)/(a*x + 1))^(13/
2))/8 - (33*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x
+ 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (1
4*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x -
 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((a*x - 1)
/(a*x + 1))^(1/2)))/(8*a) - (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.823 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}+\frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}+\frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a}+\frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{8a}-\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] 5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+27/20*c^3*(1-1/a/x)^(5/2)*(1+1/a/
x)^(3/2)/a+6/5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)/a+c^3*(1-1/a/x)^(9/2)*(1
+1/a/x)^(3/2)*x+3/8*c^3*arccsc(a*x)/a-3*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2))/a+3/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+21/8*c^3*(1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{6329, 99, 159, 163, 41, 222, 94, 214\}$

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^3x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{9/2} + \frac{6c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{5a} + \frac{27c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}}{20a} + \frac{5c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a} + \frac{3c^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{8a} + \frac{21c^3\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{8a} + \frac{3c^3\csc^{-1}(ax)}{8a}$$

```
[In] Int[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (21*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (3*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(8*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3
/2))/(4*a) + (27*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))/(20*a) + (6*c
^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2))/(5*a) + c^3*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(3/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) - (3*c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - c^3\text{Subst}\left(\int \frac{\left(-\frac{3}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int\frac{\left(-\frac{15}{a^2}-\frac{27x}{a^3}\right)\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{20}\left(a^2c^3\right)\text{Subst}\left(\int\frac{\left(-\frac{60}{a^3}-\frac{75x}{a^4}\right)\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{60}\left(a^3c^3\right)\text{Subst}\left(\int\frac{\left(-\frac{180}{a^4}-\frac{45x}{a^5}\right)\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{120}\left(a^4c^3\right)\text{Subst}\left(\int\frac{\left(-\frac{360}{a^5}+\frac{315x}{a^6}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{1}{120}\left(a^5c^3\right)\text{Subst}\left(\int\frac{\frac{360}{a^6}+\frac{45x}{a^7}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2} + \frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+}}\right.}{a}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{(3c^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2} - \frac{(3c^3)\,\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\right.}{a^2}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{3c^3\csc^{-1}(ax)}{8a} - \frac{3c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(-8+30ax-24a^2x^2-55a^3x^3+152a^4x^4+40a^5x^5)+15a^4x^4\arcsin\left(\frac{1}{ax}\right)-120a^4x^4\log\left(\left(1\right.\right.\right.}{40a^5x^4}$$

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(-8 + 30*a*x - 24*a^2*x^2 - 55*a^3*x^3 + 152*a^
4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
risch	$\frac{(ax+1)(152a^4x^4-55a^3x^3-24a^2x^2+30ax-8)c^3\sqrt{\frac{ax-1}{ax+1}}}{40x^5a^6} + \frac{\left(-\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}} + \frac{3a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8} + a^5\sqrt{(ax-1)(ax+1)}\right.}{a^6(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(-120\sqrt{a^2x^2-1}\sqrt{a^2}\,a^6x^6+120(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-15a^5x^5\sqrt{a^2}\sqrt{a^2x^2-1}-15a^5x^5\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.\right.}{40(ax-1)\sqrt{(ax-\right.}}$

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(a*x+1)*(152*a^4*x^4-55*a^3*x^3-24*a^2*x^2+30*a*x-8)/x^5*c^3/a^6*((a*x
-1)/(a*x+1))^(1/2)+(-3*a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1
/2)+3/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3/a^
6*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(40\,a^6c^3x^6 \cdots}{40\,a^6x^5}$$

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 + 192*a^5*c^3*x^5 + 97*a^4*c^3*x^4 - 79*a^3*c
^3*x^3 + 6*a^2*c^3*x^2 + 22*a*c^3*x - 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^
6*x^5)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{3a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{3a^4\sqrt{\cdots}}{\cdots} \cdots\right)}{a^6}$$

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Inte
gral(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(-3
*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(3*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(3*a**4*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-3*a**5*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**6*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1), x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.12

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+465\right.$$

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 +
(105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 465*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 - 298*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 842*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 575*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 135*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{4\,a} + \frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a} + \frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\operatorname{sgn}(ax+1)+200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\operatorname{sgn}(ax+1)-10\left(x|a|-\sqrt{a^2x^2-\right.}$$

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^3*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^3*sgn(a*x + 1)/a + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) + 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) - 10*
(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) + 720*(x*abs(a) -
sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) + 800*(x*abs(a) - sqrt(a^2*x^2 - 1)
```

```
)^4*a*c^3*sgn(a*x + 1) + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn
(a*x + 1) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 55*(x
*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 152*a*c^3*sgn(a*x +
1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{\frac{27c^3\sqrt{\frac{ax-1}{ax+1}}}{4}+\frac{115c^3\left(\frac{ax-1}{ax+1}\right)^{3/2}}{4}+\frac{421c^3\left(\frac{ax-1}{ax+1}\right)^{5/2}}{10}+\frac{149c^3\left(\frac{ax-1}{ax+1}\right)^{7/2}}{10}-\frac{93c^3\left(\frac{ax-1}{ax+1}\right)^{9/2}}{4}-\frac{21c^3\left(\frac{ax-1}{ax+1}\right)^{11/2}}{4}}{a+\frac{4a(ax-1)}{ax+1}+\frac{5a(ax-1)^2}{(ax+1)^2}-\frac{5a(ax-1)^4}{(ax+1)^4}-\frac{4a(ax-1)^5}{(ax+1)^5}-\frac{a(ax-1)^6}{(ax+1)^6}} - \frac{3c^3\operatorname{atan}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{4a}-\frac{6c^3\operatorname{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((27*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (115*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/4 + (421*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 + (149*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 - (21*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2
)))/(4*a) - (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.824 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x-\frac{c^2\csc^{-1}(ax)}{2a}-\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] -1/2*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+11/
6*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/a+4/3*c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(
1/2)/a+c^2*(1-1/a/x)^(7/2)*x*(1+1/a/x)^(1/2)+5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a
/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a} + c^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2} + \frac{4c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{3a} + \frac{11c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{6a} + \frac{5c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a} - \frac{c^2\csc^{-1}(ax)}{2a}$$

```
[In] Int[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (11*c^2*(1 - 1/(a*x))^(
3/2)*Sqrt[1 + 1/(a*x)])/(6*a) + (4*c^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
])/(3*a) + c^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x - (c^2*ArcCsc[a*x])/
(2*a) - (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c^2\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - c^2\text{Subst}\left(\int \frac{\left(-\frac{3}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$
\begin{aligned}
&= \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} \\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(-\frac{9}{a^2}-\frac{11x}{a^3}\right)\left(1-\frac{x}{a}\right)^{3/2}}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} \\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(-\frac{18}{a^3}-\frac{15x}{a^4}\right)\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} \\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{1}{6}\left(a^3c^2\right)\text{Subst}\left(\int \frac{-\frac{18}{a^4}+\frac{3x}{a^5}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} \\
&\quad + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x \\
&\quad - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} \\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2} \\
&\quad - \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2} \\
&= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} \\
&\quad + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{c^2\csc^{-1}(ax)}{2a} - \frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(2-9ax+16a^2x^2+6a^3x^3)-3a^2x^2\arcsin\left(\frac{1}{ax}\right)-18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x^
2*ArcSin[1/(a*x)] - 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*
x^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

method	result
risch	$\frac{(ax+1)(16a^2x^2-9ax+2)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1}}{a^4(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(-18\sqrt{a^2x^2-1}\sqrt{a^2}\,a^4x^4+18(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\sqrt{a^2}\,a^3x^3+3a^3x^3\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+1\right.}{6(ax-1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x+1)*(16*a^2*x^2-9*a*x+2)/x^3*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)+(-3*a^
4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/2*a^3*arctan(1/(a^2
*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)
/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^2x^4+22\,a^3c^2}{6\,a^4x^3}$$

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 7*a^2*c^2*x^2 - 7*a*c^2*x + 2*c^
2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{2a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\right.\right.}{a^4}$$

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Int
egral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(2*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-2*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17c^2\left(\frac{ax-1}{ax+1}\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax}{(ax}}\right.$$

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21
*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 37*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a}+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\operatorname{sgn}(ax+1)+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\operatorname{sgn}(ax+1)+36\left(x|a|-\sqrt{a^2x^2-1}\right)}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3}$$

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*s
gn(a*x + 1)/a + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
+ 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) + 36*(x*abs(a
) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(a^2*x^2 -
1))*c^2*abs(a)*sgn(a*x + 1) + 16*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

```
[In] int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (17*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.825 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] -3*c*arccsc(a*x)/a-3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/
x)^(3/2)*x*(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6329, 100, 12, 132, 41, 222, 94, 214}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}+cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}$$

```
[In] Int[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] c*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x - (3*c*ArcCsc[a*x])/a - (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + c\text{Subst}\left(\int \frac{3\sqrt{1-\frac{x}{a}}}{ax\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{(3c)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - \frac{(3c)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&\quad + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - \frac{(3c)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&\quad - \frac{(3c)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2}\\
&= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - \frac{3c\csc^{-1}(ax)}{a} - \frac{3c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)-3\arcsin\left(\frac{1}{ax}\right)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

[In] Integrate[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]

[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/a

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-3ax\sqrt{a^2}\,\arctan\left(\right.\right.}{(ax-1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] -(a*x+1)/x*c/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(-3*a*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)-3*arctan(1/(a^2*x^2-1)^
(1/2)))*c*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) + 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 - c)
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}-\frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a - 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

```
[In] int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

3.826 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+5/3*(1-1/a/x)^(1/2)/a/c/(1+
1/a/x)^(3/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(3/2)+14/3*(1-1/a/x)^(1/2)/a/c/(
1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 101, 157, 12, 94, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}+\frac{x\sqrt{1-\frac{1}{ax}}}{c\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(\frac{1}{ax}+1\right)^{3/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (5*Sqrt[1 - 1/(a*x)])/(3*a*c*(1 + 1/(a*x))^(3/2)) + (14*Sqrt[1 - 1/(a*x)])/
(3*a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*(1 + 1/(a*x))^(3/2)) -
 (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x^2\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}+\frac{2x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{5x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c}\\
&= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\text{Subst}\left(\int -\frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}\\
&= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}\\
&= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}x(14+19ax+3a^2x^2)}{(1+ax)^2} - \frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 + 19*a*x + 3*a^2*x^2))/(1 + a*x)^2 - (9*Log[(
1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{3a^5\left(x+\frac{1}{a}\right)^2}+\frac{13\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{3a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(-9\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3+9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+6\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}\,ax-27\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2\cdots\right.}{\cdots}$

[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)

[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+13/3/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^2/c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.67

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3\,a^2x^2+19\,ax+14)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2cx+ac)}$$

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")

[Out] -1/3*(9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^2*x^2 + 19*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x + a*c)

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx+\int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx\right)}{c}$$

[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)

[Out] a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x - 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x - 1), x))/c

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{1}{3}\,a\left(\frac{6\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+12\sqrt{\frac{ax-1}{ax+1}}}{a^2c}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(6*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) - (
((a*x - 1)/(a*x + 1))^(3/2) + 12*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c) + 9*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a)) + sqrt(a^
2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (4*((a*
x - 1)/(a*x + 1))^(1/2))/(a*c) + ((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c) + (ata
n(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c)
```

3.827 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2+6/5*(1-1/a/x)^(1/2)/a/c^2
/(1+1/a/x)^(5/2)+9/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(3/2)+x*(1-1/a/x)^(1/2
)/c^2/(1+1/a/x)^(5/2)+24/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6329, 105, 21, 101, 157, 12, 94, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}+\frac{x\sqrt{1-\frac{1}{ax}}}{c^2\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{5/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(5*a*c^2*(1 + 1/(a*x))^(5/2)) + (9*Sqrt[1 - 1/(a*x)])
/(5*a*c^2*(1 + 1/(a*x))^(3/2)) + (24*Sqrt[1 - 1/(a*x)])/(5*a*c^2*Sqrt[1 + 1
/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c^2*(1 + 1/(a*x))^(5/2)) - (3*ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}\\
&= \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{3x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}\\
&= \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{ac^2}\\
&= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{6\text{Subst}\left(\int \frac{-\frac{5}{2}+\frac{2x}{a}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5ac^2}\\
&= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{2\text{Subst}\left(\int \frac{-\frac{15}{2a}+\frac{9x}{2a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{5c^2}
\end{aligned}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{(2a)\text{Subst}\left(\int -\frac{15}{2a^2x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.43

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(24+57ax+39a^2x^2+5a^3x^3)}{5(1+ax)^3}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 57*a*x + 39*a^2*x^2 + 5*a^3*x^3))/(5*(1 +
 a*x)^3) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{2}}+\frac{\left(-\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{4}\sqrt{a^{2}}}+\frac{\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{5a^{8}\left(x+\frac{1}{a}\right)^{3}}-\frac{6\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{5a^{7}\left(x+\frac{1}{a}\right)^{2}}+\frac{24\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{5a^{6}\left(x+\frac{1}{a}\right)}\right)a^{4}\sqrt{\frac{ax}{ax}}}{c^{2}(ax-1)}$
default	$-\frac{\left(120\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{5}x^{4}-125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{4}x^{4}+480\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+85((ax-1)(ax-}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)+1/5/a^8/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2
)-6/5/a^7/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+24/5/a^6/(x+1/a)*(a^2
*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^4/c^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*
x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{2}}\,dx =$$
$$-\frac{15\left(a^{2}x^{2}+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^{2}x^{2}+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(5\,a^{3}x^{3}+39\,a^{2}x^{2}+57\right.}{5\left(a^{3}c^{2}x^{2}+2\,a^{2}c^{2}x+ac^{2}\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(15*(a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*(a^2
*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (5*a^3*x^3 + 39*a^2*
x^2 + 57*a*x + 24)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 + 2*a^2*c^2*x +
a*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\left(\int \left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int \frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx\right)}{c^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x*
*4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x**5*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 +
 a*x + 1), x))/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.89

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{1}{20}\,a\left(\frac{40\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+85\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}\right.}{a^2c^2}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/20*a*(40*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^
2) - (((a*x - 1)/(a*x + 1))^(5/2) + 10*((a*x - 1)/(a*x + 1))^(3/2) + 85*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(
a^2*c^2) - 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.33

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^2|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac^2}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^2*abs(a)) + sqrt(
a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}}+\frac{17\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^2}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^2,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + (17
*((a*x - 1)/(a*x + 1))^(1/2))/(4*a*c^2) + ((a*x - 1)/(a*x + 1))^(3/2)/(2*a*
c^2) + ((a*x - 1)/(a*x + 1))^(5/2)/(20*a*c^2) + (atan(((a*x - 1)/(a*x + 1))
^(1/2)*1i)*6i)/(a*c^2)
```

3.828 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 253

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-2/a/c^3/(1+1/a/x)^(7/2)/(
1-1/a/x)^(1/2)+x/c^3/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+11/7*(1-1/a/x)^(1/2)/a
/c^3/(1+1/a/x)^(7/2)+54/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+71/35*(1-1
/a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1
/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3}+\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}} +\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{5/2}} +\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -2/(a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (11*Sqrt[1 - 1/(a*x)])/(
7*a*c^3*(1 + 1/(a*x))^(7/2)) + (54*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x
))^(5/2)) + (71*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x))^(3/2)) + (176*Sq
rt[1 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*Sqrt[1 - 1/(a*x)]*(1
 + 1/(a*x))^(7/2)) - (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{8x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}}\\
&\quad + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^2\text{Subst}\left(\int \frac{-\frac{21}{a^3}+\frac{33x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^3}
\end{aligned}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^3\text{Subst}\left(\int \frac{-\frac{105}{a^4}+\frac{108x}{a^5}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^4\text{Subst}\left(\int \frac{-\frac{315}{a^5}+\frac{213x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{105c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^5\text{Subst}\left(\int -\frac{315}{a^6x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{105c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-176-423ax-125a^2x^2+368a^3x^3+286a^4x^4+35a^5x^5)}{35(-1+ax)(1+ax)^4} - 3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 + 2
86*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)*(1 + a*x)^4) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x])/(a*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3} + \frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}} - \frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{14a^{11}\left(x+\frac{1}{a}\right)^4} + \frac{71\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{140a^{10}\left(x+\frac{1}{a}\right)^3} - \frac{477\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{280a^9\left(x+\frac{1}{a}\right)^2} + \right.}{c^3(ax-1)}$
default	$-\frac{\left(-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7+3360\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^8x^7+2555((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-11025\sqrt{(ax-1)\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-3/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/14/a^11/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1
/2)+71/140/a^10/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-477/280/a^9/(x+
1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+2931/560/a^8/(x+1/a)*(a^2*(x+1/a)^
```

```
2-2*a*(x+1/a))^(1/2)-1/16/a^8/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^
6/c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{105\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3}{35\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
- 1) - (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2*x^2 - 423*a*x - 17
6)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x -
a*c^3)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{a^6\left(\int\left(-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\right)dx+\int\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx\right)}{c^3}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(Integral(-x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x*
*6 - 3*a**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x) +
 Integral(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 -
 3*a**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x))/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{1}{560}\,a\left(\frac{35\left(\frac{33\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+56\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+350\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2520\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1680\log}{}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
[Out] -1/560*a*(35*(33*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3
/2) - a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - (5*((a*x - 1)/(a*x + 1))^(7/2) +
 56*((a*x - 1)/(a*x + 1))^(5/2) + 350*((a*x - 1)/(a*x + 1))^(3/2) + 2520*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 1680*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{33\,(a\,x-1)}{a\,x+1}-1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{9\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}+\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{112\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^3}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3,x)
[Out] ((33*(a*x - 1))/(a*x + 1) - 1)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2) - 16*a
*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (9*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^
3) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2
)/(10*a*c^3) + ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3) + (atan(((a*x - 1)/(
a*x + 1))^(1/2)*1i)*6i)/(a*c^3)
```

3.829 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}$$

```
[Out] -4/3/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(9/2)+x/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^
(9/2)-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-5/a/c^4/(1+1/a/x)^(9
/2)/(1-1/a/x)^(1/2)+28/9*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(9/2)+139/63*(1-1/
a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+202/105*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(5
/2)+719/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1/a/x)^(1/2)/
a/c^4/(1+1/a/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -4/(3*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - 5/(a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(9/2)) + (28*Sqrt[1 - 1/(a*x)])/(9*a*c^4*(1 + 1/(a*x))
^(9/2)) + (139*Sqrt[1 - 1/(a*x)])/(63*a*c^4*(1 + 1/(a*x))^(7/2)) + (202*Sqr
t[1 - 1/(a*x)])/(105*a*c^4*(1 + 1/(a*x))^(5/2)) + (719*Sqrt[1 - 1/(a*x)])/(
315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - (3*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}
\end{aligned}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{a^2\text{Subst}\left(\int\frac{\frac{9}{a^3}-\frac{75x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{11/2}}\,dx,x,\frac{1}{x}\right)}{3c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{a^3\text{Subst}\left(\int\frac{\frac{81}{a^4}-\frac{336x}{a^5}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}\,dx,x,\frac{1}{x}\right)}{27c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{a^4\text{Subst}\left(\int\frac{\frac{567}{a^5}-\frac{1251x}{a^6}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{189c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{a^5\text{Subst}\left(\int\frac{\frac{2835}{a^6}-\frac{3636x}{a^7}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{945c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{a^6\text{Subst}\left(\int\frac{\frac{8505}{a^7}-\frac{6471x}{a^8}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2835c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{a^7\text{Subst}\left(\int\frac{8505}{a^8x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{2835c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{3\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{ac^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{3\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c^4}$$

$$=-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(1664+4047ax-339a^2x^2-7399a^3x^3-4029a^4x^4+2967a^5x^5+2669a^6x^6+315a^7x^7)}{315(-1+ax)^2(1+ax)^5}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3 -
 4029*a^4*x^4 + 2967*a^5*x^5 + 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x)
^2*(1 + a*x)^5) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.09

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{59\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{252a^{13}\left(x+\frac{1}{a}\right)^4}+\frac{1507\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{1680a^{12}\left(x+\frac{1}{a}\right)^3}-\frac{691\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{315a^{11}\left(x+\frac{1}{a}\right)^2}+\right.}{}$
default	$-\frac{\left(-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^9x^9+120960\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^7x^7-416745\right.}{}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-3/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-59/252/a^13/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2)+1507/1680/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-691/315/a^
11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+113591/20160/a^10/(x+1/a)*(a
^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+1/36/a^14/(x+1/a)^5*(a^2*(x+1/a)^2-2*a*(x+1
/a))^(1/2)-1/96/a^11/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-31/192/a^1
0/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^8/c^4*((a*x-1)/(a*x+1))^(1/2
)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.84

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{945\left(a^6x^6+2a^5x^5-a^4x^4-4a^3x^3-a^2x^2+2ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6+2a^5x^5-a^4x^4-\right.}{315\left(a^7c^4x^6+2\right.}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (31
5*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^3*x^3 - 339
*a^2*x^2 + 4047*a*x + 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{20160}a\left(\frac{105\left(\frac{29(ax-1)}{ax+1}-\frac{414(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{35\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+450\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+2961\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+14700\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\right.}{a^2c^4}\right.$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*(105*(29*(a*x - 1)/(a*x + 1) - 414*(a*x - 1)^2/(a*x + 1)^2 + 1)/(
a^2*c^4*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2))
+ (35*((a*x - 1)/(a*x + 1))^(9/2) + 450*((a*x - 1)/(a*x + 1))^(7/2) + 2961*
((a*x - 1)/(a*x + 1))^(5/2) + 14700*((a*x - 1)/(a*x + 1))^(3/2) + 95445*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{303\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{29\,(a\,x-1)}{3\,(a\,x+1)}-\frac{138\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{35\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{47\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}+\frac{5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^4}$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4,x)
[Out] (303*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((29*(a*x - 1))/(3*(a*x + 1)
) - (138*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3
/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (35*((a*x - 1)/(a*x + 1))^(3/
2))/(48*a*c^4) + (47*((a*x - 1)/(a*x + 1))^(5/2))/(320*a*c^4) + (5*((a*x -
1)/(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4)
+ (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^4)
```

3.830 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 321

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x^
2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1
-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+3/2*c
^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c
^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{6332, 6328, 90\}$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[c
 - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
- 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^
2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (c^3*Sqrt[c
- c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^3(1+ax)^4}{x^7}\,dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}-\frac{a}{x^6}+\frac{3a^2}{x^5}+\frac{3a^3}{x^4}-\frac{3a^4}{x^3}-\frac{3a^5}{x^2}+\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}\\
&\quad+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6a^7x^6}+\frac{1}{5a^6x^5}-\frac{3}{4a^5x^4}-\frac{1}{a^4x^3}+\frac{3}{2a^3x^2}+\frac{3}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
[Out] ((c - c/(a^2*x^2))^(7/2)*(1/(6*a^7*x^6) + 1/(5*a^6*x^5) - 3/(4*a^5*x^4) - 1
/(a^4*x^3) + 3/(2*a^3*x^2) + 3/(a^2*x) + x + Log[x]/a))/(1 - 1/(a^2*x^2))^(
7/2)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{(60a^7x^7+60a^6\ln(x)x^6+180a^5x^5+90a^4x^4-60a^3x^3-45a^2x^2+12ax+10)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}x}{60(ax+1)(a^2x^2-1)^3\sqrt{\frac{ax-1}{ax+1}}}$	112

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(60*a^7*x^7+60*a^6*ln(x)*x^6+180*a^5*x^5+90*a^4*x^4-60*a^3*x^3-45*a^2*
x^2+12*a*x+10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)/(a^2*x^2-1)^3/((a*x-
1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{(60\,a^7c^3x^7+60\,a^6c^3x^6\log(x)+180\,a^5c^3x^5+90\,a^4c^3x^4-60\,a^3c^3x^3-45\,a^2c^3x^2+12\,ac^3x}{60\,a^8x^6}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/60*(60*a^7*c^3*x^7 + 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 + 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)

[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

[In] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)

3.831 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x
^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/
a^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c
-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -1/4*(c^2*Sqrt[c - c/(a^2*x^2)])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqr
t[c - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^
2*x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2
*x^2)] + (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^2(1+ax)^3}{x^5}\,dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5+\frac{1}{x^5}+\frac{a}{x^4}-\frac{2a^2}{x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}\right)\,dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}\\
&\quad+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4a^5x^4}-\frac{1}{3a^4x^3}+\frac{1}{a^3x^2}+\frac{2}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(-1/4*1/(a^5*x^4) - 1/(3*a^4*x^3) + 1/(a^3*x^2) +
2/(a^2*x) + x + Log[x]/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{(12a^5x^5+12\ln(x)x^4a^4+24a^3x^3+12a^2x^2-4ax-3)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x}{12(ax+1)(a^2x^2-1)^2\sqrt{\frac{ax-1}{ax+1}}}$	96

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(12*a^5*x^5+12*ln(x)*x^4*a^4+24*a^3*x^3+12*a^2*x^2-4*a*x-3)*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*x/(a*x+1)/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{(12\,a^5c^2x^5+12\,a^4c^2x^4\log(x)+24\,a^3c^2x^3+12\,a^2c^2x^2-4\,ac^2x-3\,c^2)\sqrt{a^2c}}{12\,a^6x^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(12*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 + 12*a^2*c^2*
x^2 - 4*a*c^2*x - 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.832 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2)
/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c*ln
(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 76}

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c - c
/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/Sq
```

```
rt[1 - 1/(a^2*x^2)] + (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x
^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)(1+ax)^2}{x^3}\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^3-\frac{1}{x^3}-\frac{a}{x^2}+\frac{a^2}{x}\right)\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{3}{2a}+\frac{1}{2a^3x^2}+\frac{1}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
[Out] ((c - c/(a^2*x^2))^(3/2)*(3/(2*a) + 1/(2*a^3*x^2) + 1/(a^2*x) + x + Log[x]/
a))/(1 - 1/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{(2a^3x^3+2a^2\ln(x)x^2+2ax+1)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}x}{2(ax+1)(a^2x^2-1)\sqrt{\frac{ax-1}{ax+1}}}$	80

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(2*a^3*x^3+2*a^2*ln(x)*x^2+2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{(2\,a^3cx^3+2\,a^2cx^2\log(x)+2\,acx+c)\sqrt{a^2c}}{2\,a^4x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
[Out] 1/2*(2*a^3*c*x^3 + 2*a^2*c*x^2*log(x) + 2*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.833 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x + Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+\log(x))}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x
)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.834 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 45}

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 - a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{-1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x+\frac{\log(1-ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(x + Log[1 - a*x]/a))/Sqrt[c - c/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(ax-1)(ax+\ln(ax-1))}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,a^2}$	57

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2*(a*x+
ln(a*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{a^2c}(ax+\log(ax-1))}{a^2c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(a*x + log(a*x - 1))/(a^2*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.835 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-a
*x+1)/(c-c/a^2/x^2)^(1/2)+5/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x
^2)^(1/2)-1/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x+\frac{1}{2a-2a^2x}+\frac{5\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(x + (2*a - 2*a^2*x)^(-1) + (5*Log[1 - a*x])/(4*a)
 - Log[1 + a*x]/(4*a)))/(c - c/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{(ax-1)\left(-4a^2x^2+a\ln(ax+1)x-5a\ln(ax-1)x+4ax-\ln(ax+1)+5\ln(ax-1)+2\right)(ax+1)}{4\sqrt{\frac{ax-1}{ax+1}}\,a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$	102

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(-4*a^2*x^2+a*ln(a*x+1)*x-5*a*ln(a*x-1
)*x+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)*(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(4\,a^2x^2-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{a^2c}}{4\,(a^3c^2x-a^2c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)*sqrt(a^2*c)/(a^3*c^2*x - a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.836 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 263

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2
/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+23/16
*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-7/16*ln(a*x+1)*(1
-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + Sqrt[1 - 1/(a^2*x^2)]/(a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(8*a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*
c^2*Sqrt[c - c/(a^2*x^2)]) - (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$
= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.37

$$
\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(x-\frac{1}{8a(-1+ax)^2}+\frac{1}{a-a^2x}-\frac{1}{8a+8a^2x}+\frac{23\log(1-ax)}{16a}-\frac{7\log(1+ax)}{16a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}
$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(x - 1/(8*a*(-1 + a*x)^2) + (a - a^2*x)^(-1) - (8*
a + 8*a^2*x)^(-1) + (23*Log[1 - a*x])/(16*a) - (7*Log[1 + a*x])/(16*a)))/(c
 - c/(a^2*x^2))^(5/2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

method	result
default	$-\frac{(ax-1)(ax+1)\left(-16a^4x^4+7a^3\ln(ax+1)x^3-23a^3\ln(ax-1)x^3+16a^3x^3-7a^2\ln(ax+1)x^2+23a^2\ln(ax-1)x^2+34a^2x^2-7a\ln(ax+1)x\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-16*a^4*x^4+7*a^3*ln(a*x+1)*
x^3-23*a^3*ln(a*x-1)*x^3+16*a^3*x^3-7*a^2*ln(a*x+1)*x^2+23*a^2*ln(a*x-1)*x^
2+34*a^2*x^2-7*a*ln(a*x+1)*x+23*a*ln(a*x-1)*x-18*a*x+7*ln(a*x+1)-23*ln(a*x-
1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3\right.\right.}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
- a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.837 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 359

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}} dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3} - \frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2} - \frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/24*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2
/(c-c/a^2/x^2)^(1/2)+3/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(
1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-5/16*(1-1
/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+51/32*ln(-a*x+1)*(1-1/a^2
/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-19/32*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a
/c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}} -\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}} +\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(24*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (11*Sqrt[1 - 1/(a^2*x^2)]
)/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/
(2*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (5*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^4(1+ax)^3}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{8a^7(-1+ax)^4}+\frac{11}{16a^7(-1+ax)^3}+\frac{3}{2a^7(-1+ax)^2}+\frac{51}{32a^7(-1+ax)}-\frac{1}{16a^7(1+ax)^3}+\frac{1}{16a^7(\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}$$
$$+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}$$
$$+\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96x-\frac{4}{a(-1+ax)^3}-\frac{33}{a(-1+ax)^2}+\frac{3}{a(1+ax)^2}+\frac{144}{a-a^2x}-\frac{30}{a+a^2x}+\frac{153\log(1-ax)}{a}-\right.}{96\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*x - 4/(a*(-1 + a*x)^3) - 33/(a*(-1 + a*x)^2) +
 3/(a*(1 + a*x)^2) + 144/(a - a^2*x) - 30/(a + a^2*x) + (153*Log[1 - a*x])/
a - (57*Log[1 + a*x])/a))/(96*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)(-96a^6x^6+57\ln(ax+1)x^5a^5-153\ln(ax-1)x^5a^5+96a^5x^5-57\ln(ax+1)x^4a^4+153\ln(ax-1)x^4a^4+366a^4x^4-114a^3\ln}{}$

[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/96/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-96*a^6*x^6+57*ln(a*x+1)*x^5*a^5-153*ln(a*x-1)*x^5*a^5+96*a^5*x^5-57*ln(a*x+1)*x^4*a^4+153*ln(a*x-1)*x^4*a^4+366*a^4*x^4-114*a^3*ln(a*x+1)*x^3+306*a^3*ln(a*x-1)*x^3-222*a^3*x^3+114*a^2*ln(a*x+1)*x^2-306*a^2*ln(a*x-1)*x^2-338*a^2*x^2+57*a*ln(a*x+1)*x-153*a*ln(a*x-1)*x+122*a*x-57*ln(a*x+1)+153*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(96\,a^6x^6 - 96\,a^5x^5 - 366\,a^4x^4 + 222\,a^3x^3 + 338\,a^2x^2 - 122\,ax - 57\,(a^5x^5 - a^4x^4 - 2\,a^3}{96\,(a^7c^4x^5 - a^6c^4x^4}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")

[Out] 1/96*(96*a^6*x^6 - 96*a^5*x^5 - 366*a^4*x^4 + 222*a^3*x^3 + 338*a^2*x^2 - 122*a*x - 57*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1) + 153*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x - 1) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 - a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.838 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 372

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}+\frac{2a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\arcsin(ax)}{(1-ax)^{7/2}(1+ax)^{7/2}}+\frac{25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}$$

```
[Out] 11/30*a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a*x+1)^3-57/16*a^6*(c-c/a^2/x^2)^(7/2)*
x^7/(-a*x+1)^3/(a*x+1)^3+41/24*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a*x+1)^3/(a*x+
1)^2+57/80*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)-13/40*a^2*(c-c/a^
2/x^2)^(7/2)*x^3*(a*x+1)/(-a*x+1)^3+1/15*a*(c-c/a^2/x^2)^(7/2)*x^2*(a*x+1)/
(-a*x+1)^2+1/6*(c-c/a^2/x^2)^(7/2)*x*(a*x+1)/(-a*x+1)+2*a^6*(c-c/a^2/x^2)^(
7/2)*x^7*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(
7/2)*x^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(1-ax)}-\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}+\frac{2a^6x^7\arcsin(ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{(1-ax)^{7/2}(ax+1)^{7/2}}+\frac{25a^6x^7\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^{7/2}(ax+1)^{7/2}}-\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}+\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}+\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) - (57*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (41*a^5*(c - c/(a^2*x^2))^
(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) + (57*a^4*(c - c/(a^2*x^2))^(7/2)*x
^5)/(80*(1 - a*x)^3*(1 + a*x)) - (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 + a
*x))/(40*(1 - a*x)^3) + (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
a*x)^2) + ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) + (2*a^6*(c -
 c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (2
5*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
```

```
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx\\
&= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{2\operatorname{arctanh}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}\\
&= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{5/2}(1+ax)^{9/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}\\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{3/2}(1+ax)^{7/2}(2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}\\
&= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}\\
&\quad-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{\sqrt{1-ax}(1+ax)^{7/2}(-39a^2+33a^3x)}{x^5}dx}{30(1-ax)^{7/2}(1+ax)^{7/2}}\\
&= -\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}\\
&\quad+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1+ax)^{7/2}(132a^3-93a^4x)}{x^4\sqrt{1-ax}}dx}{120(1-ax)^{7/2}(1+ax)^{7/2}}\\
&= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}\\
&\quad+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1+ax)^{5/2}(513a^4-411a^5x)}{x^3\sqrt{1-ax}}dx}{360(1-ax)^{7/2}(1+ax)^{7/2}}
\end{aligned}
$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1+ax)^{3/2}(1230a^5-1335a^6x)}{x^2\sqrt{1-ax}}\,dx}{720(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1+ax}(1125a^6-2565a^7x)}{x\sqrt{1-ax}}\,dx}{720(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{-1125a^7+1440a^8x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{720a(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{16(1-ax)^{7/2}(1+ax)^{7/2}}+\frac{\left(2a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}+\frac{\left(25a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}+\frac{\left(2a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)} - \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)} + \frac{2a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\arcsin(ax)}{(1-ax)^{7/2}(1+ax)^{7/2}} + \frac{25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.40

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-40-96ax+70a^2x^2+352a^3x^3+105a^4x^4-736a^5x^5+240a^6\right.}{240a^6x^5\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2 +
 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{(736a^7x^7-105a^6x^6-1088a^5x^5+35a^4x^4+448a^3x^3+110a^2x^2-96ax-40)c^3\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{240x^5a^6(a^2x^2-1)} + \frac{\left(\frac{2a^7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{25a^6\ln\right.}{}}{}$
default	$-\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}x\left(-2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^9cx^7+2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,a^9x^5+375\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^8cx^6\right.}{}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(736*a^7*x^7-105*a^6*x^6-1088*a^5*x^5+35*a^4*x^4+448*a^3*x^3+110*a^2
*x^2-96*a*x-40)/x^5*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(2*a^
7*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+25/16*a^6/(-c
```

```
)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+a^6/c*(c*(a^2*x^2-1))
^(1/2))*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \left[-\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) - 375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{x^2}\right.}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
- 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^6*c
^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x
^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240
*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 1
05*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.76 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.85

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \text{Too large to display}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
```

```
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 41.99 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.51

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=$$

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2}cx-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2}\cdots}{\cdots}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.839 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 294

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}-\frac{2a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\arcsin(ax)}{(1-ax)^{5/2}(1+ax)^{5/2}}-\frac{9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}}$$

```
[Out] -5/8*a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)^2+25/8*a^4*(c-c/a^2/x^2)^(5/2)*x^
5/(-a*x+1)^2/(a*x+1)^2-17/12*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)
+1/6*a*(c-c/a^2/x^2)^(5/2)*x^2*(a*x+1)/(-a*x+1)^2+1/4*(c-c/a^2/x^2)^(5/2)*x
*(a*x+1)/(-a*x+1)-2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/
(a*x+1)^(5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1
)^(1/2))/(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules

used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2} + \frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)} - \frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2} - \frac{2a^4x^5\arcsin(ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^{5/2}(ax+1)^{5/2}} - \frac{9a^4x^5\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^{5/2}(ax+1)^{5/2}} + \frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2} - \frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (-5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) + (25*a^4*(c - c/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (17*a^3*(c - c/(a^2*x^2))^(5
/2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) + (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a
*x))/(6*(1 - a*x)^2) + ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x))
- (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x
)^(5/2)) - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1
+ a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
```

```
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{2\text{arctanh}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}(1+ax)^{5/2}(2a-5a^2x)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1+ax)^{5/2}(-15a^2+13a^3x)}{x^3\sqrt{1-ax}}dx}{12(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1+ax)^{3/2}(-34a^3+41a^4x)}{x^2\sqrt{1-ax}}dx}{24(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1+ax}(-27a^4+75a^5x)}{x\sqrt{1-ax}}dx}{24(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{27a^5-48a^6x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24a(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} + \frac{\left(9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8(1-ax)^{5/2}(1+ax)^{5/2}} - \frac{\left(2a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(9a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}} - \frac{\left(2a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{2a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\arcsin(ax)}{(1-ax)^{5/2}(1+ax)^{5/2}} - \frac{9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.46

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6+16ax-3a^2x^2-64a^3x^3+24a^4x^4)+27a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right.}{24a^4x^3\sqrt{-1+a^2x^2}}$$

[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]

[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{(64a^5x^5+3a^4x^4-80a^3x^3-9a^2x^2+16ax+6)c^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3a^4(a^2x^2-1)}+\frac{\left(\frac{2a^5\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-c}\right)}{\sqrt{a^2c}}+\frac{9a^4\ln\left(\frac{-2c+2\sqrt{-c}\sqrt{a^2cx^2-c}}{x}\right)}{8\sqrt{-c}}+\frac{a^4\sqrt{}}{}\right.}{a^4(a^2x^2-1)}$
default	$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}a^7cx^5+80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}a^7x^3+48\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^6cx^4+27\sqrt{-}\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/24*(64*a^5*x^5+3*a^4*x^4-80*a^3*x^3-9*a^2*x^2+16*a*x+6)/x^3*c^2/a^4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(2*a^5*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+9/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+a^4/c*(c*(a^2*x^2-1))^(1/2))*c^2/a^4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \left[-\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-}{x^2}\right.}{48\,a^4x^3}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2
*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^2*x
^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c
^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log{(ax)}}{a}+\frac{i\sqrt{c}\log{(a^2x^2)}}{2a}+\frac{\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log{(a^2x^2)}}{2a}-\frac{i\sqrt{c}\log{\left(\sqrt{-a^2x^2+1}+1\right)}}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}{(ax)}+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}{(ax)}-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}-\frac{2c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3}-\frac{c^2\left(\begin{cases}\frac{ia^3\sqrt{c}\operatorname{acosh}{\left(\frac{1}{ax}\right)}}{8}-\frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{8}+\frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 1.81 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.41

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=$$
$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2}\cdots}{a\cdots}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) - 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.840 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

```
[Out] a*(c-c/a^2/x^2)^(3/2)*x^2/(-a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)
/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(a*x+1)/(-a*x+1)+2*a^2*(c-c/a^2/x^2)^(3/
2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3/2)
*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{2a^2x^3\arcsin(ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}$$

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x) - (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*(1
- a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
```

```
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\operatorname{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{2\operatorname{arctanh}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1-ax}(1+ax)^{5/2}}{x^3}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1+ax)^{3/2}(2a-3a^2x)}{x^2\sqrt{1-ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1+ax}(a^2-5a^3x)}{x\sqrt{1-ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{-a^3+4a^4x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}} + \frac{\left(2a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\operatorname{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}} + \frac{\left(2a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}} + \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.54

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-1-4ax+2a^2x^2)+a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+4a^2x^2\log\left(ax+\right.\right.}{2a^2x\sqrt{-1+a^2x^2}}$$

[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]

[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(2a^4x^4-4a^3x^3-3a^2x^2+4ax+1)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x\,a^2(a^2x^2-1)}+\frac{\left(\frac{2a^3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{2\sqrt{-c}}\right)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\sqrt{c(}}{a^2(a^2x^2-1)}$
default	$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}x\left(12\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^5cx^3-12\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^5x+4\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^4cx^2-\left(\frac{c(a^2x^2-1}{a^2}\right.\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(2*a^4*x^4-4*a^3*x^3-3*a^2*x^2+4*a*x+1)/x*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(2*a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+1/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x))*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.49

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\left[-\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(}{4\,a^2x}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.77

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{c\left(\begin{cases}\frac{ia\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(3/2),x)

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=$$
$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x)}{a^2}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 + 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) - 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.841 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=-\frac{2x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}\\
&\quad-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}}\right.}{\right.}}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\right.}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.842 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2\sqrt{1-ax}\sqrt{1+ax}\arcsin(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

```
[Out] -2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2)^(1/2)-(a*x+1)^2/a^2/x/(c-c/a^2/x^2)
^(1/2)+2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6294, 6264, 79, 52, 41, 222}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{2\sqrt{1-ax}\sqrt{ax+1}\arcsin(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) - (1 + a*x)^2/(a^2*S
qrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx \\
&= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{e^{2\text{arctanh}(ax)}x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x} \\
&= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{1+ax}}{(1-ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}x} \\
&= -\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}x} \\
&= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}x} \\
&= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}x} \\
&= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{2\sqrt{1-ax}\sqrt{1+ax}\arcsin(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{-3-2ax+a^2x^2+2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
[Out] (-3 - 2*a*x + a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{a \sqrt{a^{2} c}}-\frac{2 \sqrt{a^{2} c \left(x -\frac{1}{a}\right)^{2}+2 \left(x -\frac{1}{a}\right) a c}}{a^{3} c \left(x -\frac{1}{a}\right)}\right) \sqrt{c \left(a^{2} x^{2}-1\right)}}{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}$
default	$\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \Bigg(\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \sqrt{c}\, a^{2} x +2 \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a c x -2 a \sqrt{\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}}\, \sqrt{c}-\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a \sqrt{c}-2 \ln \Bigg(\sqrt{c}\, x +\sqrt{c \cdots}}{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \,c^{\frac{3}{2}} a \left(a x -1\right)}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2*x^2-1)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a*ln(a^2*c*x/(a^2*c)^(
1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-2/a^3/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*
(x-1/a)*a*c)^(1/2))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(a^2x^2-3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac},\right.$$
$$\left.-\frac{2\,(ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(a^2x^2-3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x - a*c), -(2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)), x)
```

3.843 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -1/3*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-2*a*x+5)*(-a*x+1)*(a*x+1)^2/
a^4/(c-c/a^2/x^2)^(3/2)/x^3-2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6294, 6264, 100, 148, 41, 222}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}} - \frac{2(1-ax)^{3/2}(ax+1)^{3/2}\arcsin(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}} + \frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -1/3*(1 + a*x)^2/(a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(5 - 2*a*x)*(1 - a*x)
*(1 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 +
 a*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx \\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{2\text{arctanh}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{(1-ax)^{5/2}\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2+4ax)}{(1-ax)^{3/2}\sqrt{1+ax}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&\quad -\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&\quad -\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{10-4ax-11a^2x^2+3a^3x^3+6(-1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}x(-1+ax)}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.74

method	result
risch	$\frac{a^2x^2-1}{a^2cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^3\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{3a^6c\left(x-\frac{1}{a}\right)^2}-\frac{8\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{3a^5c\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{c(a^2x^2-1)}}{cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^3x^3+4\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c^{\frac{3}{2}}a^2x^2-15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+6\sqrt{\frac{c(a^2x^2-1)}{a^2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\frac{c(ax-1}{a}}\right.}{3\sqrt{\frac{c(ax-1)(a}{a^2}}}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2*x^2-1)/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^3*ln(a^2*c*x/(a^2*
c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/3/a^6/c/(x-1/a)^2*(a^2*c*(x-1
/a)^2+2*(x-1/a)*a*c)^(1/2)-8/3/a^5/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c
)^(1/2))*a^2/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.28

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\left[\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3-14\,a^2x^2+1\right.}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right.$$
$$\left.-\frac{6\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(3\,a^3x^3-14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(6*
(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)), x)
```

3.844 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 203

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} \\ &+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} \\ &-\frac{2(1-ax)^3(1+ax)^2(28+43ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}+\frac{2(1-ax)^{5/2}(1+ax)^{5/2}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

```
[Out] -1/5*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x+2/3*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^
2/x^2)^(5/2)/x^2-58/15*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/1
5*(-a*x+1)^3*(a*x+1)^2*(43*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+2*(-a*x+1)^(
5/2)*(a*x+1)^(5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= -\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\arcsin(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}} \\ &-\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\end{aligned}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -1/5*(1 + a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)*(1 + a*x)^2
)/(3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) - (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^
4*(c - c/(a^2*x^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/
(15*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*A
rcSin[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx\\
&= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{2\text{arctanh}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{7/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+6ax)}{(1-ax)^{5/2}(1+ax)^{3/2}}\,dx}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2(-30a-28a^2x)}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}\\
&\quad+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x(116a^2+86a^3x)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+43ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} + \frac{\left(2(1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+43ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} + \frac{\left(2(1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+43ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} + \frac{2(1-ax)^{5/2}(1+ax)^{5/2}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30(-1+ax)^2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1\right.}}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}x(-1+ax)^2}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*(-1 + a*x)^2*Sqrt
[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x
^2)]*x*(-1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.47

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}} + \frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}} + \frac{\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{8a^7c\left(x+\frac{1}{a}\right)} - \frac{41\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{60a^8c\left(x-\frac{1}{a}\right)^2} - \frac{383\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\right.}}{120a^7c\left(x-\frac{1}{a}\right)}\right.}{c^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}$
default	$\left(15c^{\frac{5}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^5x^5-45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}-16c^{\frac{5}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4-60c^{\frac{5}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^3x^3+16c^{\frac{5}{2}}\right.$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2*x^2-1)/c^2/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^5*ln(a^2*c*x/(a^
2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+1/8/a^7/c/(x+1/a)*(a^2*c*(x+1
/a)^2-2*(x+1/a)*a*c)^(1/2)-41/60/a^8/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+2*(x-1/a)
*a*c)^(1/2)-383/120/a^7/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-1/1
0/a^9/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2))*a^4/c^2/x/(c*(a^2*
x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \left[\frac{15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(15\,a^5x^5-\cdots}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}\right.$$

$$-\frac{30\,(a^4x^4-2\,a^3x^3+2\,ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(15\,a^5x^5-76\,a^4x^4+32\,a^3x^3+82\,a^2x^2-5\cdots}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 - 76*a^4*x^
4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 - 2*a^3
*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82*a^2*x
^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^2*c^3*x - a*c^3)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{a\,x+1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x-1\right)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)), x)
```

3.845 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 283

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}+\frac{142(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}+\frac{2(1-ax)^4(1+ax)^3(72+107ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}-\frac{2(1-ax)^{7/2}(1+ax)^{7/2}\arcsin(ax)}{a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] -1/7*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+2/5*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^
2/x^2)^(7/2)/x^2-124/105*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(7/2)/x^3+7
82/105*(-a*x+1)^3*(a*x+1)^2/a^5/(c-c/a^2/x^2)^(7/2)/x^4+142/35*(-a*x+1)^4*(
a*x+1)^2/a^6/(c-c/a^2/x^2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(107*a*x+72)
/a^8/(c-c/a^2/x^2)^(7/2)/x^7-2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8
/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\frac{(ax+1)^2}{7a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}} - \frac{2(ax+1)^{7/2}(1-ax)^{7/2}\arcsin(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}} - \frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{2(ax+1)^2(1-ax)}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -1/7*(1 + a*x)^2/(a^2*(c - c/(a^2*x^2))^(7/2)*x) + (2*(1 - a*x)*(1 + a*x)^2
)/(5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) - (124*(1 - a*x)^2*(1 + a*x)^2)/(105*
a^4*(c - c/(a^2*x^2))^(7/2)*x^3) + (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(
c - c/(a^2*x^2))^(7/2)*x^4) + (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/
(a^2*x^2))^(7/2)*x^5) + (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*
(c - c/(a^2*x^2))^(7/2)*x^7) - (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*
x])/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx\\
&= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{2\text{arctanh}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{9/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+8ax)}{(1-ax)^{7/2}(1+ax)^{5/2}}\,dx}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-70a-54a^2x)}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{35a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^3(496a^2+286a^3x)}{(1-ax)^{3/2}(1+ax)^{5/2}}\,dx}{105a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^2(-2346a^3-1068a^4x)}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{105a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x(-2556a^4-1926a^5x)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{315a^{10}\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{2(1-ax)^4(1+ax)^3(72+107ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} - \frac{\left(2(1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{2(1-ax)^4(1+ax)^3(72+107ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} - \frac{\left(2(1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}$$
$$+ \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}$$
$$+ \frac{2(1-ax)^4(1+ax)^3(72+107ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} - \frac{2(1-ax)^{7/2}(1+ax)^{7/2}\arcsin(ax)}{a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.47

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{432-654ax-636a^2x^2+1226a^3x^3+74a^4x^4-562a^5x^5+105a^6x^6+210(-1+ax)^3(1}{105a^2c^3\sqrt{c-\frac{c}{a^2x^2}}x(-1+ax)^3(1+ax)}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (432 - 654*a*x - 636*a^2*x^2 + 1226*a^3*x^3 + 74*a^4*x^4 - 562*a^5*x^5 + 10
5*a^6*x^6 + 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.35

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}} + \frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-c}\right)}{a^7\sqrt{a^2c}} - \frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{28a^{12}c\left(x-\frac{1}{a}\right)^4} - \frac{39\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{140a^{11}c\left(x-\frac{1}{a}\right)^3} - \frac{1753\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right.}}{1680a^{10}c\left(x-\frac{1}{a}\right)^2}\right.}{c^3x\sqrt{\frac{c\left(a\right.}{}}}$
default	$\frac{\left(105c^{\frac{7}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^7x^7-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}+96c^{\frac{7}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}a^6x^6-392c^{\frac{7}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^5x^5-96c^{\frac{7}{2}}\right.}{}$

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2*x^2-1)/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^7*ln(a^2*c*x/(a^
2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/28/a^12/c/(x-1/a)^4*(a^2*c*
(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-39/140/a^11/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+2*(
x-1/a)*a*c)^(1/2)-1753/1680/a^10/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c
)^(1/2)-3061/840/a^9/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-1/48/a
^10/c/(x+1/a)^2*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+7/24/a^9/c/(x+1/a)*(a
```

```
^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2))*a^6/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2
)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \left[\frac{105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-\right.}\right.$$

$$-\frac{210\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(105a^7x^7-562\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-a^5c^4x^4+4a^4c^4x^3-a^3c^4x^2\right.}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*a^3
*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x
+ a*c^4), -1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2
- 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^
4 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2
*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x-1)}\,dx$$

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)), x)
```

3.846 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x^
2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/(1
-1/a^2/x^2)^(1/2)-3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+2
*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+4*c^4*(c-c/a^2/x^2)^(1
/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/
2)+3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{6332, 6328, 90\}$

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx = \frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqrt
[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(a
^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^4*Sqrt[c - c/(a^2*x^2)])
/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^3) + (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^4
*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^3(1+ax)^6}{x^9}\,dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^9-\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}+\frac{6a^4}{x^5}-\frac{6a^5}{x^4}-\frac{8a^6}{x^3}+\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}\\
&\quad+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$
\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{1}{8a^9x^8}+\frac{3}{7a^8x^7}-\frac{8}{5a^6x^5}-\frac{3}{2a^5x^4}+\frac{2}{a^4x^3}+\frac{4}{a^3x^2}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}
$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*(1/(8*a^9*x^8) + 3/(7*a^8*x^7) - 8/(5*a^6*x^5) - 3
/(2*a^5*x^4) + 2/(a^4*x^3) + 4/(a^3*x^2) + x + (3*Log[x])/a))/(1 - 1/(a^2*x
^2))^(9/2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(280 a^{9} x^{9}+840 a^{8} \ln \left(x\right) x^{8}+1120 a^{6} x^{6}+560 a^{5} x^{5}-420 a^{4} x^{4}-448 a^{3} x^{3}+120 a x +35\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{9}{2}} x}{280 \left(a x +1\right)^{3} \left(a^{2} x^{2}-1\right)^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	112

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/280*(280*a^9*x^9+840*a^8*ln(x)*x^8+1120*a^6*x^6+560*a^5*x^5-420*a^4*x^4-448*a^3*x^3+120*a*x+35)*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a*x+1)^3/(a^2*x^2-1)^3/((a*x-1)/(a*x+1))^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{(280\,a^9c^4x^9+840\,a^8c^4x^8\log\left(x\right)+1120\,a^6c^4x^6+560\,a^5c^4x^5-420\,a^4c^4x^4-448\,a^3c^4x^3+12}{280\,a^{10}x^8}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fricas")

[Out] 1/280*(280*a^9*c^4*x^9 + 840*a^8*c^4*x^8*log(x) + 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqrt(a^2*c)/(a^10*x^8)

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxim
a")
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac"
)
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.847 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 324

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(
1-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+
5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(
1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2
)+3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used

= {6332, 6328, 90}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -1/6*(c^3*Sqrt[c - c/(a^2*x^2)])/(a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*S
qrt[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (c^3*Sqrt[c - c/(
a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)]
)/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*
Sqrt[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a
^2*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^3*
Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^2(1+ax)^5}{x^7}\,dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^7+\frac{1}{x^7}+\frac{3a}{x^6}+\frac{a^2}{x^5}-\frac{5a^3}{x^4}-\frac{5a^4}{x^3}+\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}\\
&\quad+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6a^7x^6}-\frac{3}{5a^6x^5}-\frac{1}{4a^5x^4}+\frac{5}{3a^4x^3}+\frac{5}{2a^3x^2}-\frac{1}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/6*1/(a^7*x^6) - 3/(5*a^6*x^5) - 1/(4*a^5*x^4)
+ 5/(3*a^4*x^3) + 5/(2*a^3*x^2) - 1/(a^2*x) + x + (3*Log[x])/a))/(1 - 1/(a^
2*x^2))^(7/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(60 a^{7} x^{7}+180 a^{6} \ln \left(x \right) x^{6}-60 a^{5} x^{5}+150 a^{4} x^{4}+100 a^{3} x^{3}-15 a^{2} x^{2}-36 a x -10\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} x}{60 \left(a x +1\right)^{3} \left(a^{2} x^{2}-1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	112

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/60*(60*a^7*x^7+180*a^6*ln(x)*x^6-60*a^5*x^5+150*a^4*x^4+100*a^3*x^3-15*a^2*x^2-36*a*x-10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)^3/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{(60\,a^7c^3x^7+180\,a^6c^3x^6\log\left(x\right)-60\,a^5c^3x^5+150\,a^4c^3x^4+100\,a^3c^3x^3-15\,a^2c^3x^2-36\,ac^3}{60\,a^8x^6}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")

[Out] 1/60*(60*a^7*c^3*x^7 + 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 + 150*a^4*c^3*x^4 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(a^2*c)/(a^8*x^6)

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")

[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")

[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

[In] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)

[Out] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)

3.848 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(
1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x
^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c/a^
2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2
/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 76}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[c
 - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*x^2
)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*Sq
rt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)
] + (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)(1+ax)^4}{x^5}\,dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}\\
&\quad -\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{5}{4a}+\frac{1}{4a^5x^4}+\frac{1}{a^4x^3}+\frac{1}{a^3x^2}-\frac{2}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(5/(4*a) + 1/(4*a^5*x^4) + 1/(a^4*x^3) + 1/(a^3*x^
2) - 2/(a^2*x) + x + (3*Log[x])/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\left(4a^5x^5+12\ln(x)x^4a^4-8a^3x^3+4a^2x^2+4ax+1\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}x}{4(ax+1)^3\left(a^2x^2-1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	96

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(4*a^5*x^5+12*ln(x)*x^4*a^4-8*a^3*x^3+4*a^2*x^2+4*a*x+1)*(c*(a^2*x^2-1)
/a^2/x^2)^(5/2)*x/(a*x+1)^3/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.31

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\left(4\,a^5c^2x^5+12\,a^4c^2x^4\log(x)-8\,a^3c^2x^3+4\,a^2c^2x^2+4\,ac^2x+c^2\right)\sqrt{a^2c}}{4\,a^6x^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 + 4*a^2*c^2*x^2
+ 4*a*c^2*x + c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.849 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3
*c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[
c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*
```

```
x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1
/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(1+ax)^3}{x^3}\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3+\frac{1}{x^3}+\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2a^3x^2}-\frac{3}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
[Out] ((c - c/(a^2*x^2))^(3/2)*(-1/2*1/(a^3*x^2) - 3/(a^2*x) + x + (3*Log[x])/a))
/(1 - 1/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\left(2a^3x^3+6a^2\ln(x)x^2-6ax-1\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x}{2(ax+1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	69

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(2*a^3*x^3+6*a^2*ln(x)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(2\,a^3cx^3+6\,a^2cx^2\log\left(x\right)-6\,acx-c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
[Out] 1/2*(2*a^3*c*x^3 + 6*a^2*c*x^2*log(x) - 6*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.850 $\int e^{3 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2 x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int e^{3 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2 x^2}} \, dx = \frac{\sqrt{c-\frac{c}{a^2 x^2}}\, x}{\sqrt{1-\frac{1}{a^2 x^2}}} - \frac{\sqrt{c-\frac{c}{a^2 x^2}} \log(x)}{a\sqrt{1-\frac{1}{a^2 x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2 x^2}} \log(1-ax)}{a\sqrt{1-\frac{1}{a^2 x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 84}

$$\int e^{3 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2 x^2}} \, dx = \frac{x\sqrt{c-\frac{c}{a^2 x^2}}}{\sqrt{1-\frac{1}{a^2 x^2}}} - \frac{\log(x)\sqrt{c-\frac{c}{a^2 x^2}}}{a\sqrt{1-\frac{1}{a^2 x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2 x^2}} \log(1-ax)}{a\sqrt{1-\frac{1}{a^2 x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/Sqrt[1 - 1/(a^2
*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{(-ax+\ln(x)-4\ln(ax-1))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+4\log(ax-1)-\log(x))}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.851 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+2*(1-1/a^2/x^2)^(1/2)/a/(-a*x+1)/
(c-c/a^2/x^2)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 78}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(1+ax)}{(-1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x+\frac{2}{a(1-ax)}+\frac{3\log(1-ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(x + 2/(a*(1 - a*x)) + (3*Log[1 - a*x])/a))/Sqrt[c -
 c/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{(ax-1)\left(a^2x^2+3a\ln(ax-1)x-ax-3\ln(ax-1)-2\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,a^2}$	85

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a
^2*(a^2*x^2+3*a*ln(a*x-1)*x-a*x-3*ln(a*x-1)-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\left(a^2x^2-ax+3\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{a^2c}}{a^3cx-a^2c}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c*x - a^2*c
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.852 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}} \, dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-a
*x+1)^2/(c-c/a^2/x^2)^(1/2)+3*(1-1/a^2/x^2)^(1/2)/a/c/(-a*x+1)/(c-c/a^2/x^2
)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}} \, dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}} \\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}} \\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{a^3(-1+ax)^3}+\frac{3}{a^3(-1+ax)^2}+\frac{3}{a^3(-1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}} \\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x+\frac{5-6ax}{2a(-1+ax)^2}+\frac{3\log(1-ax)}{a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(x + (5 - 6*a*x)/(2*a*(-1 + a*x)^2) + (3*Log[1 - a
*x])/a))/(c - c/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(ax-1)\left(2a^3x^3+6a^2\ln(ax-1)x^2-4a^2x^2-12a\ln(ax-1)x-4ax+6\ln(ax-1)+5\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$	102

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(2*a^3*x^3+6*a^2*ln(a*x-1)*x^2-4*a^2*x^
2-12*a*ln(a*x-1)*x-4*a*x+6*ln(a*x-1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(2\,a^3x^3-4\,a^2x^2-4\,ax+6\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+5\right)\sqrt{a^2c}}{2\,(a^4c^2x^2-2\,a^3c^2x+a^2c^2)}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
[Out] 1/2*(2*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 6*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) +
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.853 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3} - \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} + \frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/6*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)^2/(c
-c/a^2/x^2)^(1/2)+31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2/x^2)^(1/
2)+49/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-1/16*ln(a
*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{6332, 6328, 90\}$

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}} - \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}} + \frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^4(1+ax)}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{2a^5(-1+ax)^4}+\frac{9}{4a^5(-1+ax)^3}+\frac{31}{8a^5(-1+ax)^2}+\frac{49}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48x-\frac{8}{a(-1+ax)^3}-\frac{54}{a(-1+ax)^2}+\frac{186}{a-a^2x}+\frac{147\log(1-ax)}{a}-\frac{3\log(1+ax)}{a}\right)}{48\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*x - 8/(a*(-1 + a*x)^3) - 54/(a*(-1 + a*x)^2) +
 186/(a - a^2*x) + (147*Log[1 - a*x])/a - (3*Log[1 + a*x])/a))/(48*(c - c/(
a^2*x^2))^(5/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
default	$-\frac{(ax-1)(ax+1)\left(-48a^4x^4+3a^3\ln(ax+1)x^3-147a^3\ln(ax-1)x^3+144a^3x^3-9a^2\ln(ax+1)x^2+441a^2\ln(ax-1)x^2+42a^2x^2+9a\ln(ax\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*a^4*x^4+3*a^3*ln(a*x+1)*
x^3-147*a^3*ln(a*x-1)*x^3+144*a^3*x^3-9*a^2*ln(a*x+1)*x^2+441*a^2*ln(a*x-1)
*x^2+42*a^2*x^2+9*a*ln(a*x+1)*x-441*a*ln(a*x-1)*x-270*a*x-3*ln(a*x+1)+147*l
n(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.52

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(48\,a^4x^4-144\,a^3x^3-42\,a^2x^2+270\,ax-3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(ax+1\right)+14\right.}{48\,(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(48*a^4*x^4 - 144*a^3*x^3 - 42*a^2*x^2 + 270*a*x - 3*(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)*log(a*x + 1) + 147*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2
*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.854 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 360

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^4} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3} - \frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/16*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^4/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^3/(
c-c/a^2/x^2)^(1/2)-59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2/(c-c/a^2/x^2)
^(1/2)+75/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)-1/32*(1
-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+201/64*ln(-a*x+1)*(1-1/
a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-9/64*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)
/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}} -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}} -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)^4\sqrt{c-\frac{c}{a^2x^2}}}+\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^4) + Sqrt[1 - 1/(a^2*x^2)]/(2*
a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*
c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64
*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^5(1+ax)^2}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{4a^7(-1+ax)^5}+\frac{3}{2a^7(-1+ax)^4}+\frac{59}{16a^7(-1+ax)^3}+\frac{75}{16a^7(-1+ax)^2}+\frac{201}{64a^7(-1+ax)}+\frac{}{32a^7(}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^4}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}$$
$$-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}$$
$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.32

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{2(104-207ax-59a^2x^2+309a^3x^3-87a^4x^4-96a^5x^5+32a^6x^6)}{(-1+ax)^4(1+ax)}+201\log(1-ax)-9\log(\right.}{64a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*((2*(104 - 207*a*x - 59*a^2*x^2 + 309*a^3*x^3 - 87
*a^4*x^4 - 96*a^5*x^5 + 32*a^6*x^6))/((-1 + a*x)^4*(1 + a*x)) + 201*Log[1 -
 a*x] - 9*Log[1 + a*x]))/(64*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)(-64a^6x^6+9\ln(ax+1)x^5a^5-201\ln(ax-1)x^5a^5+192a^5x^5-27\ln(ax+1)x^4a^4+603\ln(ax-1)x^4a^4+174a^4x^4+18a^3 \ln}{}$

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/64/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*a^6*x^6+9*ln(a*x+1)*x^5*a^5-201*ln(a*x-1)*x^5*a^5+192*a^5*x^5-27*ln(a*x+1)*x^4*a^4+603*ln(a*x-1)*x^4*a^4+174*a^4*x^4+18*a^3*ln(a*x+1)*x^3-402*a^3*ln(a*x-1)*x^3-618*a^3*x^3+18*a^2*ln(a*x+1)*x^2-402*a^2*ln(a*x-1)*x^2+118*a^2*x^2-27*a*ln(a*x+1)*x+603*a*ln(a*x-1)*x+414*a*x+9*ln(a*x+1)-201*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(64\,a^6x^6 - 192\,a^5x^5 - 174\,a^4x^4 + 618\,a^3x^3 - 118\,a^2x^2 - 414\,ax - 9\,(a^5x^5 - 3\,a^4x^4 + 2}{64\,(a^7c^4x^5 - 3\,a}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")

[Out] 1/64*(64*a^6*x^6 - 192*a^5*x^5 - 174*a^4*x^4 + 618*a^3*x^3 - 118*a^2*x^2 - 414*a*x - 9*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log(a*x + 1) + 201*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.855 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(
1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-3/2*
c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-
c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{6332, 6328, 90\}$

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/6*(c^3*Sqrt[c - c/(a^2*x^2)])/(a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqr
t[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^3*Sqrt[c - c/(
a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/
(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt
[1 - 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (c^3*Sqrt
[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^4(1+ax)^3}{x^7}\,dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^7+\frac{1}{x^7}-\frac{a}{x^6}-\frac{3a^2}{x^5}+\frac{3a^3}{x^4}+\frac{3a^4}{x^3}-\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}\\
&\quad-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6a^7x^6}+\frac{1}{5a^6x^5}+\frac{3}{4a^5x^4}-\frac{1}{a^4x^3}-\frac{3}{2a^3x^2}+\frac{3}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/6*1/(a^7*x^6) + 1/(5*a^6*x^5) + 3/(4*a^5*x^4)
- 1/(a^4*x^3) - 3/(2*a^3*x^2) + 3/(a^2*x) + x - Log[x]/a))/(1 - 1/(a^2*x^2)
)^(7/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,x\left(-60a^7x^7+60a^6\ln(x)x^6-180a^5x^5+90a^4x^4+60a^3x^3-45a^2x^2-12ax+10\right)}{60(ax-1)\left(a^2x^2-1\right)^3}$	112

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-60*a^7*x^7+
60*a^6*ln(x)*x^6-180*a^5*x^5+90*a^4*x^4+60*a^3*x^3-45*a^2*x^2-12*a*x+10)/(a
*x-1)/(a^2*x^2-1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{(60\,a^7c^3x^7-60\,a^6c^3x^6\log(x)+180\,a^5c^3x^5-90\,a^4c^3x^4-60\,a^3c^3x^3+45\,a^2c^3x^2+12\,ac^3x-}{60\,a^8x^6}$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 - 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \int \left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \int \left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \int \left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.856 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x^
2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a
^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-
c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a
^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[c
 - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x
^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*
Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^
2)] - (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^3(1+ax)^2}{x^5}\,dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5-\frac{1}{x^5}+\frac{a}{x^4}+\frac{2a^2}{x^3}-\frac{2a^3}{x^2}-\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}\\
&\quad+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4a^5x^4}-\frac{1}{3a^4x^3}-\frac{1}{a^3x^2}+\frac{2}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]

[Out] ((c - c/(a^2*x^2))^(5/2)*(1/(4*a^5*x^4) - 1/(3*a^4*x^3) - 1/(a^3*x^2) + 2/(a^2*x) + x - Log[x]/a))/(1 - 1/(a^2*x^2))^(5/2)

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,x\left(-12a^5x^5+12\ln(x)x^4a^4-24a^3x^3+12a^2x^2+4ax-3\right)}{12(ax-1)\left(a^2x^2-1\right)^2}$	96

[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-12*a^5*x^5+12*ln(x)*x^4*a^4-24*a^3*x^3+12*a^2*x^2+4*a*x-3)/(a*x-1)/(a^2*x^2-1)^2

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(12\,a^5c^2x^5-12\,a^4c^2x^4\log(x)+24\,a^3c^2x^3-12\,a^2c^2x^2-4\,ac^2x+3\,c^2\right)\sqrt{a^2c}}{12\,a^6x^4}$$

[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

[Out] 1/12*(12*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 4*a*c^2*x + 3*c^2)*sqrt(a^2*c)/(a^6*x^4)

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.857 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2} dx = -\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2} + \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c*l
n(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 76}

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2} dx = \frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c
 - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)
```

```
/Sqrt[1 - 1/(a^2*x^2)] - (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^
2*x^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^2(1+ax)}{x^3}\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}} \\
&= -\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{3}{2a}-\frac{1}{2a^3x^2}+\frac{1}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
[Out] ((c - c/(a^2*x^2))^(3/2)*(-3/(2*a) - 1/(2*a^3*x^2) + 1/(a^2*x) + x - Log[x]
/a))/(1 - 1/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,x\left(-2a^3x^3+2a^2\ln(x)x^2-2ax+1\right)}{2(ax-1)(a^2x^2-1)}$	80

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-2*a^3*x^3+2*
a^2*ln(x)*x^2-2*a*x+1)/(a*x-1)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{(2\,a^3cx^3-2\,a^2cx^2\log\left(x\right)+2\,acx-c)\sqrt{a^2c}}{2\,a^4x^2}$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
[Out] 1/2*(2*a^3*c*x^3 - 2*a^2*c*x^2*log(x) + 2*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.858 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-\log{(x)})}{a^2}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.859 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} \, dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(
c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 45}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} \, dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 + a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x-\frac{\log(1+ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(x - Log[1 + a*x]/a))/Sqrt[c - c/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(-ax+\ln(ax+1))}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$	59

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)/x/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{a^2c}(ax-\log{(ax+1)})}{a^2c}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(a*x + 1))/(a^2*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x
)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Bad A
rgument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2), x)
```

3.860 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a*
x+1)/(c-c/a^2/x^2)^(1/2)+1/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^
2)^(1/2)-5/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a
*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)(1+ax)^2}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{4a^3(-1+ax)}+\frac{1}{2a^3(1+ax)^2}-\frac{5}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4x-\frac{2}{a+a^2x}+\frac{\log(1-ax)}{a}-\frac{5\log(1+ax)}{a}\right)}{4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]

[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*x - 2/(a + a^2*x) + Log[1 - a*x]/a - (5*Log[1 + a*x])/a))/(4*(c - c/(a^2*x^2))^(3/2))

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-4a^2x^2+5a\ln(ax+1)x-a\ln(ax-1)x-4ax+5\ln(ax+1)-\ln(ax-1)+2\right)(ax-1)}{4a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$	103

[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-4*a^2*x^2+5*a*ln(a*x+1)*x-a*ln(a*x-1)*x-4*a*x+5*ln(a*x+1)-ln(a*x-1)+2)*(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.38

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(4\,a^2x^2+4\,ax-5\,(ax+1)\log\left(ax+1\right)+(ax+1)\log\left(ax-1\right)-2\right)\sqrt{a^2c}}{4\,(a^3c^2x+a^2c^2)}$$

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")

[Out] 1/4*(4*a^2*x^2 + 4*a*x - 5*(a*x + 1)*log(a*x + 1) + (a*x + 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c^2*x + a^2*c^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.861 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 263

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-c/
a^2/x^2)^(1/2)-(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+7/16*l
n(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-23/16*ln(a*x+1)*(1-
1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(8*a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - Sqrt[1 - 1/(a^2*x^2)]/(a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)]) - (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^2(1+ax)^3}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{8a^5(-1+ax)^2}+\frac{7}{16a^5(-1+ax)}-\frac{1}{4a^5(1+ax)^3}+\frac{1}{a^5(1+ax)^2}-\frac{23}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$
= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$
\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(2\left(8x+\frac{1}{a(1+ax)^2}+\frac{1}{a-a^2x}-\frac{8}{a+a^2x}\right)+\frac{7\log(1-ax)}{a}-\frac{23\log(1+ax)}{a}\right)}{16\left(c-\frac{c}{a^2x^2}\right)^{5/2}}
$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(2*(8*x + 1/(a*(1 + a*x)^2) + (a - a^2*x)^(-1) - 8
/(a + a^2*x)) + (7*Log[1 - a*x])/a - (23*Log[1 + a*x])/a))/(16*(c - c/(a^2*
x^2))^(5/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-16a^4x^4+23a^3\ln(ax+1)x^3-7a^3\ln(ax-1)x^3-16a^3x^3+23a^2\ln(ax+1)x^2-7a^2\ln(ax-1)x^2+34a^2x^2-23a\ln}{16a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-16*a^4*x^4+23*a^3*ln(a*x+1)
*x^3-7*a^3*ln(a*x-1)*x^3-16*a^3*x^3+23*a^2*ln(a*x+1)*x^2-7*a^2*ln(a*x-1)*x^
2+34*a^2*x^2-23*a*ln(a*x+1)*x+7*a*ln(a*x-1)*x+18*a*x-23*ln(a*x+1)+7*ln(a*x-
1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(16\,a^4x^4+16\,a^3x^3-34\,a^2x^2-18\,ax-23\,(a^3x^3+a^2x^2-ax-1)\log(ax+1)+7\,(a^3\right.}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/16*(16*a^4*x^4 + 16*a^3*x^3 - 34*a^2*x^2 - 18*a*x - 23*(a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 7*(a^3*x^3 + a^2*x^2 - a*x - 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.862 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 358

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3} + \frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/32*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+5/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c
-c/a^2/x^2)^(1/2)-1/24*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(1
/2)+11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-3/2*(1-1/
a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+19/32*ln(-a*x+1)*(1-1/a^2/
x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-51/32*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (5*Sqrt[1 - 1/(a^2*x^2)])
/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(24*a*c
^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (11*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^
3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(2*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^3(1+ax)^4}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{16a^7(-1+ax)^3}+\frac{5}{16a^7(-1+ax)^2}+\frac{19}{32a^7(-1+ax)}+\frac{1}{8a^7(1+ax)^4}-\frac{11}{16a^7(1+ax)^3}+\frac{3}{2a^7(1-}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}$$
$$- \frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3} + \frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}$$
$$- \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96x-\frac{3}{a(-1+ax)^2}-\frac{4}{a(1+ax)^3}+\frac{33}{a(1+ax)^2}+\frac{30}{a-a^2x}-\frac{144}{a+a^2x}+\frac{57\log(1-ax)}{a}-\frac{15}{}\right.}{96\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*x - 3/(a*(-1 + a*x)^2) - 4/(a*(1 + a*x)^3) + 3
3/(a*(1 + a*x)^2) + 30/(a - a^2*x) - 144/(a + a^2*x) + (57*Log[1 - a*x])/a
- (153*Log[1 + a*x])/a))/(96*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)(-96a^6x^6+153\ln(ax+1)x^5a^5-57\ln(ax-1)x^5a^5-96a^5x^5+153\ln(ax+1)x^4a^4-57\ln(ax-1)x^4a^4+366a^4x^4-}{}$

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-96*a^6*x^6+153*ln(a*x+1)*x^
5*a^5-57*ln(a*x-1)*x^5*a^5-96*a^5*x^5+153*ln(a*x+1)*x^4*a^4-57*ln(a*x-1)*x^
4*a^4+366*a^4*x^4-306*a^3*ln(a*x+1)*x^3+114*a^3*ln(a*x-1)*x^3+222*a^3*x^3-3
06*a^2*ln(a*x+1)*x^2+114*a^2*ln(a*x-1)*x^2-338*a^2*x^2+153*a*ln(a*x+1)*x-57
*a*ln(a*x-1)*x-122*a*x+153*ln(a*x+1)-57*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(96\,a^6x^6+96\,a^5x^5-366\,a^4x^4-222\,a^3x^3+338\,a^2x^2+122\,ax-153\,(a^5x^5+a^4x^4-2\,a}{96\,(a^7c^4x^5+a^6c^4x^4}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(96*a^6*x^6 + 96*a^5*x^5 - 366*a^4*x^4 - 222*a^3*x^3 + 338*a^2*x^2 + 1
22*a*x - 153*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x
+ 1) + 57*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 + a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*
x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.863 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 375

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}-\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}-\frac{2a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\arcsin(ax)}{(1-ax)^{7/2}(1+ax)^{7/2}}+\frac{25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}$$

```
[Out] 7/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(a*x+1)^3+3/8*a^5*(c-c/a^2/x^2)
^(7/2)*x^6/(-a*x+1)^3/(a*x+1)^2-1/15*a*(c-c/a^2/x^2)^(7/2)*x^2/(a*x+1)-19/1
6*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)+2/3*a^3*(c-c/a^2/x^2)^(7/2
)*x^4/(-a*x+1)^2/(a*x+1)-23/120*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a*x+1)/(a*x+1
)+1/6*(c-c/a^2/x^2)^(7/2)*x*(-a*x+1)/(a*x+1)-2*a^6*(c-c/a^2/x^2)^(7/2)*x^7*
arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7*
arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(ax+1)}-\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}-\frac{2a^6x^7\arcsin(ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{(1-ax)^{7/2}(ax+1)^{7/2}}+\frac{25a^6x^7\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^{7/2}(ax+1)^{7/2}}+\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}-\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}$$

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (3*a^5*(
c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) - (a*(c - c/(a^2*x^
2))^(7/2)*x^2)/(15*(1 + a*x)) - (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(1
 - a*x)^3*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*(
1 + a*x)) - (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
+ ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) - (2*a^6*(c - c/(a^2*
x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (25*a^6*(c
 - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a*
x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
```

```
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-2\text{arctanh}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{9/2}(1+ax)^{5/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}(-2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{3/2}(-23a^2+37a^3x)}{x^5}dx}{30(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{3/2}(1+ax)^{3/2}(240a^3-125a^4x)}{x^4}dx}{120(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1-ax}(1+ax)^{3/2}(-855a^4+135a^5x)}{x^3}dx}{360(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1+ax)^{3/2}(270a^5+585a^6x)}{x^2\sqrt{1-ax}}\,dx}{720(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1+ax}(1125a^6+315a^7x)}{x\sqrt{1-ax}}\,dx}{720(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{-1125a^7-1440a^8x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{720a(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{16(1-ax)^{7/2}(1+ax)^{7/2}} - \frac{\left(2a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} + \frac{\left(25a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}} - \frac{\left(2a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{2a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\arcsin(ax)}{(1-ax)^{7/2}(1+ax)^{7/2}} + \frac{25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-40+96ax+70a^2x^2-352a^3x^3+105a^4x^4+736a^5x^5+240a^6\right.}{240a^6x^5\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2 -
 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.67

method	result
risch	$\frac{(736a^7x^7+105a^6x^6-1088a^5x^5-35a^4x^4+448a^3x^3-110a^2x^2-96ax+40)c^3\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{240x^5a^6(a^2x^2-1)} + \frac{\left(-\frac{2a^7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{25a^6\ln\left(\right.}{}\right.}{}$
default	$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}x\left(-2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^9cx^7+2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,a^9x^5-375\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^8cx^6+480\right.}{}$

[In] int((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/240*(736*a^7*x^7+105*a^6*x^6-1088*a^5*x^5-35*a^4*x^4+448*a^3*x^3-110*a^2*x^2-96*a*x+40)/x^5*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(-2*a^7*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+25/16*a^6/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+a^6/c*(c*(a^2*x^2-1))^(1/2))*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.17

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \left[\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2}{x^2}\right.}{}\right.$$

[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqr

```
t((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 + 10
5*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.11 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.82

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((c-c/a**2/x**2)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**2 + 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 - 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 46.81 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.50

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=$$

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2c}}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

3.864 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 293

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}+\frac{2a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\arcsin(ax)}{(1-ax)^{5/2}(1+ax)^{5/2}}-\frac{9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}}$$

```
[Out] -7/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(a*x+1)^2-1/6*a*(c-c/a^2/x^2)^(
5/2)*x^2/(a*x+1)+2*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)-7/24*a^2*
(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)/(a*x+1)+1/4*(c-c/a^2/x^2)^(5/2)*x*(-a*x+1)
/(a*x+1)+2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/(a*x+1)^(
5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/
(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules

used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = -\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)} -\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}+\frac{2a^4x^5\arcsin(ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^{5/2}(ax+1)^{5/2}} -\frac{9a^4x^5\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^{5/2}(ax+1)^{5/2}} -\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}$$

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (a*(c -
c/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)
/((1 - a*x)^2*(1 + a*x)) - (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x
)*(1 + a*x)) + ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) + (2*a^4
*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
 - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])
/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
```

```
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx \\
&= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-2\text{arctanh}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}} \\
&= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}}{x^5}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}} \\
&= \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}(-2a-5a^2x)}{x^4}\,dx}{4(1-ax)^{5/2}(1+ax)^{5/2}} \\
&= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} \\
&\quad-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}\sqrt{1+ax}(-7a^2+17a^3x)}{x^3}\,dx}{12(1-ax)^{5/2}(1+ax)^{5/2}} \\
&= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} \\
&\quad-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}\sqrt{1+ax}(48a^3-27a^4x)}{x^2}\,dx}{24(1-ax)^{5/2}(1+ax)^{5/2}} \\
&= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} \\
&\quad+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1+ax}(-27a^4-21a^5x)}{x\sqrt{1-ax}}\,dx}{24(1-ax)^{5/2}(1+ax)^{5/2}}
\end{aligned}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{27a^5+48a^6x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24a(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8(1-ax)^{5/2}(1+ax)^{5/2}} + \frac{\left(2a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \frac{\left(9a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}} + \frac{\left(2a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{2a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\arcsin(ax)}{(1-ax)^{5/2}(1+ax)^{5/2}} - \frac{9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.46

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6-16ax-3a^2x^2+64a^3x^3+24a^4x^4)+27a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2}}\right.\right.}{24a^4x^3\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 64
*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4*x
^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.80

method	result
risch	$\frac{(64a^5x^5-3a^4x^4-80a^3x^3+9a^2x^2+16ax-6)c^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3a^4(a^2x^2-1)}+\frac{\left(-\frac{2a^5\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{9a^4\ln\left(\frac{-2c+2\sqrt{-c}\sqrt{a^2c\,x^2-c}}{x}\right)}{8\sqrt{-c}}+\right.}{a^4(a^2x^2-1)}$
default	$-\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}a^7cx^5+80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}a^7x^3-48\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^6cx^4-27\right.}{}$

```
[In] int((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(64*a^5*x^5-3*a^4*x^4-80*a^3*x^3+9*a^2*x^2+16*a*x-6)/x^3*c^2/a^4*(c*(a
^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(-2*a^5*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*
c*x^2-c)^(1/2))/(a^2*c)^(1/2)+9/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2
*c*x^2-c)^(1/2))/x)+a^4/c*(c*(a^2*x^2-1))^(1/2))*c^2/a^4*(c*(a^2*x^2-1)/a^2
/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2} dx = \left[\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{48\,a^4x^3}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2*
a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2*x^
4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c^
2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.97 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.71

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{2c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3}-\frac{c^2\left(\begin{cases}\frac{ia^3\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}$$

```
[In] integrate((c-c/a**2/x**2)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**2*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)

Giac [A] (verification not implemented)

none

Time = 1.98 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.42

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=$$
$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-}}{a^2}\right.$$

[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")

[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) + 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x-1)}{a\,x+1}\,dx$$

[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1),x)

[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1), x)

3.865 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

```
[Out] -a*(c-c/a^2/x^2)^(3/2)*x^2/(a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)
/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(-a*x+1)/(a*x+1)-2*a^2*(c-c/a^2/x^2)^(3/
2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3/2)
*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=-\frac{2a^2x^3\arcsin(ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}$$

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x)) - (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*
(1 + a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
```

```
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-2\text{arctanh}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}}{x^3}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{3/2}(-2a-3a^2x)}{x^2\sqrt{1+ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1-ax}(a^2+5a^3x)}{x\sqrt{1+ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{a^3+4a^4x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}-\frac{\left(2a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}-\frac{\left(2a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.54

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-1+4ax+2a^2x^2)+a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-4a^2x^2\log\left(ax+\right.\right.}{2a^2x\sqrt{-1+a^2x^2}}$$

[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]

[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^2 *x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(2a^4x^4+4a^3x^3-3a^2x^2-4ax+1)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x\,a^2(a^2x^2-1)}+\frac{\left(-\frac{2a^3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{2\sqrt{-c}}\right)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{a^2(a^2x^2-1)}$
default	$-\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}x\left(12\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^5cx^3-12\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^5x-4\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^4cx^2+\left(\frac{c(a^2\right.\right.}{}$

[In] int((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/2*(2*a^4*x^4+4*a^3*x^3-3*a^2*x^2-4*a*x+1)/x*c/a^2*(c*(a^2*x^2-1)/a^2/x^2) ^(1/2)/(a^2*x^2-1)+(-2*a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a ^2*c)^(1/2)+1/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x))*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\left[\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(2\,a^2}{4\,a^2x}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*x
^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.77

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\,\mathrm{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\,\mathrm{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{c\left(\begin{cases}\frac{ia\sqrt{c}\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

```
[In] integrate((c-c/a**2/x**2)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=$$
$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x}{a^2}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 - 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) + 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.866 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{2x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}\\
&\quad+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}}\sqrt{\cdots}}{\cdots}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\ \cdots}{\cdots}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.867 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2\sqrt{1-ax}\sqrt{1+ax}\arcsin(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

```
[Out] -(-a*x+1)^2/a^2/x/(c-c/a^2/x^2)^(1/2)-2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2
)^(1/2)-2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6294, 6264, 79, 52, 41, 222}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\frac{2\sqrt{ax+1}\sqrt{1-ax}\arcsin(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] -((1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x)) - (2*(1 - a*x)*(1 + a*x))/(a^2
*Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^
2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx\\
&= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{e^{-2\text{arctanh}(ax)}x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{1-ax}}{(1+ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2\sqrt{1-ax}\sqrt{1+ax}\,\arcsin(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{-3+2ax+a^2x^2-2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}
$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
[Out] (-3 + 2*a*x + a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.38

method	result
risch	$\frac{a^{2}x^{2}-1}{a^{2}x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}+\frac{\left(-\frac{2\ln\left(\frac{a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-c}\right)}{a\sqrt{a^{2}c}}+\frac{2\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-2\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right)\sqrt{c\left(a^{2}x^{2}-1\right)}}{x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}$
default	$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\left(-\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\sqrt{c}\,a^{2}x+2\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)acx-2a\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}\sqrt{c}-\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a\sqrt{c}+2\ln\left(\sqrt{c}\,x+\sqrt{\right.\right.}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,c^{\frac{3}{2}}a(ax+1)}$

[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/a^2*(a^2*x^2-1)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+2/a^3/c/(x+1/a)*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2))*(c*(a^2*x^2-1))^(1/2)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \left[\frac{(ax+1)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(a^2x^2+3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx+ac}, \frac{2\,(ax+1)\sqrt{-c}\arctan\left(\frac{a^2\cdots}{}\right.}{}\right.$$

[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")

[Out] [((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*c*x + a*c), (2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*c*x + a*c)]

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{a\,x-1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)), x)
```

3.868 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-a*x+1)^2*(a*x+1)*(2*a*x+5)/
a^4/(c-c/a^2/x^2)^(3/2)/x^3+2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6294, 6264, 100, 148, 41, 222}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}} + \frac{2(ax+1)^{3/2}(1-ax)^{3/2}\arcsin(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}} + \frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -1/3*(1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(1 - a*x)^2*(1 + a*x)
*(5 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 +
 a*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\operatorname{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx\\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{-2\operatorname{arctanh}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2-4ax)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&\quad+\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&\quad+\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{-10-4ax+11a^2x^2+3a^3x^3-6(1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
[Out] (-10 - 4*a*x + 11*a^2*x^2 + 3*a^3*x^3 - 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

method	result
risch	$\frac{a^2x^2-1}{a^2cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^3\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{3a^6c\left(x+\frac{1}{a}\right)^2}+\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{3a^5c\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{c(a^2x^2-1)}}{cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^3x^3+15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}-4\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c^{\frac{3}{2}}a^2x^2-6\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\frac{c(ax-}{}}\right.}{3\sqrt{\frac{c(ax-1)}{a^2}}}$

[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/a^2*(a^2*x^2-1)/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/3/a^6/c/(x+1/a)^2*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+8/3/a^5/c/(x+1/a)*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2))*a^2/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3+14\,a^2x^2+\right.}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}\right.$$

[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")

[Out] [1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), 1/3*(6*(a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)), x)
```

3.869 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}-\frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}-\frac{2(1-ax)^{5/2}(1+ax)^{5/2}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] -(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x-2/5*(-a*x+1)^3/a^3/(c-c/a^2/x^2)^(5/2
)/x^2+2/15*(-a*x+1)^3*(a*x+1)/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/15*(-a*x+1)^3*(
a*x+1)^2*(13*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-2*(-a*x+1)^(5/2)*(a*x+1)^(
5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\arcsin(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] -((1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x)) - (2*(1 - a*x)^3)/(5*a^3*(c
- c/(a^2*x^2))^(5/2)*x^2) + (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x
^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/
(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^
6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{-2\text{arctanh}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{3/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+2ax)}{\sqrt{1-ax}(1+ax)^{7/2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}-\frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2(6a+8a^2x)}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{5a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}-\frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x(-4a^2+26a^3x)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$
\begin{aligned}
&= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} \\
&\quad - \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} - \frac{\left(2(1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} \\
&\quad - \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} - \frac{\left(2(1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} \\
&\quad - \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} - \frac{2(1-ax)^{5/2}(1+ax)^{5/2}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{-56-82ax+32a^2x^2+76a^3x^3+15a^4x^4-30(1+ax)^2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+\right.}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}
$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (-56 - 82*a*x + 32*a^2*x^2 + 76*a^3*x^3 + 15*a^4*x^4 - 30*(1 + a*x)^2*Sqrt[
-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^
2)]*x*(1 + a*x)^2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.47

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}}-\frac{41\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{60a^8c\left(x+\frac{1}{a}\right)^2}+\frac{383\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{120a^7c\left(x+\frac{1}{a}\right)}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right.}}{8a^7c\left(x-\frac{1}{a}\right)}\right.}{c^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}$
default	$\frac{\left(15c^{\frac{5}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^5x^5+45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}+16c^{\frac{5}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4-60c^{\frac{5}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^3x^3+16c^{\frac{5}{2}}\left(\frac{c(\right.\right.}{}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2*x^2-1)/c^2/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^5*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-41/60/a^8/c/(x+1/a)^2*(a^2*c
*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+383/120/a^7/c/(x+1/a)*(a^2*c*(x+1/a)^2-2*(x
+1/a)*a*c)^(1/2)-1/8/a^7/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)+1/
10/a^9/c/(x+1/a)^3*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2))*a^4/c^2/x/(c*(a^2
*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.80

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \left[\frac{15\,(a^4x^4+2\,a^3x^3-2\,ax-1)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(15\,a^5x^5}{15\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3}\right.$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 + 76*a^4*x^
4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 + 2*a^3*
x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2
*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*a^2*x^
2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)), x)
```

3.870 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 270

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}} dx = -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}+\frac{2(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{2(1-ax)^4(1+ax)^3(72+37ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^{7/2}(1+ax)^{7/2}\arcsin(ax)}{a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] -1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+10/3*(-a*x+1)^3/a^3/(c-c/a^2/x^2)
^(7/2)/x^2+12/7*(-a*x+1)^4/a^4/(c-c/a^2/x^2)^(7/2)/x^3+82/105*(-a*x+1)^4*(a
*x+1)/a^5/(c-c/a^2/x^2)^(7/2)/x^4+2/35*(-a*x+1)^4*(a*x+1)^2/a^6/(c-c/a^2/x^
2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(37*a*x+72)/a^8/(c-c/a^2/x^2)^(7/2)/
x^7+2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6302, 6294, 6264, 100, 155, 148, 41, 222\}$

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{2(ax+1)^{7/2}(1-ax)^{7/2}\arcsin(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}} + \frac{10(1-ax)^3}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] -1/3*(1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(7/2)*x) + (10*(1 - a*x)^3)/(3*a^3*
(c - c/(a^2*x^2))^(7/2)*x^2) + (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7
/2)*x^3) + (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4)
 + (2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) + (2*(1
 - a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) +
 (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/
2)*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx \\
&= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{-2\text{arctanh}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} \\
&= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{5/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+4ax)}{(1-ax)^{3/2}(1+ax)^{9/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-50a-14a^2x)}{\sqrt{1-ax}(1+ax)^{9/2}}\,dx}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^3(-144a^2-62a^3x)}{\sqrt{1-ax}(1+ax)^{7/2}}\,dx}{21a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^2(-246a^3-228a^4x)}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{105a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x(-36a^4-666a^5x)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{315a^{10}\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{2(1-ax)^4(1+ax)^3(72+37ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} + \frac{\left(2(1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{2(1-ax)^4(1+ax)^3(72+37ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} + \frac{\left(2(1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5} + \frac{2(1-ax)^4(1+ax)^3(72+37ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7} + \frac{2(1-ax)^{7/2}(1+ax)^{7/2}\arcsin(ax)}{a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{432+654ax-636a^2x^2-1226a^3x^3+74a^4x^4+562a^5x^5+105a^6x^6-210(-1+ax)(}{105a^2\sqrt{c-\frac{c}{a^2x^2}}x(-1+ax)(c+acx)^3}$$

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (432 + 654*a*x - 636*a^2*x^2 - 1226*a^3*x^3 + 74*a^4*x^4 + 562*a^5*x^5 + 10
5*a^6*x^6 - 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.37

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}} + \frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^7\sqrt{a^2c}} - \frac{\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{28a^{12}c\left(x+\frac{1}{a}\right)^4} + \frac{39\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{140a^{11}c\left(x+\frac{1}{a}\right)^3} - \frac{1753\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(\right.}}{1680a^{10}c\left(x+\frac{1}{a}\right)^{}}\right.}{c^3x\sqrt{\frac{c}{}}}$
default	$-\frac{\left(-105c^{\frac{7}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^7x^7+96c^{\frac{7}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}a^6x^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}+96c^{\frac{7}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}a^5x^5+392\right.}{}$

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2*x^2-1)/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^7*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/28/a^12/c/(x+1/a)^4*(a^2*c
*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+39/140/a^11/c/(x+1/a)^3*(a^2*c*(x+1/a)^2-2*
(x+1/a)*a*c)^(1/2)-1753/1680/a^10/c/(x+1/a)^2*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*
c)^(1/2)+3061/840/a^9/c/(x+1/a)*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)-1/48/
a^10/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-7/24/a^9/c/(x-1/a)*(
```

```
a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2))*a^6/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/
2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \left[\frac{105\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a}{}}\right.}{105\left(a^7c^4x^6+2\,a^6c^4x^5-a^5\right.}\right.$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*a^3
*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x
+ a*c^4), 1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 +
 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4
 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*
a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x+1)}\,dx$$

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)), x)
```

3.871 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x
^2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/(
1-1/a^2/x^2)^(1/2)+3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+
2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-4*c^4*(c-c/a^2/x^2)^(
1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1
/2)-3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{6332, 6328, 90\}$

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/8*(c^4*Sqrt[c - c/(a^2*x^2)])/(a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*S
qrt[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c
/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^4*Sqrt[c - c/(a^2*x^2
)])/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*
Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/
(a^2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*
c^4*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral}=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^6(1+ax)^3}{x^9}\,dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^9+\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}-\frac{6a^4}{x^5}-\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(-\frac{1}{8a^9x^8}+\frac{3}{7a^8x^7}-\frac{8}{5a^6x^5}+\frac{3}{2a^5x^4}+\frac{2}{a^4x^3}-\frac{4}{a^3x^2}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
[Out] ((c - c/(a^2*x^2))^(9/2)*(-1/8*1/(a^9*x^8) + 3/(7*a^8*x^7) - 8/(5*a^6*x^5)
+ 3/(2*a^5*x^4) + 2/(a^4*x^3) - 4/(a^3*x^2) + x - (3*Log[x])/a))/(1 - 1/(a^
2*x^2))^(9/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(-280a^9x^9+840a^8\ln(x)x^8+1120a^6x^6-560a^5x^5-420a^4x^4+448a^3x^3-120ax+35\right)x\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{280(ax-1)^3\left(a^2x^2-1\right)^3}$	112

```
[In] int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(-280*a^9*x^9+840*a^8*ln(x)*x^8+1120*a^6*x^6-560*a^5*x^5-420*a^4*x^4
+448*a^3*x^3-120*a*x+35)*x*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^3/(a^2*x^2-1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{9/2} dx = \frac{(280\,a^9c^4x^9 - 840\,a^8c^4x^8\log(x) - 1120\,a^6c^4x^6 + 560\,a^5c^4x^5 + 420\,a^4c^4x^4 - 448\,a^3c^4x^3 + 1}{280\,a^{10}x^8}$$

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/280*(280*a^9*c^4*x^9 - 840*a^8*c^4*x^8*log(x) - 1120*a^6*c^4*x^6 + 560*a^
5*c^4*x^5 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqrt(
a^2*c)/(a^10*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{9/2} dx = \text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{9/2} dx = \int \left(c - \frac{c}{a^2x^2}\right)^{\frac{9}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.872 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 324

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x^
2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1
-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-5
/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
-3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {6332, 6328, 90}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqrt
[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(
3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqr
t[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*
x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^3*Sqr
t[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^5(1+ax)^2}{x^7}\,dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^7-\frac{1}{x^7}+\frac{3a}{x^6}-\frac{a^2}{x^5}-\frac{5a^3}{x^4}+\frac{5a^4}{x^3}+\frac{a^5}{x^2}-\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}\\
&\quad-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6a^7x^6}-\frac{3}{5a^6x^5}+\frac{1}{4a^5x^4}+\frac{5}{3a^4x^3}-\frac{5}{2a^3x^2}-\frac{1}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
[Out] ((c - c/(a^2*x^2))^(7/2)*(1/(6*a^7*x^6) - 3/(5*a^6*x^5) + 1/(4*a^5*x^4) + 5
/(3*a^4*x^3) - 5/(2*a^3*x^2) - 1/(a^2*x) + x - (3*Log[x])/a))/(1 - 1/(a^2*x
^2))^(7/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(-60a^7x^7+180a^6\ln(x)x^6+60a^5x^5+150a^4x^4-100a^3x^3-15a^2x^2+36ax-10\right)x\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{60(ax-1)^3(a^2x^2-1)^2}$	112

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-60*a^7*x^7+180*a^6*ln(x)*x^6+60*a^5*x^5+150*a^4*x^4-100*a^3*x^3-15*
a^2*x^2+36*a*x-10)*x*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/
(a*x-1)^3/(a^2*x^2-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \frac{(60\,a^7c^3x^7 - 180\,a^6c^3x^6\log(x) - 60\,a^5c^3x^5 - 150\,a^4c^3x^4 + 100\,a^3c^3x^3 + 15\,a^2c^3x^2 - 36\,ac^3x}{60\,a^8x^6}$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 - 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \int \left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

```
[In] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.873 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^
(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/
x^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c/a
^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^
2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 76}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/4*(c^2*Sqrt[c - c/(a^2*x^2)])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqr
t[c - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*
x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2
*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x
^2)] - (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^4(1+ax)}{x^5}\,dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}\\
&\quad -\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{5}{4a}-\frac{1}{4a^5x^4}+\frac{1}{a^4x^3}-\frac{1}{a^3x^2}-\frac{2}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(-5/(4*a) - 1/(4*a^5*x^4) + 1/(a^4*x^3) - 1/(a^3*x
^2) - 2/(a^2*x) + x - (3*Log[x])/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(-4a^5x^5+12\ln(x)x^4a^4+8a^3x^3+4a^2x^2-4ax+1\right)x\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4(ax-1)^3\left(a^2x^2-1\right)}$	96

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*a^5*x^5+12*ln(x)*x^4*a^4+8*a^3*x^3+4*a^2*x^2-4*a*x+1)*x*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{(4\,a^5c^2x^5-12\,a^4c^2x^4\log(x)-8\,a^3c^2x^3-4\,a^2c^2x^2+4\,ac^2x-c^2)\sqrt{a^2c}}{4\,a^6x^4}$$

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 - 4*a^2*c^2*x^2
+ 4*a*c^2*x - c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

```
[In] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.874 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*
c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c -
 c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/
```

```
Sqrt[1 - 1/(a^2*x^2)] - (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a
^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^3}{x^3}\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^3-\frac{1}{x^3}+\frac{3a}{x^2}-\frac{3a^2}{x}\right)\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^3x^2}-\frac{3}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*(1/(2*a^3*x^2) - 3/(a^2*x) + x - (3*Log[x])/a))/(1
 - 1/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{\left(-2a^{3}x^{3}+6a^{2}\ln(x)x^{2}+6ax-1\right)x\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2(ax-1)^{3}}$	69

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*a^3*x^3+6*a^2*ln(x)*x^2+6*a*x-1)*x*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*(
(a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(2\,a^3cx^3-6\,a^2cx^2\log\left(x\right)-6\,acx+c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*c*x^3 - 6*a^2*c*x^2*log(x) - 6*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

```
[In] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.875 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 84}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{(-ax+4\ln(ax+1)-\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	67

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] -(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/
(a*x+1))^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.876 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-2*(1-1/a^2/x^2)^(1/2)/a/(a*x+1)/(
c-c/a^2/x^2)^(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 78}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(-1+ax)}{(1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(1+ax)^2}-\frac{3}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x-\frac{2}{a(1+ax)}-\frac{3\log(1+ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(x - 2/(a*(1 + a*x)) - (3*Log[1 + a*x])/a))/Sqrt[c -
 c/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(-a^2x^2+3a\ln(ax+1)x-ax+3\ln(ax+1)+2\right)}{(ax-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,a^2}$	87

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)*(-a^2*x^2+3*a*ln(a*x+1)*x-a*x+3*ln
(a*x+1)+2)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\left(a^2x^2+ax-3\,(ax+1)\log\left(ax+1\right)-2\right)\sqrt{a^2c}}{a^3cx+a^2c}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (a^2*x^2 + a*x - 3*(a*x + 1)*log(a*x + 1) - 2)*sqrt(a^2*c)/(a^3*c*x + a^2*c
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2), x)
```

3.877 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a*
x+1)^2/(c-c/a^2/x^2)^(1/2)-3*(1-1/a^2/x^2)^(1/2)/a/c/(a*x+1)/(c-c/a^2/x^2)^
(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}} \\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}} \\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}-\frac{1}{a^3(1+ax)^3}+\frac{3}{a^3(1+ax)^2}-\frac{3}{a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}} \\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.38

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x-\frac{5+6ax}{2a(1+ax)^2}-\frac{3\log(1+ax)}{a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(x - (5 + 6*a*x)/(2*a*(1 + a*x)^2) - (3*Log[1 + a*
x])/a))/(c - c/(a^2*x^2))^(3/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(-2a^3x^3+6a^2\ln(ax+1)x^2-4a^2x^2+12a\ln(ax+1)x+4ax+6\ln(ax+1)+5\right)}{2a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$	102

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-2*a^3*x^3+6*a^2*ln(a*x+1)*x^2-4*a^2*
x^2+12*a*ln(a*x+1)*x+4*a*x+6*ln(a*x+1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(
3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(2\,a^3x^3+4\,a^2x^2-4\,ax-6\,\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)-5\right)\sqrt{a^2c}}{2\,\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*x^3 + 4*a^2*x^2 - 4*a*x - 6*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1) -
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.878 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 264

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3} + \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2} - \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/6*(1-1/a^2/x^2)^(1/2)/a/c^2
/(a*x+1)^3/(c-c/a^2/x^2)^(1/2)+9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-c
/a^2/x^2)^(1/2)-31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+
1/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-49/16*ln(a*x+
1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

= {6332, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}} + \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int\frac{x^5}{(-1+ax)(1+ax)^4}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int\left(\frac{1}{a^5}+\frac{1}{16a^5(-1+ax)}+\frac{1}{2a^5(1+ax)^4}-\frac{9}{4a^5(1+ax)^3}+\frac{31}{8a^5(1+ax)^2}-\frac{49}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3} + \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2} - \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48x-\frac{8}{a(1+ax)^3}+\frac{54}{a(1+ax)^2}-\frac{186}{a+a^2x}+\frac{3\log(1-ax)}{a}-\frac{147\log(1+ax)}{a}\right)}{48\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*x - 8/(a*(1 + a*x)^3) + 54/(a*(1 + a*x)^2) - 1
86/(a + a^2*x) + (3*Log[1 - a*x])/a - (147*Log[1 + a*x])/a))/(48*(c - c/(a^
2*x^2))^(5/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(ax-1)\left(-48a^4x^4+147a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-144a^3x^3+441a^2\ln(ax+1)x^2-9a^2\ln(ax-1)x^2+42a^2x^2+\right.}{48a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(-48*a^4*x^4+147*a^3*ln(a*x+1
)*x^3-3*a^3*ln(a*x-1)*x^3-144*a^3*x^3+441*a^2*ln(a*x+1)*x^2-9*a^2*ln(a*x-1)
*x^2+42*a^2*x^2+441*a*ln(a*x+1)*x-9*a*ln(a*x-1)*x+270*a*x+147*ln(a*x+1)-3*l
n(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{(48\,a^4x^4+144\,a^3x^3-42\,a^2x^2-270\,ax-147\,(a^3x^3+3\,a^2x^2+3\,ax+1)\log{(ax+1)}}{48\,(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+a}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/48*(48*a^4*x^4 + 144*a^3*x^3 - 42*a^2*x^2 - 270*a*x - 147*(a^3*x^3 + 3*a^
2*x^2 + 3*a*x + 1)*log(a*x + 1) + 3*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2
*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.879 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 357

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^4} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3} + \frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2} - \frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^4/(c-
c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(1/2
)+59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-75/16*(1-1/
a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+9/64*ln(-a*x+1)*(1-1/a^2/x
^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-201/64*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}} -\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}} +\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)^4\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(16*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^4) - Sqrt[1 - 1/(a^2*x^2)]/(2*a*c^3*Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^3*Sqr
t[c - c/(a^2*x^2)]*(1 + a*x)^2) - (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64*a
*c^3*Sqrt[c - c/(a^2*x^2)]) - (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^2(1+ax)^5}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{32a^7(-1+ax)^2}+\frac{9}{64a^7(-1+ax)}-\frac{1}{4a^7(1+ax)^5}+\frac{3}{2a^7(1+ax)^4}-\frac{59}{16a^7(1+ax)^3}+\frac{75}{16a^7(1+a}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^4}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(64x+\frac{208+478ax+74a^2x^2-490a^3x^3-302a^4x^4}{a(-1+ax)(1+ax)^4}+\frac{9\log(1-ax)}{a}-\frac{201\log(1+ax)}{a}\right)}{64\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(64*x + (208 + 478*a*x + 74*a^2*x^2 - 490*a^3*x^3
- 302*a^4*x^4)/(a*(-1 + a*x)*(1 + a*x)^4) + (9*Log[1 - a*x])/a - (201*Log[1
 + a*x])/a))/(64*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(ax-1)(-64a^6x^6+201\ln(ax+1)x^5a^5-9\ln(ax-1)x^5a^5-192a^5x^5+603\ln(ax+1)x^4a^4-27\ln(ax-1)x^4a^4+174a^4x^4$

[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/64*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(-64*a^6*x^6+201*ln(a*x+1)*x^5*a^5-9*ln(a*x-1)*x^5*a^5-192*a^5*x^5+603*ln(a*x+1)*x^4*a^4-27*ln(a*x-1)*x^4*a^4+174*a^4*x^4+402*a^3*ln(a*x+1)*x^3-18*a^3*ln(a*x-1)*x^3+618*a^3*x^3-402*a^2*ln(a*x+1)*x^2+18*a^2*ln(a*x-1)*x^2+118*a^2*x^2-603*a*ln(a*x+1)*x+27*a*ln(a*x-1)*x-414*a*x-201*ln(a*x+1)+9*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(64\,a^6x^6+192\,a^5x^5-174\,a^4x^4-618\,a^3x^3-118\,a^2x^2+414\,ax-201\,(a^5x^5+3\,a^4x^4+}{64\,(a^7c^4x^5+3\,a^6}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(64*a^6*x^6 + 192*a^5*x^5 - 174*a^4*x^4 - 618*a^3*x^3 - 118*a^2*x^2 +
414*a*x - 201*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x + 1) + 9*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 -
 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.880 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2)
/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x^m}{am}+\frac{x^{1+m}}{1+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^m/(a*m) + x^(1 + m)/(1 + m)))/Sqrt[1 - 1/(a^2*x^2
)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x^{1+m}(amx+m+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{m(1+m)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{(ax-1)(ax+1)}}\,(ax-1)(amx+m+1)x^m}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{c}\,(a^2x^2-1)(1+m)m}$	101

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)/m/(1+m)/(a*x+1)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=-\frac{(amx^2+(m+1)x)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\left(a\sqrt{c}mx+\sqrt{c}(m+1)\right)(ax+1)x^m}{(m^2+m)a^2x+(m^2+m)a}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] (a*sqrt(c)*m*x + sqrt(c)*(m + 1))*(a*x + 1)*x^m/((m^2 + m)*a^2*x + (m^2 + m
)*a)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int\frac{x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.881 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/
2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{3}(2ax+3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$\frac{x^{3}(2ax+3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,ax^3+3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x+3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x + 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(a
*x - 1))
```

3.882 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6332, 6328}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)
]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x}{a}+\frac{x^2}{2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x/a + x^2/2))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x^2(ax+2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52
default	$\frac{x^2(ax+2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{a^2c}(ax^2+2\,x)}{2\,a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(a^2*c)*(a*x^2 + 2*x)/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)),
 x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+2)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,(a\,x-1)}$$

```
[In] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*x
 - 1))
```

3.883 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 45}

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x + Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+\log(x))}{a^2}$$

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")

[Out] sqrt(a^2*c)*(a*x + log(x))/a^2

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)

[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x
)

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.884 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) + (Sqrt[c - c/(a^2*x^2
)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int\frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(1/(a*x)) + Log[x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE
)
```

```
[Out] (a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{a^2c}(ax\log{(x)}-1)}{a^2x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) - 1)/(a^2*x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))
), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.885 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

```
[Out] -1/2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 37}

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2ax^2}-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/(a*x^2) - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

method	result	size
gosper	$-\frac{(2ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$-\frac{(2ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{\sqrt{a^2c}(2\,ax+1)}{2\,a^2x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(a^2*c)*(2*a*x + 1)/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}+\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((x*(c - c/(a^2*x^2))^(1/2) + (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a*
x + 1))^(1/2))/(x/a - x^2)
```

3.886 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{8a^3\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] 7/8*x*(c-c/a^2/x^2)^(1/2)/a^3+7/24*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3+1/6*x*
(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2
-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 92, 81, 52, 41, 222}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{7x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{8a^3\sqrt{1-ax}\sqrt{ax+1}}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) + (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\text{arctanh}(ax)}x^2\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(-1-2ax)(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{4a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{12a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} \\
&\quad + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{8a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} \\
&\quad + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3} \\
&\quad + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^2\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x\arcsin(ax)}{8a^3\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}(32+21ax+16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*
a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+21ax+32)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{24a^3} + \frac{7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8a^2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-6x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4-16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+27c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-48c^{\frac{3}{2}}\ln\right.}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\,a^4}$

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*a^3*x^3+16*a^2*x^2+21*a*x+32)/a^3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+7
/8/a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*
x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.39

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\left[\frac{2\left(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\frac{(6\,a^4x^4}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
[Out] [1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 +
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2\,x\left(\frac{3\,x\mathrm{sgn}(x)}{a^2}+\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x+\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x-\right.\right.}{a^4}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.887 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 123

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+
1)^2*(c-c/a^2/x^2)^(1/2)/a^2-x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+1)
^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6294, 6264, 81, 52, 41, 222}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=-\frac{x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-ax}\sqrt{ax+1}}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\operatorname{arctanh}(ax)}x\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{3a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(5+3ax+a^2x^2)+3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

method	result
risch	$\frac{(a^2x^2+3ax+5)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{3a^2}+\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)x\sqrt{c(a^2x^2-1)}\,\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{a\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+3c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)-\right.}{3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c}$

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a^2*x^2+3*a*x+5)/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+1/a*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)*(c*(
a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \left[\frac{2\,(a^3x^3+3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\ \frac{(a^3x^3+3\,a^2x^2+5\,ax}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}+\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.888 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] 3/2*x*(c-c/a^2/x^2)^(1/2)/a+1/2*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arcsi
n(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6302, 6294, 6264, 52, 41, 222}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = -\frac{3x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-ax}\sqrt{ax+1}}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2*
a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\operatorname{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

$$
=-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}
$$

$$
=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}
$$

$$
=\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}
$$

$$
=\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}
$$

$$
=\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\arcsin(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}
$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(ax+4)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2+\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)-4\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a\right)}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2}$

```
[In] int(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x+4)/a*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+3/2*ln(a^2*c*x/(a^2*c)^(1/2)+
(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^
2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\left[\frac{2\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\frac{\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-}}{2\,a^{}}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{(3\,a\sqrt{c}\log\left(|c|\right)-$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.889 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=-\frac{2x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}\\
&\quad-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c}{}}}{}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.890 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x
+1)^(1/2)+2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6294, 6264, 100, 163, 41, 222, 94, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{ax\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}-2ax\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] - 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

method	result
risch	$\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}+\frac{\left(\frac{a^2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}-\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{a^2x^2-1}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c\,x^2+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}\,ax-2c^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,\ln \cdots\right.}{a\sqrt{\frac{c\left(a^2x^2 \cdots\right)}{a^2}}\cdots}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(
1/2))/(a^2*c)^(1/2)-2*a/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2
))/x))*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.15

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}, -2\,\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \frac{1}{2}\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arctan(
a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*
log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) +
sqrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x}\,dx$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x), x)

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a} - \frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{|a|} + \frac{2c^{\frac{3}{2}}\mathrm{sg}}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2c}\right.\right.}\right.$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")

[Out] (4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a - sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) + 2*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs(a)

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)

[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)

3.891 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] 3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arcta
nh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)
^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6294, 6264, 96, 94, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{3a^2x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-ax}\sqrt{ax+1}}+\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) + (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} - \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left((1+4ax)\sqrt{-1+a^2x^2}-3a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*((1 + 4*a*x)*Sqrt[-1 + a^2*x^2] - 3*a^2*x^2*ArcTan[1
/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.28

method	result
risch	$\frac{(4a^3x^3+a^2x^2-4ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(a^2x^2-1)} - \frac{3a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c\,x^3+4\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3x+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)a\,x^2-4\sqrt{-\frac{c}{a^2}}\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/2*(4*a^3*x^3+a^2*x^2-4*a*x-1)/x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)
-3/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x
^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\ -\frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^2\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)),
x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^2}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(91) = 182.

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= \left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\,\mathrm{sgn}(x) - 4\left(\sqrt{a^2c}x-\sqrt{a^2c\right.}}{\left(\left(\sqrt{a^2\right.\right.}}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^2\,(a\,x-1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

3.892 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}+\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] a^2*(c-c/a^2/x^2)^(1/2)+1/3*a*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(a*x+1)^2*(
c-c/a^2/x^2)^(1/2)/x^2+a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2
/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6294, 6264, 98, 96, 94, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\operatorname{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^4\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}\\
&\quad-\frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}\\
&\quad+\frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}x\right)\operatorname{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}\\
&\quad+\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(1+3ax+5a^2x^2)-3a^3x^3\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]

[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 + 3*a*x + 5*a^2*x^2) - 3*a^3*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(5a^4x^4+3a^3x^3-4a^2x^2-3ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{3x^2(a^2x^2-1)}-\frac{a^3\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a\left(-6\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c\,x^4+6\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3x^2+6\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\,a\,x^3-6\sqrt{\frac{c(}{}}\right.}{}$

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)

[Out] 1/3*(5*a^4*x^4+3*a^3*x^3-4*a^2*x^2-3*a*x-1)/x^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-a^3/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.47

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$
$$=\left[\frac{3\,a^2\sqrt{-c}x^2\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(5\,a^2x^2+3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2},\right.$$
$$\left.-\frac{3\,a^2\sqrt{c}x^2\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(5\,a^2x^2+3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,x^2}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/x^2]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^3\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)),
x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^3}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.69

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$
$$=\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5ac\operatorname{sgn}(x)-3\left(\sqrt{a^2c}x-\sqrt{\right.}\right.}{}\right)$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^3)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^3\,(a\,x-1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

3.893 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] 4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3+2/3*a*(c-c/a^2/x^2)
^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)*(
a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{8\sqrt{1-ax}\sqrt{ax+1}}+\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*S
```

qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*x]*Sqrt[1 + a*x])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^5\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{21a^2+16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{21a^4}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}$$

$$
= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6+16ax+21a^2x^2+32a^3x^3)-21a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^
3*x^3) - 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*x^
2])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(32a^5x^5+21a^4x^4-16a^3x^3-15a^2x^2-16ax-6)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{24x^3(a^2x^2-1)} - \frac{7a^4\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-48\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c\,x^5+48\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3x^3+48\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a\,x^4-4\right.}{}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(32*a^5*x^5+21*a^4*x^4-16*a^3*x^3-15*a^2*x^2-16*a*x-6)/x^3*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-7/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^
2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a
^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\right.$$
$$\left.-\frac{21\,a^3\sqrt{c}x^3\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{24\,x^3}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 21*a^
2*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^4\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)),
x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^4}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(128) = 256.

Time = 2.08 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.03

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7 a^2c\mathrm{sgn}(x) + 45\left(\sqrt{a^2}\right.}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^4\,(a\,x-1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

3.894 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] 6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4+1/2*a*(c-c/a^2/x^2)
^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2+3/4*a^3*(c-c/a^2/x^2)^(1/2)/x+3/
4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^
(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^5x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-ax}\sqrt{ax+1}}+\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}$$

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^6\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{36a^2+30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{-90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}\\
&\quad+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{144a^4+90a^5x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{120\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}\\
&\quad+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int -\frac{90a^5}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{120\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} \\
&\quad +\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{\left(3a^5\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} \\
&\quad +\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{\left(3a^6\sqrt{c-\frac{c}{a^2x^2}}x\right)\mathrm{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} \\
&\quad +\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}x\,\mathrm{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$
\int\frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx
=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(4+10ax+12a^2x^2+15a^3x^3+24a^4x^4)-15a^5x^5\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}
$$

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*a^
3*x^3 + 24*a^4*x^4) - 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92

method	result
risch	$\frac{(24a^6x^6+15a^5x^5-12a^4x^4-5a^3x^3-8a^2x^2-10ax-4)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{20x^4(a^2x^2-1)}-\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2cx^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{4\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-40\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^4cx^6+40\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4x^4+40\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)a^2x^5-\right.}{}$

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(24*a^6*x^6+15*a^5*x^5-12*a^4*x^4-5*a^3*x^3-8*a^2*x^2-10*a*x-4)/x^4*(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-3/4*a^5/(-c)^(1/2)*ln((-2*c+2*(-c)^
(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))
^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4},\ -\frac{15\,a^4\sqrt{c}x^4\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{20\,x^4}\right]$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^4*a
rctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (24*a^
4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/x^4]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^5\,(ax-1)}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)),
x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^5}\,dx$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(149) = 298.

Time = 2.36 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\operatorname{sgn}(x)+70\left(\sqrt{a^2}\right.}{}\right.$$

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac")
[Out] 1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2
*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*c
^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2 + c)^5)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^5\,(a\,x-1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

3.895 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/a
^2/(1-1/a^2/x^2)^(1/2)+x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^
4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/
a^4/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a^
2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*S
qrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2)
]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1-ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((4*x)/a^3 + (2*x^2)/a^2 + x^3/a + x^4/4 + (4*Log[1
- a*x])/a^4))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(a^4x^4+4a^3x^3+8a^2x^2+16ax+16\ln(ax-1)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax-1)}{4a^3(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	89

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\left(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log\left(ax-1\right)\right)\sqrt{a^2c}}{4\,a^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(a^2*c
)/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.896 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x^2*(c-c/a^2/x^2)^(1/2)
/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 78}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
```

```
qrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^3*Sqrt[1
 - 1/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(24+9ax+2a^2x^2)+24\log(1-ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6
*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{(2a^3x^3+9a^2x^2+24ax+24\ln(ax-1))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{6a^2(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	82

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBO
SE)
[Out] 1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/
2)*(a*x-1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log{(ax-1)})\sqrt{a^2c}}{6\,a^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(a^2*c)/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.897 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^
2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])
/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(3+ax+\frac{4}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/Sqrt[1 - 1
/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x-1\right)}{2 a \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	73

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a/(
a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{a^2c}}{2\,a^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.898 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 84}

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/Sqrt[1 - 1/(a^2
*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{(-ax+\ln(x)-4\ln(ax-1))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{a^2c}(ax+4\log(ax-1)-\log(x))}{a^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.899 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (3*Sqrt[c - c/(a^2*x^2)
]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sq
rt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}-3\log(x)+4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}
$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(a*x) - 3*Log[x] + 4*Log[1 - a*x]))/Sqrt[1 - 1/(a
^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{(3a\ln(x)x-4a\ln(ax-1)x-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	67

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE
)
```

```
[Out] -(3*a*ln(x)*x-4*a*ln(a*x-1)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x
+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{a^2c}(4\,ax\log(ax-1)-3\,ax\log(x)+1)}{a^2x}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.900 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/(
1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2*
x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[
1 - 1/(a^2*x^2)] + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2
*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2ax^2}+\frac{3}{x}-4a\log(x)+4a\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*a*x^2) + 3/x - 4*a*Log[x] + 4*a*Log[1 - a*x]))
/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{(8a^2\ln(x)x^2-8a^2\ln(ax-1)x^2-6ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{2(ax+1)^2x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	82

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*a^2*ln(a*x-1)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x-1)/(a*x+1)^2/x/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{8\,a^3\sqrt{c}x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\sqrt{a^2c}(6\,ax+1)}{2\,a^2x^2}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
1)*sqrt(c) + a*c)/(a*x^2 - x)) + sqrt(a^2*c)*(6*a*x + 1))/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.901 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/x
^2/(1-1/a^2/x^2)^(1/2)+4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^2*
ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(c-c/a^2/x^2
)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2*
x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^
2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&\quad-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3ax^3}+\frac{3}{2x^2}+\frac{4a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(3*a*x^3) + 3/(2*x^2) + (4*a)/x - 4*a^2*Log[x] +
4*a^2*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result	size
default	$-\frac{\left(24a^3\ln(x)x^3-24a^3\ln(ax-1)x^3-24a^2x^2-9ax-2\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax-1)}{6(ax+1)^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	90

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*a^3*ln(a*x-1)*x^3-24*a^2*x^2-9*a*x-2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{24\,a^4\sqrt{c}x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\left(24\,a^2x^2+9\,ax+2\right)\sqrt{a^2c}}{6\,a^2x^3}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="m
axima")
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="g
iac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.902 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 222

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/(
1-1/a^2/x^2)^(1/2)+2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*(c
-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{6332, 6328, 90\}$

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^2
)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) + (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (
4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^3*Sqrt[c -
 c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} - \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.35

$$\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4ax^4}+\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(4*a*x^4) + x^(-3) + (2*a)/x^2 + (4*a^2)/x - 4*a^
3*Log[x] + 4*a^3*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\left(16\ln(x)x^4a^4-16\ln(ax-1)x^4a^4-16a^3x^3-8a^2x^2-4ax-1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax-1)}{4(ax+1)^2x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	98

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/4*(16*ln(x)*x^4*a^4-16*ln(a*x-1)*x^4*a^4-16*a^3*x^3-8*a^2*x^2-4*a*x-1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{16\,a^5\sqrt{c}x^4\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+(16\,a^3x^3+8\,a^2x^2+4\,ax+1)\sqrt{a^2c}}{4\,a^2x^4}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*sqrt
(a^2*c))/(a^2*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.903 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 264

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/x
^4/(1-1/a^2/x^2)^(1/2)+4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+2*
a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*(c-c/a^2/x^2)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*
a^4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

= {6332, 6328, 90}

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2*
x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt[
1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)
]*x^2) + (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*S
qrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^2
*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x^6(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-\frac{1}{x^6}-\frac{3a}{x^5}-\frac{4a^2}{x^4}-\frac{4a^3}{x^3}-\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.34

$$\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5ax^5}+\frac{3}{4x^4}+\frac{4a}{3x^3}+\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(5*a*x^5) + 3/(4*x^4) + (4*a)/(3*x^3) + (2*a^2)/x
^2 + (4*a^3)/x - 4*a^4*Log[x] + 4*a^4*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{(240a^5\ln(x)x^5-240\ln(ax-1)x^5a^5-240a^4x^4-120a^3x^3-80a^2x^2-45ax-12)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{60(ax+1)^2x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	106

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/60*(240*a^5*ln(x)*x^5-240*ln(a*x-1)*x^5*a^5-240*a^4*x^4-120*a^3*x^3-80*a
^2*x^2-45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^4/((a*x
-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{240\,a^6\sqrt{c}x^5\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+(240\,a^4x^4+120\,a^3x^3+80\,a^2x^2+45\,ax+12)\sqrt{a^2c}}{60\,a^2x^5}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
 - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (240*a^4*x^4 + 120*a^3*x^3 + 80*a^2*x^2
 + 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.904 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2
)/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a
^2*x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x^m\left(-\frac{1}{am}+\frac{x}{1+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m*(-(1/(a*m)) + x/(1 + m)))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{1+m}(amx-m-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{m(1+m)(ax-1)}$	65
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax+1)(amx-m-1)x^{m}}{\sqrt{c}\,(a^{2}x^{2}-1)(1+m)m}$	103

```
[In] int(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x^(1+m)/m/(1+m)/(a*x-1)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/
(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = -\frac{(amx^2-(m+1)x)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\left(a\sqrt{c}mx-\sqrt{c}(m+1)\right)(ax-1)x^m}{(m^2+m)a^2x-(m^2+m)a}$$

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] (a*sqrt(c)*m*x - sqrt(c)*(m + 1))*(a*x - 1)*x^m/((m^2 + m)*a^2*x - (m^2 + m
)*a)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int \sqrt{c-\frac{c}{a^2x^2}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.905 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a
^2*x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(-3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(-3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{3}(2ax-3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53
default	$\frac{x^{3}(2ax-3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*
x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,ax^3-3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

```
[In] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(a
*x - 1))
```

3.906 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6332, 6328}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{x}{a}+\frac{x^2}{2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(x/a) + x^2/2))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x^2(ax-2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52
default	$\frac{x^2(ax-2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{a^2c}(ax^2-2\,x)}{2\,a^2}$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(a^2*c)*(a*x^2 - 2*x)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}\,(a\,x-2)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,(a\,x-1)}$$

```
[In] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*x
 - 1))
```

3.907 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-\log{(x)})}{a^2}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.908 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 45}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (Sqrt[c - c/(a^2*x^2)]*
Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(a*x) + Log[x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	50

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(ax\log(x)+1)}{a^2x}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) + 1)/(a^2*x)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/x,
 x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.909 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

```
[Out] 1/2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 37}

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2ax^2}-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*a*x^2) - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result	size
gosper	$-\frac{(2ax-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)x}$	53
default	$-\frac{(2ax-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)x}$	53

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{a^2c}(2\,ax-1)}{2\,a^2x^2}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(a^2*c)*(2*a*x - 1)/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)

[Out] ((x*(c - c/(a^2*x^2))^(1/2) - (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a*x + 1))^(1/2))/(x/a - x^2)

3.910 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{8a^3\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] -7/8*x*(c-c/a^2/x^2)^(1/2)/a^3-7/24*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3-1/6*
x*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)
/a^2-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 92, 81, 52, 41, 222}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{7x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{8a^3\sqrt{ax+1}\sqrt{1-ax}}-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) + (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\operatorname{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\operatorname{arctanh}(ax)}x^2\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}(-1+2ax)}{\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{12a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3} \\
&\quad + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{8a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3} \\
&\quad + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8a^2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3} \\
&\quad + \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^2\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x\arcsin(ax)}{8a^3\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}(-32+21ax-16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

method	result
risch	$\frac{(6a^3x^3-16a^2x^2+21ax-32)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{24a^3}+\frac{7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8a^2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-6x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4+16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+27c^{\frac{3}{2}}\ln\left(\sqrt{c}x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-48c^{\frac{3}{2}}\ln\left(\cdots\right)\right.}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\,a^4}$

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*a^3*x^3-16*a^2*x^2+21*a*x-32)/a^3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+7
/8/a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*
x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.36

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\left[\frac{2\left(6\,a^4x^4-16\,a^3x^3+21\,a^2x^2-32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\frac{(6\,a^4x\right.$$

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 -
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax+1}\,dx$$

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\mathrm{sgn}(x)}{a^2}-\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\right.\right.}{a^4|a}\right.$$

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.911 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*
x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2+x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6294, 6264, 81, 52, 41, 222}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{ax+1}\sqrt{1-ax}}+\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\text{arctanh}(ax)}x\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{3a\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(5-3ax+a^2x^2)-3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}
$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

method	result
risch	$\frac{(a^2x^2-3ax+5)x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{3a^2}-\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)x\sqrt{c(a^2x^2-1)}\,\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{a\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+3c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)+6\sqrt{\frac{c(}{}}\right.}{3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c}$

[In] int(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/3*(a^2*x^2-3*a*x+5)/a^2*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)-1/a*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.65

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \left[\frac{2\,(a^3x^3-3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\frac{(a^3x^3-3\,a^2x^2+5\,ax)}{}\right.$$

[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^3, 1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^3]

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax+1}\,dx$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}-\right.$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.912 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] -3/2*x*(c-c/a^2/x^2)^(1/2)/a-1/2*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arc
sin(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6302, 6294, 6264, 52, 41, 222}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{3x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-ax}\sqrt{ax+1}}-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2
*a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx \\ &= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \end{aligned}$$

$$=-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{1+ax}(4-5ax+a^2x^2)-6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt
[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2
])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(ax-4)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2+\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)+4\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a\right)}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2}$

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

[Out] 1/2*(a*x-4)/a*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+3/2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.90

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\left[\frac{2\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\ \frac{\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}}{2\,a^2}\right.$$

[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

[In] integrate(x*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)

[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x}{ax+1}\,dx$$

[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{(3\,a\sqrt{c}\log\left(|c|\right)}{}\right.$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.913 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{2x\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}}}{}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.914 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x
+1)^(1/2)-2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6294, 6264, 100, 163, 41, 222, 94, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = -\frac{ax\arcsin(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}} - \frac{2ax\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}} + \sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&\quad -\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}+2ax\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

method	result
risch	$\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}+\frac{\left(\frac{a^2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{a^2x^2-1}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c\,x^2+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}\,ax-2c^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,\ln \cdots\right.}{a\sqrt{\frac{c\left(a^2 \cdots\right.}{\cdots}}}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(
1/2))/(a^2*c)^(1/2)+2*a/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2
))/x))*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}, 2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \frac{1}{2}\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right]$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arctan(a
*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*l
og(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + s
qrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx= -\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2c\cdots}{\left(\left(\sqrt{a^2c}x-\cdots\right.\right.}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] -(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 + sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) - 2
*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs
(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.915 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] -3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arc
tanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+
1)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6294, 6264, 96, 94, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{3a^2x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-ax}\sqrt{ax+1}} - \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) +
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((-1+4ax)\sqrt{-1+a^2x^2}+3a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*Ar
cTan[1/Sqrt[-1 + a^2*x^2]]))/(x*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{(4a^3x^3-a^2x^2-4ax+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(a^2x^2-1)}-\frac{3a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{-c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3cx^3+4\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3x+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)a\,x^2-4\sqrt{-\frac{c}{a^2}}\right.}{}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(4*a^3*x^3-a^2*x^2-4*a*x+1)/x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1
)-3/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*
x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.57

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\right.$$
$$\left.-\frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^2\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)),
x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(91) = 182$.

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$= \left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\,\mathrm{sgn}(x) + 4\left(\sqrt{a^2c}x-\sqrt{a^2cx\right.}{\left(\left(\sqrt{a^2cx\right.\right.}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.916 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} - \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] a^2*(c-c/a^2/x^2)^(1/2)-1/3*a*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(-a*x+1)^2
*(c-c/a^2/x^2)^(1/2)/x^2-a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a
^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6294, 6264, 98, 96, 94, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{a^3x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^4\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}\\
&\quad+\frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}\\
&\quad-\frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2}\\
&\quad-\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(1-3ax+5a^2x^2)+3a^3x^3\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

method	result
risch	$\frac{(5a^4x^4-3a^3x^3-4a^2x^2+3ax-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{3x^2(a^2x^2-1)}+\frac{a^3\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a\left(-6\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c\,x^4+6\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3x^2+6\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a\,x^3-6\sqrt{\cdot}\right.}{}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(5*a^4*x^4-3*a^3*x^3-4*a^2*x^2+3*a*x-1)/x^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/
2)/(a^2*x^2-1)+a^3/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)
*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{-c}x^2\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(5\,a^2x^2-3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2}, \frac{3\,a^2\sqrt{c}x^2\arctan\left(\frac{a\sqrt{c}x\sqrt{\cdot}}{a^2cx^2\cdots}\right.}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^3\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)),
x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^3}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.65

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=$$
$$-\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5ac\,\mathrm{sgn}(x)+3\left(\sqrt{a^2c}x-\right.}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.917 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] -4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3-2/3*a*(c-c/a^2/x^2
)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)*
(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{8\sqrt{1-ax}\sqrt{ax+1}}-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^5\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{21a^2-16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} \\
&\quad +\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{21a^4}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} \\
&\quad +\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-6+16ax-21a^2x^2+32a^3x^3)+21a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -1/24*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2
+ 32*a^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^3*Sqrt[-1 + a^
2*x^2])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3(a^2x^2-1)} - \frac{7a^4\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8\sqrt{-c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-48\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c\,x^5+48\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3x^3+48\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)a\,x^4-48\right.}{}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(32*a^5*x^5-21*a^4*x^4-16*a^3*x^3+15*a^2*x^2-16*a*x+6)/x^3*(c*(a^2*x^
2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-7/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a
```

```
^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(
a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\right.$$
$$\left.-\frac{21\,a^3\sqrt{c}x^3\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{24\,x^3}\right]$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 21*a^
2*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^4\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)),
x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^4}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(128) = 256$.

Time = 1.77 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.03

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7a^2c\mathrm{sgn}(x)+45\left(\sqrt{c}\right.}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.918 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}}$$

```
[Out] 6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4-1/2*a*(c-c/a^2/x^2)
^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2-3/4*a^3*(c-c/a^2/x^2)^(1/2)/x-3/
4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^
(1/2)/(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{3a^5x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-ax}\sqrt{ax+1}}+\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\text{integral} = -\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^6\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{36a^2-30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} -\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{144a^4-90a^5x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{120\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} -\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{90a^5}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{120\sqrt{1-ax}\sqrt{1+ax}}$$

$$
\begin{aligned}
&= \frac{6}{5} a^4 \sqrt{c - \frac{c}{a^2 x^2}} + \frac{\sqrt{c - \frac{c}{a^2 x^2}}}{5x^4} - \frac{a\sqrt{c - \frac{c}{a^2 x^2}}}{2x^3} + \frac{3a^2\sqrt{c - \frac{c}{a^2 x^2}}}{5x^2} \\
&\quad - \frac{3a^3\sqrt{c - \frac{c}{a^2 x^2}}}{4x} + \frac{\left(3a^5\sqrt{c - \frac{c}{a^2 x^2}}\, x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{6}{5} a^4 \sqrt{c - \frac{c}{a^2 x^2}} + \frac{\sqrt{c - \frac{c}{a^2 x^2}}}{5x^4} - \frac{a\sqrt{c - \frac{c}{a^2 x^2}}}{2x^3} + \frac{3a^2\sqrt{c - \frac{c}{a^2 x^2}}}{5x^2} \\
&\quad - \frac{3a^3\sqrt{c - \frac{c}{a^2 x^2}}}{4x} - \frac{\left(3a^6\sqrt{c - \frac{c}{a^2 x^2}}\, x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{6}{5} a^4 \sqrt{c - \frac{c}{a^2 x^2}} + \frac{\sqrt{c - \frac{c}{a^2 x^2}}}{5x^4} - \frac{a\sqrt{c - \frac{c}{a^2 x^2}}}{2x^3} + \frac{3a^2\sqrt{c - \frac{c}{a^2 x^2}}}{5x^2} \\
&\quad - \frac{3a^3\sqrt{c - \frac{c}{a^2 x^2}}}{4x} - \frac{3a^5\sqrt{c - \frac{c}{a^2 x^2}}\, x \operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c - \frac{c}{a^2 x^2}}}{x^5}\,dx = \frac{\sqrt{c - \frac{c}{a^2 x^2}}\left(\sqrt{-1 + a^2 x^2}\left(4 - 10ax + 12a^2 x^2 - 15a^3 x^3 + 24a^4 x^4\right) + 15a^5 x^5 \arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1 + a^2 x^2}}
$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a^
3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\left(24a^6x^6-15a^5x^5-12a^4x^4+5a^3x^3-8a^2x^2+10ax-4\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{20x^4\left(a^2x^2-1\right)}+\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{4\sqrt{-c}\,\left(a^2x^2-1\right)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-40\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^4c\,x^6+40\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4+40\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a^2x^5\right.}{}$

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/20*(24*a^6*x^6-15*a^5*x^5-12*a^4*x^4+5*a^3*x^3-8*a^2*x^2+10*a*x-4)/x^4*(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+3/4*a^5/(-c)^(1/2)*ln((-2*c+2*(-c)^
(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))
^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.28

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, \frac{15}{}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4*ar
ctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24*a^4
*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^5\,(ax+1)}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)),
x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^5}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(149) = 298$.

Time = 2.57 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=$$
$$-\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\operatorname{sgn}(x)+70\,(\cdots}{\cdots}\right.$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
[Out] -1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*
c^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2 + c)^5)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.919 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=-\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/
a^2/(1-1/a^2/x^2)^(1/2)-x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x
^4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/
a^4/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a
^2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*
Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2
)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4x}{a^3}+\frac{2x^2}{a^2}-\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1+ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^3 + (2*x^2)/a^2 - x^3/a + x^4/4 + (4*Log[1
 + a*x])/a^4))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(a^4x^4-4a^3x^3+8a^2x^2-16ax+16\ln(ax+1)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4a^3(ax-1)^2}$	89

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\left(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log\left(ax+1\right)\right)\sqrt{a^2c}}{4\,a^5}$$

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(a^2*c
)/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

```
[In] int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.920 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x^2*(c-c/a^2/x^2)^(1/2)
/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln
(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 78}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
```

```
qrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]

[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*a^3*Sqrt[1 - 1/(a^2*x^2)])

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(-2a^3x^3+9a^2x^2-24ax+24\ln(ax+1)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6a^2(ax-1)^2}$	82

[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log(ax+1))\sqrt{a^2c}}{6\,a^4}$$

[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(a^2*c)/a^4

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

```
[In] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.921 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x]
)/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-3+ax+\frac{4}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}
$$

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/Sqrt[1 -
1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x +8 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{2} a}$	73

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a
*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{(a^2x^2-6\,ax+8\,\log{(ax+1)})\sqrt{a^2c}}{2\,a^3}$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \text{Timed out}$$

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.922 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 84}

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{(-ax+4\ln(ax+1)-\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	67

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] -(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/
(a*x+1))^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.923 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) - (3*Sqrt[c - c/(a^2*x
^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])
/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{ax}-3\log(x)+4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(1/(a*x)) - 3*Log[x] + 4*Log[1 + a*x]))/Sqrt[1 - 1
/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\left(4a\ln\left(ax+1\right)x-3a\ln\left(x\right)x-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(ax-1\right)^{2}}$	66

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] (4*a*ln(a*x+1)*x-3*a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x
-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{a^2c}(4\,ax\log\left(ax+1\right)-3\,ax\log\left(x\right)-1)}{a^2x}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.924 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/
(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*a*l
n(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] -1/2*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a
^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sq
rt[1 - 1/(a^2*x^2)] - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(
a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2ax^2}+\frac{3}{x}+4a\log(x)-4a\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/(a*x^2) + 3/x + 4*a*Log[x] - 4*a*Log[1 + a*x
]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\left(8a^2\ln(ax+1)x^2-8a^2\ln(x)x^2-6ax+1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2(ax-1)^2x}$	82

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(8*a^2*ln(a*x+1)*x^2-8*a^2*ln(x)*x^2-6*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{8\,a^3\sqrt{c}x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\sqrt{a^2c}(6\,ax-1)}{2\,a^2x^2}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
1)*sqrt(c) + a*c)/(a*x^2 + x)) + sqrt(a^2*c)*(6*a*x - 1))/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="max
ima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^2}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.925 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} -\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)-4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^2
*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(c-c/a^2/x^2
)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}} +\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/3*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a
^2*x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt
[1 - 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2
*x^2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&\quad -\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3ax^3}+\frac{3}{2x^2}-\frac{4a}{x}-4a^2\log(x)+4a^2\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/3*1/(a*x^3) + 3/(2*x^2) - (4*a)/x - 4*a^2*Log[x]
 + 4*a^2*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(24a^3\ln(ax+1)x^3-24a^3\ln(x)x^3-24a^2x^2+9ax-2\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6(ax-1)^2x^2}$	90

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*(24*a^3*ln(a*x+1)*x^3-24*a^3*ln(x)*x^3-24*a^2*x^2+9*a*x-2)*(c*(a^2*x^2-
1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{24\,a^4\sqrt{c}x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{a^2c}}{6\,a^2x^3}$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="max
ima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.926 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/
(1-1/a^2/x^2)^(1/2)-2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*(
c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{6332, 6328, 90\}$

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] -1/4*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*
x^2)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) - (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1
/(a^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x)
+ (4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] - (4*a^3*Sqrt[
c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4ax^4}+\frac{1}{x^3}-\frac{2a}{x^2}+\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/4*1/(a*x^4) + x^(-3) - (2*a)/x^2 + (4*a^2)/x + 4
*a^3*Log[x] - 4*a^3*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{(16\ln(ax+1)x^4a^4-16\ln(x)x^4a^4-16a^3x^3+8a^2x^2-4ax+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4(ax-1)^2x^3}$	98

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4*(16*ln(a*x+1)*x^4*a^4-16*ln(x)*x^4*a^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{16\,a^5\sqrt{c}x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+(16\,a^3x^3-8\,a^2x^2+4\,ax-1)\sqrt{a^2c}}{4\,a^2x^4}$$

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")

[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x + 1)*sqrt(c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(a^2*c))/(a^2*x^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \text{Timed out}$$

[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.927 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 263

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/
x^4/(1-1/a^2/x^2)^(1/2)-4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+2
*a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*(c-c/a^2/x^2)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4
*a^4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{6332, 6328, 90\}$

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/5*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a
^2*x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sq
rt[1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x
^2)]*x^2) - (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^
4*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(
a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{x^6(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{1}{x^6}-\frac{3a}{x^5}+\frac{4a^2}{x^4}-\frac{4a^3}{x^3}+\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.34

$$\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5ax^5}+\frac{3}{4x^4}-\frac{4a}{3x^3}+\frac{2a^2}{x^2}-\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/5*1/(a*x^5) + 3/(4*x^4) - (4*a)/(3*x^3) + (2*a^2
)/x^2 - (4*a^3)/x - 4*a^4*Log[x] + 4*a^4*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2
)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(240 \ln \left(a x+1\right) x^{5} a^{5}-240 a^{5} \ln \left(x \right) x^{5}-240 a^{4} x^{4}+120 a^{3} x^{3}-80 a^{2} x^{2}+45 a x-12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{60 \left(a x-1\right)^{2} x^{4}}$	106

[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE
)

[Out] 1/60*(240*ln(a*x+1)*x^5*a^5-240*a^5*ln(x)*x^5-240*a^4*x^4+120*a^3*x^3-80*a^
2*x^2+45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3
/2)/(a*x-1)^2/x^4

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{240\,a^{6}\sqrt{c}x^{5}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx+\sqrt{a^{2}c}(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)-(240\,a^{4}x^{4}-120\,a^{3}x^{3}+80\,a^{2}x^{2}-45\,ax+12)\sqrt{a^{2}c}}{60\,a^{2}x^{5}}$$

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fri
cas")

[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x
 + 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (240*a^4*x^4 - 120*a^3*x^3 + 80*a^2*x^2
 - 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Timed out}$$

[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.928 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 154

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)} - \frac{2^{1+\frac{n}{2}}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

```
[Out] 4*c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*
n],(a-1/x)/(a+1/x))/a/(2-n)-2^(1+1/2*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([-1
/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/a/(2-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6329, 130, 71, 133}

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)} - \frac{c2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (4*c*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2
, 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n)) - (2^(1 + n/2)*
c*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a -
x^(-1))/(2*a)])/(a*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 130

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_Symbol] :> Dist[b*(d/f^2), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)
, x], x] + Dist[(b*e - a*f)*((d*e - c*f)/f^2), Int[(a + b*x)^(m - 1)*((c +
d*x)^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right) + \frac{c\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= \frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)} - \frac{2^{1+\frac{n}{2}}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$
$$= \frac{ce^{n\coth^{-1}(ax)}\left(2ax+anx+e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1, 1+\frac{n}{2}, 2+\frac{n}{2}, e^{2\coth^{-1}(ax)}\right)+(2+n)\,\text{Hyper}\right.}{a(2}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(2*a*x + a*n*x + E^(2*ArcCoth[a*x])*n*Hypergeometric2
F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])] + 4*E^(2*ArcCoth[a*x])*Hypergeometric2F1[
2, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]))/(a*(2 + n))
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*x^2), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\int a^2e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-\frac{e^{n\operatorname{acoth}(ax)}}{x^2}\right)\,dx\right)}{a^2}$$

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2),x)
[Out] c*(Integral(a**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x**2,
x))/a**2
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)\,dx$$

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)), x)
```

3.929 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}+\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}$$

```
[Out] -(1+n)*(1+1/a/x)^(1/2*n)/a/c/n/((1-1/a/x)^(1/2*n))+(1+1/a/x)^(1/2*n)*x/c/((
1-1/a/x)^(1/2*n))+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a+1/x
)/(a-1/x))/a/c/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6329, 105, 160, 12, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}-\frac{(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{acn}+\frac{x\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{c}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(((1 + n)*(1 + 1/(a*x))^(n/2))/(a*c*n*(1 - 1/(a*x))^(n/2))) + ((1 + 1/(a*x
))^(n/2)*x)/(c*(1 - 1/(a*x))^(n/2)) + (2*(1 + 1/(a*x))^(n/2)*Hypergeometric
```

```
2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a*c*(1 - 1/(a*x))^(n/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\text{integral}=-\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}+\frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}-\frac{a\text{Subst}\left(\int \frac{n^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{a^2x}\,dx,x,\frac{1}{x}\right)}{cn}$$

$$=-\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}-\frac{n\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{ac}$$

$$=-\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}+\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+n\,\text{Hy}\right.\right.}{acn(2+n)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + n*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{c - \frac{c}{a^2x^2}} dx$$

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - \frac{c}{a^2x^2}} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c - \frac{c}{a^2x^2}} dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - \frac{c}{a^2x^2}} dx = \frac{a^2 \int \frac{x^2 e^{n \operatorname{acoth}(ax)}}{a^2x^2-1} dx}{c}$$

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2),x)
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - \frac{c}{a^2x^2}} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c - \frac{c}{a^2x^2}} dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{a^2x^2}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a^2\,x^2}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)), x)
```

3.930 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 289

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/(2+n)+(-n^3-n^2+4*n+
6)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(-n^2+4)-(n^2+4*n+6)*(1
+1/a/x)^(-1+1/2*n)/a/c^2/n/(2+n)/((1-1/a/x)^(1/2*n))+(1-1/a/x)^(-1-1/2*n)*(
1+1/a/x)^(-1+1/2*n)*x/c^2+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n
],(a+1/x)/(a-1/x))/a/c^2/((1-1/a/x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6329, 105, 160, 12, 133}

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx = \frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2} - \frac{(n^2+4n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}}{ac^2n(n+2)} + \frac{(-n^3-n^2+4n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{ac^2(2-n)n(n+2)} - \frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)} + \frac{x\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{c^2}$$

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(a*c^2*(2 +
 n))) + ((6 + 4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 +
 n)/2))/(a*c^2*(2 - n)*n*(2 + n)) - ((6 + 4*n + n^2)*(1 + 1/(a*x))^((-2 + n
)/2))/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)) + ((1 - 1/(a*x))^(-1 - n/2)*(1
+ 1/(a*x))^((-2 + n)/2)*x)/c^2 + (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1
, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a*c^2*(1 - 1/(a*x))^(n/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
```

```
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{c^2} \\
&= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{3x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{c^2} \\
&= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}\left(\frac{n(2+n)}{a^2}+\frac{2(3+n)x}{a^3}\right)}{x}\,dx, x, \frac{1}{x}\right)}{c^2(2+n)}
\end{aligned}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{a^2\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}\left(-\frac{n^2(2+n)}{a^3}-\frac{(6+4n+n^2)x}{a^4}\right)}{x}\,dx, x, \frac{1}{x}\right)}{c^2n(2+n)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} - \frac{a^3\text{Subst}\left(\int \frac{n^2(4-n^2)\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{a^4x}\,dx, x, \frac{1}{x}\right)}{c^2n\left(4-n^2\right)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} - \frac{n\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1, \frac{n}{2}, \frac{2+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{e^{n\coth^{-1}(ax)}\Big(-6+n^2+6anx-an^3x+6a^2x^2-2a^2n^2x^2-4a^3nx^3+a^3n^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n^2(-1\cdots}{ac^2(\cdots}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-6 + n^2 + 6*a*n*x - a*n^3*x + 6*a^2*x^2 - 2*a^2*n^2*x
^2 - 4*a^3*n*x^3 + a^3*n^3*x^3 + E^(2*ArcCoth[a*x])*(-2 + n)*n^2*(-1 + a^2*
x^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + n*(-4 + n
^2)*(-1 + a^2*x^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))
/(a*c^2*(-2 + n)*n*(2 + n)*(-1 + a^2*x^2))
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4e^{n\,\mathrm{acoth}\,(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**2,x)
[Out] a**4*Integral(x**4*exp(n*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a^2\,x^2}\right)^2}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2,x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2, x)
```

3.931 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 295

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} \\ &\quad+\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}} \\ &\quad-\frac{2^{\frac{1+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

```
[Out] (1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/
x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1, 1
/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2
/x^2)^(1/2)-2^(1/2+1/2*n)*(1-1/a/x)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1/2-1
/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2/x^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6332, 6329, 130, 71, 98, 133}

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}$$
$$-\frac{2^{\frac{n+1}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}$$
$$+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[1 - 1/(a^2*x^2)] + (2*n*Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[1 - 1/(a^2*x^2)]) - (2^((1 + n)
/2)*Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*Hypergeometric2F1[(1 -
n)/2, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a)])/(a*(1 - n)*Sqrt[1 - 1/(a^2
*x^2)])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 130

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_Symbol] :> Dist[b*(d/f^2), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)
, x], x] + Dist[(b*e - a*f)*((d*e - c*f)/f^2), Int[(a + b*x)^(m - 1)*((c +
d*x)^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2^{\frac{1+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\left(n\sqrt{c-\frac{c}{a^2x^2}}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2^{\frac{1+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.49

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}x^2\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}x+2e^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\right.\right.}{(1+n)\left(-1+a^2x^2\right)}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2*(a*(1
 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E^ArcCoth[a*x]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, -E^(2*ArcCoth[a*x])] + 2*E^ArcCoth[a*x]*n*Hypergeometric2F
1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]))/((1 + n)*(-1 + a^2*x^2))
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \sqrt{c - \frac{c}{a^2 x^2}} dx$$

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{a^2 x^2}} \, dx = \int \sqrt{c - \frac{c}{a^2 x^2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{a^2 x^2}} \, dx = \int \sqrt{-c \left(-1 + \frac{1}{ax}\right) \left(1 + \frac{1}{ax}\right)} e^{n \operatorname{acoth}(ax)} \, dx$$

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{a^2 x^2}} \, dx = \int \sqrt{c - \frac{c}{a^2 x^2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a^2\,x^2}}\,dx$$

[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2),x)

[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2), x)

3.932 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/
x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1, 1
/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a/(1-n)/(c-c/a^2
/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{6332, 6329, 98, 133\}$

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}} + \frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[c - c/(a^2*x^2)] + (2*n*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[c - c/(a^2*x^2)])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&\quad -\frac{\left(n\sqrt{1-\frac{1}{a^2x^2}}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&\quad +\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{e^{n\coth^{-1}(ax)}(-1+a^2x^2)\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}x+2e^{\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax)}\right)\right)}{a^3(1+n)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}x^2}$$

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)*(a*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E
^ArcCoth[a*x]*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x
])]))/(a^3*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2)
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2), x)
```

3.933 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}\operatorname{AppellF1}\left(1+\frac{n}{2}+p,\frac{1}{2}(n-2p),2,2+\frac{n}{2}+p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{a x}\right)}{a(2+n+2p)}$$

```
[Out] -2^(1-1/2*n+p)*(c-c/a^2/x^2)^p*(1+1/a/x)^(1+1/2*n+p)*AppellF1(1+1/2*n+p,1/2
*n-p,2,2+1/2*n+p,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6329, 141}

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = -\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\operatorname{AppellF1}\left(\frac{n}{2}+p+1,\frac{1}{2}(n-2p),2,\frac{n}{2}+p+2,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{a x}\right)}{a(n+2p+2)}$$

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF1
[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x)]
)/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}\text{AppellF1}\left(1+\frac{n}{2}+p,\frac{1}{2}(n-2p),2,2+\frac{n}{2}+p,\frac{a+\frac{1}{x}}{2a}\right.}{a(2+n+2p)}$$

Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p dx$$

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p dx = \int \left(c-\frac{c}{a^2x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p dx = \int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{n \operatorname{acoth}(ax)} dx$$

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p dx = \int \left(c-\frac{c}{a^2x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^p \, dx = \int \left(c - \frac{c}{a^2x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^p \, dx = \int e^{n \operatorname{acoth}(a\,x)} \left(c - \frac{c}{a^2\,x^2}\right)^p \, dx$$

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

3.934 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1-\frac{1}{ax}\right)}{a(1+2p)}$$

```
[Out] (c-c/a^2/x^2)^p*(1-1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2*p+2],1-1/a/x)/a/(
1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6332, 6329, 67}

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1-\frac{1}{ax}\right)}{a(2p+1)}$$

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p\,dx\\
&= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1-\frac{1}{ax}\right)}{a(1+2p)}
\end{aligned}$$

Mathematica [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
[Out] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]), x]
```

Maple [F]

$$\int \left(c-\frac{c}{a^2x^2}\right)^p \mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}dx$$

```
[In] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
[Out] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

Fricas [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{-2p\,\mathrm{acoth}\,(ax)}\,dx$$

```
[In] integrate((c-c/a**2/x**2)**p/exp(2*p*acoth(a*x)),x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \mathrm{e}^{-2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

```
[In] int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

3.935 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{1}{ax}\right)}{a(1+2p)}$$

```
[Out] -(c-c/a^2/x^2)^p*(1+1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2*p+2],1+1/a/x)/a/
(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6332, 6329, 67}

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1+\frac{1}{ax}\right)}{a(2p+1)}$$

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p
, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p\,dx\\
&= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{2p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{1}{ax}\right)}{a(1+2p)}
\end{aligned}$$

Mathematica [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F]

$$\int e^{2p\ \mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

[In] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)

[Out] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)

Fricas [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")

[Out] integral(((a*x + 1)/(a*x - 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)

Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{2p\,\mathrm{acoth}\,(ax)}\,dx$$

[In] integrate(exp(2*p*acoth(a*x))*(c-c/a**2/x**2)**p,x)

[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)

Maxima [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")

[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)

Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \mathrm{e}^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

```
[In] int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                          convert(ExpnType_result,string)," vs. order ",
                          convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-ty
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```